Electroconvection in a horizontal layer of poorly conducting liquid in the presence of heat or matter fluxes

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Mechanisms are compared for the excitation of convective structures in liquid semiconductors and insulators due to heat or matter fluxes within them. A deep analogy is explored between these mechanisms under conditions in which excitation of electroconvection is possible. These mechanisms can be useful in explaining effects observed during the melting of solids by radiation, and during the development of electromechanical instability. © *1995 American Institute of Physics.*

1. The term "electroconvection" usually refers to the cellular motion[1] that appears when a liquid at rest is subjected to a sufficiently strong electric field. However, it is known[2] that electroconvection can also occur without application of an external field, due to an "internal" thermoelectric field that appears when the liquid is heated. In this case, the fundamental property of electroconvection is preserved, i.e., the conditions of excitation are independent of the direction of the force of gravity, and, consequently, of the direction of heating. Therefore, thermoelectric convection is also possible when the fluid is heated from above through a free surface; in this case, as is well known, ordinary thermal convection is impossible. It is important to include this fact in analyzing experiments in which materials are melted by radiation.[3]

However, the excitation of electroconvective cellular motion by a thermal flux is possible only in poorly conducting media, i.e., in liquid media with large dielectric constants. In practice, any bulk electric charge generated during the nonequilibrium heating of a medium that is a good conductor can relax before electroconvection can operate. Thus, thermoelectric convection is a characteristic of liquid semimetals (or semiconductors).

However, it is worth emphasizing once more that this kind of electroconvection is still a species of thermal convection (due to heating).

The excitation of instability by a heat flux is analogous[4] to excitation by a matter flux, i.e., thermal and concentration-induced convection are analogous. For example, the presence of a matter flux in a liquid insulator subjected to an electric field can, in and of itself, lead to the appearance of cellular structures.[5] In principle, a matter flux, e.g., a flux of one insulator in another that leads to nonuniformity of the dielectric constant, can also lead to the appearance of a "concentration-electric" field; however, in practice, this effect is weak.

Thus, the presence of fluxes of heat or matter, and the presence of an electric field ("external" or created by the same flux) in weakly conducting liquids, can excite both a cellular structure (convection) and structures involving the electric field within the liquid. In this paper, the excitation of structures when various kinds of fluxes are present in a weakly conducting liquid will be analyzed with regard to similarities and differences.

This paper is organized as follows. In Sec. 2, qualitative physical considerations are set forth and results are given; in Sec. 3, the problem is stated; in Sec. 4, criteria are obtained for the excitation of instabilities by solving linearized problems; in Sec. 5, the amplitudes of the structures that appear under excitation are computed by solving nonlinear problems. Finally, in Sec. 6, the influence of rotation on the excitation conditions is discussed. Throughout the paper, close attention is paid to analysis of available experiments. These are the subject of Sec. 7.

2. In order to obtain qualitative results, it is sufficient to assume that the problem to be solved can be treated using the simplest possible model, i.e., a planar layer of thickness h, infinite in two directions (x and y), with the z axis directed perpendicular to the layer.

Let a flux of heat or matter flow transverse to the layer. This implies that in the transverse direction there is either a temperature gradient A_t or a concentration gradient A_c of the light impurity c:

$$A_t=|\nabla T|=\frac{T_h-T_c}{h}, \quad A_c=|\nabla c|=\frac{c_s-c_f}{h}, \tag{1}$$

where T_h and T_c are the temperatures at the hot and cold surfaces respectively, and c_s and c_f are the concentrations at the starting and ending surfaces, respectively.

Fluctuations in the motion ($\mathbf{v}$ is the velocity) of the liquid in the presence of a flux also give rise to small departures in the temperature T_1 and concentration c_1 from their equilibrium values T_0 and c_0. We may assume that these deviations are, to order of magnitude,

$$T_1\simeq\frac{v_z A_t\lambda^2}{\varkappa_t}, \quad c_1\simeq\frac{v_z A_c\lambda^2}{\varkappa_c}, \tag{2}$$

$\varkappa_t$ and $\varkappa_c$ are transport coefficients that correspond to the coefficients of thermal conductivity and diffusion respectively, while λ is the characteristic distance over which the motion changes significantly. It is obvious that λ and h are of

