Bremsstrahlung in intermediate-energy electron scattering by noble gas atoms

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Within the low-energy approximation scheme an explanation is given of the experimental data on the total bremsstrahlung cross section in the scattering of 300–600 eV electrons by xenon atoms. It is shown that the Born approximation gives a qualitatively incorrect description of the bremsstrahlung cross section for electrons up to an energy of 4 keV scattered by xenon and krypton atoms.

1. INTRODUCTION

It is customary to assume that the bremsstrahlung cross section in the scattering of a nonrelativistic electron by an atom decreases as a function of electron energy.[1] Such behavior of the cross section is assumed in studies of many applied problems, such as the theory of radiative losses in the passage of particles through matter and the theory of emission and absorption of radiation by plasma.

Lately, however, bremsstrahlung measurements in the ultrasoft x-ray region in electron scattering by xenon atoms[2] have shown that the number of bremsstrahlung photons increases as the electron energy grows. The present paper is devoted to explaining this fact.

In the traditional Bethe–Heitler and Sauter theory of bremsstrahlung an atom is considered the source of a potential field consisting of the Coulomb field of the nucleus and that of the screening electrons. Allowing for the dynamic electron degrees of freedom led to the idea of polarized bremsstrahlung,[3] whose direct experimental study in electron scattering by xenon atoms is described in Ref. 4.

The dynamic properties of the electron system also manifest themselves in the description of the growth in the bremsstrahlung cross section with the incident-electron energy observed by Tkachenko *et al.*[2] As the electron energy increases, so does the number of open atomic excitation channels accompanying the bremsstrahlung. In other words, in bremsstrahlung from an electron scattered by an atom the latter does not remain in the initial state but can become excited or ionized. As a result the total bremsstrahlung cross section grows.

Of course, the possibility of exciting or ionizing atoms in bremsstrahlung has been repeatedly mentioned in the literature. However, there are no systematic calculations of the respective cross sections. More than that, the Born approximation of collision theory is insufficient for intermediate-energy electrons, which are electrons whose energies exceed the first atomic ionization potential by a factor of 10–100. Hence obtaining theoretical estimates of the cross section of bremsstrahlung accompanied by atomic excitation or ionization requires a considerable effort.

2. STARTING EQUATIONS

To solve the problem we use the low-energy theorem.[1,5] For soft photons, whose energy is much lower than that of the incident electron, the bremsstrahlung cross section σ^γ can be expressed in terms of the "nonbraking" scattering cross section σ^e. In the nonrelativistic approximation,

$$d\sigma^\gamma=\alpha[(\mathbf{v}'-\mathbf{v})\cdot\mathbf{k}]^2\frac{d\omega\, d\Omega}{4\pi^2c^2k^2\omega}d\sigma^e. \tag{1}$$

Here α is the fine-structure constant, $\mathbf{v}$ and $\mathbf{v}'$ are the electron velocities before and after scattering, ω, $\mathbf{k}$, and Ω are the frequency, wave vector, and solid angle of bremsstrahlung photon emission, and we sum over the bremsstrahlung photon polarizations.

We expand the cross section σ^e in Legendre polynomials, which depend on the angle between the vectors $\mathbf{v}$ and $\mathbf{v}'$. In view of conservation of spatial parity in electromagnetic interactions, the expansion contains only polynomials with even indices:

$$d\sigma^e=\frac{1}{4\pi}\sum_{l=0} \sigma_{2l}^e P_{2l}(\cos\theta')\sin\theta'\, d\theta'\, d\varphi'. \tag{2}$$

Here θ' and φ' are the angles specifying $\mathbf{v}'$, and the direction of $\mathbf{v}$ is chosen as the z axis.

Substituting Eq. (2) into Eq. (1) and integrating with respect to θ' and φ', we obtain

$$\frac{d\sigma^\gamma}{d\omega\, d\Omega}=\frac{\alpha}{4\pi^2c^2\omega}\left\{[v^2\sin^2\theta+\tfrac{2}{3}v'^2(1+\cos^2\theta)]\sigma_0^e + \frac{v'^2}{15}(1+\cos^2\theta)\sigma_2^e\right\}, \tag{3}$$

where θ is the polar angle of photon emission with respect to the incident electron.

