The quantum-field renormalization group in the problem of a growing phase boundary

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(Submitted 7 February 1995)
Zh. Éksp. Teor. Fiz. **108**, 885–893 (September 1995)

Within the quantum-field renormalization-group approach we examine the stochastic equation discussed by S. I. Pavlik (JETP **79**, 303 (1994)) in describing a randomly growing phase boundary. We show that, in contrast to Pavlik's assertion, the model is not multiplicatively renormalizable and that its consistent renormalization-group analysis requires introducing an infinite number of counterterms and the respective coupling constants ("charges"). An explicit calculation in the one-loop approximation shows that a two-dimensional surface of renormalization-group points exists in the infinite-dimensional charge space. If the surface contains an infrared stability region, the problem allows for scaling with the nonuniversal critical dimensionalities of the height of the phase boundary and time, Δ_h and Δ_t, which satisfy the exact relationship $2\Delta_h=\Delta_t+d$, where d is the dimensionality of the phase boundary.

1. INTRODUCTION

Over the years a problem that has constantly aroused interest is the deposition of a substance on a surface and the growth of the corresponding phase boundary (see, e.g., Refs. 1–15 and the literature therein). The results of numerical experiments suggest the presence of infrared (large times and distances) scaling with universal critical dimensionalities.[4,5,10,11] The common starting point in the attempts to build an analytical theory is the stochastic equation

$$\partial_t h=\nu_0\partial^2 h+V(h)+f, \quad \langle f(x)f(x')\rangle=2D_0\delta(x-x'), \quad (1)$$

in which $h(x)=h(t,\mathbf{x})$ is the random component of the height of the phase boundary, f is the Gaussian random force with a correlator specified in Eq. (1) and proportional to $\delta(x-x')\equiv\delta(t-t')\delta^{(d)}(x-x')$, d is the dimensionality of space $\mathbf{x}$, ν_0 and D_0 are positive initial parameters, $V(h)$ is a nonlinearity parameter, and $\partial^2\equiv\Delta$ is the Laplace operator. The linear contribution $\nu_0\partial^2 h$ models surface tension. (Strictly speaking, f is expected to have a certain constant component $\langle f(x)\rangle=\text{const}$ that guarantees $\langle h\rangle=0$, which follows from the meaning of h, but in actual calculation one can ignore $\langle f\rangle$ and $\langle h\rangle$ simultaneously and assume they are zero.)

Kardar, Parisi, and Zhang[6] studied a model with the simplest nonlinearity $V(h)=(\partial h)^2/2$. Such an interaction is logarithmic for $d=\frac{1}{2}(\partial h)^2$. Such an interaction is logarithmic for $d=2$, nonessential for $d>2$, and can be studied by the standard $d=2-\varepsilon$ renormalization-group (RG) technique for $d\leqslant 2$. The RG-analysis done in Ref. 6 (which actually reproduces the results of an old paper by Forster, Nelson, and Stephen, who discussed an d-dimensional generalization of Burgers' stochastic equation) showed that in the $(2-\varepsilon)$ expansion this model contains no infrared (IR) stable fixed RG-points in the physical region $D_0,\nu_0>0$, i.e., the justification of IR-scaling customarily employed in the theory of the behavior of the critical state[17] is not valid here. Note that the results of Refs. 6 and 16 recalculated in terms of the more convenient minimal subtraction calculation scheme lead to the expression $\beta(u)=-\varepsilon u-u^2$ for the one-loop β-function of the renormalized charge u corresponding to the bare charge $u_0=D_0\nu_0^{-3}/4\pi$. It is the "wrong" sign in front of u^2 that forced the fixed point $u_*\sim\varepsilon$ to be in the nonphysical region $u<0$. But if we still assume (without rigorous substantiation, because the contributions of the higher powers of u in the expansion of the β-function are unknown) that somewhere in the range of positive u of order unity the β-function has the necessary fixed point, i.e., $\beta(u_*)=0$ and $\beta'(u_*)>0$, then the relationship between the renormalization constants of the model of Ref. 6 leads to an equation $\Delta_h+\Delta_t+2=0$ for the critical dimensionalities of field and time, and for $d=1$ the fluctuation–dissipation theorem makes it possible to find these dimensionalities exactly: $\Delta_h=-\frac{1}{2}$ and $\Delta_t=-\frac{3}{2}$. But since the very fact of the existence of the necessary fixed point is an unprovable hypothesis, the situation cannot, naturally, be considered satisfactory.

