Resonant electron tunneling in the field of a strong electromagnetic wave

N. V. Kornyakov

Microelectronics Institute of the Russiian Academy of Sciences, 150051 Yaroslavl', Russia
(Submitted 16 November 1994)
Zh. Éksp. Teor. Fiz. **107**, 1983–1995 (June 1995)

The problem of resonant tunneling of electrons in the field of a strong electromagnetic wave is solved by diagonalizing the Hamiltonian of the electron system, which includes the Coulomb interaction between free and localized electrons, with a unitary transformation. The solution obtained is asymptotically exact in the limit of weak electron–electron interactions. A distinctive feature of this problem is the fact that the "electron–electron" and "electron–hole" singularities that appear in the amplitude for scattering of electrons by a resonance level are separated in frequency; therefore, there is no linkage of "ladder" diagrams for the Green's function into a "parquet." This makes it possible to describe the electron–electron correlations correctly by means of the generalized self-consistent field approximation. Matrix elements are calculated that correspond to various types of tunneling transitions. It is found that the strong electromagnetic field mediates a renormalization of the matrix elements for direct tunneling of electrons through the barrier by the processes of inelastic resonant tunneling, which leads to important changes in the probabilities for these tunneling processes. A closed expression is obtained for the tunneling current.

1. INTRODUCTION

There are a number of practical applications of the tunneling effect whose analysis involves the resonant tunneling of electrons in the field of a strong electromagnetic wave. These applications range from investigating solid surfaces by scanning tunneling microscopy to designing new quantum semiconductor devices based on heterostructures, in particular optoelectronic devices.

The combined use of lasers and scanning tunneling microscopy (STM) to investigate semiconductors and metals, and also to probe the semiconductor surface locally, has opened up new perspectives in the physics of surfaces. The resonant character of electron tunneling in STM is due to the presence within the tunneling barrier of quasilocalized electronic states of various kinds. In particular, these states can be associated with atoms or molecules that are adsorbed at the surface. However, from a practical point of view the more important situation is one where a thin film of insulator (e.g., a polymer or Langmuir–Blodgett film) is deposited on the surface of the semiconductor artificially. In this case, some of the quasilocalized surface states belong to bonding and antibonding orbitals of the polymer molecules. If this is the case, it should be possible to modify the properties of the film in a purposeful way by using a laser to change the degree of occupation of these states. However, in order to accomplish this goal it is necessary to have an adequate theoretical description of tunneling processes in the presence of a strong electromagnetic field. The author has reported certain results in this direction in a previous publication (see Ref. 1). In recent years, numerous experimental investigations have stimulated further developments in the theory. Thus, Molotkov[2] developed a theory of resonance spectroscopy of adsorbates exposed to electromagnetic radiation in order to explain the experimental measurements of steady-state photocurrents in STM by Volcker *et al.*[3]

There are many potential applications of the phenomenon of resonant tunneling in the presence of a strong electromagnetic field in the design of quantum devices, including those based on new physical principles.[4–6] In Ref. 6, for example, Kopaev *et al.* discussed the possibility of using light to induce switching between various quantized states in a tunneling quantum nanostructure. In this case a relocation of the electron wave function occurs in space with a corresponding change in the system conductivity. Since the tunneling probability for penetration of a tunnel-coupled system of quantum wells depends resonantly on the position of the energy levels in the quantum wells, it becomes possible to develop quantum optoelectronic devices that are switched by optical pulses from one logic state to another.

In a recent paper,[7] Inarrea *et al.* discuss the effect of resonant tunneling in a heterostructure with one quantum well (a type of resonant tunnel diode) under the action of laser illumination within the context of the experimental infrared-absorption measurements of Chitta *et al.*[8]

2. HAMILTONIAN OF THE PROBLEM

The complete Hamiltonian of the problem is a sum of the electron Hamiltonian $\hat{H}_e$ and a term $\hat{H}_{eA}$ that describes the interaction of the electrons with the field of the strong electromagnetic wave. Let us write the electron Hamiltonian in the form:

$$\hat{H}_e=\hat{H}_0+\hat{H}_T+\hat{H}_R+\hat{H}_{ee}. \quad (1)$$

Here $\hat{H}_0$ describes electronic states in the separate parts of the tunnel contact; $\hat{H}_T$ and $\hat{H}_R$ give rise to the processes of direct tunneling of electrons through the barrier and of elastic resonant tunneling; and $\hat{H}_{ee}$ describes the Coulomb interaction between free and localized electrons.

Usually there is a whole set of localized states within the tunnel barrier with various energies; however, only a small portion of them will turn out to be in resonance with the field of the electromagnetic wave. Therefore, for simplicity we will investigate a case where the tunneling barrier contains only one localized state with energy E_0:

$$\hat{H}_0=\sum_{\sigma,\mathbf{p}} \varepsilon(\mathbf{p})(a^+_{\mathbf{p}\sigma}a_{\mathbf{p}\sigma}+b^+_{\mathbf{p}\sigma}b_{\mathbf{p}\sigma})$$

$$+\sum_{\sigma} (E_0 d^+_\sigma d_\sigma + W d^+_\sigma d^+_\sigma d_{-\sigma} d_{-\sigma}), \tag{2}$$

$$\hat{H}_T=\sum_{\sigma,\mathbf{p},\mathbf{p}'} [T(\mathbf{p},\mathbf{p}')a^+_{\mathbf{p}\sigma}b_{\mathbf{p}'\sigma}+\text{h.c.}]), \tag{3}$$

$$\hat{H}_R=\sum_{\sigma,\mathbf{p}} [V_1(\mathbf{p})a^+_{\mathbf{p}\sigma}d_\sigma+V_2(\mathbf{p})b^+_{\mathbf{p}\sigma}d_\sigma+\text{h.c.}], \tag{4}$$

$$\hat{H}_{ee}=\sum_{\mathbf{p},\mathbf{q},\sigma} [g_1(\mathbf{q})a^+_{\mathbf{p}+\mathbf{q}\sigma}d^+_\sigma d_\sigma a_{\mathbf{p}\sigma}$$

$$+g_2(\mathbf{q})b^+_{\mathbf{p}+\mathbf{q}\sigma}d^+_\sigma d_\sigma b_{\mathbf{p}\sigma}]. \tag{5}$$

