Formation of polarization-squeezed states of light in spatially periodic nonlinear-optical media

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A new possibility for generating a nonclassical state of a vector electromagnetic field is considered. It is shown that polarization-squeezed light, in which the quantum fluctuations of one of the Stokes parameters characterizing the polarization properties of the light are smaller than in the coherent state, can form as a result of a nonlinear interaction between two waves with orthogonal polarizations in a spatially periodic medium. The region of uncertainty of the polarization state for coherent light in the space of the Stokes parameters is a sphere, and the region for polarization-squeezed light is an ellipsoid. A definite analogy between suppression of the quantum fluctuations of the Stokes parameters of polarization-squeezed light and the components of quadrature-squeezed light is traced. The optimal conditions for obtaining polarization-squeezed light with respect to the parameters of the problem are elucidated. It is shown, in particular, that the relationship between the phases of the original waves at the entrance to the nonlinear medium and the linear coupling coefficient of the waves are of great importance here. © *1995 American Institute of Physics.*

1. INTRODUCTION

The quantum analysis of the polarization structure of light has aroused definite interest in recent years.[1–6] The discussion has generally focused on calculations of the Stokes parameters, which are associated with Hermitian Stokes operators and have an explicit physical interpretation. A general description of the polarization structure of a multimode field was developed in Refs. 5 and 6. In particular, unpolarized light having rather specific properties, i.e., having both mean values of the Stokes parameters and their variances equal to zero, was defined.

From the standpoint of applying quantum squeezed light in high-precision polarimetry and ellipsometry, polarization-squeezed light[4] is of special interest. Such light is completely polarized. However, the quantum fluctuations of one of its Stokes parameters are smaller than in the coherent state. It was shown in Ref. 4 that polarization-squeezed light can form in cubically nonlinear uniform media when there is anisotropy of the nonlinear correction to the refractive index. The latter circumstance plays a major role in shaping a nonclassical polarization state in uniform nonlinear media.

In this paper we consider another possibility for obtaining polarization-squeezed light in spatially periodic nonlinear media. In such media, one specific feature of the interaction process is the occurrence of both linear and nonlinear power conversion between different modes, i.e., polarization components.[7–9] Owing to the linear power conversion, an additional channel of fluctuation transfer for the quadrature field components appears and quadrature-squeezed light forms in such media.[9] It is shown below that the linear coupling between modes enables the generation of light with a nonclassical polarization state in spatially nonuniform nonlinear media.

Before proceeding to an analysis of the problem that we have posed, let us briefly dwell on the description of the polarization characteristics of radiation in quantum optics.

2. POLARIZATION STATES OF LIGHT

From the quantum standpoint, the difference between polarized (elliptically in the general case) monochromatic light and unpolarized light is that in the former case there is a pure state, i.e., a coherent mixture of light polarized in two perpendicular directions (the amplitudes of these components are summed), while unpolarized light represents a mixed state, i.e., an incoherent mixture (the intensities of the components are summed).[10]

In the language of wave functions (for photons they exist only in the momentum representation[11]), for pure states we have linear combinations, which can be used to calculate the total probability, and for mixed states we can sum the squares of the absolute values of the wave functions, i.e., the individual probabilities. Moreover, although no type of polarization is dominant, right- and left-circularly polarized photons are most easily defined in terms of spin operators.[11]

The presence of fluctuations, which are unavoidable in quantum theory and result in uncertainty in the polarization state of light, calls for a quantum-mechanical treatment of partial polarization. The directly measurable (observable) quantities that characterize the polarization state of light are the Stokes parameters S_j, where $j=0, 1, 2, 3$. The parameter S_0 specifies the total intensity of the light field, and the other three parameters specify the polarization state proper.[10] They

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are measured experimentally using a combination of optical elements, viz., polarizers and phase shifters.

A graphic geometric interpretation of the Stokes parameters can be given using the Poincaré sphere in the Stokes space of S_1, S_2, S_3. Each point on the sphere corresponds to a definite polarization state, whose variation is characterized by movement of the image point. Here the fluctuational polarization uncertainty can be associated with a certain region of uncertainty in the Stokes parameters $S_{1,2,3}$ around their mean values.[4] In a quantum treatment there are operators corresponding to the Stokes parameters [see the expressions (2) below].

