Neutrino oscillations in the magnetic field of the sun, supernovae, and neutron stars

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We examine the feasibility of oscillations of Dirac and Majorana neutrinos in a strong magnetic field (assuming a nonvanishing neutrino magnetic moment). We determine the critical magnetic field $\tilde{B}_{cr}(\Delta m_\nu^2, \theta, n_{eff}, E_\nu, \dot{\phi}(t))$ as a function of the neutrino mass difference, the vacuum mixing angle, the effective mass density, the neutrino energy, and the angle specifying the variation of the magnetic field in the plane transverse to the neutrino's motion. The conditions under which magnetic field-induced neutrino oscillations are significant are discussed. We study the possibility that such oscillations come about in supernova explosions, neutron stars, the sun, and the interstellar medium. We analyze the possible conversion of half the active neutrinos in a beam into sterile neutrinos when the beam emerges from the surface of a neutron star (cross-boundary effect), as well as when it crosses the interface between internal layers of a neutron star.

1. INTRODUCTION

The experimental confirmation of neutrino oscillations would go far toward providing an explanation of a variety of astrophysical and cosmological phenomena. Studies in this field have been motivated, above all, by the desire for a solution to the solar neutrino problem that makes use of neutrino conversion in the presence of matter and a magnetic field. Another important reason for studying neutrino conversion and oscillations has been the suggestion that these processes may play a significant role in supernova explosions and neutron star cooling.

The possibility of interconversion and oscillations among the various types of (massive) neutrinos was broached in its most general form by Pontecorvo;[1] it was further developed and bolstered with actual calculations of neutrino beam evolution.[2,3] Wolfenstein[4] examined the effect on oscillations of neutrino interactions with the ambient medium, and Mikheev and Smirnov[5,6] predicted the existence of resonant amplification of oscillations when a beam propagates through a medium of varying density. A multitude of attempts to resolve the solar neutrino problem have since been based upon this effect, which has come to be known as the Mikheev–Smirnov–Wolfenstein effect (see, for example, the surveys in Refs. 7–11, which comprise a reasonably complete account of the present status of the problem).

The next significant step in the study of neutrino oscillations was taken by Voloshin and coworkers[12–15] (see also Ref. 16), who carefully discussed neutrino spin precession in a magnetic field, and used the results to analyze the solar neutrino problem. Akhmedov[17] and Lim *et al.*[18] examined the resonant amplification of neutrino spin–flavor conversion; in the latter paper, in addition to neutrino oscillations in the convective zone of the sun, Lim *et al.* studied the consequences of neutrino spin–flavor conversion in supernova explosions. We emphasize once again that the description of the neutrino flux from a supernova explosion or neutron star[19–24] is (along with the solar neutrino problem) an important application of neutrino oscillation studies.

Note that the magnetic field strengths considered in most of the papers cited above, which were concerned with neutrino oscillations in a magnetized medium, were not very high—they were typically held at the solar level ($B \leqslant 10^5$ G). There have also been studies[18,19,25] of the resonant conversion of neutrinos in matter subject to strong magnetic fields in a supernova explosion. Nevertheless, a number of recent papers[20–23] devoted to neutrino conversion and oscillations in the vicinity of supernovae and neutron stars have totally disregarded the influence of a strong magnetic field on neutrino conversion.

Recent investigations[26–32] indicate that the magnetic fields in supernovae and neutron stars ($B \sim 10^{12}-10^{14}$ G) at various evolutionary stages) may not just give rise to new types of interacting particles that begin to play an important role under such extreme conditions (see also Ref. 33), but may also significantly affect neutrino oscillations.[28,29,32]

In our own work,[28,29] which addressed oscillations of Dirac and Majorana neutrinos in strong magnetic fields (such as those that can exist in neutron stars), we derived a value of the critical magnetic field $B_{cr}(\Delta m_\nu^2, \theta, n_{eff}, E_\nu)$ (where $\Delta m_\nu^2 = m_2^2 - m_1^2 > 0$, m_1 and m_2 are the masses of neutrino states ν_1 and ν_2, θ is the vacuum mixing angle, n_{eff} is the effective matter density, and E_ν is the neutrino energy), which determines the magnetic field values ($B \geqslant B_{cr}$) at which neutrino oscillations become significant. In those papers, however, we neglected field variations along the neutrino trajectory.

Continuing the same line of inquiry,[1)] in the present paper we study oscillations engendered in the presence of matter by a strong magnetic field among various types of Dirac and Majorana neutrinos (we naturally assume that the neutrinos have a nonvanishing magnetic moment), taking account of the vacuum mixing of neutrinos in various astrophysical objects (supernovae, neutron stars, the sun, the interstellar medium).

