Overcoming the limit on electron thermal transport in a weakly collisional plasma

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(Submitted 31 January 1995)
Zh. Éksp. Teor. Fiz. **108**, 193–205 (July 1995)

Under nonlinear inverse bremsstrahlung conditions, when the electron distribution function is far from Maxwellian, a nonlocal linear theory is developed for the action on the plasma of relatively weak radiation with strong spatial variation. In the weakly collisional limit an analytical description is given for both the collisionless thermal electrons and the collisional cold electrons. It is shown how the nonlocal law for the coupling between the perturbed electron density and temperature and the radiation perturbing them depends on the radiation power. The form of the effective electron thermal conductivity is determined and the law describing how the limit on electron thermal conductivity is overcome as a function of heating power is derived.

1. The efficiency with which a plasma absorbs strong electromagnetic radiation depends nonlinearly on the intensity in consequence of both the effect of the field on the electron velocities[1,2] and the considerable modification of the electron distribution function under conditions such that the electron heating velocity becomes comparable with their thermalization velocity.[3–6] The latter holds when

$$Zv_E^2 > v_{Te}^2. \tag{1.1}$$

Here $v_{Te} = \sqrt{\kappa_B T_e/m_e}$ is the electron thermal velocity, $Z=|e_i/e|$ is the ionization rate, and $v_E = eE/m_e\omega_0$ is the oscillation amplitude of an electron in the high-frequency electromagnetic field with electric field E and frequency ω_0, which is substantially greater than the electron-ion collision frequency. In what follows we will be interested in situations in which condition (1.1) holds, while at the same time we have

$$v_{Te}^2 > v_E^2, \tag{1.2}$$

which in some sense allows us to treat the electromagnetic field as weakly affecting the electron velocity. Conditions (1.1) and (1.2) hold simultaneously in plasmas typically found in laser inertial confinement fusion (ICF), where the ions have a high charge state

$$Z \gg 1, \tag{1.3}$$

which we will assume to be the case in our treatment. We emphasize that conditions (1.1) and (1.3) are of particular interest in the case of ICF laser-produced plasmas specifically when it is necessary to achieve high radiation absorption efficiency through the inverse bremsstrahlung effect.

When the electron distribution differs greatly from Maxwellian under conditions (1.1)–(1.3), the nature of transport in the plasma changes considerably. This change in a highly collisional plasma was discovered relatively early.[7] We will be interested in the case of a weakly collisional plasma, when the mean free path of a thermal electron is much longer than the characteristic length scale L on which the electron distribution function varies,

$$l_{ei} = v_{Te}/\nu_{ei} \gg L. \tag{1.4}$$

Here

$$\nu_{ei} = \frac{4\sqrt{2\pi}\, e^2 e_i^2 n_i \Lambda}{3 m_e^2 v_{Te}^3} \tag{1.5}$$

is the frequency with which a thermal electron collides with ions, n_i is the ion number density, and Λ is the Coulomb logarithm. One usually refers to the plasma as collisionless when condition (1.4) is satisfied. We, however, will refer to it as a weakly collisional plasma. The reason is that the mean free path $l_e(v)$ for an electron with velocity v for collisions with ions,

$$l_e(v) \sim v^4 (l_{ei}/v_{Te}^4) \tag{1.6}$$

is shorter than the characteristic spatial length scale L for velocities

$$v < v^* = v_{Te}(L/l_{ei})^{1/4}. \tag{1.7}$$

Hence there are always cold electrons that are subject to strong collisions. Then the important question becomes: In what processes does the effect of these cold electrons dominate? References 8 and 9 use the idea of separating the electrons into thermal collisionless and subthermal collisional groups to understand the essential cause of phenomena which have long appeared paradoxical. The latter include the limit on the electron thermal transport in a laser plasma, long ago determined experimentally,[10,11] and the noncollinearity of the electron thermal flux and the temperature gradient, discovered in numerical simulations (see, e.g., Refs. 12 and 13). It is worth emphasizing that many of the prevalent paradoxical ideas about laser-produced plasmas result from work using numerical simulation of collisional effects, which frequently obscure the fundamental physical concepts obscure. In contrast to numerical experiments, analytical kinetic transport theory allows one to see the essential physics with relative ease. In this connection we note the work of Maksimov and Silin,[14] in which an analytical approach was developed using an asymptotic expansion in inverse powers of the large Knudsen number $\mathrm{Kn}=L/l_{ei}$ and the decisive role of collisional subthermal electrons was revealed. The fundamental ideas introduced by Ref. 14 enabled the paradoxical properties of laser plasma to be explained in Refs. 8 and 9.

