Giant quantum oscillations of the cyclotron damping in cadmium

V. G. Skobov and A. S. Chernov

International Institute of Physics, 125040 Moscow, Russia; A. F. Ioffe Physicotechnical Institute, Russian Academy of Sciences, 194021 St. Petersburg, Russia

(Submitted 20 March 1995)
Zh. Éksp. Teor. Fiz. **108**, 732–740 (August 1995)

The influence of the quantum gyration of electrons in a magnetic field on the propagation of radio-frequency waves in cadmium is studied theoretically. It is shown that in a quantizing magnetic field the damping of the hole doppleron caused by the cyclotron absorption of waves by electrons exhibits a set of high sharp quantum peaks separated by deep minima. Such a quantum structure of the cyclotron damping results in giant oscillations of the surface impedance of cadmium as a function of the magnetic field. This effect should be observed in fields of the order of 100 kOe at a temperature of 1 K and a frequency of the exciting field of the order of 10 GHz.

1. INTRODUCTION

It was shown in Ref. 1 that quantization of electronic energy levels in a magnetic field can result in giant quantum oscillations of the magnetic Landau damping[2] and in corresponding oscillations of the surface impedance of a compensated metal when the magnetic field is at an angle to the symmetry axis of the Fermi surface. Besides magnetic Landau damping, another type of collisionless absorption of rf and microwave waves, viz., cyclotron absorption by electrons whose displacement during a cyclotron period is equal to the wavelength of the electromagnetic wave, can occur in metals. Such a situation is observed, for example, in cadmium. Besides the electron doppleron caused by the Doppler-shifted cyclotron resonance of the lens reference-point electrons, in cadmium there is a hole doppleron associated with the Doppler-shifted cyclotron resonance of the "monster" holes which undergo maximum displacement during a cyclotron period.[3] Since this displacement of the holes is approximately four times smaller than the displacement of the lens reference-point electrons, the hole doppleron propagates under conditions under which there is cyclotron absorption by electrons whose displacement is equal to the wavelength of the hole doppleron, i.e., is four times smaller than the displacement of the reference-point electrons. Thus, the damping of the hole doppleron is caused by collisional cyclotron absorption by electrons in a section of the lens located considerably closer to its equator than to the reference point. This raises the question of whether quantization of the gyration of the electrons in a magnetic field can cause a significant change in the character of cyclotron absorption. This question is treated from a theoretical standpoint in the present paper.

2. HOLE DOPPLERON IN THE CLASSICAL LIMIT

The electron Fermi surface of cadmium has the form of a lens, whose shape is described well by the equation[3]

$$\varepsilon(p)\equiv\frac{p_x^2+p_y^2}{2m}+\frac{p_0}{p_1}\left(1-\frac{|p_z|}{3p_1}\right)\frac{p_z^2}{m}=\frac{2p_0p_1}{3m}\equiv\varepsilon_F, \quad (1)$$

where

$$p_0=1.5\hbar\ \text{Å}^{-1},\quad p_1=0.28\hbar\ \text{Å}^{-1},\quad m=1.25m_0, \quad (2)$$

m_0 is the mass of the free electron, and the z axis is parallel to the hexagonal axis of the crystal.

If the constant magnetic field **H** and the propagation vector of the electromagnetic wave **k** are parallel to the z axis, the Fourier transform of the electronic part of the nonlocal conductivity in the absence of quantum effects is given by the expression

$$\sigma_\pm^{(e)}(k,\omega,H)=-\frac{2e^2}{(2\pi\hbar)^3}\int_0^\infty d\varepsilon\,\frac{df(\varepsilon)}{d\varepsilon}\times\int dp_z\frac{S(\varepsilon,p_z)/m}{\nu-i[\omega\pm\omega_c-kv_z(p_z)]}, \quad (3)$$

where

$$S(\varepsilon,p_z)=\pi\left[2m\varepsilon-\frac{2p_0}{p_1}\left(1-\frac{|p_z|}{3p_1}\right)p_z^2\right], \quad (4)$$

$$v_z(p_z)\equiv\frac{\partial\varepsilon(\mathbf{p})}{\partial p_z}=\frac{2p_0}{p_1}\left(1-\frac{|p_z|}{2p_1}\right)\frac{p_z}{m},\quad \omega_c=\frac{eH}{mc}, \quad (5)$$

ω is the angular frequency of the wave, e is the charge of the electron, c is the velocity of light, ω_c is the cyclotron frequency of the electrons, ν is the frequency of their collisions with impurity atoms, S is the area of the section of the isoenergetic surface $\varepsilon=$const formed by the $p_z=$const plane, and $f(\varepsilon)$ is the Fermi function.