To quantitatively describe the polarization state of light, we introduce the degree of polarization P, which is equal to the ratio of the intensity of the polarized part of the radiation I_{pol} to the total intensity I_{tot}:

$$P=\frac{I_{\text{pol}}}{I_{\text{tot}}}=\frac{\sqrt{S_1^2+S_2^2+S_3^2}}{S_0}. \tag{1}$$

For completely polarized light $P=1$, while for partially polarized light $0<P<1$, and a transition to unpolarized light corresponds to squeezing the Poincaré sphere to zero radius.[10] The description of partial polarization using real Stokes parameters corresponds to the intimate relationship between the classical and quantum approaches for treating polarization properties. However, quantum theory permits the existence of nonclassical polarization states, for which a general analysis is presented in the next section.

3. NONCLASSICAL POLARIZATION STATES, POLARIZATION-SQUEEZED LIGHT

Let us consider two modes of a light field with orthogonal polarizations interacting in a nonlinear medium. The polarization state of the light is described by the Stokes operators (see, for example, 2–4):

$$S_0(z)=n_1(z)+n_2(z), \tag{2a}$$

$$S_1(z)=n_1(z)-n_2(z), \tag{2b}$$

$$S_2(z)=a_1^+(z)a_2(z)+a_2^+(z)a_1(z), \tag{2c}$$

$$S_3(z)=i(a_2^+(z)a_1(z)-a_1^+(z)a_2(z)), \tag{2d}$$

where $n_j(z)=a_j^+(z)a_j(z)$ is the photon number of the operator in the jth mode, a_j (a_j^+) is the photon annihilation (creation) operator, and z is the coordinate in the direction of propagation of the radiation. The operators a_j and a_j^+ obey the known commutation relations $[a_j,a_k^+]=\delta_{jk}$, where δ_{jk} is the Kronecker delta. The operators S_j satisfy the commutation relations of $SU(2)$ algebra:

$$[S_2(z),S_3(z)]=2iS_1(z), \tag{3a}$$

$$[S_1(z),S_2(z)]=2iS_3(z), \tag{3b}$$

$$[S_3(z),S_1(z)]=2iS_2(z). \tag{3c}$$

In addition, the operators $S_j(z)$ ($j=1, 2, 3$) commute with $S_0(z)$.

The noncummutation conditions (3) lead to the uncertainty relations:

$$\langle\Delta S_j^2(z)\rangle\langle\Delta S_k^2(z)\rangle\geqslant|\langle S_m(z)\rangle|^2, \tag{4}$$

$$j,k,m=1,2,3, \quad j\neq k\neq m.$$

The quantities $\langle\Delta S_i^2(z)\rangle\equiv\langle S_i^2(z)\rangle-\langle S_i(z)\rangle^2$ are the fluctuational variances of the Stokes parameters. Relations (4) imply that in quantum optics the Stokes parameters cannot be measured simultaneously to arbitrarily high accuracy. As already mentioned above, it is convenient to represent the state of the quantum vector field in the form of a certain uncertainty volume with the central coordinates $\langle S_{1,2,3}\rangle$ in a Poincaré sphere. In the case of a coherent state, such a region has the form of a sphere.[4] In fact, the coherent state for a two-mode field is $|\alpha\rangle=|\alpha_1\rangle|\alpha_2\rangle$, where $|\alpha_j\rangle$ is an eigenstate of the operator a_j:

$$a_j|\alpha_j\rangle=\alpha_j|\alpha_j\rangle \quad (j=1,2).$$

Using the definitions (2), we can easily obtain

$$\langle\Delta S_k^2\rangle=\langle n_1\rangle+\langle n_2\rangle, \ \langle n_j\rangle=|\alpha_j|^2 \ (j=1,2, \ k=1,2,3). \tag{5}$$

Hence it is seen that the level of fluctuations of the Stokes parameters for two-mode coherent radiation is determined by the sum of the mean numbers of photons $\langle n_j\rangle$ in the modes: $N_+\equiv\langle n_1\rangle+\langle n_2\rangle$.