In the present work we report results of an analytical theory of nonlocal transport in a weakly collisional plasma subject to a relatively strong radiation field (1.1). The purpose of this exposition is partly to provide a theoretical explanation for the phenomenon found in the numerical experiments of Ref. 15, whose authors were far from using the physical picture of a weakly collisional plasma associated with the division of particles into thermal collisionless and subthermal collisional classes. At the same time, the phenomenon observed in Ref. 15 is unquestionably of interest. Specifically, Fig. 3 of Ref. 15 shows that as the radiation intensity increases there is a decrease in the inhibition of the electron thermal transport, which has long been known in laser plasmas[10,11] and which for a weak electromagnetic field has recently been studied in detail (see, e.g., Refs. 16–20). In Ref. 15, in analogy with the strongly collisional reduction in thermal transport found theoretically in Ref. 7, it is suggested that the effect of collisions increases even in the weakly collisional case because the role of the slow particles is enhanced. As will be shown below, this idea is correct. However, Ref. 15 was far from establishing the range of electron velocities which determine the electron thermal transport anomaly observed in the numerical experiment. In the present work a physical problem is posed which makes possible an analytical description of the phenomenon observed numerically in Ref. 15. We obtain analytical results which reveal the physical nature of the decrease in the limit on the electron thermal transport as the intensity of the radiation source increases. Note that at the present time there is no clear evidence in an actual physical experiment for the weakening of the electron thermal transport inhibition. The material presented below can therefore be regarded as a theory developed in order to understand the numerical simulation of Ref. 15, which in turn allows us to better understand actual physical experiments.

2. As is customary in the kinetics of collisional plasmas subjected to electromagnetic radiation at high frequency ω_0, much larger than the electron collision frequency, we will use the approach of Perel and Pinskiĭ,[21] in which the electron distribution function is divided into a slowly varying part f_0 and a rapidly varying part $\delta\tilde{f}$. To within terms of order ω_0^{-2} the latter is linear in the strength of the high-frequency electric field,

$$\tilde{\mathscr{E}}=\frac{1}{2}\,\mathbf{E}\exp(-i\omega_0 t)+\text{c.c.} \tag{2.1}$$

We use the kinetic equation for the slowly varying function f_0 derived in Ref. 22:

$$\frac{\partial f_0}{\partial t}+\mathbf{v}\,\frac{\partial f_0}{\partial \mathbf{r}}+\frac{e\mathbf{E}_0}{m_e}\,\frac{\partial f_0}{\partial \mathbf{v}}-J_{ei}[f_0]-J_{ee}[f_0,f_0]=\frac{e^2}{4\omega_0^2 m_e^2}$$
$$\times\left\{\frac{\partial|\mathbf{E}|^2}{\partial \mathbf{r}}\,\frac{\partial f_0}{\partial \mathbf{v}}+\frac{1}{2}\,\frac{\partial^2 f_0}{\partial v_i\partial v_j}\left(\frac{\partial}{\partial t}+\mathbf{v}\,\frac{\partial}{\partial \mathbf{r}}\right)(E_iE_j^*+E_i^*E_j)\right.$$
$$+(E_iE_j^*+E_i^*E_j)\left(\frac{\partial^2 f_0}{\partial r_i\partial v_j}+\left(\frac{\partial}{\partial t}+\mathbf{v}\,\frac{\partial}{\partial \mathbf{r}}\right)\frac{\partial^2 f_0}{\partial v_i\partial v_j}\right.$$
$$\left.\left.-\frac{\partial}{\partial v_i}\,J_{ei}\left[\frac{\partial f_0}{\partial v_j}\right]-J_{ee}\left[\frac{\partial f_0}{\partial v_i},\frac{\partial f_0}{\partial v_j}\right]\right)\right\}, \tag{2.2}$$

where $\mathbf{E}_0$ is the strength of the quasisteady electric field. In the electron-ion collision integral we neglect small terms of order the electron-ion mass ratio and use the expression

$$J_{ei}[f_0]=\nu(v)\,\frac{\partial}{\partial v_r}\left\{[v^2\delta_{rs}-v_rv_s]\,\frac{\partial f_0}{\partial v_s}\right\}, \tag{2.3}$$

where

$$\nu(v)=\frac{3\sqrt{\pi}\;\nu_{ei}v_{Te}^3}{2\sqrt{2}\;v^3}=\frac{2\pi e^2e_i^2n_i\Lambda}{m_e^2v^3}\equiv\frac{A}{v^3}. \tag{2.4}$$

Finally, for the electron-electron collision integral we use the standard Landau collision integral (see, e.g., Ref. 23).

In the special case of a spatially uniform pump field, where we have, first,

$$\frac{\partial}{\partial \mathbf{r}}\,E_iE_j^*=0, \tag{2.5}$$

second, $Z\gg 1$, and third, condition (1.2) is satisfied, Eq. (2.2) assumes the following relatively simple form:

$$\frac{\partial f_0}{\partial t}-J_{ei}[f_0]-J_{ee}[f_0,f_0]$$
$$=-\frac{e^2}{4\omega_0^2m_e^2}\,(E_iE_j^*+E_i^*E_j)\,\frac{\partial}{\partial v_i}\,J_{ei}\left[\frac{\partial f_0}{\partial v_j}\right]. \tag{2.6}$$