In the absence of quantum effects, the derivative $df(\varepsilon)/d\varepsilon$ can be assumed to be equal to $-\delta(\varepsilon_F-\varepsilon)$, and Eq. (3) takes the form

$$\sigma_\pm^{(e)}=\frac{2e^2}{(2\pi\hbar)^3}\int_{-p_1}^{p_1}\frac{S(\varepsilon_F,p_z)dp_z}{\nu-i[\omega\pm\omega_c-kv_z(p_z)]}. \quad (6)$$

Substitution of the expressions in (4) and (5) into (6) and integration with respect to p_z give

$$\sigma_\pm^{(e)}=\pm\frac{2i}{5q_e}\left[\ln\frac{1+q_e}{1-q_e}-\frac{1}{2q_e}\ln(1-q_e^2)\right]\frac{n_eec}{H}, \quad (7)$$

where

$$q_e = \frac{kp_0}{m(\pm\omega_c + \omega + i\nu)}, \quad n_e = \frac{2\pi p_0 p_1^2}{(2\pi\hbar)^3}, \tag{8}$$

n_e is the electron density. The logarithmic singularity of $\sigma_{\pm}^{(e)}$ at $q_e^2 = 1$ represents the Doppler-shifted cyclotron resonance of the lens reference-point electrons.

In this paper we are interested in the case of a strong magnetic field, in which the cyclotron frequency exceeds ω and ν:

$$\omega \ll \omega_c, \quad \nu \ll \omega_c. \tag{9}$$

In this case

$$q_e \simeq \pm \frac{ku_e}{2\pi}, \quad u_e = 2\pi \frac{p_0 c}{eH}, \tag{10}$$

where u_e is the displacement of the lens reference-point electrons during a cyclotron period.

In the limit $q_e^2 \gg 1$ the expression for $\sigma^{(e)}$ is simplified:

$$\sigma^{(e)} \simeq \frac{2\pi n_e e^2}{5p_0|k|}, \tag{11}$$

i.e., the electron conductivity associated with cyclotron absorption in this case has the same form as in the case of an anomalous skin effect in the absence of a magnetic field.

The Fermi surface of the holes in cadmium (a so-called "monster") has a third-order symmetry axis parallel to the hexagonal axis (the z axis) and has a very complicated form, which precludes calculating the hole contribution to the nonlocal conductivity in an analytical form. The displacement u of the holes during a cyclotron period reaches its maximum value $u_1 = u_e/4$ in a section of the monster close to the central section. Then the Doppler-shifted cyclotron resonance of the holes is expressed by the fact that the hole part of the nonlocal conductivity has a root-type singularity. The dispersion of the dielectric function in the vicinity of this resonance causes the existence of a hole doppleron, whose field rotates in the same direction as the holes ("plus" polarization). A calculation of the surface impedance of the metal in the case in which the nonlocal conductivity has two different branching points corresponding to the Doppler-shifted cyclotron resonance of the electrons and holes is a complicated mathematical problem. To simplify it we consider a model in which the hole Fermi surface has the form of a parabolic lens[4] with an axis parallel to the C_6 axis. In this case the displacements of all the holes during a cyclotron period are identical in magnitude and the hole conductivity has a pole-type, rather than root-type, singularity:

$$\sigma_{\pm} = \frac{n_e e^2}{2m_1}\left[\frac{1}{\nu_1 - i(\omega \mp \omega_{c1} - kv_1)} + \frac{1}{\nu_1 - i(\omega \mp \omega_{c1} + kv_1)}\right], \tag{12}$$