Here $a^+_{\mathbf{p}\sigma}$, $b^+_{\mathbf{p}\sigma}$, d^+_σ, are, respectively, operators that create an electron with spin σ tunneling through the barrier from the left, tunneling from the right, or resident in the localized state; $\varepsilon(\mathbf{p})$ is the energy dispersion law for the electrons, which for simplicity we will assume is the same on both sides of the barrier; W is the Coulomb interaction energy between two electrons on the same center; $T(\mathbf{p},\mathbf{p}')$ is the matrix element for direct tunneling of an electron through the barrier; $V_{1,2}(\mathbf{p})$ are matrix elements for tunneling transitions between localized states and the corresponding band states; and $g_{1,2}(\mathbf{p})$ are constants for the electron–electron interaction.

The overlap of the wave functions for band and localized states, expressed here by the appearance of the nonzero matrix elements $V_1(\mathbf{p})$ and $V_2(\mathbf{p})$ for elastic tunneling, also allows the electrons to make inelastic radiative transitions between these same states.

In order to correctly describe the interaction of tunneling electrons with the field of an electromagnetic wave, it is necessary to transform the Hamiltonian (1). By introducing new Fermi operators via the relations[9]

$$c_{\mathbf{p}\sigma}=ua_{\mathbf{p}\sigma}+vb_{\mathbf{p}\sigma}, \quad \tilde{c}_{\mathbf{p}\sigma}=ub_{\mathbf{p}\sigma}-va_{\mathbf{p}\sigma}, \tag{6}$$

$$u=V_1(\mathbf{p})/V(\mathbf{p}), \quad v=V_2(\mathbf{p})/V(\mathbf{p}),$$

$$V(\mathbf{p})=\sqrt{V_1^2(\mathbf{p})+V_2^2(\mathbf{p})}, \quad u^2+v^2=1, \tag{7}$$

we can change to a representation in which only those quasiparticles described by the operators $c^+_{\mathbf{p}\sigma}$ interact with the localized state. For simplicity let us consider the symmetric case: $V_1(\mathbf{p})=V_2(\mathbf{p})$, $g_1(\mathbf{p})=g_2(\mathbf{p})$. Then the Hamiltonian (1) takes the form

$$\hat{H}_e=\sum_{\sigma,\mathbf{p}} \varepsilon(\mathbf{p})(c^+_{\mathbf{p}\sigma}c_{\mathbf{p}\sigma}+\tilde{c}^+_{\mathbf{p}\sigma}\tilde{c}_{\mathbf{p}\sigma})$$

$$+\sum_{\sigma} (E_0 d^+_\sigma d_\sigma + W d^+_\sigma d_\sigma d^+_{-\sigma} d_{-\sigma})$$

$$+\sum_{\sigma,\mathbf{p},\mathbf{p}'} T(\mathbf{p},\mathbf{p}')(c^+_{\mathbf{p}\sigma}c_{\mathbf{p}'\sigma}+\tilde{c}^+_{\mathbf{p}\sigma}\tilde{c}_{\mathbf{p}'\sigma})$$

$$+\sum_{\sigma,\mathbf{p}} [V(\mathbf{p})c^+_{\mathbf{p}\sigma}d_\sigma+\text{h.c.}]+\sum_{\sigma,\mathbf{p},\mathbf{q}} [g(\mathbf{q})c^+_{\mathbf{p}+\mathbf{q}\sigma}d^+_\sigma d_\sigma c_{\mathbf{p}\sigma}$$

$$+g(\mathbf{q})\tilde{c}^+_{\mathbf{p}+\mathbf{q}\sigma}d^+_\sigma d_\sigma \tilde{c}_{\mathbf{p}\sigma}]. \tag{8}$$

According to Eqs. (6), the wave functions of quasiparticles described by the operators $c^+_{\mathbf{p}\sigma}$ and $\tilde{c}^+_{\mathbf{p}\sigma}$ are, respectively, in-phase and out-of-phase superpositions of the wave functions for electrons on the right and left sides of the tunnel contact. Therefore, we may use the operator for interaction with an electromagnetic field in its usual form for the $c^+_{\mathbf{p}\sigma}$ quasiparticles. However, the terms that contain the operator $\tilde{c}^+_{\mathbf{p}\sigma}$ will be discarded in what follows.