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In Sec. 2, we engage in a general discussion of the influence that (Dirac and Majorana) neutrino interactions with matter and with a variable magnetic field have on neutrino oscillations. We determine in Sec. 3 the critical field $\tilde{B}_{\rm cr}(\Delta m_\nu^2,\ \theta,\ n_{\rm eff},\ E_\nu,\ \dot{\phi}(t))$, where the phase $\phi(t)$ characterizes the varying direction of the field $\mathbf{B}(t)=\mathbf{B}e^{i\phi(t)}$ transverse to the neutrino motion. We show that when the field varies, the critical field $\tilde{B}_{\rm cr}$ can be weaker than the corresponding value $B_{\rm cr}$ obtained by neglecting field variations $(\dot{\phi}=0)$.[32] This means, in particular, that neutrino oscillations become appreciable in a variable magnetic field at lower field values and neutrino magnetic moments than in a constant field, as can be painstakingly demonstrated (Sec. 7) by looking at oscillations in the convective zone of the sun.

We have analyzed the effects of a strong magnetic field on neutrino oscillations in a neutron star (Sec. 4), and have studied the so-called cross-boundary effect,[28,29] which can lead to the "loss" of a significant number of neutrinos (conversion of up to half the total number of neutrinos from the active phase to the sterile phase) as they emerge from the surface of a neutron star or cross between two layers in the star's interior. This then involves us in a discussion of the validity of the adiabatic approximation. We have also investigated the possible role played by magnetic field-induced neutrino oscillations in supernova explosions (Sec. 6), and have set bounds on the neutrino's magnetic moment. In Sec. 8, we consider neutrino oscillations in the Galactic magnetic field, and we briefly summarize our principal results in Sec. 9.

Note that we have primarily concentrated on those situations in which multiple transformations from one neutrino state to another become possible in a strong magnetic field, and we do not require that the conditions for resonant amplification of these transformations be met continuously.

2. GENERAL ANALYSIS OF NEUTRINO OSCILLATIONS IN A MAGNETIC FIELD

We restrict our discussion of neutrino interactions to ν_e and ν_μ, each of which is a superposition of the two mass eigenstates ν_1 and ν_2:

$$\begin{aligned} \nu_e &= \nu_1 \cos\theta + \nu_2 \sin\theta, \\ \nu_\mu &= -\nu_1 \sin\theta + \nu_2 \cos\theta. \end{aligned} \tag{1}$$

Allowing for a variable magnetic field, the evolution of a neutrino beam passing through matter (consisting of electrons, protons, and neutrons) can be described by a Schrödinger-type differential equation:[9]

$$i\frac{d}{dt}\nu(t)=H\nu(t). \tag{2}$$

The Hamiltonian H consists of four terms,

$$H=H_V+H_{\rm int}+H_F+H_{\dot{\phi}}, \tag{3}$$

where H_V accounts for vacuum oscillations, $H_{\rm int}$ and H_F for neutrino interactions with matter and the magnetic field, and $H_{\dot{\phi}}$ for the rotation of the magnetic field $\mathbf{B}$ in a plane perpendicular to the direction of neutrino travel.

It we consider two Dirac neutrinos, we may use the basis $\nu^D=(\nu_{e_L},\nu_{\mu_L},\nu_{e_R},\nu_{\mu_R})$ to write

$$H^D=\begin{pmatrix} -\frac{\Delta m_\nu^2}{4E_\nu}c+V_{\nu_e}^0-\frac{\dot{\phi}}{2} & \frac{\Delta m_\nu^2}{4E_\nu}s & \mu_{ee}B & \mu_{e\mu}B \\ \frac{\Delta m_\nu^2}{4E_\nu}s & \frac{\Delta m_\nu^2}{4E_\nu}c+V_{\nu_\mu}^0-\frac{\dot{\phi}}{2} & \mu_{\mu e}B & \mu_{\mu\mu}B \\ \mu_{ee}B & \mu_{\mu e}B & -\frac{\Delta m_\nu^2}{4E_\nu}+\frac{\dot{\phi}}{2} & 0 \\ \mu_{e\mu}B & \mu_{\mu\mu}B & 0 & \frac{\Delta m_\nu^2}{4E_\nu}+\frac{\dot{\phi}}{2} \end{pmatrix}. \tag{4a}$$

We assume in Eq. (4a) that right-handed neutrinos (ν_{e_R} and ν_{μ_R}) are sterile, i.e., do not interact with matter, so they can carry off energy unimpeded.