where $\omega_{c1} = eH/m_1 c$. Here m_1 is the hole cyclotron mass, v_1 is the velocity of the holes parallel to the z axis, and ν_1 is the frequency of collisions with impurities. The vanishing of the denominator in (12) corresponds to the Doppler-shifted cyclotron resonance of the holes. Neglecting the small quantities ω and ν_1 in (12), we can write

$$\sigma_{\pm}(q) = -i\frac{n_e ec}{H}\frac{1}{1-q^2}, \tag{13}$$

where

$$q = \frac{kv_1}{\omega_{c1}} \equiv \frac{ku_1}{2\pi} \simeq \frac{ku_e}{8\pi}. \tag{14}$$

Replacement of the root-type singularity $\sigma_{\pm}$ by a pole-type singularity causes the behavior of the wave field in the model under consideration to differ from that observed in cadmium. One of the differences is that along with the hole doppleron with plus polarization there is another hole doppleron with minimum polarization [in our model $\sigma_{\pm}$ is imaginary not only for $q^2 < 1$, but also for $q^2 > 1$, so that the dielectric function $4\pi\sigma_{\pm}/(-i\omega)$ can be real and positive for both circular polarizations]. To avoid the problem associated with the appearance of a "false" hole doppleron with minus polarization, we shall consider only plus polarization below.

A second difference is that the amplitude of the hole doppleron drops more slowly with increasing H than in the case of a root-type singularity. However, if a comparatively small range of values of H near the doppleron threshold (where its amplitude is largest) is considered, this difference becomes insignificant. On the other hand, our model has an important advantage: the absence of a branch point in (13) makes it possible to calculate the surface impedance of a metal in an analytical form and to analyze the role of quantization of the electronic energy levels.

The dispersion equation for a wave with plus polarization has the form

$$k^2c^2 = 4\pi i\omega[\sigma_+(q) + \sigma_+^{(e)}]. \tag{15}$$

Substituting the expressions for σ_+ and $\sigma_+^{(e)}$ into Eq. (15), we can write it in the form

$$q^2 = \xi\left(\frac{1}{1-q^2} + \frac{i\pi}{10|q|}\right), \quad \xi = \frac{\pi\omega n_e p_0^2 c}{eH^3}. \tag{16}$$

In (16) we used the asymptotic expression (11) for $\sigma_+^{(e)}$ at large q_e and took into account the relation $q_e = 4q$.

For small values of ξ, i.e., for strong magnetic fields ($\xi \sim H^{-3}$), Eq. (16) has roots whose imaginary parts are small compared with their real parts. These roots correspond to propagating modes, their spectrum being determined by the holes and their damping being determined by the electrons. Approximate expressions for the roots can be obtained, if the equation is first solved omitting the small imaginary term on the right-hand side of (16) and then, after substitution of the values of q obtained into the imaginary term, the roots are found with cyclotron damping already included. This gives

$$q_{1,2} = q'_{1,2} + iq''_{1,2}, \quad q'_{1,2} = \pm\left(\tfrac{1}{2} \mp \sqrt{\tfrac{1}{4} - \xi}\right), \tag{17}$$

$$q''_1 = \frac{\pi\xi}{20}\frac{|q'_2|}{\sqrt{q'_1}}\left(\tfrac{1}{4} - \xi\right)^{-1/2},$$

$$q''_2 = \frac{\pi\xi}{20}\frac{q'_1}{\sqrt{a'_2}}\left(\tfrac{1}{4} - \xi\right)^{-1/2}. \tag{18}$$

These roots specify propagating modes in the range of magnetic fields $H>H_L$, where H_L is the threshold value of the field corresponding to the condition $\xi=\xi_L\equiv 1/4$. The root q_1 specifies the wave vector of the hole helicon, and q_2 specifies the hole doppleron. This helicon has considerable damping and exists in the narrow range of values $H>H_L$, since q_1' decreases with increasing H, while q_1'' increases. This corresponds to the real situation in cadmium, in which a helicon is not observed. Nevertheless, it must be taken into account, since its contribution to the smooth part of the impedance exceeds the contribution of the doppleron. As will be seen from the content of the next section, the quantum effects are associated mainly with the doppleron, rather than the helicon. Therefore, even though (17)–(18) give a very rough approximation for q_1, we use it to evaluate the classical (nonoscillating) part of the surface impedance.