3. INTERACTION WITH THE FIELD OF A STRONG ELECTROMAGNETIC WAVE

The interaction of electrons with the electromagnetic field is taken into account by introducing the following term into the Hamiltonian:

$$\hat{H}_{eA}(t)=\sum_{\sigma,\mathbf{p}} \lambda(\mathbf{p})c^+_{\sigma\mathbf{p}}d_\sigma \exp(-i\Omega t)+\text{h.c.} \tag{9}$$

Here Ω is the frequency of the electromagnetic field, while $\lambda(\mathbf{p})$ is the matrix element for the corresponding transition ($\hbar=c=1$):

$$\lambda(\mathbf{p})=\frac{e}{m_0\Omega}\int \psi^*_{\mathbf{p}}(\mathbf{r})\mathbf{E}(\mathbf{r})\varphi_0(\mathbf{r})d^3\mathbf{r}, \tag{10}$$

e and m_0 are the charge and mass of a free electron, $\mathbf{E}(\mathbf{r})$ is the field intensity of the electromagnetic wave, $\varphi_0(\mathbf{r})$ are the wave functions of electrons in the localized states, and the wave functions $\psi_{\mathbf{p}}(\mathbf{r})$ are represented in accordance with (6) as linear combinations

$$\psi_{\mathbf{p}}(\mathbf{r})=u\psi_{1\mathbf{p}}(\mathbf{r})+v\psi_{2\mathbf{p}}(\mathbf{r})$$

of band wave functions

$$\psi_{j\mathbf{p}}(\mathbf{r})=\xi_{j\mathbf{p}}(\mathbf{r})\exp(i\mathbf{pr}), \quad j=1,2$$

on the right and left sides of the contact; here $\xi_{j\mathbf{p}}(\mathbf{r})$ are Bloch amplitudes. Furthermore, Eq. (10) can be transformed to the form

$$\lambda(\mathbf{p})=\frac{e}{m_0\Omega}(\mathbf{E}(\mathbf{p})\mathbf{p}_0), \tag{11}$$

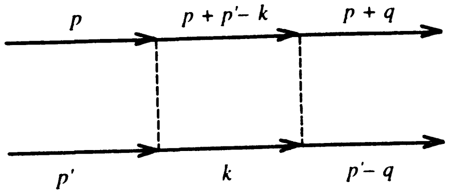

FIG. 1.

where $\mathbf{E}(\mathbf{p}) = \int \mathbf{E}(\mathbf{r}) \exp(i\mathbf{p}\mathbf{r}) d^3r$ and $\mathbf{p}_0 = \int \psi_{\mathbf{p}}^*(r)(-i\nabla)\varphi(\mathbf{r}) d^3r$ are, respectively, the Fourier transform of the electromagnetic wave electric field intensity and the matrix element of the momentum operator between the corresponding states.

Thus, Eqs. (8) and (9) specify the Hamiltonian of the problem:

$$\hat{H} = \hat{H}_e + \hat{H}_{eA} = \sum_{\sigma,\mathbf{p}} \varepsilon(\mathbf{p}) c_{\mathbf{p}\sigma}^+ c_{\mathbf{p}\sigma} + \sum_{\sigma} (E_0 d_\sigma^+ d_\sigma + W d_\sigma^+ d_\sigma d_{-\sigma}^+ d_{-\sigma}) + \sum_{\sigma,\mathbf{p},\mathbf{q}} T(\mathbf{p},\mathbf{p}') c_{\mathbf{p}\sigma}^+ c_{\mathbf{p}'\sigma} + \sum_{\sigma,\mathbf{p}} [V(\mathbf{p}) c_{\mathbf{p}\sigma}^+ d_\sigma + \text{h.c.}] + \sum_{\sigma,\mathbf{p},\mathbf{q}} g(\mathbf{q}) c_{\mathbf{p}+\mathbf{q}\sigma}^+ d_\sigma^+ d_\sigma c_{\mathbf{p}\sigma} + \sum_{\mathbf{p},\sigma} [\lambda(\mathbf{p}) c_{\mathbf{p}\sigma}^+ d_\sigma \exp(-i\Omega t) + \text{h.c.}]. \quad (12)$$

In general, the quasilocalized state is located far from the Fermi surface and the contribution of resonant processes to the tunneling current is insignificant. However, in the presence of a strong electromagnetic field with frequency $\Omega \geqslant |E_0 - E_F|$ (where E_F is the Fermi energy) a coherent state of band and localized electrons can appear.[10] This creates conditions that allow inelastic resonant tunneling transitions of electrons through the barrier. The character of the elastic tunneling will also change in this case.

The electromagnetic field is considered to be strong if the electron–hole pair creation rate exceeds the pair recombination rate, i.e., a "saturation effect" occurs. In the saturated state, the distribution functions of electrons in the band and localized states may be considered to be quasi-Fermilike, with quasi-Fermi levels μ and μ_0, respectively, where $\Omega = \mu_0 - \mu$. In order to define the values of the parameters μ and μ_0, we must use the equation for conservation of particle number. However, under conditions of exact resonance, i.e., $\Omega = E_0 - E_F$ (for definiteness we take $E_0 > E_F$), then $\mu = E_F$, while $\mu_0 = E_F + \Omega$.

The term $\hat{H}_{eA}$ in the Hamiltonian (12) contains an explicit dependence on time. By using the unitary transformation

$$U(t) = \exp\left[-it\left(\mu \sum_{\sigma,\mathbf{p}} c_{\mathbf{p}\sigma}^+ c_{\mathbf{p}\sigma} + \mu_0 \sum_{\sigma} d_\sigma^+ d_\sigma\right)\right], \quad (13)$$

we can eliminate this dependence. However, when we do this, a time-dependent factor appears in the term connected with elastic tunneling:

$$\hat{H}_U = U^+ \hat{H}(t) U + U^+ (i\partial/\partial t) U = \sum_{\sigma,\mathbf{p}} [\varepsilon(\mathbf{p}) - \mu] c_{\mathbf{p}\sigma}^+ c_{\mathbf{p}\sigma} + \sum_{\sigma} [(E_0 - \mu_0) d_\sigma^+ d_\sigma + W d_\sigma^+ d_\sigma d_{-\sigma}^+ d_{-\sigma}] + \sum_{\sigma,\mathbf{p},\mathbf{q}} g(\mathbf{q}) c_{\mathbf{p}+\mathbf{q}\sigma}^+ d_\sigma^+ d_\sigma c_{\mathbf{p}\sigma} + \sum_{\sigma,\mathbf{p}} [\lambda(\mathbf{p}) c_{\mathbf{p}\sigma}^+ d_\sigma + \text{h.c.}] + \sum_{\sigma,\mathbf{p},\mathbf{p}'} T(\mathbf{p},\mathbf{p}') c_{\mathbf{p}\sigma}^+ c_{\mathbf{p}'\sigma} + \sum_{\sigma,\mathbf{p}} [V(\mathbf{p}) c_{\mathbf{p}\sigma}^+ d_\sigma \exp(i\Omega t) + \text{h.c.}]. \quad (14)$$