The relations (3) can serve as a basis for writing inequalities which describe the conditions for the existence of squeezed (with respect to fluctuations of the Stokes parameters) states of a light field:

$$\langle\Delta S_j^2(z)\rangle\gtrless|\langle S_m(z)\rangle|,$$

$$\langle\Delta S_k^2(z)\rangle\lessgtr|\langle S_m(z)\rangle|, \quad j,k,m=1,2,3, \quad j\neq k\neq m. \tag{6}$$

In Eqs. (6) either the upper or lower signs should be taken. Physically, the inequalities (6) mean that the uncertainty region of the Stokes parameters transforms and takes the form of an ellipsoid with suppressed fluctuations of one of the parameters and increased or, at least, undiminished fluctuations of the other parameters. Such a polarization state of a light field is customarily termed a polarization-squeezed state.[4] It is nonclassical. In fact, in this case one of the quantities

$$\langle N(S_j-\langle S_j\rangle)^2\rangle=\langle\Delta S_j^2\rangle-N_+, \quad (j=1,2,3), \tag{7}$$

where N is the normal ordering operator, turns out to be negative. The expression (7) is nonnegative for classical states.

In many cases, orthogonally polarized modes can represent light waves with linear polarization, for example, along the x and y axes, and with the amplitudes A_x and A_y. The amplitudes $A_\pm\equiv A_x\pm iA_y$ then specify modes with circular polarization.

Let us discuss which nonclassical states studied have properties which are similar to those of light with squeezed fluctuations of the Stokes parameters. We first of all consider the parameters $S_0(z)$ and $S_1(z)$, which are defined by the relations (2a) and (2b). It is well known that fluctuations of these parameters can be suppressed to values below the level corresponding to the coherent state, owing to a correlation or anticorrelation among the photons in the two modes. Such a situation is realized, for example, in parametric processes and four-wave mixing,[12] as well as for other schemes of multiwave scattering,[13] where correlated photons are created in pairs.

The physical properties of squeezed light with suppression of the fluctuations of $\langle \Delta S_2^2(z)\rangle$ or $\langle \Delta S_3^2(z)\rangle$ can be elucidated in the simplest case by assuming that the field of one of the modes is classical. Then, for example, replacing a_1 by the classical quantity $|A_1|\exp(i\varphi_1)$ (φ_1 is the phase), we have

$$S_2(z)=[\exp(i\varphi_1)a_2(z)+\exp(-i\varphi_1)a_2^+(z)]|A_1| \equiv Q_2(z)|A_1|, \tag{8a}$$

$$S_3(z)=i[\exp(i\varphi_1)a_2^+(z)-\exp(-i\varphi_1)a_2(z)]|A_1| \equiv P_2(z)|A_1|, \tag{8b}$$

where

$$Q_2(z)\equiv\exp(i\varphi_1)a_2(z)+\exp(-i\varphi_1)a_2^+(z),$$

$$P_2=i[\exp(i\varphi_1)a_2^+-\exp(-i\varphi_1)a_2]$$

are Hermitian quadrature components for the mode described by a_2.

It is seen from (8a) and (8b) that in this case polarization-squeezed light is closest to quadrature-squeezed states of a field (compare with Ref. 14). Experiments devised to obtain light with such characteristics are well known (see, for example, Ref. 12).

When polarization-squeezed light is compared with quadrature-squeezed light, the specific features associated with the vector character of the field must, of course, be borne in mind. For example, in a cubically nonlinear medium, quadrature-squeezed states can be obtained in practice only in processes involving the self-interaction and interaction of waves,[14] while polarization-squeezed light can form in such media only when there is anisotropy in the nonlinear correction to the refractive index.[4] In the latter case, there is no power conversion between polarization modes propagating in the nonlinear medium. Therefore, both the total number of photons and the difference between the numbers of photons in the modes are maintained. The main purpose of the present paper is to demonstrate the possibility of obtaining a new class of polarization-squeezed light, in which there is power conversion between the polarization modes.

4. BASIC EQUATIONS AND RELATIONS FOR POLARIZATION-SQUEEZED LIGHT UNDER THE CONDITIONS OF POWER CONVERSION BETWEEN MODES

The high efficiency of devices for generating nonclassical light on the basis of distributed-feedback systems has been established (see, for example, Ref. 15). At the same time, the realization of distributed feedback seems most promising either in optical fibers with a dielectric constant that varies periodically along the propagation coordinate, or in tunneling-coupled twin-core fiber waveguides.[9] We show that the presence of two orthogonally polarized coupled modes in such systems makes it possible to obtain polarization-squeezed light with new properties in them. Let us consider the formation of polarization-squeezed light in a spatially periodic optical fiber. We begin the analysis with the classical equations.