The impedance of a semi-infinite metal with diffuse reflection of the carriers from the surface is given by the well known formula

$$Z_+=\frac{4\pi\omega}{c^2}\times\left[-\frac{1}{2\pi i}\int_{-\infty}^{\infty}dk\,\ln\frac{k^2-4\pi i\omega c^{-2}(\sigma_++\sigma_+^{(e)})}{k^2}\right]^{-1}. \tag{19}$$

Substituting expressions (13) and (11) into (19) and making the transition to integration with respect to the dimensionless variable $q=ku_e/8\pi$, we obtain

$$Z_+^{-1}=\frac{ic^2}{\pi\omega u_e}\int_{-\infty}^{\infty}dq\ln\left[1-\frac{\xi}{q^2}\left(\frac{1}{1-q^2}+\frac{i\pi}{10\sqrt{q^2}}\right)\right]. \tag{20}$$

In the approximation in which Eq. (16) was solved, the integral in (20) equals $-2\pi i(q_1+q_2+1)$ and, therefore,

$$Z_+=\frac{\omega u_e}{2c^2}(q_1+q_2+1)^{-1}, \tag{21}$$

where q_1 and q_2 are assigned by Eqs. (17)–(18).

For $H=H_L$ ($\xi=1/4$), the real part of the second root satisfies $q_2'=1/\sqrt{2}$ and asymptotically tends to -1 as H increases, so that the sum $1+q_2$ decreases with increasing field strength proportionally to ξ. In strong fields, however, q_1 is proportional to $\sqrt{\xi}$, i.e., q_1 is the main term.

3. QUANTUM CASE

Let us now proceed to a study of the influence of quantization of the electronic energy levels on the properties of the propagating modes. We first note that q_1 and q_2 are always less than unity, i.e., the wave vectors of the hole helicon and doppleron are found in a range of values where there is no cyclotron absorption by holes. Therefore, quantization of the hole energy levels cannot have any influence on the properties of the helicon and doppleron, and we assume that the motion of the holes is described, as before, by the classical expressions.

In the case of strong fields and low temperatures, in which $\hbar\omega_c\gg k_BT$ holds (T is the temperature, and k_B is the Boltzmann constant), we must take into account the quantization of the electron gyration around the magnetic field lines, as a result of which the energy of the transverse motion assigned by the first term in (1) takes only the discrete values $\hbar\omega_c(n+1/2)$, where $n=0,1,2,\ldots$. In other words, the energy levels have the form

$$\varepsilon_{np_z}=\hbar\omega_c(n+\tfrac{1}{2})+\varepsilon_\parallel(p_z),$$

$$\varepsilon_\parallel(p_z)=\frac{p_0}{p_1}\left(1-\frac{|p_z|}{3p_1}\right)\frac{p_z^2}{m}, \tag{22}$$

where, for simplicity, we disregard the spin splitting of the levels.

The area of the electron orbit S is quantized along with the transverse energy. Therefore, to obtain the conductivity $\sigma_+^{(e)}$ in the quantum case, $S(\varepsilon,p_z)$ must be replaced by $S_n=2\pi m\hbar\omega_c(n+1/2)$ in the classical expression (3), and ε must be replaced by ε_{np_z}. In addition, the integration with respect to ε must be replaced by summation over $\hbar\omega_c n$, i.e.,

$$\sigma_+^{(e)}=\frac{2e^2}{(2\pi\hbar)^3}\hbar\omega_c\sum_{n=0}^{\infty}\int_{-\infty}^{\infty}dp_z\times\frac{df(\varepsilon_{np_z})}{d\varepsilon_f}\frac{2\pi\hbar\omega_c(n+1/2)}{\nu-i[\omega+\omega_c-kv_z(p_z)]}. \tag{23}$$

It is assumed in (23) that $\hbar\omega\ll k_BT$. Otherwise, the following additional replacement must be made:

$$\frac{df(\varepsilon_{np_z})}{d\varepsilon_f}\rightarrow\frac{1}{\hbar\omega}[f(\varepsilon_{np_z})-f(\varepsilon_{np_z}+\hbar\omega)]. \tag{24}$$

This formula can be rigorously derived by a method similar to the method developed in Refs. 5 and 6 to derive the quantum theory of the absorption of ultrasound and magnetic Landau damping.