The solution of this equation has a symmetric part $\bar{f}$ which is independent of the direction of $\mathbf{v}$ and a numerically small antisymmetric part which vanishes when averaged over angle. The latter is important for the tensor of the electron momentum flux density, but negligible in the treatment of the thermal transport. Below we will disregard this small antisymmetric part. Then the function $\bar{f}$ satisfies the equation

$$\frac{\partial \bar{f}}{\partial t}-J_{ee}[\bar{f},\bar{f}]=\frac{e^2E^2}{3\omega_0^2m_e^2}\,\nu(v)v\,\frac{\partial}{\partial v}\left(\frac{1}{v}\,\frac{\partial \bar{f}}{\partial v}\right). \tag{2.7}$$

For real values of the Langdon parameter[3]

$$\alpha=\frac{Ze^2|\mathbf{E}|^2}{\omega_0^2m_e^2v_{Te}^2}\equiv 0.042\,\frac{I_0}{10^{14}\,\mathrm{W\cdot cm^{-2}}}\cdot\frac{\lambda_0^2}{(1.06\,\mu\mathrm{m})^2}$$
$$\times\frac{Z}{K_BT_e(\mathrm{keV})} \tag{2.8}$$

this equation was studied numerically in detail in Ref. 24. There it was shown that in accordance with the suggestion of Ref. 7, solutions of Eq. (2.7) can be written as

$$\bar{f}(v,t)=f_\mu(v,t)=C_\mu\exp[-(v/v_\mu(t))^\mu] \tag{2.9}$$

or in the form

$$f_\mu(v,t)=C_\mu\exp\left[-\left(\frac{m_ev^2}{2K_BT_e(t)}\,a_\mu\right)^{\mu/2}\right], \tag{2.10}$$

where

$$C_\mu=\frac{n_e}{4\pi}\,\frac{\mu}{\Gamma(3/\mu)v_\mu^3},\qquad a_\mu=\frac{2\Gamma(5/\mu)}{3\Gamma(3/\mu)},$$

$$v_\mu^2=\frac{3K_BT_e}{m_e}\frac{\Gamma(3/\mu)}{\Gamma(5/\mu)}. \tag{2.11}$$

Here $\Gamma(x)$ is the Γ function, and the electron temperature T_e is defined by

$$\frac{3}{2}n_eK_BT_e=2\pi m_e\int_0^\infty f_\mu v^4dv. \tag{2.12}$$

Reference 24 determined the following relationship between the parameter α and the exponent μ:

$$\mu(\alpha)=2+\frac{3}{1+1.66\alpha^{-0.724}}. \tag{2.13}$$

Here μ varies between two and five. In particular, for $\mu=2$ we have a Maxwellian distribution, while for $\mu=5$ we obtain the Langdon distribution,[3] where v_5 varies as a function of time according to

$$v_5^4\frac{dv_5}{dt}=\frac{5}{3}Av_E^2. \tag{2.14}$$

Equation (2.9) has been used repeatedly to treat transport processes (see, e.g., Ref. 7). The distributions (2.9) for different values of μ were also used in Ref. 15 in order to determine numerically the effect of strong electromagnetic fields on nonlocal transport in the plasma. The results of the numerical experiment reveal (see Fig. 3 of Ref. 15) that as μ (and consequently α) increases, corresponding to a rise in the intensity I_0 of the pump field, the magnitude of the characteristic gradient length scale decreases, whereupon the limit in the electron heat flux begins to be important. Here, following Ref. 15, we use the distribution (2.9) in order to analytically exhibit the effect of the high-frequency field strength on the nonlocal behavior of electron transport associated with inverse bremsstrahlung absorption of the laser radiation by the plasma.

3. In order to develop a theory of nonlocal transport we will assume that in addition to the spatially uniform heating radiation there is a small component of spatially varying pump field:

$$E_iE_j^*+\delta(E_iE_j^*)e^{i\mathbf{kr}}. \tag{3.1}$$

Accordingly, we will look for the electron distribution function in the form

$$f_0=f_\mu+\delta f e^{i\mathbf{kr}}. \tag{3.2}$$

In what follows we will neglect the time dependence of δf, since in accordance with Ref. 15 we are taking the heating rate to be less than the thermal transport rate on a scale κ^{-1}, which holds for $\kappa l_e(v_\mu)\gg 1$, where $l_e(v_\mu)$ is defined in Eq. (1.6). Then, disregarding the effect of the quasisteady electric field $\mathbf{E}_0$, noting that we have $Z\gg 1$, and keeping in mind the inequality (1.2) we find from (2.2)

$$i\mathbf{k}\boldsymbol{v}\delta f-J_{ei}[\delta f]-J_{ee}[\delta f]$$
$$=\frac{e^2}{4\omega_0^2m_e^2}\left\{i\mathbf{k}\frac{\partial f_\mu}{\partial \boldsymbol{v}}\,\delta\left|\mathbf{E}\right|^2+\left(\frac{1}{2}\frac{\partial^2 f_\mu}{\partial v_s\partial v_r}i\mathbf{k}\boldsymbol{v}\right.\right.$$
$$\left.\left.-\frac{\partial}{\partial v_s}J_{ei}\left[\frac{\partial f_\mu}{\partial v_r}\right]\right)\delta(E_rE_s^*+E_r^*E_s)\right\}. \tag{3.3}$$