Recently Pavlik[1] suggested another variant of problem (1) with the equation

$$\partial_t h=\nu_0\partial^2 h+\partial^2 h^2/2+f. \quad (2)$$

The nonlinearity $\partial^2 h^2=2(\partial h)^2+2h\partial^2 h$ contains the ordinary interaction[6] and an additional term interpreted in Ref. 1 as a random correction to $\nu_0\partial^2 h$ (in the presence of nonlinearity Pavlik[1] introduced an additional factor α, but this factor can be eliminated by a stretching transformation $\alpha h\to h$). In his paper Pavlik used the minimal subtraction calculation technique and claimed that the model (2), in contrast to that studied in Ref. 6, contains the necessary fixed point in the $(2-\varepsilon)$ expansion (one of the equations in (16) in Ref. 1 is equivalent to the expression $\beta_g=-\varepsilon g+28g^2+\cdots$ for the one-loop β-function of the renormalized charge g with the "proper" sign in front of g^2).

In this paper we wish to focus attention on several mistakes made in Ref. 1 and perform a consistent RG-analysis of the model of type (2). The principal remark is the following: Pavlik[1] failed to notice that the model (2) is actually not multiplicatively renormalizable, since the interaction $\partial^2 h^2$ necessarily generates an infinite number of new counterterms

 1063-7761/95/090485-05$10.00

TABLE I.

F	h	h'	ν_0,ν	m_0,m,μ	g_0	g_{n0}	g,g_n
d_F^k	$d/2$	$d/2$	-2	1	$1-d/2\equiv\varepsilon/2$	$\varepsilon(n-1)/2$	0
d_F^ω	$-1/2$	$1/2$	1	0	0	0	0
d_F	$d/2-1$	$d/2+1$	0	1	$1-d/2\equiv\varepsilon/2$	$\varepsilon(n-1)/2$	0

of the form $\partial^2 h^n$ with any $n=1,2,3,\ldots$, with the result that the complete renormalized theory contains an infinite number of parameters ("charges") g_n and corresponding β-functions in the RG-equations.

The second mistake is the incorrect treatment of the IR divergences present in the model (2) (in contrast to that of Ref. 6), which Pavlik erroneously interpreted in the same way as he did the ultraviolet (UV) divergences, not distinguishing the IR and UV poles in ε when he calculated the renormalization constants. In reality the IR and UV divergences are of different origin, and only the latter can be removed by the standard renormalization procedure (see, e.g., Refs. 18 and 19). If there are IR divergences, they must be first removed by appropriate regularization. In the model (2) this can be done by substituting $(\partial^2-m^2)h$ for $\partial^2 h$, where m is the "infrared mass" (by its very meaning $1/m$ is the maximum characteristic dimension in the system). Introduction of the "regularizer" m removes the IR poles in ε but does not influence the UV poles, which are the only ones that must be taken into account in calculating the renormalization constants Z. The fact that the Z in the minimal subtraction scheme are independent of mass m in no way means that these constants can be directly calculated for $m=0$ when the model has IR singularities at $\varepsilon=0$; this would mean unjustified merging of the UV and IR poles in ε.

To be sure, modifying the minimal subtraction scheme by introducing a mass m into (2) dramatically changes Pavlik's result, leading to a β-function with a minus in front of g^2, as in the model of Ref. 6. But if the first remark (see above) is taken into account, the result cannot be considered final, since a rigorous RG-analysis requires allowing for all the necessary counterterms.