When losses are neglected, the propagation of a linearly polarized wave in such spatially periodic fibers is described by the contracted equations for the slowly varying complex amplitudes A_x and A_y (Ref. 8) (the components of the polarization parallel to the x and y axes):

$$\begin{aligned} -i\frac{dA_x}{dz}&=k_xA_x+2\beta\cos(k_0z)A_y+R\left[|A_x|^2+\frac{2}{3}|A_y|^2\right]A_x+\frac{R}{3}A_x^*A_y^2, \\ -i\frac{dA_y}{dz}&=k_yA_y+2\beta\cos(k_0z)A_x+R\left[|A_y|^2+\frac{2}{3}|A_x|^2\right]A_y+\frac{R}{3}A_y^*A_x^2, \end{aligned} \tag{9}$$

where β is the linear coupling coefficient of the waves, $R=2k_0n_2/n_1s$ is the nonlinear coupling coefficient ($k_0=\omega/c$), n_1 and n_2 are the linear and nonlinear refractive indices, s is the effective area of the fiber filled by the radiation, and $k_{x,y}$ are the wave numbers of the waves polarized along the x and y axes, respectively.

In (9) we perform the following replacement:

$$\begin{aligned} A_x(z)&=B_x(z)\exp[0.5i(k_x+k_y+k_0)z], \\ A_y(z)&=B_y(z)\exp[0.5i(k_x+k_y-k_0)z]. \end{aligned} \tag{10}$$

As a result, for $B_{x,y}$ we obtain the equations

$$\begin{aligned} -i\frac{dB_x}{dz}&=\tfrac{1}{2}\delta B_x+\beta B_y+R[|B_x|^2+\tfrac{2}{3}|B_y|^2]B_x, \\ -i\frac{dB_y}{dz}&=-\tfrac{1}{2}\delta B_y+\beta B_x+R[|B_y|^2+\tfrac{2}{3}|B_x|^2]B_y, \end{aligned} \tag{11}$$

where $\delta\equiv k_x-k_y-k_0$. In deriving (11) we neglected the rapidly oscillating terms containing the factors $\exp(-2ik_0z)$, i.e., we assumed that $k_0z\gg 1$ (compare with Ref. 8). In the case of $R=0$, Eqs. (11) describe the propagation of two modes of radiation in optical fields with linear power conversion: tunneling-coupled or spatially periodic modes.[7–9]

Upon passage to the quantum description, the complex classical amplitudes $B_{x,y}$ in Eqs. (11) should be replaced by operators in the standard manner, i.e.,

$$B_{x,y}\rightarrow i\sqrt{\frac{2\pi\hbar\omega}{\varepsilon_0V}}a_{x,y} \tag{12}$$

(V is the quantization volume), and the right-hand sides of the equations should be normal-ordered. As a result, we arrive at the quantum equations of motion for the operators $a_{x,y}$ (in the Heisenberg representation):

$$\begin{aligned} \frac{da_x}{dz}&=i\beta a_y+i\tilde{R}[a_x^+a_x+\tfrac{2}{3}a_y^+a_y]a_x, \\ \frac{da_y}{dz}&=i\beta a_x+i\tilde{R}[a_y^+a_y+\tfrac{2}{3}a_x^+a_x]a_y, \end{aligned} \tag{13}$$

where $\tilde{R}=2\pi\hbar\omega R/\varepsilon_0 V$.

Equations (13) can also be obtained from the Heisenberg equations of evolution for the operators a_x and a_y

$$i\hbar\frac{da_j}{dt}=[a_j,H_{\rm int}]\quad(j=x,y)$$

with the interaction Hamiltonian

$$H_{\rm int}=\frac{\hbar c}{n_1}\left[\beta a_x^+a_y+\beta a_y^+a_x+\frac{\tilde{R}}{2}(a_x^{+2}a_x^2+a_y^{+2}a_y^2)+\frac{2}{3}\tilde{R}a_x^+a_xa_y^+a_y\right]\tag{14}$$

when d/dt is replaced by $-(c/n_1)d/dz$, where c is the velocity of light in vacuum.