For two Majorana particles in the basis $\nu^M=(\nu_e,\nu_\mu,\bar{\nu}_e,\bar{\nu}_\mu)$, the corresponding Hamiltonian is

$$H^M=\begin{pmatrix} -\frac{\Delta m_\nu^2}{4E_\nu}c+V_{\nu_e}^0-\frac{\dot{\phi}}{2} & \frac{\Delta m_\nu^2}{4E_\nu}s & 0 & \mu B \\ \frac{\Delta m_\nu^2}{4E_\nu}s & \frac{\Delta m_\nu^2}{4E_\nu}c+V_{\nu_\mu}^0-\frac{\dot{\phi}}{2} & -\mu B & 0 \\ 0 & \mu B & -\frac{\Delta m_\nu^2}{4E_\nu}c-V_{\nu_e}^0+\frac{\dot{\phi}}{2} & \frac{\Delta m_\nu^2}{4E_\nu}s \\ \mu B & 0 & \frac{\Delta m_\nu^2}{4E_\nu}s & \frac{\Delta m_\nu^2}{4E_\nu}c-V_{\nu_\mu}^0 \end{pmatrix}. \tag{4b}$$

In Eqs. (4a) and (4b),

$$s=\sin 2\theta, \quad c=\cos 2\theta.$$

The terms $V^0_{\nu_l}$ account for contributions to the effective neutrino energy due to interactions with matter[4,6,9]:

$$V^0_{\nu_e}=\sqrt{2}G_F\left(n_e-\frac{1}{2}n_n\right), \quad V^0_{\nu_\mu}=-\frac{1}{\sqrt{2}}G_F n_n. \tag{5}$$

Assuming that the original neutrino beam consists solely of left-handed neutrinos ν_{e_L}, and making use of Eq. (4), we may consider the following neutrino conversion processes $(\nu_i\to\nu_j)$ and their corresponding neutrino oscillations $(\nu_i\leftrightarrow\nu_j)$ engendered by a magnetic field:

$$\nu_{e_L}\to\nu_{e_R}, \quad \nu_{e_L}\to\nu_{\mu_R}, \quad \nu_{e_L}\to\bar{\nu}_\mu. \tag{6}$$

The probability of a change in neutrino type $\nu_i\to\nu_j$ (i.e., the probability of detecting a neutrino of type ν_j at a distance x from a source of ν_i neutrinos) is given (see, e.g., Ref. 9) by the general formula

$$P(\nu_i\to\nu_j)=\sin^2 2\theta_{\text{eff}}\sin^2\frac{\pi x}{L_{\text{eff}}}, \quad i\neq j, \tag{7}$$

and the probability that the neutrino type remains unchanged upon traversing a distance x is

$$P(\nu_i\to\nu_i)=1-P(\nu_i\to\nu_j). \tag{8}$$

If $\sin^2 2\theta_{\text{eff}}$ is nonnegligible, then under certain circumstances (which we carefully analyze below) the neutrino type can change repeatedly, i.e., neutrino oscillations can occur.

The effective mixing angle θ_{eff} and the effective oscillation length L_{eff} derive from the form of the Hamiltonian [see Eq. (4)]:

$$\tan 2_{\text{eff}}=\frac{2\tilde{\mu}B}{\frac{\Delta m^2_\nu}{2E_\nu}A-\sqrt{2}G_F n_{\text{eff}}+\dot{\phi}}, \tag{9}$$

$$L_{\text{eff}}=2\pi\left[\left(\frac{\Delta m^2_\nu}{2E_\nu}A-\sqrt{2}G_F n_{\text{eff}}+\dot{\phi}\right)^2+(2\tilde{\mu}B)^2\right]^{-1/2}, \tag{10}$$

where for the various neutrino conversion processes (6), the neutrino magnetic moment $\tilde{\mu}$ and the quantities A and n_{eff} take the form

$$\mu=\begin{cases}\mu_{ee} & \text{for } \nu_{e_L}\to\nu_{e_R}\\ \mu_{e\mu} & \text{for } \nu_{e_L}\to\nu_{\mu_R},\\ \mu & \text{for } \nu_{e_L}\to\bar{\nu}_\mu\end{cases} \tag{11}$$

$$A=\begin{cases}(\cos 2\theta-1)/2 & \text{for } \nu_{e_L}\to\nu_{e_R}\\ (\cos 2\nu+1)/2 & \text{for } \nu_{e_L}\to\nu_{\mu_R},\\ \cos 2\theta & \text{for } \nu_{e_L}\to\bar{\nu}_\mu\end{cases} \tag{12}$$

$$n_{\text{eff}}=\begin{cases}n_e-n_n & \text{for } \nu_{e_L}\to\nu_{\mu_R}\\ n_e-n_n/2 & \text{for } \nu_{e_L}\to\nu_{e_R},\bar{\nu}_\mu\end{cases}. \tag{13}$$