Nevertheless, since the elastic processes take place far from resonance, the Hamiltonian (14) can be diagonalized by dropping the last two terms. In what follows, we will take the process of elastic tunneling into account by perturbation theory. Note that without the last two terms, Eq. (14) is formally equivalent to the Anderson Hamiltonian, which describes anomalies in the resistance of metals with magnetic impurities. This allows us to solve the problem of resonant tunneling by making use of methods developed in the theory of metals to investigate impurity states.

4. THE ROLE OF ELECTRON–ELECTRON INTERACTIONS

In the steady coherent state, a primary role is played by the Coulomb interaction $\hat{H}_{ee}$ between free and localized electrons. This assertion is supported by the results of recent experiments[11] on resonant tunneling in heterostructures. Although this interaction cannot mediate transitions of electrons through the tunnel barrier by itself, in what follows we will show that it changes the probability of inelastic tunneling by interfering with the interaction $\hat{H}_{eA}$.

Let us compute the amplitude $\Gamma_1(\mathbf{p},\mathbf{q})$ for scattering of a free electron described by the operators $c_{\mathbf{p}\sigma}^+$ by the localized state d_σ^+ to second order in perturbation theory with respect to the electron–electron interaction. Graphically, this expression can be represented by the diagram of Fig. 1, where the upper line corresponds to a free electron and a bottom line to an electron in the localized state; the dashed line denotes the Coulomb interaction. Here we have adopted the notation $p = \{\mathbf{p},\omega\}$, $p' = \{\mathbf{p}',\omega'\}$, $k = \{\mathbf{k},\omega_0\}$, $q = \{\mathbf{q},\omega_1\}$.

Assuming that the dispersion law for free electrons is isotropic with an effective mass m, and replacing the magnitude of the density of states by the quantity $\nu_0 = mp_F/\pi^2$, for $E = E_F$ we obtain:

$$\Gamma_1(\mathbf{p},\mathbf{q})=g^2\int\frac{d\omega_0}{2\pi}\frac{d^3k}{2\pi^3}\frac{1}{\omega_0-(E_0-\mu_0)+i\delta\ \mathrm{sign}(E_0-\mu_0)}$$

$$\times\frac{1}{\omega+\omega'-\omega_0-[\varepsilon(\mathbf{p}+\mathbf{p}'-\mathbf{k})-\mu]+i\delta\ \mathrm{sign}[\varepsilon(\mathbf{p}+\mathbf{p}'-\mathbf{k})-\mu]}$$

$$=\frac{1}{2}g^2\nu_0\ln\frac{\omega_0}{E_0-E_F-\Omega+2(E_F-\mu)},\qquad(15)$$

where ω_0 is a cutoff energy for the integral. This expression is singular when $\Omega=E_0-E_F$. The pole of the scattering amplitude reveals the existence of a bound state, analogous to the Cooper-pairing state for electrons in superconductors. In order to take this feature into account, let us introduce anomalous averages into the discussion of the form $\langle c^+_{\mathbf{p}\sigma}d^+_\sigma\rangle$.

Meanwhile, the amplitude $\Gamma_2(\mathbf{p},\mathbf{q})$ for scattering of a free electron by a localized hole characterized by the operator d also has a (logarithmic) singularity. Graphically, the corresponding process is represented by the diagram of Fig. 2; the analytic expression has the form

$$\Gamma_2(\mathbf{p},\mathbf{q})=g^2\int\frac{d\omega_0}{2\pi}\frac{d^3k}{2\pi^3}$$

$$\times\frac{1}{\omega_0-(E_0-\mu_0)+i\delta\ \mathrm{sign}(E_0-\mu_0)}$$

$$\times\frac{1}{\omega+\omega_1-[\varepsilon(\mathbf{k}+\mathbf{q})-\mu]+i\delta\ \mathrm{sign}[\varepsilon(\mathbf{k}+\mathbf{q})-\mu]}$$

$$=\frac{1}{2}g^2\nu_0\ln\frac{\omega_0}{E_0-E_F-\Omega}.\qquad(16)$$

The pole of the scattering amplitude $\Gamma_2(\mathbf{p},\mathbf{q})$, which corresponds to a bound state of an electron and hole analogous to the excitonic-insulator pairing state in semimetals,[12] is characterized by anomalous nonzero averages of the form $\langle c^+_{\mathbf{p}\sigma}d^+_\sigma\rangle$.

The presence of two types of singularity in the scattering amplitude significantly complicates the situation, since it becomes necessary to discuss diagrams of the parquet type in addition to the ordinary ladder diagrams,[13] which corresponds to going beyond the self-consistent field approximation. A distinctive feature of the problem under study here, i.e., pairing of quasiparticles in the field of a strong electromagnetic wave, is the fact that these singularities of the scattering amplitude are separated in frequency[14]; therefore, a meshing of the ladder diagrams into a parquet does not take place. This makes it possible to describe electron correlations correctly by means of a generalized self-consistent-field approximation.