Expressing the derivative of the Fermi function in (23) in terms of the hyperbolic cosine and neglecting the frequency ω in comparison with ω_c, we write the expression for $\sigma_+^{(e)}$ in the form

$$\sigma_+^{(e)}=\frac{e^2\omega_c^2}{4\pi^2\hbar k_BT}\int_{-\infty}^{\infty}\sum_{n=0}^{\infty}\left(n+\frac{1}{2}\right)\cosh^{-2}\left(\frac{\varepsilon_{np_z}-\varepsilon_F}{2k_BT}\right)\times\frac{dp_z}{\nu-i[\omega_c-kv_z(p_z)]}. \tag{25}$$

The integrand (25) is a product of two rapidly varying factors. The first represents a set of narrow high maxima separated by deep minima. The maxima are located at values of $p_z=p_n$ which correspond to the conditions

$$\varepsilon_\parallel(p_n)=\varepsilon_F-\hbar\omega_c(n+\tfrac{1}{2}),\quad n=0,1,2,\ldots,N_F,$$

$$N_F=\left[\frac{\varepsilon_F}{\hbar\omega_c}\right]. \tag{26}$$

It follows from (26) and the second equation in (22) that for $n\sim N_F$,

$$p_n^2=2m_\parallel[\varepsilon_F-\hbar\omega_c(n+\tfrac{1}{2})], \tag{27}$$

$$m_{\|}=\frac{mp_1}{2p_0}\simeq 0.1m, \tag{28}$$

and the distances between successive maxima are

$$\Delta p_n \equiv p_n - p_{n+1} \simeq m_{\|}\hbar\omega_c/p_n. \tag{29}$$

The second factor, i.e., the fraction with a frequency-dependent denominator, has a maximum at $p_z=P$, which corresponds to the condition

$$v_z(P)\equiv \frac{2P_0}{m}\left(1-\frac{|P|}{2p_1}\right)\frac{P}{p_1}=\frac{\omega_c}{k}\equiv\frac{p_0}{mq_e}, \tag{30}$$

whence

$$|P|=p_1(1-\sqrt{1-|q_e|^{-1}}). \tag{31}$$

Since the value of $|q_e|$ for the hole doppleron is close to 4, P is close to $-p_1/8$ and depends weakly on H. The value of P for the helicon is considerably greater, however, i.e., the cyclotron damping of the helicon is caused by electrons which are located closer to the reference point of the lens.

The width of the maximum of the second factor is

$$\delta p_z \simeq m_{\|}\nu/k. \tag{32}$$

We are interested in the case in which δp_z is small compared with the distance between neighboring maxima of the first factor: $\delta p_z \ll \Delta p_n$. Taking into account (29) and the fact that $\delta p_z \simeq \nu p_1/2\omega_c q_e$, this inequality can be represented in the form

$$\eta q_e^{-2}\ll 1, \quad \eta\equiv\frac{\varepsilon_F\nu}{\hbar\omega_c^2}. \tag{33}$$

For a collision frequency $\nu=10^9\ s^{-1}$ and a field $H=10^5$ Oe, we have $\varepsilon_F/\hbar\omega_c\simeq 2\times 10^3$ and $\nu/\omega_c\simeq 10^{-3}$, so that the quantity on the left-hand side of (33) amounts to approximately 0.1.

When the inequality in (33) holds, $\sigma_+^{(e)}$ depends on whether the maximum of the second factor falls on one of the maxima of the first factor. If the maxima coincide, the value of the integral is maximal. If the maximum of the second function falls in the middle of the interval between the maxima of the first function, the integral is minimal. As the field H varies, the values of p_n vary, and the maxima of the first function alternately pass through the maximum of the second function. As a result, the cyclotron absorption specified by $\mathrm{Re}(\sigma_+^{(e)})$ will have the form of a set of sharp peaks which are periodic in $1/H$ and are separated by gently sloping and deep minima.