We can write Eqs. (9) and (10) for the effective mixing angles and oscillation lengths in terms of the characteristic lengths L_V, L_{int}, L_F, and $L_{\dot{\phi}}$:

$$\tan 2\theta_{\text{eff}}=L^{-1}\left(\frac{A}{L_V}-\frac{1}{L_{\text{int}}}+\frac{1}{L_{\dot{\phi}}}\right)^{-1}, \tag{14}$$

$$L_{\text{eff}}=\left[\left(\frac{A}{L_V}-\frac{1}{L_{\text{int}}}+\frac{1}{L_{\dot{\phi}}}\right)^2+\left(\frac{1}{L_F}\right)^2\right]^{-1/2}. \tag{15}$$

Here, the vacuum oscillation length is

$$L_V=\frac{2\pi}{\Delta E}=\frac{4\pi E_\nu}{\Delta m^2_\nu}, \tag{16}$$

the contribution of the effective oscillation length due to neutrino interactions with the medium is

$$L_{\text{int}}=\frac{2\pi}{\sqrt{>2}}G_F n_{\text{eff}}, \tag{17}$$

[(n_{eff} is the combined density of electrons n_e and neutrons n_n; see Eq. (13)]; the contribution to the effective oscillation length due to neutrino interactions with the magnetic field is

$$L_F=\frac{\pi}{\tilde{\mu}B}, \tag{18}$$

and that due to field variations is

$$L_{\dot{\phi}}=\frac{2\pi}{\dot{\phi}}. \tag{19}$$

Based on the general formula (7), we obtain for the probability of neutrino oscillations in special cases corresponding to various relationships among the characteristic lengths L_F, L_{int}, $L_{\dot{\phi}}$, and L_V

$$P(\nu_i\to\nu_j)=\begin{cases}\left(\frac{L_V}{L_F A}\right)^2\sin^2\left(\frac{\pi x A}{L_V}\right), & \text{for } L_F^{-1}\ll -L_{\text{int}}^{-1}+L_{\dot\phi}^{-1}\ll AL_V^{-1},\\ \left(\frac{L_{\text{int}}}{L_F}\right)^2\sin^2\left(\frac{\pi x}{L_{\text{int}}}\right), & \text{for } L_F^{-1}\ll AL_V^{-1}+L_{\dot\phi}^{-1}\ll L_{\text{int}}^{-1}\\ \left(\frac{L_{\dot\phi}}{L_F}\right)^2\sin^2\left(\frac{\pi x}{L_{\dot\phi}}\right), & \text{for } L_F^{-1}\ll AL_V^{-1}-L_{\text{int}}^{-1}\ll L_{\dot\phi}^{-1},\\ \sin^2\left(\frac{\pi x}{L_F}\right), & \text{for } AL_V^{-1}-L_{\text{int}}+L_{\dot\phi}^{-1}=0,\\ \to\sin^2\left(\frac{\pi x}{L_F}\right), & \text{for } L_F^{-1}\gg AL_V^{-1}-L_{\text{int}}^{-1}+L_{\dot\phi}^{-1}.\end{cases} \tag{20}$$

We also find from (7) that the probability of neutrino conversion can be close to unity when two conditions are satisfied simultaneously: the "oscillation amplitude" $\sin^2 2\theta_{\text{eff}}$ must be of order unity ($\sin^2 2\theta_{\text{eff}}\sim 1$), and the mean free path of the neutrino beam must be at least half the oscillation length ($x\geqslant L_{\text{eff}}/2$).

We find from (7) and (20) that the first condition is satisfied if one of two relations holds between the characteristic lengths: either

$$\frac{A}{L_V}-\frac{1}{L_{\text{int}}}+\frac{1}{L_{\dot\phi}}=0, \tag{21a}$$

(if $L_F^{-1}\neq 0$), or

$$\frac{1}{L_F}\geqslant\left|\frac{A}{L_V}-\frac{1}{L_{\text{int}}}+\frac{1}{L_{\dot\phi}}\right|. \tag{21b}$$

When (21a) holds, we are justified (by analogy with the resonant amplification of neutrino conversion in matter[5]) in speaking of the resonant nature of neutrino conversion amplification in a magnetic field (see also Refs. 17 and 18). When (21b) holds, the amplification of neutrino conversion in a magnetic field takes place in a "nonresonant" manner.