In the classical problem, the solutions of the system (11) can be expressed in terms of elliptic integrals.[7] However, in the quantum description of the process, certain difficulties arise in solving Eqs. (13); these are related to the operator algebra. We solve the system (13) in the secondary-contraction approximation (see, for example, Ref. 15). In this context we write the solutions of Eqs. (13) in the form

$$a_{x,y}(z)=\frac{1}{\sqrt{2}}[C_1(z)e^{i\beta z}\pm C_2(z)e^{-i\beta z}],\tag{15}$$

where $C_{1,2}(z)$ are slowing varying operators. After substituting (15) into (13) and neglecting the rapidly oscillating terms that are proportional to exp$(2i\beta z)$, we obtain the following simplified system of nonlinear equations:

$$\frac{dC_{1,2}}{dz}=i\tilde{R}[\tfrac{5}{6}C_{1,2}^+C_{1,2}+C_{2,1}^+C_{2,1}]C_{1,2}.\tag{16}$$

The approximations under which the system (16) was obtained are applicable under the condition

$$1/k_0\ll L_L<L_{NL},\tag{17}$$

where $L_L\equiv 1/\beta$ and $L_{NL}\equiv 1/\tilde{R}|a_x|^2$ are the linear and nonlinear spatial scales of the interaction of the polarization components. The right-hand side of the inequality (17) physically means that we neglected the rapidly oscillating terms during the transition from (9) to (16) using the transformations (10).

The solutions of Eqs. (16) are written in the form

$$C_{1,2}(z)=\exp[i\gamma C_{1,2}^+C_{1,2}+i\sigma C_{2,1}^+C_{2,1}]C_{1,2},\tag{18}$$

where $C_{1,2}\equiv C_{1,2}(z=0)$ are the values of the operators at the entrance to the nonlinear medium, $\gamma\equiv 5\tilde{R}z/6$, and $\sigma\equiv\tilde{R}z$.

We note that the introduction of $C_{1,2}(z)$ in accordance with relation (15) essentially signifies the selection of independent variables, as is usually done in the treatment of coupled oscillators (the transition to independent modes).

It is easy to see that $a_{x,y}$ and $C_{1,2}(z)$ satisfy the commutation relations for a boson system:

$$[a_i(z),a_j^+(z)]=\delta_{ij},$$

$$[C_k(z),C_m^+(z)]=\delta_{km},\quad i,j=x,y,\quad k,m=1,2.\tag{19}$$

5. FLUCTUATIONS OF THE STOKES PARAMETERS

We define the operators of the Stokes parameters by the expressions [compare with (2)]

$$S_1=a_x^+a_x-a_y^+a_y,$$

$$S_2=a_x^+a_ye^{i\theta}+a_y^+a_xe^{-i\theta},$$

$$S_3=i(a_y^+a_xe^{-i\theta}-a_x^+a_ye^{i\theta}),\tag{20}$$

where the phase $\theta\equiv\Phi-k_0z$ takes into account both the initial phase difference Φ between the orthogonally polarized waves and the phase difference associated with the difference between the wave numbers of these waves [see (10)].

Let the modes a_x and a_y initially be in a coherent state. Then the initial operators $C_{1,2}$ also correspond to coherent states, i.e., satisfy the relations

$$C_{1,2}|\alpha_{1,2}\rangle=\alpha_{1,2}|\alpha_{1,2}\rangle,\tag{21}$$

where $\alpha_{1,2}=(\alpha_x\pm\alpha_y)/\sqrt{2}$ [$\alpha_{x,y}$ is an eigenvalue of the operator $a_{x,y}(z=0)$] and the total state vector of the field under consideration is $|\xi\rangle=|\alpha_1\rangle|\alpha_2\rangle$. We assume that $\alpha_1=\alpha_2=\alpha_x/\sqrt{2}$, i.e., at $z=0$ the polarization component α_y is in the vacuum state $|0\rangle_y$ and $\alpha_y=0$.