Clearly, reconstructing the bremsstrahlung cross section from the cross section of the "nonbraking" scattering cross section requires knowing the angular distribution of the scattered electrons, which determines σ_2^e. This quantity, however, has a small numerical coefficient, hence to within $\sim$10% we have

$$\frac{d\sigma^\gamma}{d\omega\, \delta\Omega}\approx\frac{\alpha v^2}{4\pi^2c^2\omega}[1+\tfrac{2}{3}(1-\tfrac{1}{2}\cos^2\theta)]\sigma_0^e. \tag{4}$$

Here we have put $v'\approx v$. Note that the difference between v' and v is related not only to photon emission but also to atomic excitation. However, this approximation is apparently valid if we wish to obtain only qualitative results.

TABLE I. The coefficients of the interpolation formulas (6) and (7) in units of 10^{-16} cm^2.

	A_1	A_2	A_3	B_1	B_2
Xe	−2.12	62.4	−66.9	139	−292
Kr	−0.65	32	0	71.4	−80.4

3. COMPARISON WITH EXPERIMENTAL DATA

Let us now compare Eq. (4) with the experimental data. In Ref. 2 the angle θ is close to 90°. Writing Eq. (4) for this case, we get

$$\frac{d\sigma^\gamma}{d\omega\, d\Omega} \approx \frac{5\alpha v^2}{12\pi^2 c^2 \omega}\sigma_0^e. \tag{5}$$

We use the data of Zecca *et al.*,[6] who in experiments measured the total cross sections of electron scattering by xenon and krypton atoms and discussed in detail the contributions of elastic and inelastic processes to the total cross section, with the inelastic processes often determining the size of the total cross section. The total cross sections were approximated in Ref. 6 roughly to within 5% by the following formulas:

$$\sigma_0^e = A_1 + \frac{A_2}{\sqrt{E/\mathrm{Ry}}} + \frac{A_3}{E/\mathrm{Ry}} \tag{6}$$

for incident-electron energies E in the 80–1000 eV range, and

$$\sigma_0^e = \frac{B_1}{E/\mathrm{Ry}} \ln\frac{E}{\mathrm{Ry}} + \frac{B_2}{E/\mathrm{Ry}} \tag{7}$$

for E in the 800–4000 eV range. The coefficients A and B for xenon and krypton are listed in Table I.

Tkachenko *et al.*[2] did not obtain the absolute values of the bremsstrahlung cross sections, only their variation with electron energy. If these data are described on the hyperbolic sections of the bremsstrahlung spectra, where $\sigma^\gamma \propto \omega^{-1}$, by Eq. (5), we arrive at the pattern depicted in Fig. 1. Clearly, theoretical formulas based on the low-energy theorem provide a correct qualitative picture of the behavior of the experimental data, although quantitatively certain differences are apparent.

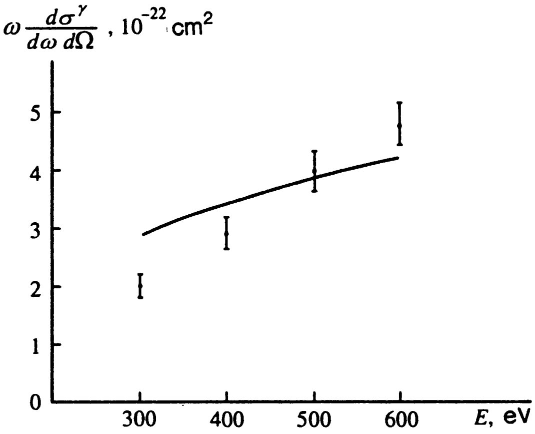

FIG. 1. Comparison of the theoretical results (solid curve) with the experimental data of Ref. 2.

4. DISCUSSION

As Fig. 1 shows, the experimental data vary faster with electron energy than predicted by the theory. This suggests that there is an additional source of bremsstrahlung. Recombination radiation emerging in the interaction volume is a possible source. In addition, radiation may appear directly at the time secondary electrons are knocked out of the atom, in full analogy with internal bremsstrahlung accompanying nuclear beta decay.[7] The theory of the latter phenomenon can easily be developed, but the absence of fairly complete experimental data on the energy distribution of the secondary electrons makes it impossible to use these ideas in describing the experimental results in total bremsstrahlung. Also, the quantitative discrepancy between theory and experiment may be due to the approximations used in deriving Eq. (4).

The data of Zecca *et al.*[6] in conjunction with the low-energy theorem also make it possible to establish the limit of "inapplicability" of the Born approximation when electrons are scattered by krypton and xenon atoms. Equations (5) and (7) show that at energies up to 4 keV the bremsstrahlung cross section still increases with E, but only logarithmically. Since in accordance with the Born approximation the cross section must decrease, we can conclude that up to 4 keV the Born approximation of customary bremsstrahlung theory is inapplicable for krypton and xenon atoms.

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