As was done in Ref. 14, we represent the perturbation of the electron distribution function in the form

$$\delta f=\delta f_1+\delta f_c, \tag{3.4}$$

where

$$\delta f_1=\frac{e^2}{4\omega_0^2m_e^2}\left\{\delta\left|\mathbf{E}\right|^2\frac{1}{v}\frac{\partial f_\mu}{\partial v}+\frac{1}{2}\frac{\partial^2 f_\mu}{\partial v_s\partial v_r}\right.$$
$$\left.\times\,\delta(E_rE_s^*+E_r^*E_s)\right\}. \tag{3.5}$$

For the function δf_c in the limit $Z\gg 1$, which allows us to disregard the contribution coming from substitution of (3.5) into the electron-electron collision integral, we have the following equation:

$$+i\mathbf{k}\mathbf{v}\delta f_c-J_{ei}[\delta f_c]-J_{ee}[\delta f_c]=Y_0+Y_a, \tag{3.6}$$

where

$$Y_0=B\frac{\partial}{\partial v_s}\left(\frac{1}{v^3}\frac{\partial f_\mu}{\partial v_s}\right), \tag{3.7}$$

$$Y_a=-C_{ij}\left(v_iv_j-\frac{1}{3}v^2\delta_{ij}\right)$$
$$\times\left[\frac{1}{2v^4}\frac{d}{dv}\left(\frac{1}{v}\frac{df_\mu}{dv}\right)+\frac{3}{v^6}\frac{df_\mu}{dv}\right]. \tag{3.8}$$

Here

$$B=\frac{e^2A\delta|\mathbf{E}|^2}{3m_e^2\omega_0^2}, \tag{3.9}$$

$$C_{ij}=\frac{e^2A}{2m_e^2\omega_0^2}\delta\left(E_iE_j^*+E_i^*E_j-\frac{2}{3}\delta_{ij}\left|\mathbf{E}\right|^2\right), \tag{3.10}$$

and the quantity A is defined in (2.4). Equation (3.6) is written in a form such that for $\mu=2$, i.e., in the case of a Maxwellian distribution, it goes over to the distribution studied in Refs. 14 and 8. Equations (3.7) and (3.8) are written in a form which makes the passage to the case of a Maxwellian distribution most transparent. The results obtained by solving Eq. (3.6) dictate the subsequent conclusions. However, before going on to study Eq. (3.6), we point out some consequences of the correction term (3.5) in the electron distribution function. Thus, for the corresponding perturbation of the electron density we have

$$\delta n_1=-\frac{\pi e^2\delta|\mathbf{E}|^2}{\omega_0^2m_e^2}\int_0^\infty dv f_\mu=-\frac{e^2\delta|\mathbf{E}|^2n_e\Gamma(1/\mu)}{4\omega_0^2m_e^2v_\mu^2\Gamma(3/\mu)}. \tag{3.11}$$

For $\mu=2$ this expression yields the usual density variation caused by the ponderomotive force of the high-frequency electromagnetic field. For the perturbation of the kinetic energy density we have

$$\delta\left(\frac{3}{2}n_eK_BT_e\right)_1=-\frac{e^2\delta|\mathbf{E}|^2n_e}{8\omega_0^2m_e}. \quad (3.12)$$

This expression does not depend on μ. It corresponds to the time average of the energy density associated with the electron oscillations due to the spatially nonuniform correction to the electromagnetic field heating the plasma.

4. We start by considering the implications of Eq. (3.6) for collisionless thermal electrons with velocity greater than the value v^* determined by Eq. (1.7). The perturbation $\delta f_{c,T}(\mathbf{v})$ of the thermal electron distribution takes the form (cf. Ref. 8)

$$\delta f_{c,T}(\mathbf{v})=-i(Y_0+Y_a)\left[\frac{P}{\mathbf{kv}}+i\pi\delta(\mathbf{kv})\right], \quad (4.1)$$

where P denotes the Cauchy principal value and $\delta(\mathbf{kv})$ is a Dirac δ function. It is not difficult to see that Eq. (4.1) yields a thermal electron density perturbation and a perturbation in their kinetic energy density. These are found to be smaller by a factor

$$\frac{1}{kl(v_\mu)}\ll 1 \quad (4.2)$$

than expressions (3.11) and (3.12). Small terms of order (4.2) can be neglected. Here

$$l(v_\mu)=\frac{v_\mu^4m_e^2}{2\pi e^2e_i^2n_i\Lambda}. \quad (4.3)$$

Next we note that the average over angle of the electron velocity yields

$$\langle\mathbf{kv}\delta f_{c,T}(\mathbf{v})\rangle=-iY_0. \quad (4.4)$$

We will be interested in the case $\mu>2$, since the case of weak laser intensities corresponding to a Maxwell distribution, i.e., $\mu=2$, was treated previously.[8] Expression (3.7) therefore does not give rise to a $\delta(v)$ term, which simplifies our treatment. In particular, we find immediately that

$$\operatorname{div}\mathbf{j}_T=i\mathbf{kj}_T=e\int d\mathbf{v}(i\mathbf{kv})\delta f_{c,T}=0. \quad (4.5)$$