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the same order of magnitude. Because of thermal (or concentration) bulk expansion, a buoyancy force $\beta_t g T_1$ (or $\beta_c g c_1$) appears, consisting of the difference between the force of gravity and the Archimedean force [$\beta_t = -(1/\rho)\partial\rho/\partial T$, $\beta_c = -(1/\rho)\partial\rho/\partial c$, where ρ is the density and g the acceleration]. At the free surface (if there is one), a surface tension force $\sigma_t T_1/\lambda^2$ (or $\sigma_c c_1/\lambda^2$) appears as a consequence of the dependence of the surface tension coefficient α on temperature [here $\sigma_t = -\partial\alpha/\partial T$ is the coefficient of thermocapillarity; likewise, $\sigma_c = -\partial\alpha/\partial c$ is the coefficient of concentration-capillarity].

In the presence of a steady electric field (with potential φ_0 and voltage $E_0 \simeq \varphi_0/\lambda$), electrical forces are also present. We note that the field can itself be created by a flow. Thus, heating can cause a thermoelectric field $E = \gamma_t A_t$ to appear (where γ_t is the thermopower coefficient). Electrical forces of two kinds are possible: first of all, a Coulomb force $en_1 E_0/\rho$, because the flux can lead to the appearance of bulk charge (n_1 is a fluctuation in the concentration of carriers with charge e); secondly, an electrophoretic force $(\varepsilon_1/\varepsilon_0)E_0^2|\nabla\varepsilon_0|/\rho$ due to variation in the dielectric constant ε.

Each of these forces (buoyant, capillary, and electrical) can enhance the fluctuations that appear randomly in the velocity and lead to instability. The condition for excitation, i.e., the condition for such a force to overcome the force of dissipation, can be expressed in terms of a threshold value for the corresponding dimensionless number that indicates by what factor the minimally exciting force should exceed the dissipative force $\upsilon\nu/\lambda^2$ (where ν is the coefficient of kinematic viscosity or simple viscosity). Let us write these numbers, omitting labels corresponding to the various fluxes:

$$\mathscr{R} = \frac{\beta g A h^4}{\nu\varkappa}, \quad \mathscr{M} = \frac{\sigma A h^2}{\rho\nu\varkappa}, \quad \mathscr{E} = \frac{en\varphi h^2}{\rho\nu\varkappa} \tag{3}$$

(for the case of motion of a heat flux, $\mathscr{R}$ and $\mathscr{M}$ are the usual Rayleigh and Marangoni numbers[4]). The deep difference between the "hydrodynamic" numbers $\mathscr{R}$ and $\mathscr{M}$ and the "electric" number $\mathscr{E}$ consists of the fact that the numbers $\mathscr{R}$ and $\mathscr{M}$ are proportional to the first power of the gradient A (the quantity that characterizes the flux). This implies that it is impossible to excite any motion using a downward flux (for $\beta>0$) or from the free surface (for $\sigma>0$), and in fact the numbers $\mathscr{R}$ and $\mathscr{M}$ must be positive. However, electroconvection is possible for any direction of the flux.[1]

In the absence of an external field, i.e., if a steady field is produced by the flux, electroconvection is still possible and

$$\mathscr{E}_t = \frac{\varepsilon\gamma_t^2 A_t^2 h^2}{\rho\nu\varkappa_t}. \tag{4}$$

It is this form that is most convenient to use in analyzing the criteria for stability of thermoelectric convection.[2]

A flux of neutral particles does not create an electric field. Therefore, even in the presence of a matter flux, excitation is possible only in an external field. The corresponding dimensionless number can be written as

$$\mathscr{E}_c = \frac{\varepsilon\varphi^2 A_c^2 h^2}{\rho\nu\varkappa_c}. \tag{5}$$

Comparing the numbers $\mathscr{R}$ and $\mathscr{M}$ with the number $\mathscr{E}$ in Eqs. (4) and (5), we see that at the instant of excitation the necessary fluxes depend differently on the characteristic length, i.e., in the layer model these numbers are different functions of the layer thickness. It is obvious that electroconvection is more important in thin layers, i.e., films. Specific estimates will be obtained after an analysis of the exact solutions, to which we now turn.