An additional simplification is available if one of the following strong inequalities holds: $\lambda(\mathbf{p})\gg W$ or $\lambda(\mathbf{p})\ll W$. For tunneling contacts both cases are possible (depending on the nature of the localized state), and a solution exists for both. We will consider the first case, because it allows a simpler mathematical description. However, we should keep in mind that in this case we lose a solution that corresponds to the triplet type of electron–hole pairing. Nevertheless, this does not lead to any serious consequences, since singlet pairing is ordinarily more favorable energetically.[15]

In our case, the energy of the localized state does not depend on the electron spin, so that the spin indices on the operator need not be written; a summation over the spin variables is taken in the final answer. In what follows, we will assume that the interaction takes the form of an electron–electron repulsion; then, the "superconducting" average $\langle c^+_{\mathbf{p}}d^+\rangle$ equals zero. Let us decouple the electron-electron interaction Hamiltonian as follows:

$$\hat{H}_{ee}=\sum_{\mathbf{p},\mathbf{q}}g(\mathbf{p})c^+_{\mathbf{p}+\mathbf{q}}d^+dc_{\mathbf{p}}$$

$$=\sum_{\mathbf{p}}[\Delta(\mathbf{p})d^+c_{\mathbf{p}}+\Delta^*(\mathbf{p})c^+_{\mathbf{p}}d],\qquad(17)$$

where

$$\Delta(\mathbf{p})=g(\mathbf{p})\langle c^+_{\mathbf{p}}d\rangle.\qquad(18)$$

From Eq. (17) it is clear that the Coulomb interaction between free and localized electrons leads to a renormalization of the matrix element $\lambda(\mathbf{p})$ for inelastic tunneling in the Hamiltonian (14):

$$\lambda(\mathbf{p})\to\Sigma(\mathbf{p})=\lambda(\mathbf{p})+\Delta(\mathbf{p}).$$

We now diagonalize the Hamiltonian with the following canonical transformation:

$$c_{\mathbf{p}}=\alpha_{\mathbf{p}}+(\cos\gamma-1)\eta_{\mathbf{p}}\sum_{\mathbf{p}'}\eta_{\mathbf{p}'}\alpha_{\mathbf{p}'}-\eta_{\mathbf{p}}\beta\sin\gamma,\qquad(19)$$

$$d=\beta\cos\gamma+\sum_{\mathbf{p}}\eta_{\mathbf{p}}\alpha_{\mathbf{p}}\sin\gamma,\qquad(20)$$

where $\alpha_{\mathbf{p}}$ and β are new quasiparticle operators. The parameters of the transformation are as follows:

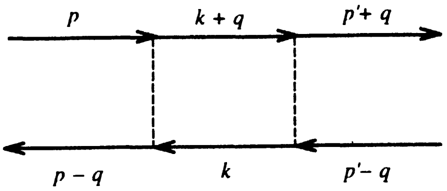

FIG. 2.

$$\cos^2\gamma=\frac{1}{2}\left[1+\frac{E_0-Z-\Omega}{\sqrt{(E_0-Z-\Omega)^2+4\Sigma^2}}\right],$$

$$\eta_{\mathbf{p}}=\frac{\Sigma(\mathbf{p})}{\varepsilon(\mathbf{p})-Z+g\gamma+\Sigma},$$

$$Z=\sum_{\mathbf{p}}\eta_{\mathbf{p}}^2\varepsilon(\mathbf{p}),\quad \Sigma=\sum_{\mathbf{p}}\eta_{\mathbf{p}}\Sigma(\mathbf{p}),\quad \sum_{\mathbf{p}}\eta_{\mathbf{p}}^2=1. \tag{21}$$

In this case, the parameter $\Delta(\mathbf{p})$ is determined from the self-consistency equation

$$\Delta(\mathbf{p})=\int g(\mathbf{p}-\mathbf{q})\frac{\Delta(\mathbf{q})}{E_0'-\varepsilon(\mathbf{q})}\frac{d^3q}{2\pi^3}, \tag{22}$$

where the renormalized position of the energy level E_0' for the localized state is determined from the expression

$$E_0'=\frac{1}{2}[(E_0+Z+\Omega)\pm\sqrt{(E_0-Z-\Omega)^2+4\Sigma^2}]. \tag{23}$$

Analogous transformations were used previously in Refs. 14 and 16 to describe impurity states in semiconductors.

Thus, the Hamiltonian for the problem of resonant tunneling of electrons in the field of a strong electromagnetic wave can be diagonalized by the unitary transformations (6), (13), and (19), (20) if we make two basic assumptions: that the interaction of two electrons at the same resonance level is weak, and that we are allowed to neglect elastic tunneling processes compared to inelastic processes. The Coulomb interaction between free and localized electrons is taken into account. Our solution is asymptotically exact in the limit of weak electron–electron interactions.

In what follows, we will include elastic processes via perturbation theory and compute the tunneling current.

5. MATRIX ELEMENTS FOR TUNNELING TRANSITIONS

The coherent nature of the interaction of tunneling electrons with the electromagnetic field leads to the appearance of so-called "coherence factors" in the matrix elements for elastic processes after the transformations (19), (20), which we neglected previously and will now take into account via perturbation theory. Doing so transforms the Hamiltonian (14) as follows:

$$\begin{aligned}\hat{H}'=&\sum_{\mathbf{p}}[\varepsilon(\mathbf{p})-\mu]\alpha_{\mathbf{p}}^+\alpha_{\mathbf{p}}+(E_0'-\mu_0)\beta^+\beta\\&+\sum_{\mathbf{p},\mathbf{p}'}T'(\mathbf{p},\mathbf{p}')\alpha_{\mathbf{p}}^+\alpha_{\mathbf{p}'}+\sum_{\mathbf{p}}[V'(\mathbf{p})\alpha_{\mathbf{p}}^+\beta+\text{h.c.}]\\&+\sum_{\mathbf{p}}[V''(\mathbf{p})\alpha_{\mathbf{p}}^+\beta\exp(-i\Omega t)+\text{h.c.}]\\&+\sum_{\mathbf{p},\mathbf{p}'}[T''(\mathbf{p},\mathbf{p}')\alpha_{\mathbf{p}}^+\alpha_{\mathbf{p}}\exp(-i\Omega t)+\text{h.c.}].\end{aligned} \tag{24}$$