Let us estimate the maximum value of $\sigma'=\mathrm{Re}(\sigma_+^{(e)})$. Let the magnetic field H be such that for $n=N$, the maxima of the two functions coincide: $p_N=P$. In this case it is sufficient to retain only the term with $n=N$ in the sum (25), the main contribution to the integral with respect to p_z being made by a small neighborhood of the point $p_z=P$. Therefore, the difference $\varepsilon_{Np_z}-\varepsilon_F$ in the argument of the hyperbolic cosine and the difference ω_c-kv_z in the denominator of the integrand in (25) can be expanded in powers of p_z-P, and we can restrict ourselves to the linear terms of the expansion

$$\varepsilon_{Np_z}-\varepsilon_F\simeq\frac{d\varepsilon_{\|}(P)}{dP}(p_z-P)=v_z(P)(p_z-P)=\frac{\omega_c}{k}(p_z-P),$$

$$\omega_c-kv_z\simeq -k\frac{dv_z(P)}{dP}(p_z-P)=-\frac{k}{m_{\|}}\sqrt{1-|q_e|^{-1}}(p_z-P). \tag{34}$$

As a result, the expression for σ' conductivity, which describes the cyclotron absorption of the wave by electrons, can be brought into the form

$$\sigma'_M=\frac{e^2\omega_c N|k|}{4\pi^2\hbar\nu}\int_{-\infty}^{\infty}\frac{dx}{1+\alpha^2x^2}\cosh^{-2}x, \tag{35}$$

where

$$\alpha\simeq\frac{2k_BTk^2}{m_{\|}\nu\omega_c}=\frac{8}{3}\frac{k_BT\omega_c}{\varepsilon_F\nu}q_e^2, \tag{36}$$

and the subscript M refers to the fact that the value of σ' at the maximum is being calculated.

It is convenient to characterize the quantum oscillations of the cyclotron absorption by the ratio of the conductivity σ' to its classical limiting value $\sigma^{(e)}$ given by Eq. (11), rather than by σ' itself. Neglecting the difference between N and $N_F=[\varepsilon_F/\hbar\omega_c]$ in (35) and taking into account that the electron density equals $n=5p_0p_1^2/12\pi^2\hbar^3$ in the model under consideration, we can represent $Q_M=\sigma'_M/\sigma_{\mathrm{cl}}^{(e)}$ in the form

$$Q_M=\frac{2q^{e2}}{3\pi\eta}\int_{-\infty}^{\infty}\frac{dx}{1+\alpha^2x^2}\cosh^{-2}x. \tag{37}$$

In the case of a strong field and a long electron mean free path, when the inequality (33) holds, the multiplier in front of the integral in (37) is large. The value of the integral depends on α. When α is large, i.e., when the inequalities

$$\hbar\omega_c\eta q_e^{-2}\ll k_BT\ll\hbar\omega_c \tag{38}$$

hold, the integral equals π/α, and, therefore,

$$Q_M\simeq\hbar\omega_c/4k_BT. \tag{39}$$

This is the case in which the height of the quantum cyclotron absorption peaks is determined by the temperature and does not depend on the electron mean free path.

When α is small, i.e., when the inequalities

$$k_BT\ll\hbar\omega_c\eta q_e^{-2}\ll\hbar\omega_c \tag{40}$$

hold, the integral in (37) equals 2, and

$$Q_M\simeq 64q^2/3\pi\eta. \tag{41}$$

In this case the height and shape of the quantum peaks are determined by electron scattering.