3. CRITICAL MAGNETIC FIELD

If the right-hand side of (21b) is small or equal to zero, (21b) effectively reduces to (21a). We now proceed by considering the more general case of a strong magnetic field for arbitrary values of the right-hand side of (21b), and we determine the conditions under which the probability of neutrino conversion is close to unity. For the sake of convenience, we write (21b) in the form

$$2\tilde\mu B\geqslant\left|\frac{\Delta m_\nu^2}{2E_\nu}A-\sqrt{2}G_F n_{\text{eff}}+\dot\phi\right|, \tag{22}$$

and we define the critical magnetic field $\tilde B_{\text{cr}}$ (by requiring equality of the left– and right–hand sides of (22)),

$$\tilde B_{\text{cr}}=\left|\frac{1}{2\tilde\mu}\left(\frac{\Delta m_\nu^2}{2E_\nu}A-\sqrt{2}G_F n_{\text{eff}}+\dot\phi\right)\right|, \tag{23}$$

which is a lower bound on the magnetic fields ($B\geqslant\tilde B_{\text{cr}}$) for which the oscillation amplitude is close to unity (i.e., at least 1/2). In a strong magnetic field with $B>\tilde B_{\text{cr}}$, $\sin^2\theta_{\text{eff}}\approx 1$, and for long enough neutrino flight paths in a magnetized medium, with $x\approx L_{\text{eff}\cdot k/2}$, $k=1,2,\ldots$, the neutrino conversion probability will be close to unity: $P(\nu_i\to\nu_j)\sim 1$.

At field strengths much above the critical value ($B\gg\tilde B_{\text{cr}}$), the oscillation amplitude will be close to unity ($\sin^2\theta_{\text{eff}}\to 1$); for $B\geqslant\tilde B_{\text{cr}}$, we have $\sin^2\theta_{\text{eff}}\geqslant 1/2$; and for weak fields ($B<\tilde B_{\text{cr}}$), $\sin^2\theta_{\text{eff}}$ is small, and no appreciable oscillations can arise. Thus, the neutrino oscillations considered in this paper are significant only in a strong enough magnetic field.

For high field values ($B\gg\tilde B_{\text{cr}}$), the effective oscillation length L_{eff} given by (15) is $L_{\text{eff}}\approx L_F$. For $B<B_{\text{cr}}$, the field has no significant influence on neutrino oscillations, while the effective oscillation length (if there are any oscillations at all) is essentially independent of B, and is governed by L_{int} and L_V.

Equation (23) for the critical field $\tilde B_{\text{cr}}$ can also be put in the form

$$\tilde B_{\text{cr}}\,[\text{G}]=43\left(\frac{\mu_B}{\tilde\mu}\right)\left|A\left(\frac{\Delta m_\nu^2}{1\text{eV}^2}\right)\left(\frac{\text{MeV}}{E_\nu}\right)-2.5\times10^{-31}\left(\frac{n_{\text{eff}}}{\text{cm}^{-3}}\right)+2.5\left(\frac{1\text{ m}}{L_{\dot\phi}}\right)\right|, \tag{24}$$

(where $\mu_B=e/2m_e$ – Bohr magnetron) which we will find convenient for numerical estimates.

From here on, in considering neutrino oscillations under neutron star conditions, in a supernova explosion, or in the interstellar medium (Secs. 4–6 and 8), we neglect effects associated with any possible change in the direction (rotation) of the magnetic field vector in a plane transverse to the neutrino trajectory. In Sec. 7, however, when we analyze oscillations in the convective zone of the sun, effects due to rotation of the magnetic field vector will be taken into consideration, and will be seen to be important.

4. NEUTRINO OSCILLATIONS IN THE MAGNETIC FIELD OF A NEUTRON STAR

We now consider neutrino oscillations that might arise in neutron stars. In contrast to a number of previous treatments,[18,19,25] we do not limit our considerations to the resonant amplification of neutrino conversion effects, i.e., we do not require that (21a) hold. Instead, we consider a more general case, assuming that (21b) holds in the presence of a suitably strong magnetic field.

Current astrophysical data indicate that the surface magnetic field of a neutron star is $B \approx 10^{12}-10^{14}$ G, and that the internal field can reach $B \sim 10^{15}$ G or even higher.[34–36] In producing our numerical estimates, we start with the information that the density of neutron star matter is very high (ranging from $\rho \sim 1.6\times 10^{9}$ g/cm^3 at the surface to $\rho \sim 1.6\times 10^{14}-1.6\times 10^{15}$ g/cm^3 near the center of the star).

For the sake of definiteness, we assume that left-handed neutrinos ν_{e_L} are created in the stellar interior, and we examine one of the possible oscillation processes listed in (6), $\nu_{e_L} \leftrightarrow \nu_{e_R}$.

Making use of (24), assuming for numerical purposes a neutrino magnetic moment $\tilde{\mu} \approx 10^{-10}\mu_B$ ($\mu_B = e/2m_e$ is the Bohr magneton), and taking $\Delta m_\nu^2 \approx 10^{-4}$ eV^2, $\sin 2\theta = 0.1$, $E_\nu \approx 20$ MeV, and $n_{\text{eff}} \sim 10^{33}$ cm^{-3}, we obtain $\tilde{B}_{\text{cr}}(\dot{\phi}(t) = 0) = B_{\text{cr}}$

$$B_{\text{cr}} \approx 1.11\times 10^{14} \text{ G}. \tag{25}$$

As noted above, magnetic fields this strong can exist at the surface of a neutron star. We also point out that the principal contribution to $\tilde{B}_{\text{cr}}$ comes from the "matter term," which is proportional to n_{eff}.

