Effective Lagrangian and the nonlinear interaction of a nonuniform electromagnetic field in QED_{2+1} in a dense fermion medium

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We study the one-loop expansion of the effective action for an electromagnetic field in QED_{2+1} in a dense fermion medium. The properties of multiphoton vertices are used to build an expression for the effective Lagrangian of fields that vary arbitrarily in space. As an application we allow for the effect of nonlinear effective interaction on the field of a charge at rest.

1. INTRODUCTION

The method of effective Lagrangians is widely used in studying various problems of quantum field theory that involve external fields. The method has been thoroughly developed for the cases of uniform fields[1] and fields that vary smoothly in space.[2] However, in some problems, say, of photons splitting in an electron–positron plasma[3,4] the need arises to study strongly nonuniform or rapidly varying fields. In this case constructing effective Lagrangians is a technically nontrivial problem. Generally the effective Lagrangian of a gauge field can be calculated from multiphoton vertex functions in the form of an infinite series.[5] In Ref. 6, when studying two-dimensional quantum electrodynamics, QED_{2+1}, in a dense medium, we discovered an important property of multiphoton vertex functions that makes it possible to build the effective action of an electromagnetic field in a dense medium in the one-loop approximation for arbitrary configurations of the static fields by using only a finite number of expansion terms. It was found that in the one-loop approximation the vertex functions are proportional to a certain power of the chemical potential μ and the exponent decreases as the number of external photon lines grows, and even the three-photon vertex function depends on the zeroth power of μ. This enables one to construct the effective action for an arbitrary static field in a dense medium by taking into account only the first three terms of the expansion. This approximation correctly represents the properties of an electromagnetic field in a medium with polarization taken into account.

The aim of the present work is to construct the effective action of an electromagnetic field in the one-loop approximation in a dense medium in QED_{2+1} using the multiphoton vertices calculated in Ref. 6. For an example of the application of the resulting effective Lagrangian we study the interaction of charged fermions and derive an expression that allows for the nonlinear terms for the field generated by a charge at rest in the dense medium.

2. THE EFFECTIVE LAGRANGIAN OF AN ARBITRARY STATIC ELECTROMAGNETIC FIELD

To derive the effective action of an electromagnetic field we write the nth term of its one-loop expansion[5] and perform the necessary transformations:

$$\begin{aligned}\mathscr{L}^{(n)}(A)&=\int \hat{A}(x_1)G(x_2-x_1)\hat{A}(x_2)\cdots\hat{A}(x_n)\\&\quad\times G(x_1-x_n)\,d^3x_1\,d^3x_2\cdots d^3x_n\\&=\int \hat{A}(k_1)G(p_1)\cdots\hat{A}(k_n)G(p_n)e^{-ik_1x_1}\\&\quad\times e^{-ip_1(x_2-x_1)}\cdots e^{-ik_nx_n}e^{-ip_n(x_1-x_n)}\\&\quad\times d^3x_1\cdots d^3x_n\,d^3k_1\cdots d^3k_n\,d^3p_1\cdots d^3p_n\\&=\int A_{\mu_1}(k_1)\cdots A_{\mu_n}(k_n)\Pi^{\mu_1\cdots\mu_n}(k_1\cdots k_n)\delta\left(\sum_{i=1}^{n}k_i\right)\\&\quad\times d^3k_1\cdots d^3k_n\\&=\int A_{\mu_1}(k_1)\cdots A_{\mu_n}(k_n)\Pi^{\mu_1\cdots\mu_n}(k_1\cdots k_n)\\&\quad\times\exp\left\{ix\sum_{i=1}^{n}k_i\right\}d^3k_1\cdots d^3k_n\,d^3x,\end{aligned}\tag{1}$$

where A_μ are the potentials, G are the Green's functions of the electromagnetic field, and $\Pi^{\mu_1\cdots\mu_n}(k_1\cdots k_n)$ are the vertex functions that are defined in the momentum space and correspond to diagrams with n external photon lines.