This means that there is no electric current in the direction of the vector $\mathbf{k}$. The solenoidal electric current density resulting from inverse bremsstrahlung absorption is found to be

$$\mathbf{j}_T=-\frac{ien_e\mu\Gamma(1-2/\mu)}{k^2l(v_\mu)\Gamma(3/\mu)}\frac{e^2}{m_e^2\omega_0^2v_\mu}\times\delta\left(\mathbf{E}(\mathbf{kE}^*)+\mathbf{E}^*(\mathbf{kE})-2\frac{\mathbf{k}}{k^2}\left|\mathbf{kE}\right|^2\right). \quad (4.6)$$

In contrast with the electric current, the electron thermal flux density has an irrotational component, which can easily be discerned from the relation

$$\operatorname{div}\mathbf{q}_T=i\mathbf{kq}_T=\int d\mathbf{v}\,\frac{m_ev^2}{2}(i\mathbf{kv})\delta f_{c,T}=4\pi m_eBf_\mu(0). \quad (4.7)$$

This permits us to write the following expression for the irrotational part $\mathbf{q}_T$:

$$\mathbf{q}_0=-\frac{i\mathbf{k}\mu n_ev_\mu e^2\delta|\mathbf{E}|^2}{3k^2l(v_\mu)\Gamma(3/\mu)m_e\omega_0^2}. \quad (4.8)$$

It follows from (4.7) that (4.8) describes the transport of only that part of the heat which is transferred to the plasma due to inverse bremsstrahlung absorption from the spatially nonuniform part of the laser radiation [see Eq. (3.1)].

The solenoidal part of the electron thermal flux density is determined by $f_\mu(0)$ as in (4.7). Here we have

$$\mathbf{q}_a=-\frac{4\pi ie^2Af_\mu(0)}{3k^2m_e\omega_0^2}\delta\left(\mathbf{E}(\mathbf{kE}^*)+\mathbf{E}^*(\mathbf{kE})-2\frac{\mathbf{k}}{k^2}\left|\mathbf{kE}\right|^2\right)=-\frac{i\mu n_ev_\mu e^2}{3\Gamma(3/\mu)k^2l(v_\mu)m_e\omega_0^2}\times\delta\left(\mathbf{E}(\mathbf{kE}^*)+\mathbf{E}^*(\mathbf{kE})-2\frac{\mathbf{k}}{k^2}\left|\mathbf{kE}\right|^2\right). \quad (4.9)$$

This is just the part of the electron heat flux which is noncollinear with the temperature gradient. Attention has long been focused on this noncollinearity in numerical experiments[12,13] on heat transport in laser plasmas. Until recently, however, it has not been discussed theoretically.

To summarize the results of the present section, we note that thermal collisionless electrons are primarily responsible for heat transport in a weakly collisional plasma.

5. We now proceed to treat the electron distribution for subthermal electrons with velocities less than v^*, when the collision integrals play the dominant role in Eq. (3.6). Then breaking δf_c into an isotropic part δf_0 and anisotropic part δf_a, we have

$$\delta f_c=\delta f_0+\delta f_a,\quad \delta f_0=\langle\delta f_c\rangle. \quad (5.1)$$

Then we obtain the following two equations:

$$i\langle\mathbf{kv}\delta f_a\rangle-J_{ee}[\delta f_0]=Y_0, \quad (5.2)$$

$$i\mathbf{kv}\delta f_0+i(\mathbf{kv}\delta f_a-\langle\mathbf{kv}\delta f_a\rangle)=J_{ei}[\delta f_a]+Y_a, \quad (5.3)$$

In Eq. (5.3) we have assumed $Z\gg 1$. Since we have taken $v\ll v^*$ and collisions play the principal role in determining δf_c, the terms containing δf_a in the left-hand side of (5.3) are found to be small compared with $J_{ei}[\delta f_a]$. As a result, Eq. (5.3) is readily solved, and Eq. (5.2) can therefore be written in the form (cf. Ref. 14)

$$\frac{k^2v^2}{6\nu(v)}\delta f_0-J_{ee}[\delta f_0]=Y_0. \quad (5.4)$$

Here

$$J_{ee}[\delta f_0]=\frac{16\pi^2e^4\Lambda}{3m_e^2}\left\{\int_0^v dv'v'^4\frac{1}{v^2}\frac{\partial}{\partial v}F(v,v')\right.$$

$$+\int_v^\infty dv'v'\left[3F(v,v')+v\frac{\partial}{\partial v}F(v,v')\right]\Bigg\}. \tag{5.5}$$

And

$$F(v,v')=\frac{1}{v}\frac{\partial}{\partial v}[f_\mu(v)\delta f_0(v')+f_\mu(v')\delta f_0(v)] -\frac{1}{v'}\frac{\partial}{\partial v'}[f_\mu(v')\delta f_0(v)+f_\mu(v)\delta f_0(v')]. \tag{5.6}$$