The matrix elements in (24) are not all the same order of magnitude. The dominant matrix element is

$$T'(\mathbf{p},\mathbf{p}')=T_0(\mathbf{p},\mathbf{p}')+T_1(\mathbf{p},\mathbf{p}'), \tag{25}$$

where the terms are defined by the expressions

$$\begin{aligned}T_0(\mathbf{p},\mathbf{p}')=&T(\mathbf{p},\mathbf{p}')-(1-\cos\gamma)[\eta_{\mathbf{p}}T(\mathbf{p}')+T(\mathbf{p})\eta_{\mathbf{p}'}]\\&+(1-\cos\gamma)^2\eta_{\mathbf{p}}\eta_{\mathbf{p}'}T,\end{aligned} \tag{26}$$

$$T_1(\mathbf{p},\mathbf{p}')=\frac{\Sigma(\mathbf{p})\Sigma(\mathbf{p}')}{E_0'-\varepsilon(\mathbf{p})-\Omega},\quad T=\sum_{\mathbf{p}}\eta_{\mathbf{p}}T(\mathbf{p}),$$

$$T(\mathbf{p})=\sum_{\mathbf{p}}\eta_{\mathbf{p}'}T(\mathbf{p},\mathbf{p}'). \tag{27}$$

From (25) it is clear that the matrix element $T'(\mathbf{p},\mathbf{p}')$ can be interpreted physically as a sum of two terms, one due to elastic tunneling processes $T_0(\mathbf{p},\mathbf{p}')$ (including scattering by the resonant state) and the other due to inelastic resonant tunneling $T_1(\mathbf{p},\mathbf{p}')$. This latter term arises from the interaction of the electrons with the electromagnetic field.

Since $\eta_{\mathbf{p}}\cong 1$, while $\sin\gamma\approx\cos\gamma$ near resonance, the remaining matrix elements in the Hamiltonian (24) can be grouped as follows:

$$\begin{aligned}V'(\mathbf{p})=&[\eta_{\mathbf{p}}T-T(\mathbf{p})]\sin\gamma-\eta_{\mathbf{p}}T\sin\gamma\cos\gamma\\&\approx-\frac{T}{2}\frac{\Sigma(\mathbf{p})}{E_0'-\varepsilon(\mathbf{p})-\Omega},\end{aligned} \tag{28}$$

$$V''(\mathbf{p})=\eta_{\mathbf{p}}V(\cos^2\gamma-\sin^2\gamma)+[V(\mathbf{p})-\eta_{\mathbf{p}}V]\cos\gamma\approx 0, \tag{29}$$

$$\begin{aligned}T''(\mathbf{p},\mathbf{p}')=&\eta_{\mathbf{p}}[V(\mathbf{p}')-\eta_{\mathbf{p}'}V]\sin\gamma+\eta_{\mathbf{p}}\eta_{\mathbf{p}'}V\sin\gamma\cos\gamma\\&\approx\frac{V}{2}\frac{\Sigma(\mathbf{p})\Sigma(\mathbf{p}')}{[E_0'-\varepsilon(\mathbf{p})-\Omega][E_0'-\varepsilon(\mathbf{p}')-\Omega]},\end{aligned}$$

$$V=\sum_{\mathbf{p}}\eta_{\mathbf{p}}V(\mathbf{p}). \tag{30}$$

The matrix element $V(\mathbf{p})$, which must be included to second order of perturbation theory in calculating the tunneling current, has a further exponential smallness compared to $T'(\mathbf{p},\mathbf{p}')$, and can be discarded. The remaining matrix element $T''(\mathbf{p},\mathbf{p}')$ corresponds to processes that take electrons far from the Fermi surface. In order of magnitude we have $T''(\mathbf{p},\mathbf{p}')\cong(\Sigma/\Omega)V$, that is, it also can be neglected.

Thus, after weighting the matrix elements that correspond to various types of elastic and inelastic tunnel transitions, we can assert that a strong electromagnetic field causes important changes in the probability of tunneling processes, which is expressed by renormalization (25) of the original matrix elements for direct electron tunneling through the barrier by the processes of inelastic resonant tunneling.

6. TUNNELING CURRENT

The tunneling current density, according to (25)–(27), consists of two components:

$$j=j_0+j_1, \tag{31}$$

arising from the matrix elements $T_0(\mathbf{p},\mathbf{p}')$ for elastic nonresonant processes and the matrix elements $T_1(\mathbf{p},\mathbf{p}')$ for inelastic resonant processes. The first term is the simplest to write down:

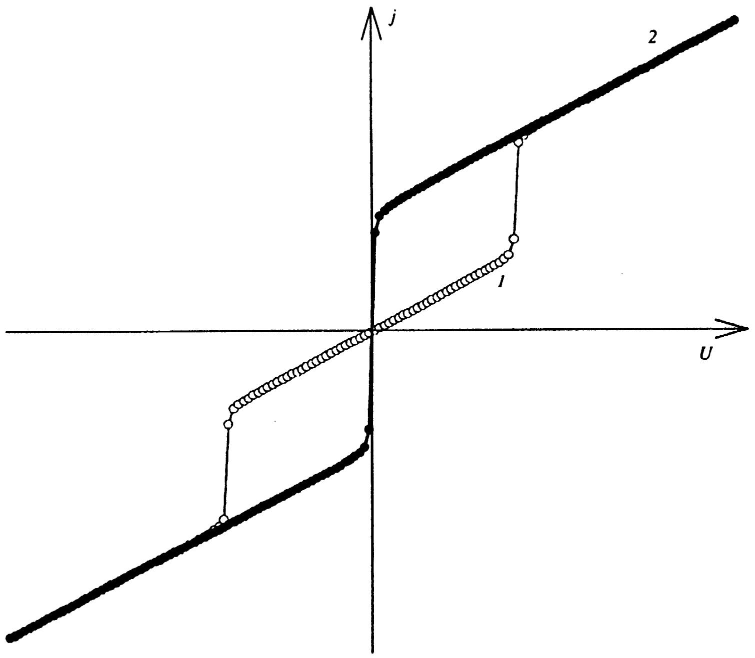

FIG. 3. Current–voltage characteristics of a tunneling contact in the field of a strong electromagnetic wave for two characteristic cases: (*1*) approximate resonance $[\Omega > E_0 - E_F, \Omega/(E_0 - E_F) \cong 1]$, and (*2*) exact resonance $(\Omega = E_0 - E_F)$.

$$j_0 = 2\pi e \sum_{\mathbf{p},\mathbf{p}'} |T_0(\mathbf{p},\mathbf{p}')|^2 [f(\mathbf{p}) - f(\mathbf{p}')] \delta[\varepsilon(\mathbf{p}) - \varepsilon(\mathbf{p}') + eU]$$
$$= 8\pi e^2 \nu_0^2 T^2 \cos^4 \gamma U = 8\pi e^2 \nu_0^2 T^2 U$$
$$\times \left\{ \frac{1}{2} \left(1 + \frac{E_0 - Z - \Omega}{\sqrt{(E_0 - Z - \Omega)^2 + 4\Sigma^2}} \right) \right\}^2, \qquad (32)$$

where U is the applied voltage and $\nu_0 = m p_F / \pi^2$ is the density of states at the Fermi level. We have taken into account the fact that for small applied voltages $U \ll E_F$ the distribution function of electrons depends only on energy, i.e., $f(\mathbf{p}) = \theta(E - E_F)$. In addition, near resonance we may set $T = T(\mathbf{p}) = T(\mathbf{p},\mathbf{p}')$.

The expression in (32) enclosed by curly brackets is a coefficient that reflects the change in the contribution to the tunneling current from elastic nonresonant processes. Because the scattering of electrons by the impurity state and by one another decreases the electron tunneling probability, this coefficient is always smaller than unity. Since the renormalization of the tunneling probability takes place primarily near the Fermi surface, it plays an important role only for small voltages $U \leqslant \Sigma$, and can be included by introducing an effective tunneling probability into the usual expression for the tunneling current (i.e., in the absence of a resonant electromagnetic field).

The contribution to the tunneling current (31) from the second term, which is connected with inelastic resonant tunneling, is considerably more important. It is purely a consequence of the interaction between the electrons and the electromagnetic field, and has the following form:

$$j_0 = 2\pi e \sum_{\mathbf{p},\mathbf{p}'} |T_1(\mathbf{p},\mathbf{p}')|^2 [f(\mathbf{p}) - f(\mathbf{p}')] \delta(\varepsilon(\mathbf{p}) - \varepsilon(\mathbf{p}') + eU)$$
$$= 2\pi e \sum_{\mathbf{p},\mathbf{p}'} \frac{\Sigma^2(\mathbf{p})\Sigma^2(\mathbf{p}')}{(E_0 - \varepsilon(\mathbf{p}) - \Omega)^2 + \Sigma^2}$$
$$\times [f(\mathbf{p}) - f(\mathbf{p}')] \delta(\varepsilon(\mathbf{p}) - \varepsilon(\mathbf{p}') + eU). \qquad (33)$$

Replacing the parameter $\Sigma(\mathbf{p})$ by its value Σ_0 for $E = E_F$, we obtain:

$$j_1 = 8\pi e \nu_0^2 \int_{E_F - eU}^{E_F + eU} \frac{\Sigma_0^4}{(E_0 - E - \Omega)^2 + \Sigma^2} dE$$
$$= 8\pi e \nu_0^2 \frac{\Sigma_0^4}{\Sigma} \left\{ \arctan\left(\frac{E_0 - E_F - \Omega - eU}{\Sigma} \right) \right.$$
$$\left. - \arctan\left(\frac{E_0 - E_F - \Omega + eU}{\Sigma} \right) \right\}. \qquad (34)$$

This expression has singularities when $\Omega \pm eU = E_0 - E_F$, when the electromagnetic wave frequency equals the distance from the resonance level to the Fermi level of one of the contacts. If there is an exact correspondence $\Omega = E_0 - E_F$, then both singularities coincide, and a resonant situation arises even when $U = 0$. In the opposite case, a resonance is possible only for nonzero applied voltage, which causes a shift of the Fermi level with respect to its equilibrium position. In this case a resonance occurs when $eU = \pm 2(E_0 - E_F - \Omega)$, which leads to an increase in the tunneling current and to the appearance of two steps in the current–voltage characteristic. The current–voltage characteristic of a tunnel contact in the field of a strong electromagnetic wave is shown schematically in Fig. 3 for two characteristic cases: exact resonance $(\Omega = E_0 - E_F)$, and approximate resonance $(\Omega > E_0 - E_F, (E_0 - E_F)/\Omega \cong 1)$. As the voltage is increased further up to values comparable to the Fermi energy, the discrete level moves out of resonance. This effect should manifest itself in the appearance of a decreasing segment of the current–voltage characteristic (this decreasing segment is not shown in Fig. 3).