3. The linearized system of equations that describes excitation of the instability must first include the equation of motion (Navier–Stokes). Because the set of system quantities that describes the phenomenon does not include a pseudovector, excitation takes place in steady state. Then this equation can be written as follows (the Boussinesq approximation[1]):

$$\nu\Delta\mathbf{v} - \frac{\nabla p_1}{\rho_0} = \mathbf{F} + \mathbf{f}, \tag{6}$$

where p is the pressure, $\mathbf{F}$ is the buoyancy force, and $\mathbf{f}$ is the electrical force. The equation includes pressure and dissipative forces. The forces of surface tension must be taken into account in the boundary conditions. The equation of motion must be supplemented by the equation of heat transfer

$$\varkappa_t \Delta T_1 = \pm v_z A_t$$

or the analogous equation for mass transfer

$$\varkappa_c \Delta c_1 = \pm v_z A_c .$$

The upper sign on the right side of these equations corresponds to the case where the flux and the z axis are in the same direction, while the lower sign is for when they are opposite. If the z axis is directed along the force of gravity, then we may speak of an "upward" flux (lower sign) or a "downward" flux (upper sign). A further requirement is incorporation of the equation of electrostatics. In the presence of a thermoelectric effect it is easy to show that $en_1 = \varepsilon\gamma\Delta T_1$; in the presence of a matter flux, this equation gives the relation $E_1 = -\varepsilon_1 E_0/\varepsilon_0$ under conditions of quasineutrality. For small changes in the concentration of impurities, fluctuations in dielectric constant are $\varepsilon_1 = ac_1$, where a is the number of order unity. Finally, we treat the liquid as incompressible, i.e., div $\mathbf{v}=0$.

This system of homogeneous equations must be supplemented by homogeneous boundary conditions at the surfaces $z=0$ and $z=h$. Of course, for arbitrary (physically achievable) types of boundary conditions, this double eigenvalue problem can be solved only numerically. However, in practice this is necessary only when we are required to explain the experimental data. To obtain the qualitative effects, many calculations have shown (see Ref. 1 for electroconvection in external fields, Ref. 4 for thermal convection, and Ref. 2 for thermoelectric convection) that it is sufficient to discuss the conditions for conversion of damped internal waves into undamped standing waves, i.e., to look for solutions in the form

$$v_z = V \sin\left(k_z \frac{z}{h}\right)\cos\left(k_x \frac{x}{h} + k_y \frac{y}{h}\right), \tag{7}$$

where V is a constant amplitude.

Because of translational symmetry, the model of an infinite layer requires that the longitudinal components of the wave vector be real: $k_{x,y}=2\pi h/\lambda_{x,y}$. This definition of the wave vector $k_\perp^2=k_x^2+k_y^2$, $k_\perp=2\pi h/\lambda$ corresponds to arranging the system of coordinates so that the boundaries of the central cell are located at $x,y=\pm\lambda_{x,y}/2$ (where $\lambda_{x,y}$ are the dimensions of the structure along the layer). Generally speaking, the quantity k_z remains complex in this case; however, we can always find a boundary condition for which k_z is also real. It is just this solution that corresponds to the transition point from damped internal waves to undamped. In fact, it turns out that after eliminating all variables except v_z, the equation contains only even derivatives $\partial^{2n}v_z/\partial z^{2n}$, where $n=1,2,3,\ldots$. In this case, it can be shown that $k_z=\pi$. For T_1 (or c_1) the condition for k_z to be real corresponds to $T_1=0$ (or $c_1=0$) at $z=0$ and $z=h$, i.e., the conditions for maintaining the boundary at fixed temperature (or impurity concentration). At such a boundary the surface tension does not affect the conditions of excitation of the instability. In the presence of a thermal flux at the surface $z=h$ it is preferable to satisfy the condition of thermal isolation $\partial T_1/\partial z=0$, and it is necessary to take into account the presence of thermocapillarity at this boundary. (This problem was solved in Ref. 2.) In the presence of a matter flux, a more realistic picture is one in which the flux is produced by layers of liquid with different dielectric constants atop one another[1]; then the preferred model is one in which there are free boundaries at which the concentration is constant.

The boundary conditions for the electric field do not depend on the type of boundary and are as follows: the boundaries are kept at constant field intensity, and there are no fluctuation-induced deviations of the field along them, i.e., $E_{1x}=E_{1y}=0$.

In what follows, we will solve the problems posed in this paper with the specific real value $k_z=\pi$ (except in Sec. 7). In this case it is possible to obtain exact analytic solutions.