For arbitrary $\Pi^{\mu_1\cdots\mu_n}(k_1...k_n)$ transition to coordinate space in constructing the effective Lagrangian is impossible. If, however, the components of the polarization tensors prove to be constant in a fairly broad momentum interval, Eq. (1) yields the following effective Lagrangian in the coordinate representation:

$$\begin{aligned}\mathscr{L}(x)&=\mathscr{L}_0(x)+\mathscr{L}'(x)\\&=\mathscr{L}_0(x)+\sum_{i=1}^{\infty}\Pi^{\mu_1\cdots\mu_i}A_{\mu_1}(x)\cdots A_{\mu_i}(x),\end{aligned}\tag{2}$$

where $\mathscr{L}_0(x)=-F^{\mu\nu}F_{\mu\nu}/4$.

In Ref. 6 we calculated the polarization tensors for the first terms in the expansion (2). In Minkowski space the results for the leading terms in the asymptotic behavior of the tensors corresponding to the momentum interval

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$|\mathbf{k}|\subset[0;2\sqrt{\mu^2-m^2}\,]$ (m is the fermion mass) and a zero sum of the external momenta at the three-photon vertex have the following form:[1)]

$$\Pi_i=0,\quad \Pi_0=\frac{e}{2\pi}(\mu^2-m^2)\theta(\mu^2-m^2), \tag{3}$$

$$\Delta\Pi_{00}=-\frac{e^2}{2\pi}\theta(\mu^2-m^2)(\mu-m), \tag{4}$$

$$\Delta\Pi_{ij}=\frac{e^2}{2\pi}\theta(\mu^2-m^2)(\mu-m)\left(\frac{k_ik_j}{\mathbf{k}^2}-\delta_{ij}\right), \tag{5}$$

$$\Delta\Pi_{i0}=0, \tag{6}$$

$$\Pi_{000}=\frac{e^3}{\pi}\theta(\mu^2-m^2), \tag{7}$$

$$\Pi_{i0j}=-\frac{e^3}{4\pi}\theta(\mu^2-m^2)\times\left(\frac{k_ik_j\mathbf{k}'^2+k_i'k_j'\mathbf{k}^2-k_ik_j'(\mathbf{k}\mathbf{k}')}{\mathbf{k}^2\mathbf{k}'^2}-\delta_{ij}\right), \tag{8}$$

$$\Pi_{i00}=0, \tag{9}$$

where $\Delta\Pi_{ij}$ is the statistical part of the polarization operator, which in a dense medium determines the operator completely, and $\theta(\mu^2-m^2)$ is the Heaviside step function.

We note that the above tensors are transverse, which guarantees the gauge invariance of the effective Lagrangian $\mathscr{L}'$.

Clearly, the above asymptotic expressions are constants, and for large chemical potentials the momentum interval may be large enough to ensure the applicability of the expansion (2). At the same time there is a certain pattern in the dependence of the polarization tensors on the chemical potential: namely, μ decreases as the number of external photon lines grows. This allows us to take into account only the first three terms in (2) when μ is large.

Substituting into (1) the asymptotic expressions for the polarization tensors (3)–(9), we find that in the current approximation the correction $\mathscr{L}'(x)$ to the Lagrangian $\mathscr{L}_0(x)$ in (2) has the form

$$\mathscr{L}'(x)=B_1A_0(x)+C_1A_0^2(x)+C_2A_i(x)(\delta^{ij}\Delta-\partial^i\partial^j)\tilde{A}_j(x)+D_1A_0^3(x)+D_2A_0(x)[(\delta^{ij}\Delta-\partial^i\partial^j)\tilde{A}_j(x)]^2, \tag{10}$$

where we have introduced the following notation:

$$B_1=\Pi_0,\qquad C_1=-\frac{e^2}{2\pi}\theta(\mu^2-m^2)(\mu-m),$$

$$C_2=\frac{e^2}{2\pi}\theta(\mu^2-m^2)(\mu-m),\qquad D_1=\frac{e^3}{\pi}\theta(\mu^2-m^2),$$

$$D_2=-\frac{e^3}{4\pi}\theta(\mu^2-m^2),\quad \tilde{A}_j(x)=\int\frac{A_j(\mathbf{k})}{\mathbf{k}^2}e^{-i\mathbf{k}\mathbf{x}}\,d^3\mathbf{k}.$$