Averaging the expression (20) with respect to the state $|\xi\rangle$ and taking into account the relations (15), for the mean values of the Stokes parameters we obtain

$$\langle S_0(z)\rangle=|\alpha_x|^2+|\alpha_y|^2=n,\tag{22a}$$

$$\langle S_1(z)\rangle=n\exp\{n[\cos(\gamma-\sigma)-1]\}\cos(2\beta z),\tag{22b}$$

$$\langle S_2(z)\rangle=-n\exp\{n[\cos(\gamma-\sigma)-1]\}\sin\theta\sin(2\beta z),\tag{22c}$$

$$\langle S_3(z)\rangle=n\exp\{n[\cos(\gamma-\sigma)-1]\}\cos\theta\sin(2\beta z),\tag{22d}$$

where $n\equiv|\alpha_x|^2$ is the initial mean number of photons in the mode a_x. The presence of the exponential factors in (22b)–(22d) is due to a purely quantum effect. However, in real situations the oscillatory behavior of the parameters (22) due to variation of the value of the exponent is not observed, since $|\gamma-\sigma|\ll 1$. In this case we can restrict ourselves in the calculations to the approximation

$$\exp\{n[\cos(\gamma-\sigma)-1]\}\approx\exp\{-n(\gamma-\sigma)^2/2\}.$$

The harmonic factors in (22) are associated with power conversion between the interacting ($\beta\neq 0$) modes and their phase difference θ. We note that when $2\beta z=m\pi$ ($m=1$, 2, 3, ...), the Stokes parameters are such that $\langle S_2\rangle=\langle S_3\rangle=0$ and $\langle S_1\rangle$ takes its maximum value. We note that the degree of polarization (1) does not depend on the coupling coefficient β or the phase θ, and equals

$$P=\exp[-n(\gamma-\sigma)^2/2].\tag{23}$$

Under real conditions, however, the value of P differs only very slightly from unity.

For the variances $\langle\Delta S_{2,3}^2(z)\rangle$ the calculations give

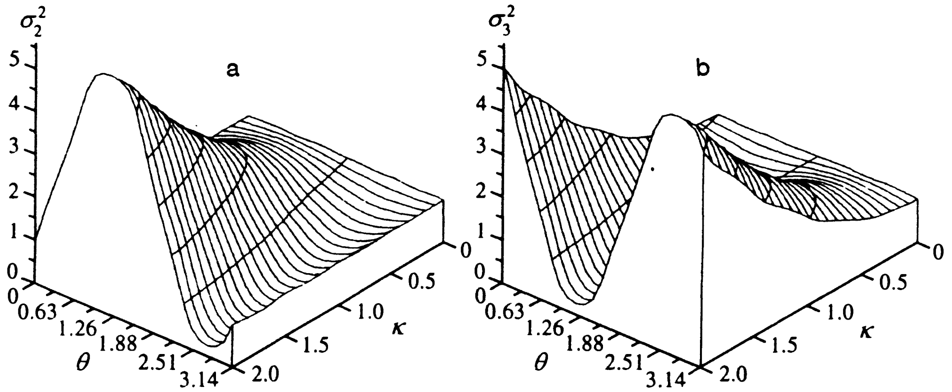

FIG. 1. Normalized variances $\sigma_j^2 \equiv \langle \Delta S_j^2(z) \rangle / n$ ($j=2, 3$) of the Stokes parameters a) $S_2(z)$ and b) $S_3(z)$ as a function of the effective nonlinear parameter κ and the phase θ. The value $\sigma_j^2=1$ corresponds to the coherent level of dispersion of the fluctuations of the Stokes parameters.

$$\langle \Delta S_2^2(z) \rangle = n\{1+\psi[\psi\cos(2\beta z)\sin\theta + 2\cos\theta]\cos(2\beta z)\sin\theta\}, \tag{24}$$

$$\langle \Delta S_3^2(z) \rangle = n\{1+\psi[\psi\cos(2\beta z)\cos\theta - 2\sin\theta]\cos(2\beta z)\cos\theta\},$$

where $\psi=(-\gamma+\sigma)n \equiv \tilde{R}nz/6$ is a nonlinear parameter. In deriving (24), we used the fact that $(-\gamma+\sigma)^2 n \ll 1$.

It follows from expression (24) that in the general case the variances of the Stokes parameters at the exit from the nonlinear medium can be either smaller or greater than those at the entrance. The variances at the exit depend on βz, θ, and ψ. The nonlinear parameter ψ determines the extrema of the variances.