It is known (see, e.g., Refs. 1 or 4) that changing the conditions at a boundary can interfere with the excitation, but cannot block it completely, as reversing the direction of the flux can, for example.

Note that it is possible to solve the problem of excitation in the combined presence of both thermal and impurity fluxes. This problem was solved by Gershuni *et al.*[4] for the case where only bulk (Archimedean) forces act (see paragraphs 18 and 19). However, their approach leads to tedious computations, which are not needed to analyze the experimental data.

4. The solution to the problem as posed leads to the following excitation condition, i.e., the condition for existence of a nontrivial solution: for the case where a thermal flux is present:

$$k^6\mp\mathscr{R}_t k_\perp^2-\mathscr{E}_t k_\perp^2 k^2=0, \tag{8}$$

and for the case where a matter flux is present:

$$k^8\mp\mathscr{R}_c k^2 k_\perp^2-\mathscr{E}_c k_\perp^2 k_z^2=0. \tag{9}$$

Of course, the excitation conditions implied by these equations apply only when the forces act separately. Thus, in the presence of bulk expansion alone, the threshold value $\mathscr{R}\geqslant\mathscr{R}^*=27\pi^4/4\approx 660$ for $\lambda\simeq 3h$ is independent of the type of flux, but only for a flux directed opposite the force of gravity. When only a heat flux acts (thermoelectric effect), the instability is independent of the direction of flux when $\mathscr{E}_t\geqslant\mathscr{E}_t^*=4\pi^2\simeq 40$; a cell is excited with the corresponding dimensions $\lambda\simeq 2h$. When only a matter flux is present (electroconcentration effect), cellular motion appears for $\mathscr{E}_c\geqslant\mathscr{E}_c^*=3\pi^4(4/3)^4\simeq 1000$ when $\lambda\simeq 3.5h$; what is more, this condition is independent of the direction of the flux. In what follows it will be convenient to use the number $\mathscr{T}=\sqrt{\mathscr{E}}$; then $\mathscr{T}_t^*\simeq 6.3$, $\mathscr{T}_c^*\simeq 3.2$.

It is easy to find a range of parameters for which the effect of the fluxes is greater than that of the buoyancy (Archimedean) forces. This will occur for layers with thicknesses

$$h<h_t^*\simeq\left(\frac{\varepsilon\gamma^2\varkappa_t\nu}{\beta_t^2 g^2\rho}\right)^{1/6},\quad h<h_c^*\simeq\left(\frac{\varepsilon\varphi^2\varkappa_c\nu}{\beta_c^2 g^2\rho}\right)^{1/6} \tag{10}$$

for fluxes of heat and matter, respectively.

Equations (8) and (9) allow us to analyze how the Rayleigh mechanism (due to the buoyancy force) affects a flux-induced excitation. For a downward flux, this effect can be important in analyzing the experimental data, since the buoyancy forces stabilize the action of the fluxes in this case. Using the obvious relation $\mathscr{T}=\mathscr{R}(h^*/h)^3$, we find that under these conditions

$$\mathscr{T}_t=\pi\frac{1+w}{\sqrt{w}}\left[\sqrt{1+\frac{4w}{(1+w)^4}\frac{h^6}{h_t^6}}-\frac{1}{1+w}\frac{h^3}{h_t^3}\right], \tag{11}$$

$$\mathscr{T}_c=\pi^2\frac{(1+w)^2}{\sqrt{w}}\left[\sqrt{1+\frac{w}{(1+w)^2}\frac{1}{4\pi^2}\frac{h^6}{h_c^6}}+\frac{\sqrt{w}}{1+w}\frac{1}{2\pi^2}\frac{h^3}{h_c^3}\right]. \tag{12}$$

These values of the dimensionless numbers are once more subject to minimization with respect to $w=k_\perp^2/k_z^2$.

The analysis shows that for a downward flux, for which the action of the Archimedean force is stabilizing overall and leads to an increase in the values of $\mathscr{T}_t^*$ and $\mathscr{T}_c^*$ required for excitation, the ratio of the cell dimensions at the instant of excitation varies differently for the different fluxes. Thus, when only a thermoelectric force acts, $w_t^*>1$ instead of $w_t^*=1$, while $w_c^*<1/3$ instead of $w_c^*=1/3$ when only the concentration varies.