Let us evaluate Q at the absorption minima. Let the value of H be such that P decreases exactly to the midpoint between p_N and p_{N+1}:

$$P=(p_N+p_{N+1})/2.$$

In this case it is sufficient to retain only the terms with $n=N$ and $n=N+1$ in the sum in (25) and to set v_z in them equal to $v_z(p_N)$ and $v_z(p_{N+1})$, respectively. This gives

$$\sigma'_{\min}\simeq\frac{2\pi e^2}{(2\pi\hbar)^3}\frac{(\hbar\omega_c)^2N\nu}{k_BT(k\Delta p_N/2m_\parallel)^2}\int_0^\infty\cosh^{-2}\left(\frac{p_z^2-p_N^2}{4m_\parallel k_BT}\right)dp_z. \tag{42}$$

Taking into account the approximate relations $p_N\simeq P\simeq p_1/2q_e$, $\Delta p_N\simeq m_\parallel\hbar\omega_c/p_N$, $N\simeq\varepsilon_F/\hbar\omega_c$, and the fact that the integral with respect to p_z is approximately equal to $4m_\parallel k_BT/p_N$, we obtain the estimate

$$Q_{\min}\equiv\frac{\sigma'_{\min}}{\sigma_{\rm cl}^{(e)}}\simeq\frac{3\eta}{8\pi q^2}\ll 1. \tag{43}$$

Thus, the function $Q(H)$ actually describes a set of narrow high quantum peaks separated by deep minima. Here the value of $Q_{\min}$ is inversely proportional to the electron mean free path, $Q_{\min}\propto\nu$, while the maximum value of Q in the case of (40) is directly proportional to the mean free path, $Q_M\propto 1/\nu$.

It should be noted that relations (38)–(41) and (43) are strongly dependent on q. Since $|q|\simeq 1$ holds for the hole doppleron and its value is considerably smaller for the helicon, the quantum effects for the doppleron begin in weaker fields and are displayed more strongly than those for the helicon. The case in which the quantum oscillations of the helicon damping are insignificant is the simplest case. The quantum oscillations of the doppleron damping can be described by introducing the additional factor $Q(q_2')$ on the right-hand side of the second formula in (18):

$$q_2''=\frac{\pi\xi}{20}\left(\tfrac{1}{2}-\sqrt{\tfrac{1}{4}-\xi}\right)^{1/2}\left[\left(\tfrac{1}{4}-\xi\right)|q_2'|\right]^{-1/2}Q(q_2'). \tag{44}$$

This equation can fail only in the immediate vicinity of the summits of the quantum peaks, where q_2'', as given by (44), can be greater than or of order of $|q_2'|$. In the vicinity of the summits of the pronounced quantum absorption peaks, the calculation of the conductivity $\sigma^{(e)}$ requires considerably greater efforts and accuracy. Such a calculation and the corresponding analysis of the dispersion equation are beyond the scope of the present work.

In conclusion we discuss the conditions under which the observation of quantum oscillations of the cyclotron absorption in cadmium is possible. Since the transverse mass of the lens electrons is fairly large ($m=1.25m_0$), the inequality $\hbar\omega_c\gg k_BT$ is very hard to satisfy. For a field strength $H=100$ kOe and a temperature $T=1$ K, the ratio $\hbar\omega_c/k_BT$ equals 10. In this case condition (33) is satisfied with some margin. The frequency of the exciting field remains to be selected. It must be chosen so that the hole doppleron threshold would be in a 90–100 kOe field. This occurs at a frequency equal to 5–8 GHz.

This research was performed with partial financial support from the International Science Foundation (Grant No. N3F000) and the Fund of the Russian Government.

[1] S. N. Savel'eva, V. G. Skobov, and A. S. Chernov, Zh. Éksp. Teor. Fiz. **107**, 1731 (1995) [JETP **80**, 905 (1995)].
[2] E. A. Kaner and V. G. Skobov, Adv. Phys. **17**, 605 (1968).
[3] O. B. Konstantinov, V. G. Skobov, V. V. Lavrova *et al.*, Zh. Éksp. Teor. Fiz. **63**, 224 (1972) [Sov. Phys. JETP **36**, 118 (1973)].
[4] R. G. Chambers and V. G. Skobov, J. Phys. F **1**, 202 (1971).
[5] V. G. Skobov, Zh. Éksp. Teor. Fiz. **40**, 1446 (1961) [Sov. Phys. JETP **13**, 1014 (1961)].
[6] É. A. Kaner and V. G. Skobov, Zh. Éksp. Teor. Fiz. **53**, 375 (1967) [Sov. Phys. JETP **26**, 251 (1968)].

Translated by P. Shelnitz