We move on to a more detailed analysis of the behavior of the fluctuations of the Stokes parameters. Note that the effective nonlinear parameter $\kappa \equiv \psi_{\text{eff}} = \psi\cos 2\beta z$ actually plays a major role here. When κ is used, the expressions (24) take the simpler forms

$$\langle \Delta S_2^2(z) \rangle = n[1+\kappa^2\sin^2\theta + \kappa\sin 2\theta], \tag{25a}$$

$$\langle \Delta S_3^2(z) \rangle = n[1+\kappa^2\cos^2\theta - \kappa\sin 2\theta]. \tag{25b}$$

The expressions (25a) and (25b) transform into one another as a result of the replacement $\theta \rightarrow \theta+\pi/2$. In other words, the behavior of the fluctuations of the Stokes parameters as a function of the phase θ is displaced by $\pi/2$. Three-dimensional plots of the functions (25a) and (25b) are presented in Figs. 1a and 1b. A comparison of Figs. 1a and 1b reveals that if the variance of one of the Stokes parameters is greater than that in the coherent state, the variance of the other parameter is less than the latter. In particular, the maximum value of the normalized variance $\sigma_2^2 \equiv \langle \Delta S_2^2(z) \rangle / n$ (Fig. 1a) corresponds to the minimum value of $\sigma_3^2 \equiv \langle \Delta S_3^2(z) \rangle / n$ when $\kappa=2$ and $\theta=1.2$ rad. Thus, the state of the field is polarization-squeezed.

Expression (25a) reaches an extremum when $\tan 2\theta = -2/\kappa$. The minimum and maximum values are

$$\langle \Delta S_2^2(z) \rangle_{\min} = n[\sqrt{1+\kappa^2/4} - |\kappa|/2]^2, \tag{26a}$$

$$\langle \Delta S_3^2(z) \rangle_{\max} = n[\sqrt{1+\kappa^2/4} + |\kappa|/2]^2, \tag{26b}$$

Figure 2 presents plots of the dependence of the minimum variances, which decrease with increasing values of the effective nonlinear parameter $|\kappa|$. Identical values of $|\kappa| \equiv \psi|\cos(2\beta z)|$, which has its largest absolute values when $\cos(2\beta z) = \pm 1$, correspond to levels of equal variance. When $2\beta z = \pi/2 + \pi m$ ($m=0, 1, 2, \ldots$), the effective nonlinear parameter $|\kappa|$ equals zero, and the variances of the Stokes parameters at the exit from the medium have the same values as in the case of fields in the coherent state [compare with (5)]. Such a situation is characteristic of the behavior of the fluctuations of observed (quadratures, numbers of photons) light fields in media with linear power conversion between the waves.[9]

We note that in the case of (26a) and (26b) under consideration, the product

$$\langle \Delta S_2^2(z) \rangle_{\min} \langle \Delta S_3^2(z) \rangle_{\max} = n^2$$

is minimal, and we are dealing with ideal squeezing.

Let us now examine the expressions (25) from the standpoint of achieving extrema as κ varies. When $\kappa = -\cot\theta$, the expression (25a) takes the minimum value

$$\langle \Delta S_2^2(z) \rangle_{\min} = n\sin^2\theta. \tag{27a}$$

In this case the variance of S_3 has the maximum

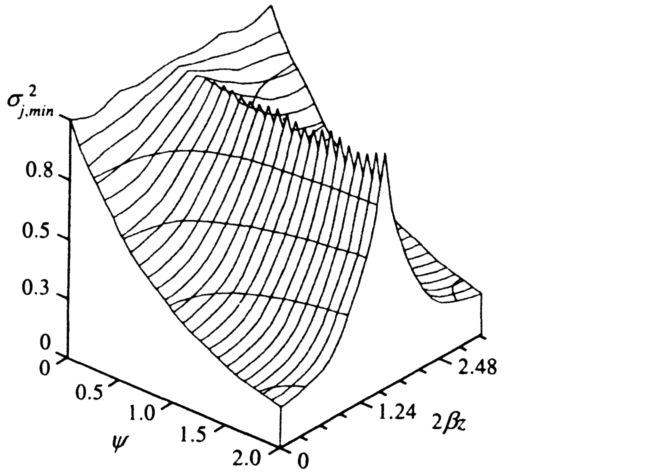

FIG. 2. Minimum normalized variance $\sigma^2_{j,\min} \equiv \langle \Delta S_j^2(z) \rangle_{\min}/n$ of the Stokes parameters $S_j(z)$ ($j=2, 3$) as a function of the nonlinear parameter ψ and the reduced linear coefficient $2\beta z$.