In order to compare (32) and (34), let us also calculate the contribution to the current associated with elastic resonant tunneling. The question of how to correctly write down an expression for the current cannot be answered without further analysis using the kinetic equation. However, when $\Omega \gg eU$ we may use the expression

$$j_0'=2\pi e\sum_{\mathbf{p},\mathbf{p}'}|T''(\mathbf{p},\mathbf{p}')|^2[f(\mathbf{p})-f(\mathbf{p}')]$$
$$\times\delta[\varepsilon(\mathbf{p})-\varepsilon(\mathbf{p}')-\Omega+eU]. \qquad (35)$$

In contrast to (32), (33), the argument of the δ-function here contains an additional frequency term. This is connected with the fact that the initial electron transition is a coherent one from the left side of the contact to the localized state; from there, the electron tunnels elastically to the right side of the contact. Electrons that have tunneled in this manner possess energies $E=E_F+\Omega$, and their distribution function for this energy has a local maximum. This makes it possible to observe this effect experimentally by spectroscopic methods, despite its smallness. After the transformations, (35) takes the following form:

$$j_1'=8\pi e\nu_0^2\left(\frac{V}{2\Omega}\right)^2\int_{E_F-eU}^{E_F+eU}\frac{\Sigma_0^4}{(E_0-E-\Omega)^2+\Sigma^2}\,dE$$
$$=8\pi e\nu_0^2\left(\frac{V}{2\Omega}\right)^2\frac{\Sigma_0^4}{\Sigma}\left\{\arctan\left(\frac{E_0-E_F-\Omega-eU}{\Sigma}\right)\right.$$
$$\left.-\arctan\left(\frac{E_0-E_F-\Omega+eU}{\Sigma}\right)\right\}. \qquad (36)$$

Comparing with (34), we see that while (36) also possesses singularities at the resonant frequencies, their contribution to the density of tunneling current has an additional factor of smallness $(V/\Omega)^2$.

7. CONCLUSIONS

By reducing the problem of resonant tunneling of electrons in the field of a strong electromagnetic wave to the exactly solvable problem of scattering of electrons by a resonant impurity in a metal, we not only improve our understanding of the deep analogy between the descriptions of equilibrium and nonequilibrium stationary processes, but also observe explicit features of the latter that do not reduce to a simple transposition of known effects to a new area. In particular, in contrast to the equilibrium case,[9] the specifics of the nonequilibrium situation allow us to include the Coulomb interaction of free and localized charged carriers within the self-consistent field approximation to logarithmic accuracy.

Another feature of our solution is the presence of a local maximum in the distribution function of electrons that have tunneled at the energy $E=E_F+\Omega$. The cause of this feature is the electromagnetic field, which launches electrons into the high-energy region.

Meanwhile, the approach of Ref. 9, which uses the formal correspondence between the tunneling Hamiltonian and the Anderson Hamiltonian for the problem of resonant scattering in metals, itself requires more detailed justification if the principal assumptions of this paper are to remain in force.

There are many problems with investigating resonant tunneling effects in a strong electromagnetic field from the point of view of experimental feasibility, associated primarily with the strong heating of the sample surface by the laser. Nevertheless, the considerable progress made in nanoelectronics and scanning tunneling microscopy in recent years allows us to hope not only that these effects may be observed, but also that they may be put to practical use.

[1] N. V. Kornyakov, Preprint No. 20, Inst. of Microelectronics, Russ. Acad. Sci., 1990.
[2] S. N. Molotkov, JETP Lett. **54**, 481 (1991).
[3] M. Volcker, W. Krieger, and H. Walther, Phys. Rev. Lett. **66**, 1717 (1991).
[4] A. A. Gorbatsevich, V. V. Kapaev, Yu. V. Kopaev, and V. Ya. Kremlev, Phys. Low-Dim. Struct. **4/5**, 57 (1994).
[5] V. V. Kapaev, Yu. V. Kopaev, and N. V. Kornyakov, JETP Lett. **58**, 843 (1993).
[6] Yu. V. Kopaev, and N. V. Kornyakov, in *Proc. International Symp. on Nanostructures: Physics and Technology*, St. Petersburg, 1994.
[7] J. Inarrea, G. Platero, and C. Tejador, Semicond. Sci. Technol. **9**, 515 (1994).
[8] V. A. Chitta, R. E. M. de Bekker, J. C. Maan, S. J. Hawkworth, J. Chamberlain, M. Henini, and G. Hill, Surf. Sci. **263**, 227 (1992).
[9] L. I. Glazman, and M. É. Raikh, JETP Lett. **47**, 452 (1988).
[10] V. M. Galitskii and B. F. Elesin, *Resonant Interaction of Electromagnetic Waves with Semiconductors*. Energoatomizdat, Moscow, 1986.
[11] R. Magno and M. G. Spenser, J. Appl. Phys. **75**, 368 (1994).
[12] L. V. Keldysh and Yu. V. Kopaev, Fiz. Tverd. Tela **6**, 2791 (1964) [Sov. Phys. Solid State **6**, 2219 (1964)].
[13] A. A. Abrikosov, Physics **2**, 21 (1965).
[14] Yu. V. Kopaev and N. V. Kornyakov, Fiz. Tverd. Tela **24**, 72 (1986) [Sov. Phys. Solid State **28**, 37 (1986)].
[15] V. F. Elesin and Yu. V. Kopaev, Fiz. Tverd. Tela **14**, 669 (1972) [Sov. Phys. Solid State **14**, 570 (1972)].
[16] K. A. Kikoin and V. N. Flerov, Zh. Éksp. Teor. Fiz. **77**, 1062 (1979) [Sov. Phys. JETP **50**, 535 (1979)].

Translated by Frank J. Crowne