At the field strength given by (25), Eq. (15) yields for the effective oscillation length

$$L_{\text{eff}} \approx 1 \text{ cm}, \tag{26}$$

which is much less than either the size of the neutron star, $r_0 \approx 10$ km, or the thickness of its crust, $l \sim 0.1 r_0 \approx 1$ km.

Based on these estimates, we can conclude that if strong fields $(B > \tilde{B}_{\text{cr}})$ exist over long enough distances $(L \gg 1$ cm), neutrino conversion and oscillations can arise when a neutrino emerges from within a neutron star and moves toward its surface.

If we are concerned with an ensemble (beam) of ν_{e_L} particles emitted at various points within a neutron star, rather than just a single neutrino, we can determine the probability of detecting a neutrino of the other type (ν_{e_R}) in the beam at some distance by averaging over x in Eq. (7). The probability of a ν_{e_R} appearing in a beam that initially consists of ν_{e_L} is therefore

$$\bar{P}_{\nu_R} = \frac{1}{2}\sin^2 2\theta_{\text{eff}}. \tag{27}$$

Thus, by virtue of neutrino oscillations engendered by a strong magnetic field $(B > \tilde{B}_{\text{cr}})$, a beam that initially consists solely of left-handed neutrinos ν_{e_L} will consist of an equal number of left-handed (active, i.e., interacting with matter) ν_{e_L} and right-handed (sterile) ν_{e_R} neutrinos $(\sin^2 2\theta_{\text{eff}} \sim 1)$. In other words, it is possible for the number of left-handed neutrinos in the beam to be halved.

5. THE CROSS-BOUNDARY EFFECT

Consider now a more moderate magnetic field, i.e., one in which all neutrino paths within a neutron star experience only a subcritical field: $(B < \tilde{B}_{\text{cr}})$. Even under these circumstances, it is possible for neutrino conversion and oscillations to be significant as a result of the so-called cross-boundary effect,[29,32] which comes into play when a neutrino beam emerges from a neutron star. The density of matter at the neutron star surface falls rapidly to its vacuum value (corresponding to $n_{\text{eff}} \to 0$), while the magnetic field remains effective even relatively far from the surface. We will assume that the fall off in the field is given (see also Ref. 19) by

$$B(r) = B_0\left(\frac{r_0}{r}\right)^3, \tag{28}$$

where r_0 is the neutron star radius and B_0 is the field strength at the surface.

The principal contribution to the critical field in Eq. (24) is now the "vacuum" term $(\sim \Delta m_\nu^2)$, and we obtain for the critical field itself

$$B'_{\text{cr}} \approx 5.4\times 10^3 \ G. \tag{29}$$

Thus, when the neutrino beam emerges from the neutron star, it runs into a "strong-field" region $(B > B'_{\text{cr}})$. The net result is that a beam that initially consists solely of ν_{e_L} winds up with an equal number of ν_{e_L} and ν_{e_R} by the time it has traversed a relatively small distance Δr $(\Delta r \gg 1$ m) from the surface. From (28) and (29) (taking the surface field to be $B_0 = 10^{12}$ G), we find that at distances

$$r \leqslant r_{\text{cr}} \approx 600 r_0 \tag{30}$$

from the neutron star surface, the field strength exceeds the critical value: $B \geqslant B'_{\text{cr}}$.

From the estimated effective oscillation length at the neutron star surface,

$$L_{\text{eff}}(B \sim B_0) \approx \frac{\pi}{\tilde{\mu} B_0} \simeq 10^2 \frac{\mu_B}{\tilde{\mu}}\left(\frac{1\ G}{B_0}\right) \approx 1 \text{ m}, \tag{31}$$

we observe that this region is considerably larger than L_{eff} across.