$$\langle \Delta S_3^2(z) \rangle_{\max} = n(1+\cos^2\theta + \cot^2\theta). \tag{27b}$$

It follows from (27a) and (27b) that if $\langle \Delta S_2^2(z) \rangle_{\min} \to 0$, $\langle \Delta S_3^2(z) \rangle_{\max} \to \infty$ in accordance with the uncertainty relations (4).

According to the expression (27a), it is easy to achieve suppression of the fluctuations in S_2. However, large values of the nonlinear parameter κ are needed to obtain small values of θ that satisfy the extremum condition $\kappa = -\cot\theta$. In fact, the expressions (27a) and (27b) can be written in the form

$$\langle \Delta S_2^2(z) \rangle_{\min} = n/(1+\kappa^2), \tag{28a}$$

$$\langle \Delta S_3^2(z) \rangle_{\max} = n[1+\kappa^2+\kappa^2/(1+\kappa^2)]. \tag{28b}$$

These expressions clearly reveal the dependence of the variance of the Stokes parameters on κ (see also Fig. 2).

One characteristic feature of the polarization-squeezed states of light under consideration is the occurrence of power conversion between waves with orthogonal polarizations. The linear coupling coefficient β in this case can be an additional parameter for regulating the functions $\langle \Delta S_{2,3}^2 \rangle$ (Fig. 2). As β varies smoothly, oscillations of the variances (24) of the Stokes parameters appear in the general case.

Let us now consider the fluctuations of the Stokes parameter S_1. The expression for $\langle \Delta S_1^2 \rangle$ obtained under the same approximations as (24) has the form

$$\langle \Delta S_1^2(z) \rangle = n[1+\psi^2 \sin^2(2\beta z)]. \tag{29}$$

It is seen that $\langle \Delta S_1^2(z) \rangle$ is also greater than the variance for the coherent state with the sole exception being values of the distance z satisfying the relation $2\beta z = \pi(1+m)$, $m=0, 1, 2, \ldots$.

6. CONCLUSIONS

Thus, the possibility of generating polarization-squeezed states of light in spatially nonuniform, nonlinear optical media with effective power conversion between the polarization modes has been demonstrated in the present work.

The expressions (24) obtained for the fluctuations of the Stokes parameters reveal the occurrence of redistribution of the quantum fluctuations, which is largely similar to that occurring in the quadrature components.

The optimal conditions for obtaining polarization-squeezed light with respect to the parameters of the problem, viz., the coupling coefficient of the waves and the nonlinear parameter, have been disclosed. Here the initial relationship between the phases of the modes, which determines the character of the power conversion in the system, is of fundamental importance.

Let us dwell briefly on the question of the realization of nonclassical polarization states of light. The possibility of observing them is essentially determined by the value of the nonlinear parameter ψ, which is linearly dependent on the intensity of the coherent radiation. Optical fibers of a special type ("twisted" fibers) can serve as spatially periodic media. Considerable progress has been achieved in the technology for fabricating such optical fibers with Bragg lattices. Such highly efficient lattices (with approximately 90% reflection) are induced even in a cw laser field (two beams) with a radiant flux density of the order of W/cm^2 in quartz optical fibers with implanted germanium ions and a length of several centimeters (see, for example, Ref. 16). In this case the light-induced modulation of the refractive index in such structures can reach $\beta \approx 10^{-2} - 10^{-3}$ cm^{-1}. Other materials which would be promising in this area are possible (see, for example, Refs. 17 and 18). Therefore, experimental observation of the effects considered in the present work has been fully achieved.

Finally, some possible applications of polarization-squeezed light should be stressed. In addition to high-precision polarimetry and ellipsometry, this effect also seems promising for obtaining light with sub-Poissonian photon statistics[19] and for performing nondemolition quantum measurements of polarization parameters.[20]

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