In Eq. (5.4) we systematically take into account the smallness of the electron velocity in comparison with v^*. Then, for example, for $\mu>2$ the right-hand side of (5.4) can be written as follows:

$$Y_0=-\frac{e^2\delta|\mathbf{E}|^2}{m_e^2\omega_0^2v_\mu^2}\frac{2\pi e^2e_i^2n_i\Lambda}{3m_e^2v_\mu^3}\mu(\mu-2)\left(\frac{v}{v_\mu}\right)^{\mu-5}f_\mu(v). \tag{5.7}$$

Simplification of the collision integral (5.5) yields the expressions

$$J_{ee}[\delta f_0]=\frac{4\pi^2e^4\Lambda}{3m_e^2}D_\mu\frac{1}{v^2}\frac{d}{dv}\left[v^2\frac{d}{dv}\delta f_0(v)\right], \tag{5.8}$$

where

$$D_\mu=2\int_0^\infty dv\, vf_\mu(v)=\frac{n_e}{2\pi v_\mu}\frac{\Gamma(2/\mu)}{\Gamma(3/\mu)}. \tag{5.9}$$

Using Eqs. (5.7) and (5.8) we can write

$$\delta f_0(v)=\psi_\mu\left(\frac{v^2}{v_\mu^2}\right)f_\mu(v)\frac{\mu(\mu-2)}{k^2l^2(v_\mu)}\frac{2e^2\delta|\mathbf{E}|^2}{m_e^2\omega_0^2v_\mu^2}. \tag{5.10}$$

Then from Eq. (5.4) we obtain

$$\frac{1}{N_\mu}\left[y^{3/2}\psi_\mu''(y)+\frac{3}{2}y^{1/2}\psi_\mu'(y)\right]-y^3\psi_\mu(y)=y^{(\mu-4)/2}, \tag{5.11}$$

where under the conditions we have assumed

$$N_\mu=\frac{Z}{16}\frac{\Gamma(3/\mu)}{\Gamma(2/\mu)}k^2l^2(v_\mu)\gg 1. \tag{5.12}$$

Equation (5.11) differs from that studied in Ref. 14 on account of the right-hand side, which in our treatment depends on the intensity of the radiation heating the plasma because of Eqs. (2.13) and (2.8). This dependence is the primary reason for the occurrence of the change in the nonlocal law relating the electron density and temperature perturbations to the radiation field causing them as a function of the intensity of the latter, and also for the corresponding nonlocal effective electron thermal conductivity.

The solution of Eq. (5.11) satisfying regular boundary conditions can be written in the form[25]

$$\psi_\mu(y)=-N_\mu^{(10-\mu)/7}\Psi\left(\xi,[\zeta^{(\mu-4)/2}]\right), \tag{5.13}$$

where

$$\xi=N_\mu^{2/7}y, \tag{5.14}$$

$$\Psi(\xi,[\varphi(\zeta)]) =\frac{4}{7\xi^{1/4}}\left\{I_{1/7}\left(\frac{4}{7}\xi^{7/4}\right)\int_\xi^\infty\frac{d\zeta}{\zeta^{1/4}}\varphi(\zeta)K_{1/7}\left(\frac{4}{7}\zeta^{7/4}\right) +K_{1/7}\left(\frac{4}{7}\xi^{7/4}\right)\int_0^\xi\frac{d\zeta}{\zeta^{1/4}}\varphi(\zeta)I_{1/7}\left(\frac{4}{7}\zeta^{7/4}\right)\right\}. \tag{5.15}$$

Here $I_{1/7}$ and $K_{1/7}$ are Bessel functions of imaginary argument. The solution (5.13) implies that the main contribution to the perturbation in the distribution comes from cold electrons with subthermal velocities

$$v\sim v_\mu N_\mu^{-1/7}<v^*. \tag{5.16}$$

This justifies the approximations (5.7) and (5.8) used above.

For $\mu>2$ the solution (5.13) enables us to determine the electron density perturbation due to redistribution of the subthermal electrons:

$$\delta n_c=-n_e\frac{e^2\delta|\mathbf{E}|^2}{m_e^2\omega_0^2v_\mu^2}\frac{Z}{N_\mu^{\mu/7}}\frac{\mu^2(\mu-2)}{16\Gamma(2/\mu)}b_\mu, \tag{5.17}$$

where

$$b_\mu=\int_0^\infty d\xi\xi^{1/2}\Psi(\xi,[\zeta^{(\mu-4)/2}])=I_\mu\left(\frac{4}{7}\right)^{3-2\mu/7}. \tag{5.18}$$

Here we introduce for the integral the expression

$$I_\mu=\int_0^\infty dzK_{1/7}(z)\int_0^z dxI_{1/7}(x)\left[\left(\frac{x^{\mu-6}}{z}\right)^{2/7}+\left(\frac{z^{\mu-6}}{x}\right)^{2/7}\right]. \tag{5.19}$$

Numerical evaluation for some specific cases yields the results $I_3=7.9$; $I_4=4.5$; $I_5=3.7$.