In this paper we do such a calculation in the quantum field setting.[20,21] In Sec. 2 we analyze the structure of the UV divergences of the massive generalization of the model (2) and substantiate the need to go over to a model with an infinite number of vertices and corresponding charges. In Sec. 3 we explicitly calculate in the one-loop approximation the renormalization constants and the RG-functions of an infinite-charge model that generalizes Eq. (2), and analyze the corresponding RG-equations. It appears that this model contains a complete two-dimensional equation of fixed points in the infinite dimensional charge space. We were unable to do a complete investigation of the stability of these points, but we believe this situation allows for IR scaling with nonuniversal (i.e., depending on the choice of parameters) critical exponents.

2. ANALYSIS OF THE UV DIVERGENCES AND COUNTERTERMS OF MODEL (2)

According to the general theory,[20,21] any stochastic problem of type (1) is equivalent to the quantum field model with a double number of fields $\Phi=h,h'$ and the action functional

$$S(\Phi)=D_0 h'h'+h'[-\partial_t h+\nu_0\partial^2 h+V(h)]$$

(integration with respect to $x\equiv t,\mathbf{x}$ in this and similar equations is implied). Applying this theorem to the massive analog of problem (2) and, for convenience, stretching the fields, $h\to D_0^{1/2}h$ and $h'\to D_0^{-1/2}h'$, we arrive at a model with the action

$$S(\Phi)=h'h'+h'[-\partial_t h+\nu_0(\partial^2-m^2)h+D_0^{1/2}\partial^2 h^2/2]. \quad (3)$$

The action is assumed unrenormalized and all its parameters have their bare values. For this reason they have a subscript "0" to distinguish them from the renormalized analogs without such a subscript, which will be discussed shortly. The role of the bare charge (the coupling constant) in the model (3) is taken by the combination $g_0=D_0^{1/2}\nu_0^{-3/2}$. The Green's functions $G_n=\langle\Phi(x_1)\cdots\Phi(x_n)\rangle$ of model (3) are represented by a functional integral of products of the corresponding fields with a weight function $\exp S(\Phi)$. The standard way to represent such objects is to use Feynman diagrams, with the lines determined by the free (quadratic in the fields) part of $S(\Phi)$ and the vertices by the interaction (the contribution of the nonlinearity in (1)).

As is known, an analysis of the UV divergences is related to an analysis of canonical dimensionalities.[18,19] Dynamical models of the type (3) are two-scale, i.e., with each quantity F we can associate here two independent canonical dimensionalities,[22] the momentum dimensionality d_F^k and the frequency dimensionality d_F^ω, and hence the total dimensionality $d_F=d_F^k+2d_F^\omega$ (in the free theory $\partial_t\sim\partial^2$). By definition, $d_k^k=-d_{\mathbf{x}}^k=d_\omega^\omega=-d_t^\omega=1$ and $d_k^\omega=d_{\mathbf{x}}^\omega=d_\omega^k=d_t^k=0$, while the dimensionalities of the other quantities are determined by the requirement that all terms in the action be dimensionless (the momentum and frequency components separately). The dimensionality data for model (3) are listed in Table I, where for convenience we include the leading coupling constants and renormalization parameters, which appear later in our discussion.

Table I shows that the theory becomes logarithmic ($d_{g_0}=0$) for $d=2$. The UV divergences emerge as poles in $\varepsilon\equiv 2-d$ in the Green's functions. The total canonical dimensionality of an arbitrary 1-irreducible diagram (the "formal divergence index") is $\delta=d+2-d_h N_h-d_{h'}N_{h'}$, where N_Φ is the number of corresponding external lines. Counter-

terms are generated only by "surface-divergent" 1-irreducible diagrams: a diagram is surface-divergent if its real divergence index (see below) in the logarithmic theory ($\varepsilon=0$) is a nonnegative integer, with δ at $\varepsilon=0$ determining the degree of homogeneity of the counterterm in momenta and frequencies. Also, when analyzing divergences in the model (3), we must take into account the following additional considerations:[22]

(1) All 1-irreducible Green's functions with $N_{h'}=0$ vanish.