It is pertinent to point out here that the adiabatic approximation is indeed applicable to the present case, and it underlies the entire development in this paper. In the most general case, the prerequisite for it to hold is that

$$(H_{jj} - H_{ii})\frac{\partial}{\partial r}(H_{ij} + H_{ji}) - (H_{ij} + H_{ji})\frac{\partial}{\partial r}(H_{jj} - H_{ii}) \ll 2[(H_{jj} - H_{ii})^2 + (H_{ij} + H_{ji})^2]^{3/2}, \tag{32}$$

where H_{ij} is an element of the matrix (4). Making use of the expression for H_{ij}, which corresponds to the neutrino conversion process $\nu_i \to \nu_j$, we can bring this inequality to the form

$$\left|\frac{\partial B}{\partial r}\tilde{B}_{\text{cr}} - B\frac{\partial \tilde{B}_{\text{cr}}}{\partial r}\right| \ll 4\tilde{\mu}\,[\tilde{B}_{\text{cr}}^2 + B^2]^{3/2}. \tag{33}$$

When the adiabatic condition is written in this form, it is immediately clear that it reduces to the requirement that the field B and density ρ [recalling that $\tilde{B}_{\text{cr}} = \tilde{B}_{\text{cr}}(n_{\text{eff}})$] vary slowly with distance r. At the neutron star surface, the density drops abruptly from $\rho \sim 10^9$ g/cm^3 to the vacuum value.

Assuming that the field varies according to (28), and substituting $\tilde{B}_{cr}$ from (24) into (33), we find that adiabaticity holds if this density change takes place over a distance $\Delta r \geqslant 10$ cm. If the density change occurs over $\Delta r \leqslant 10$ cm, adiabaticity will not hold (in that region). The transition value ($\Delta r \sim 10$ cm), however, turns out not only to be much less than typical neutron star dimensions, but also much less than the effective oscillation length in that region: $\Delta r \ll L^0_{\text{eff}} \sim 1\ m$.

Thus, even if we assume a very sharp drop in the matter density at the neutron star surface, resulting in a breakdown of adiabaticity at distances ~ 10 cm from the surface, there are no repercussions for neutrino oscillations, and effects related to nonadiabaticity can be neglected.

We now note that there is another phenomenon resembling the cross-boundary effect at the surface (in which a neutrino leaves the surface and takes off into the vacuum, where the densities $n_e,\ n_n,\ n_p \rightarrow 0$), this time transpiring inside the neutron star. The cross-boundary effect becomes observable, for example, in Majorana neutrino oscillations $\nu_{e_L} \rightarrow \bar{\nu}_{\mu_R}$ when the neutrinos cross interior regions of the neutron star consisting of helium, carbon, oxygen, nitrogen, and silicon (isotopically neutral regions;[24] see also Refs. 37 and 38). The relationship between neutrons and electrons in these layers is $n_e \approx n_n$, so that $n_{\text{eff}} = n_e - n_n \rightarrow 0$, and the matter term in the expressions (23) and (24) for $\tilde{B}_{cr}$ turns out to be much smaller than the corresponding vacuum term (see also Ref. 32).

To conclude this section, we note that we have examined the cross-boundary effect in more detail elsewhere,[39] making use of a realistic equation of state for neutron star matter.

6. SUPERNOVA EXPLOSIONS AND BOUNDS ON THE NEUTRINO MAGNETIC MOMENT

The reduction in the number of left-handed electron neutrinos (or other active neutrinos) engendered by neutrino oscillations in a magnetic field can play an important role in the cooling phase of a supernova. In particular, this makes it possible to bound the product $\tilde{\mu}B$.

As an example, we consider the model advanced by Fuller *et al.*,[21] who allowed for neutrino oscillations in matter (but disregarded effects due to a strong magnetic field), resulting in a 60% increase in the overall explosion energy and thereby making up an existing energy shortfall. When the strong magnetic fields of a neutron star are also included, new and efficient channels come into being for transforming active neutrinos into sterile ones (i.e., neutrinos that do not interact with matter, and therefore carry off energy unimpeded), exerting a profound influence on the energy balance of a supernova explosion.[2)]

We assume that magnetic field-induced neutrino oscillations do not reduce the magnitude of the increase in explosion energy predicted by Fuller *et al.*[21] Taking the magnetic field to be frozen into the matter, we find that at $r = 45$ km from the center of a hot protoneutron star (the density of matter in this region is estimated to be $\rho \sim 6\times 10^{12}$ g/cm^3), the magnetic field strength will be $B \sim 10^{14}$ G. If the field falls off with distance from the center according to (20), then at $r \sim 160$ km, the field strength will be $B \sim 6\times 10^{12}$ G. This is precisely the value of the critical field for a matter density of $\rho \sim 6\times 10^8$ g/cm^3 (which is perfectly consistent with a distance of 160 km from the center) and magnetic moment $\tilde{\mu} \sim 10^{-10}\ \mu_B$.

The probability of detecting right-handed neutrinos ν_{e_R} in a flux that initially consists solely of left-handed neutrinos ν_{e_L} will then be $\bar{P}_{\nu_{e_L} \rightarrow \nu_{e_R}} = 0.25$ (corresponding to an effective length $L_{\text{eff}} \sim 10$ cm). Consequently, to avoid the possibility of losing a significant number of neutrinos (conversion of 25% from active to sterile), with associated energy losses, we assume a more stringent bound on the neutrino magnetic moment, $\tilde{\mu} \leqslant 10^{-11}\ \mu_B$.