Because the electron velocities determining the perturbed distribution (5.13) are small, we find that

$$\delta\left(\frac{3}{2}n_eK_BT_e\right)_c\ll k_BT_e\delta n_c. \tag{5.20}$$

Hence for the temperature perturbation of the subthermal (cold) electrons we find

$$\delta T_{e,c}=T_e\frac{e\delta|\mathbf{E}|^2}{m_e^2\omega_0^2v_\mu^2}\frac{Z}{N_\mu^{\mu/7}}\frac{\mu^2(\mu-2)}{16\Gamma(2/\mu)}b_\mu. \tag{5.21}$$

The density perturbation (5.17) and temperature perturbation (5.21) are found to be larger than (3.11) and the corresponding increment in the electron oscillation energy in the field of a nonuniform pump under the condition

$$Z^{7-\mu}>[kl(v_\mu)]^{2\mu}\gg 1. \tag{5.22}$$

It is clear that as μ increases, i.e., as the intensity of the uniform electromagnetic field heating the plasma rises, it becomes more difficult to meet this condition. This means that the efficiency with which a real plasma can be heated by a nonuniform radiation field decreases.

Finally, we point out that the smallness of the subthermal electron velocities implies that the thermal flux associated with them is small in comparison with the thermal flux of the bulk of the electrons, given by (4.8) and (4.9) (cf. Refs. 8 and 9).

6. In this section we briefly discuss how the intense field heating the plasma affects the nonlocal thermal transport. We begin by emphasizing that the presence of the factor $N_\mu^{-\mu/7} \sim k^{-2\mu/7}$ in Eqs. (5.17) and (5.21) is associated, first, with the asymptotic expansion in inverse nonintegral powers of the large Knudsen parameter and, second, with the nonlocal relation between the electron density and temperature perturbations and the spatially nonuniform laser radiation $\delta|\mathbf{E}|^2$ responsible for heating the plasma. At the same time, we should recall here that the concept of temperature is rather vague when we talk about heating the cold subthermal electrons (cf. Ref. 8). It is also rather imprecise to talk about a variety of nonlocal thermal transport mechanisms. Specifically, while we have seen that the cold subthermal electrons can be heated up, the transport of the energy absorbed by the electrons from the radiation is due to the thermal electrons. Recalling this, we use Eq. (5.21) in order to express $\delta|\mathbf{E}|^2$ in Eq. (4.7) in terms of the temperature perturbation δT_e. In doing this we will not distinguish between the essentially identical values of $\mathbf{q}_T$ and the total electron thermal flux density. Then we can write (cf. Refs. 20 and 8)

$$i\mathbf{kq} = i n_e k_B v_\mu l(v_\mu) k^2 \delta T_e \, d_\mu \, [Z k^2 l^2(v_\mu)]^{\mu/7-1}, \quad (6.1)$$

where

$$d_\mu = \frac{\Gamma(3/\mu)}{\mu(\mu-2)\Gamma(5/\mu) b_\mu} \left[\frac{16\Gamma(2/\mu)}{\Gamma(3/\mu)} \right]^{1-\mu/7}. \quad (6.2)$$

If now we use the relations

$$\mathbf{q}_\parallel = -i\mathbf{k}\kappa_{\text{eff}} \delta T_e \quad (6.3)$$

for the longitudinal (irrotational) component of the electron thermal flux density to determine the effective electron thermal transport κ_{eff} (cf. Ref. 8), then Eq. (6.1) yields

$$\kappa_{\text{eff}} = \frac{n_e k_B v_\mu l(v_\mu) d_\mu}{[Z k^2 l^2(v_\mu)]^{1-\mu/7}}. \quad (6.4)$$

Although everything presented thus far presumes that the condition $\mu > 2$ holds, our treatment in the case of a Maxwellian distribution ($\mu = 2$) is inapplicable only because the numerical coefficient (6.2) cannot be used for $\mu = 2$. At the same time, the dependence of the effective thermal conductivity $\varkappa_{\text{eff}}$ on the wave vector k is given correctly by Eq. (6.4) (cf. Ref. 14, where the numerical coefficient needed for the case of a Maxwellian distribution is also derived).

The denominator of Eq. (6.4),

$$[Z k^2 l^2(v_\mu)]^{1-\mu/7}, \quad (6.5)$$

which in our treatment is large compared with unity, corresponds to the effect of electron thermal flux limiting due to nonlocal transport. As the intensity of the spatially uniform field heating the plasma increases, Eq. (2.13) shows that the exponent $1-(\mu/7)$ decreases from 5/7 to the relatively small value 2/7. Consequently, expression (6.5) decreases as the radiation intensity increases for a fixed gradient length scale $L \sim \kappa^{-1}$. This implies that the electron thermal flux limiting effect becomes weaker as the intensity of the laser radiation increases in a weakly collisional plasma. Here we emphasize that we are making these assertions in accordance with the usual assumptions in laser plasma theory, which deals only with the longitudinal component of the thermal flux and neglects its rotational part.