For an upward flux, the effects of the buoyancy forces and fluxes always add, and it is possible to relax the conditions for excitation. In fact, however, under these conditions everything is determined by the action of the buoyancy forces.

Numerical calculations show that at the instant of excitation the dimensionless numbers are related by a function that in all cases is very close to the function

$$\mathscr{T}^*=\sqrt{a\mp bR^*}$$

with constants a and b. This function can also describe the excitation conditions for boundary conditions that do not maintain reality of k_z, e.g., when the surface $z=0$ is solid. Furthermore, the function (referred to as the stability function)

$$\mathscr{T}^* = \sqrt{a \mp b\mathscr{R}^* \mp c\mathscr{M}^*} \tag{13}$$

(where c is also a constant) can specify the relation between the dimensionless numbers that characterize the various excitation mechanisms in a situation where surface tension forces act at the free surface.

For the case of heating, where the lower boundary is solid and isothermal while the upper is free (with a vacuum) and thermally isolated, we find the values $a \simeq 43$, $b \simeq 0.037$, and $c \simeq 0.53$, while for the case of a flux of light particles that disperse from the upper free surface into a half-space, the values $a \simeq 33$, $b \simeq 0.051$, $c \simeq 0.44$ yield values of the numbers $\mathscr{T}^*$, $\mathscr{R}^*$, $\mathscr{M}^*$ that are closest to the results of numerical calculations. In both of these cases we need to take the lower sign in Eqs. (13).

5. When the conditions for excitation are fulfilled, structures in the velocity are also accompanied by structures in the other quantities. The coordinate dependence of the excitation amplitudes other than (7) are determined by the expressions

$$\left.\begin{matrix} T_1 \\ c_1 \end{matrix}\right\} = \frac{A\varkappa h^2}{\nu^2}\frac{v_z}{k^2}, \tag{14}$$

$$v_{x,y} = -V\frac{k_x k_y}{k_\perp^2}\cos\left(k_z\frac{z}{h}\right)\sin\left(k_x\frac{x}{h}+k_y\frac{y}{h}\right), \tag{15}$$

$$\left.\begin{matrix} E_{1z} \\ k^2 E_{1z} \end{matrix}\right\} = \pm VE_0\frac{h}{\nu}\cos\left(k_z\frac{z}{h}\right)\cos\left(k_x\frac{x}{h}+k_y\frac{y}{h}\right), \tag{16}$$

$$\left.\begin{matrix} E_{1x,y} \\ k^2 E_{1x,y} \end{matrix}\right\} = \mp VE_0\frac{h}{\nu}\sin\left(k_z\frac{z}{h}\right)\sin\left(k_x\frac{x}{h}+k_y\frac{y}{h}\right). \tag{17}$$

The lower quantities in brackets on the left sides of Eqs. (14), (16), and (17) above indicate quantities obtained in the presence of a matter flux. All the remaining relations hold for both fluxes under discussion. It is clear that the boundaries of structures that arise in the corrections to the components of the electric field coincide with the boundaries of cells (structures) in the velocity; the temperature (or concentration) depends on the coordinate in the same way as v_z. The coordinate dependence of all the "hydrodynamic" quantities $(v_z, v_x, v_y, T_1, c_1)$ is the same as that of ordinary thermal convection for free surfaces. However, in the presence of the fluxes there also arise structures in the electric field. At the surface of a solution, an additional electric charge εE_{1z} appears under the action of a flux, which depends only on the amplitude and shape of the structure at the time of excitation.

In order to calculate the amplitude V it is necessary to use the same equations from Sec. 2, now taking into account the nonlinear terms as well. In writing the nonlinear equations, it is necessary, as before, to satisfy the Boussinesq approximation.[1,4] In particular, this requires that we discard nonlinear terms that contain the gradients of equilibrium quantities (A_t or A_c) everywhere except in terms that are also present in the linearized equations.