In closing this section, we note that if the resonance condition (21a) holds over some region during a supernova explosion, then magnetic fields can alter the energetics of the explosion substantially.[19,25]

7. NEUTRINO OSCILLATIONS IN THE VARIABLE MAGNETIC FIELD OF THE SUN

Consider now neutrino oscillations in the convective zone of the sun, allowing for possible magnetic field variations[40,41,24] in a plane perpendicular to the direction of neutrino motion. For the sake of definiteness, we again consider one possible type of transformation, $\nu_{e_L} \rightarrow \nu_{e_R}$. To estimate the critical field $\tilde{B}_{cr}$ via (23) or (24), we set $\Delta m_\nu = 10^{-4}$ eV^2, $\sin 2\theta = 0.1$, $E_\nu = 20$ MeV, and assume that $n_{\text{eff}} \sim n_e \approx 10^{23}$ cm^{-3} in the solar convective zone.

To describe effects associated with varying fields, we invoke results derived in Refs. 24 and 41: $L_{\dot{\phi}} \sim 0.1\ R_\odot \approx 7\times 10^7$ m, where $R_\odot = 7\times 10^8$ m is the radius of the sun. Plugging these numerical values into (24), we have (see also Ref. 32)

$$\tilde{B}_{cr}\,[G] \approx \left(\frac{\mu_B}{\tilde{\mu}}\right) |-10^{-6} - 5\times 10^{-7} + 1.43\times 10^{-6}| = 7\times 10^{-8} \left(\frac{\mu_B}{\tilde{\mu}}\right). \tag{34}$$

Thus, magnetic field variations [associated with the third term in Eq. (34)] can significantly reduce the critical field $\tilde{B}_{cr}$ below its uncorrected value B_{cr} ($\tilde{B}_{cr}/B_{cr} \approx 0.047$).

Assuming that typical fields in the convective zone are $B_{\text{conv}} \sim 10^5$ G, we find from Eq. (24) that $B_{\text{conv}} \geqslant \tilde{B}_{cr}$ when $\tilde{\mu} \geqslant 10^{-12}\ \mu_B$.

According to the second prerequisite for the existence of appreciable magnetic field–induced oscillation effects, the effective oscillation length should not exceed half the depth of the convective zone: $L_{\text{eff}} \leqslant L_{\text{conv}}/2$. This condition holds when $\tilde{\mu} \sim 10^{-11}\ \mu_B$ (for fields $B \sim 10^5$ G).

8. NEUTRINO OSCILLATIONS IN THE GALACTIC MAGNETIC FIELD

There is some interest in considering neutrino oscillations induced by the ambient magnetic field in the interstellar medium, with mean value $B_G \sim 10^{-6}$ G. It is worth noting that a galactic magnetic field $B \sim 10^{-6}$ G can give rise to oscillations in neutrinos traversing interstellar space. Indeed, we can estimate the critical field for superhigh–energy neutrinos ($E_\nu \geqslant 10^{17}$ eV) in interstellar space to be $\tilde{B}_{\rm cr} \leqslant 10^{-6}$ G. Bearing in mind that the corresponding effective oscillation length $L_{\rm eff}(B \sim B_G) \approx 10^{20}$ cm is less than the size of the Galaxy ($R_G \approx 3\times10^{22}$ cm), we conclude that field–induced neutrino oscillations can in this case be appreciable.

9. CONCLUSION

Our principal result is a formulation of the criterion ($B \geqslant \tilde{B}_{\rm cr}(\Delta m_\nu^2,\ \theta, n_{\rm eff},\ E_\nu,\ \dot{\phi}(t))$) governing the conditions under which the probability of magnetic field–induced conversion between the two types of Dirac or Majorana neutrinos can become large. This criterion holds both for the resonant enhancement of the transition probability between neutrino states and for nonresonant effects.

Making use of this criterion in specific examples of neutrino motion through magnetized media (neighborhood of a neutron star; supernova explosion; solar convective zone; interstellar medium), we have shown that when the magnetic fields are strong enough, neutrino oscillations can be substantial, even when resonant amplification of neutrino conversion does not come into play. We have relied upon these results in discussing bounds on the magnetic moment of the neutrino.

Having considered a flux of neutrinos crossing either the surface or interior boundary layers of a neutron star, we predict the conversion of up to half the neutrinos from the active phase to the sterile phase by virtue of other cross–boundary effect.[29]

[1)]Some of these results have also been described in Ref. 34.
[2)]We thank B. Meyer for discussions of this possibility.

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