7. Let us now discuss the results obtained above. We have treated a plasma subjected to a relatively uniform heating field, such that the characteristic scale of the spatial variation of this field is relatively large. For a relatively powerful radiation field, when condition (1.1) holds at the same time as condition (1.2), inverse bremsstrahlung absorption causes the electron distribution to depart significantly from Maxwellian, which is consistent with the nonlinear nature of inverse bremsstrahlung. When the electron velocity distribution is altered in this way in response to the radiation producing the heating, we have shown that a component of the radiation field that changes relatively rapidly in space has a nonlocal effect on the electrons of a weakly collisional plasma. The analytical theory presented here supports the qualitative remark in Ref. 15 about the inhibition of electron thermal flux limiting in a weakly collisional plasma as the strength of the laser radiation field heating it increases. We have derived an analytical law for the nonlocal thermal conduction as a function of the power of the laser radiation. The observed behavior may be the reason why different experimental groups have reported very different values for the effective coefficient of the electron thermal flux limiting (see, e.g., Refs. 11 and 26). It can therefore also be asserted that the theory presented here provides an understanding of the reason for the new phenomenon indicated by the numerical simulation[15] and opens up a possible approach to understanding the discrepancy in the various experiments on electron heat transport in laser plasmas. At the same time it should be acknowledged that we have thus far made only the first steps in understanding this paradoxical situation, which has prevailed for a long time in the exploration of inverse bremsstrahlung absorption and the transport associated with it in laser plasmas. In particular, it is obvious that the linear approximation we have used to derive the perturbed electron distribution, despite its demonstrated usefulness, is still not adequate to answer a great number of important questions, including the redistribution of electron energy between different velocity ranges.

The present work was supported under project number 94-02-03631 of the Russian Fund for Fundamental Research.

[1] V. P. Silin, Zh. Éksp. Teor. Fiz. **47**, 2254 (1964) [Sov. Phys. JETP **20**, 1510 (1965)].
[2] V. P. Silin and S. A. Uryupin, Zh. Éksp. Teor. Fiz. **81**, 910 (1981) [Sov. Phys. JETP **54**, 485 (1981)].
[3] A. B. Langdon, Phys. Rev. Lett. **44**, 575 (1980).
[4] R. Balescu, J. Plasma Phys. **23**, 553 (1982).
[5] R. D. Jones and K. Lee, Phys. Fluids **25**, 2307 (1982).
[6] B. N. Chichkov, S. A. Shumsky, and S. A. Uryupin, Phys. Rev. A **45**, 7475 (1992).
[7] P. Mora and H. Yahi, Phys. Rev. A **26**, 2259 (1982).
[8] V. P. Silin, Zh. Éksp. Teor. Fiz. **106**, 1398 (1994) [Sov. Phys. JETP **79**, 756 (1994)].
[9] V. P. Silin, Pis'ma Zh. Éksp. Teor. Fiz. **60**, 766 (1994) [JETP Lett. **60**, 775 (1994)].
[10] J. F. Kephart, R. P. Godwin, and G. H. McCall, Appl. Phys. Lett. **25**, 108 (1974).
[11] R. Benattar, C. Popovic, R. Sigel, and J. Virmont, Phys. Rev. Lett. **42**, 766 (1979).

[12] M. Strauss, G. Hazak, D. Shvarts, and R. S. Caxton, Phys. Rev. A **39**, 2627 (1984).
[13] G. J. Rickard, A. R. Bell, and K. M. Epperlein, Phys. Rev. Lett. **62**, 2687 (1989).
[14] A. V. Maksimov and V. P. Silin, Zh. Éksp. Teor. Fiz. **103**, 73 (1993) [Sov. Phys. JETP **76**, 39 (1993)].
[15] E. M. Epperlein and R. W. Short, Phys. Rev. E **50**, 1697 (1994).
[16] E. M. Epperlein, Phys. Rev. Lett. **65**, 2145 (1990).
[17] E. M. Epperlein and R. W. Short, Phys. Fluids **B4**, 2211 (1992).
[18] E. M. Epperlein and R. W. Short, Phys. Fluids **B4**, 4190 (1992).
[19] R. L. Berger, B. F. Lasinski, T. B. Kaiser *et al.*, Phys. Fluids **B5**, 2243 (1993).
[20] A. V. Maksimov and V. P. Silin, Phys. Lett. A **173**, 83 (1993).
[21] V. I. Perel and Ya. M. Pinskiĭ, Zh. Éksp. Teor. Fiz. **54**, 1889 (1968) [Sov. Phys. JETP **27**, 1014 (1968)].
[22] A. V. Maksimov, V. P. Silin, and M. V. Chegotov, Fiz. Plazmy **16**, 575 (1990) [Sov. J. Plasma Phys. **16**, 331 (1990)].
[23] V. P. Silin, *Introduction to the Kinetic Theory of Gases* [in Russian], Nauka, Moscow (1971).
[24] J. P. Matte, M. Lamoureux, C. Möller, R. Y. Yin, J. Delettrez, J. Virmont, and T. W. Johnston, Plasma Phys. and Controlled Fusion **30**, 1665 (1988).
[25] A. V. Maksimov and V. P. Silin, Zh. Éksp. Teor. Fiz. **105**, 1242 (1994) [Sov. Phys. JETP **78**, 669 (1994)].
[26] W. L. Kruer, Comments Plasma Phys. **5**, 69 (1979).

Translated by David L. Book