All quantities acquire additional terms proportional to the second, third, etc. powers of V. Furthermore, by carrying out the computations described in Secs. 3 and 4, we find that v_z has no corrections of second order, while T_1 (or c_1) does have these corrections. To second order, the condition for excitation has the form

$$\mp(\mathscr{R}-\mathscr{R}^*) - (\mathscr{E}-\mathscr{E}^*)k^2 = \mp\frac{1}{4k^2}\frac{V^2h^2}{\nu^2}(\mathscr{R}^*-\mathscr{E}^*k^2), \tag{18}$$

which allows us to find the required amplitudes of quantities that characterize the state of the liquid immediately after excitation when the numbers $\mathscr{R}$ and $\mathscr{E}$ are somewhat larger than the threshold values for the onset of instability. This result is consistent with the fact that the perturbation amplitudes are proportional to the square root of the "degree of supercriticality," i.e., the excess above the threshold value needed to excite the quantity that is the cause of the instability. Thus, in thin layers ($h<h^*$) we have $V \propto \sqrt{\mathscr{T}-\mathscr{T}^*}$, independent of the direction of flux.

By iterating the solution process we can easily compute corrections that determine the change in cell shape. However, these computations are too tedious.

Thus, we have the following relations for the amplitudes:

$$V_t \simeq 4\pi\frac{\nu}{h}\sqrt{\frac{T_h-T_h^*}{T_h^*-T_c}}, \quad V_e \simeq 2\sqrt{2}\frac{\nu}{h}\sqrt{\frac{c_s-c_s^*}{c_s^*-c_f}}. \tag{19}$$

We note that the amplitudes of structures that appear under the action of a thermal flux are 1.63 times the corresponding amplitudes for structures that arise under the action of a matter flux, and are 2.5 times smaller than the amplitudes when only the force of bulk expansion acts, computed within the same approximation.

Using Eq. (18), we can also find out how the mechanism mediated by bulk expansion affects the amplitude of motion driven by the flux. This effect makes itself felt through the appearance in Eqs. (19) of a factor (for an downward flux)

$$\sqrt{1-\frac{\mathscr{E}}{\mathscr{R}-\mathscr{E}}}. \tag{20}$$

The amplitude resulting from this stabilizing action of the buoyancy force is somewhat reduced.

6. The excitation of cellular motion and structures by fluxes of heat or matter is more convenient to observe and study experimentally when the liquid is rotated at an angular velocity Ω parallel to the z axis. Under these conditions, quantities are present in the problem that are given by a pseudovector, so that excitation is possible both with aperiodic and with oscillating growth. In fact, however, the oscillating branch of the excitation (arising for small A_t or A_c) occurs only when $\nu \ll \varkappa$, and in reality this relation does not hold at all in liquid semiconductors and liquid insulators under laboratory conditions. Therefore, we can analyze the branch of excitations with aperiodic growth as before, but

with a correction when a heat flux acts, i.e., the term $-k_z^2\mathscr{T}$ in Eq. (8), while a term $-k_z^2k^2\mathscr{T}$ is present in Eq. (9) when a matter flux acts; here the Taylor number $\mathscr{T}=4\Omega^2h^4/\nu^2$. Note that the excitation conditions for the branch of oscillating growth in fact consists of the same terms as in the case of aperiodic growth, but multiplied by a factor of order unity.

We will not pause to discuss the case of an upward flux, but only point out that for downward fluxes the following asymptotic values are obtained:

$$\mathscr{T}_t\simeq\sqrt{\frac{2}{\pi}}\sqrt{\frac{2\mathscr{T}}{\pi^4}},\quad w_t\simeq\left(\frac{2\mathscr{T}}{\pi}\right)^{1/3},$$

$$\mathscr{T}_c\simeq\sqrt{T},\quad w_c\simeq\pi\left(\frac{\mathscr{T}}{3}\right)^{1/3}. \tag{21}$$

Note that excitation of structures by fluxes in a magnetic field can also be investigated completely.

7. We will also not pause to discuss experiments on the excitation of instability by an upward flux (see, however, Refs. 6 and 7 for fluxes of heat and matter respectively). Instead, we will discuss experiments in which the direction of the fluxes coincides with the direction of the force of gravity (downward fluxes). Experiments on excitation of cellular motion by downward heat fluxes created by laser illumination are described in detail in Ref. 3. Here, however, the results we present will be compared with experimental data on electroconvection of binary mixtures.[1]