This Lagrangian leads to nonlinear equations of motion and can be employed for an arbitrary dependence of the potentials $A_\mu(x)$ on position.

3. NONLINEAR INTERACTION OF STATIC CHARGES IN A DENSE MEDIUM

As an example of how Eq. (10) can be applied we examine the simplest case where both A_1 and A_2 vanish. From the total Lagrangian $\mathscr{L}_0+\mathscr{L}'$ we obtain the equation for the electric potential generated in the medium by an electric charge at rest, $A_0\equiv\phi$:

$$\phi''+\frac{1}{\rho}\phi'+\frac{e^2}{\pi}[3e\phi^2-(\mu-m)\phi]=0. \tag{11}$$

The equation gives a correct description of the modification of Coulomb law introduced by the effective multiphoton interaction in the centrally symmetric case in the range of distances $\rho\subset[(2\sqrt{\mu^2-m^2})^{-1};\infty]$, which increases with μ. Nondimensionalizing, $\phi\to\phi/e$, and introducing the parameter $\lambda=3e^2/(\mu-m)$, we can write Eq. (11) as

$$\phi''+\frac{1}{\rho}\phi'+\frac{e^2(\mu-m)}{\pi}(\lambda\phi^2-\phi)=0. \tag{12}$$

Regarding λ as a small parameter, we arrive at the perturbation-theoretic solution of (12):

$$\phi\simeq K_0(\alpha\rho)(1-\lambda\pi), \tag{13}$$

where K_0 is a modified Bessel function, and $\alpha=e^2(\mu-m)\pi$.

Clearly, allowing for the three-photon matrix has somewhat weakened the screening of the electrostatic interaction in the medium. This weakening, however, was obtained in an approximation in which λ is assumed small. In two-dimensional models of quantum field theory in which the coupling constant is not small, the three-photon interaction obviously plays an important role. In this connection it would be interesting to account more rigorously for the effect of the nonlinear term introduced by the three-photon vertex.

4. CONCLUSION

The main result of this paper is the derivation of a closed expression for the effective Lagrangian of an electromagnetic field in a dense fermion medium, Eq. (10). This became possible because of the dependence of multiphoton vertex functions on the chemical potential that we find: the decrease in the power of μ as the number of external photon lines grows. This ensures the rapid convergence of the one-loop expansion owing to the smallness of the parameter $1/\mu$. Such a dependence is, obviously, of a general nature and in no way is related to the number of dimensions of space; it is caused by the μ-dependence of the fermion propagator. A similar procedure for building the effective Lagrangian can be carried out in other gauge theories: QED_{3+1} and QCD. The only requirement is the presence of a dense fermion medium, which ensures the emergence of a new parameter of the expansion of the effective action of the gauge field.

The main expression for $\mathscr{L}$ obtained here can be used to solve various problems involving a dense medium. With a high accuracy, up to $1/\mu$ at high densities, it yields the effective Lagrangian on an electromagnetic field arbitrarily varying in space.

Also, since Eq. (10) contains terms that mix the zeroth and spatial components of A_μ, the effective Lagrangian constructed can be used for a more detailed study of the spontaneous generation of a magnetic field, symmetry breaking,[7] and Lorentz invariance[8] in two-dimensional systems. These problems will be studied separately.

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[1]In Ref. 6 we give asymptotic expressions for the case of an arbitrary sum of external momenta at the three-photon vertex.

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