The theory developed above best describes an experiment in which a component with a large dielectric constant ε_g (for example, transformer oil) is placed above a layer of insulator with a small dielectric permittivity ε_m (for example, psilomethane). Near the boundary a thin layer of binary mixture forms, with a gradient of dielectric constant on the average equal to $(\varepsilon_g-\varepsilon_m)/h$. It is obvious that this gradient is proportional to the quantity A_c [see (1)]. If this mixture is placed in an electric field, then all the conditions are satisfied for the appearance of cellular motion. Of course, for this to occur the quantity $\mathscr{E}_c$ must exceed a value $\mathscr{E}_c^*\simeq 1000$ (see below). If we substitute in the values $\rho\simeq10^2$–10^3 kg/m^3, $\nu\simeq10^{-6}$–10^{-7} m^2/s, $\kappa_c\simeq10^{-8}$–10^{-9} m^2/s, $\varepsilon_g=10^{-9}$ F/m, $\varepsilon_m\simeq10^{-10}$ F/m, which are typical of dielectrics, we obtain the estimate

$$\varphi>\varphi^*\simeq Eh(c_s-c_f)\simeq 1\ \text{V}. \tag{22}$$

From this, we find that for $c\simeq10\%$ and $h=0.1$ to 0.01 mm the required field $E_0\simeq10^5$ to 10^7 V/m. This value is achievable in realistic high-power electrical devices, e.g., in switches. Of course, in real devices the conditions for this model are not satisfied, either for the medium geometry or the uniformity of the concentration gradient $|\nabla c|$; however, in principle these changes cannot "shut down" the excitation.

This state of affairs is analogous to the case where cellular motion is excited in a fluid heated by laser radiation from above. For this system, the experimental conditions are also far from those of the layer model; nevertheless, there is qualitative agreement between the depth of penetration observed in reality and its value calculated theoretically.

In liquid semiconductors, the conditions for excitation by heating from above are determined by the ratio of the exciting force (electrical) to the forces that stabilize it. In the presence of heating, the stabilizing force is the force of surface tension, because the illumination enters in through the free surface. We have for the conditions of excitation[6]

$$A_t>\frac{\sigma_t}{\varepsilon\gamma_t^2}\frac{\mathscr{E}_t^*}{\mathscr{M}_t^*}. \tag{23}$$

The condition for excitation in the binary mixture described above is determined by competition between the electrical and buoyancy forces. This condition can be written in the form

$$A_c>\frac{en\varphi}{\rho\beta_c gh^2}\frac{\mathscr{E}_c^*}{\mathscr{R}_c^*}, \tag{24}$$

where, of course, we must also rewrite the expression for the potential φ.

Excitation by matter fluxes in an electric field is clearly possible under conditions prior to the appearance of electromechanical isothermal convection.[1] The experimental data show that for supercritical field intensities the surface is distorted in such a way that the layer with smaller dielectric constant forms elongated spikes that penetrate the less conductive phase. This phenomenon can be explained by a word-for-word translation of the explanation for the appearance of cells when the layers are heated from above.[3]

At first, the operating mechanism is clearly caused by the flux in the direction perpendicular to the layer. The conditions for the instability (24) are surely fulfilled in an external field strong enough to satisfy the estimate (22). According to the results of Sec. 5, the resulting motion has a velocity that to order of magnitude equals

$$V=\frac{\nu}{10h}\sqrt{\frac{\varepsilon E^2(c_s-c_f)h^4}{\rho\nu\varkappa_c}-\mathscr{E}_c^*}, \tag{25}$$

i.e., about 0.1 to 0.01 mm/s.

Penetration of the material with ε_g into the lower layer results in a gradient in the dielectric constant, and likewise in the concentration, in the direction along the layer. As in the case of thermoelectric instability,[3] an abrupt expansion (by an order of magnitude) of the cell takes place in the plane along the boundary in this case. In fact, there is no need even to repeat the calculation: we can simply use the fact that under conditions of two solid boundaries $\mathscr{E}_c^*\simeq 5.6/k_x$, where for $\mathscr{E}_c^*$ we should substitute the field E along the layer. Since $\mathscr{E}_c^*\simeq 1000$, $\lambda/h\simeq1100$.

Analysis of the photographs of the evolution of the electromechanical isothermal instability given in Ref. 1 (Fig. 43) shows that the typical size of a structure in a psilomethane-transformer oil suspension perpendicular to the boundary changes from 16 to 40 μm as the applied field changes from 10 to 20 kV. The dimension along the layer is $\simeq$16 mm. It is obvious that the ratio of the dimensions is around (less than) 1000, which qualitatively confirms the calculation.

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