(2) Integration by parts allows shifting the operator ∂^2 at a vertex in model (3) to the field h'. Hence in any 1-irreducible diagram each field h' "releases" the square of the corresponding external momentum, and the real divergence index δ' is smaller than the formal index δ by the corresponding number of units: $\delta'=\delta-2N_{h'}$.

Allowing for these ideas, one can easily see that in the model considered here surface UV-divergences are present only in the 1-irreducible diagrams of the Green's functions $\langle h'h\cdots h\rangle$ with any finite number of fields h. For all these diagrams $\delta=2$ and $\delta'=0$ hold, and the corresponding counterterms are certain to contain $\partial^2 h'$, which means that they can be reduced to $h'\partial^2 h^n$ (other ways of placing the gradients or for ∂_t and m to participate are forbidden). Hence to ensure multiplicative renormalization, to the action (3) we must add all the vertices $h'\partial^2 h^n$, which means a transition to a theory with an infinite number of charges. One can easily verify that the new vertices do not generate counterterms that differ from $h'\partial^2 h^n$, so that a model incorporating all such vertices with independent coefficients (charges) is already multiplicatively renormalizable.

3. RG-ANALYSIS OF A MODEL WITH AN INFINITE NUMBER OF CHARGES

We examine the generalization of the model (3) that includes in the unrenormalized action all renormalization-generated vertices with independent coefficients:

$$S(\Phi)=h'h'+h'[-\partial_t h+\nu_0(\partial^2-m_0^2)h + \partial^2\sum_{n=2}^{\infty}\lambda_{n0}h^n/n!]. \tag{4}$$

The corresponding renormalized action is

$$S_R(\Phi)=h'h'+h'[-\partial_t h+\nu(Z\partial^2-m^2)h + \partial^2\sum_{n=2}^{\infty}Z_n\lambda_n h^n/n!], \tag{5}$$

where all the Z are dimensionless renormalization constants, which in the maximal subtraction scheme have the form "1 + poles in ε." The bare (initial) charges $g_0\equiv\{g_{n0}, n=2,3,...\}$ and the completely dimensionless renormalized charges $g\equiv\{g_n, n=2,3,\cdots\}$ are expressed in terms of the parameters λ_0 and λ in Eqs. (4) and (5) as follows:

$$\lambda_{n0}=g_{n0}\nu_0^{(n+1)/2}, \quad \lambda_n=g_n\nu^{(n+1)/2}\mu^{\varepsilon(n-1)/2}, \tag{6}$$

where the renormalization mass μ is an additional parameter of the renormalized theory.

The action (5) is obtained from (4) by the following renormalization of parameters (no renormalization of the fields ϕ is required):

$$\nu_0=\nu Z_\nu, \quad m_0=mZ_m, \quad g_{n0}=\mu^{\varepsilon(n-1)/2}g_n Z_{g_n}. \tag{7}$$

The constants Z in Eqs. (7) and (5) are related as follows:

$$Z_\nu=Z, \quad Z_m=z_\nu^{-1/2}, \quad Z_{g_n}=Z_n Z_\nu^{-(n+1)/2}. \tag{8}$$

The relationship $S(\Phi,e_0)=S_R(\Phi,e,\mu)$, where e_0 is the set of all initial parameters and e the set of all renormalized parameters, yields the following RG-equation for the Green's functions $G_n=\langle\Phi\cdots\Phi\rangle$ of model (5):

$$\mathscr{D}_{\mathrm{RG}}G_n=0, \quad \mathscr{D}_{\mathrm{RG}}\equiv\mathscr{D}_\mu-\sum_{F=m,\nu}\gamma_F\mathscr{D}_F+\sum_{n=2}^{\infty}\beta_n\partial_{g_n}, \tag{9}$$

where we have introduced the notation $\mathscr{D}_y\equiv y\partial_y$ for every variable y, and the RG-functions (the β-function and the anomalous dimensionalities γ) are specified as

$$\gamma_F\equiv\tilde{\mathscr{D}}_\mu\ln Z_F, \quad \beta_n\equiv\tilde{\mathscr{D}}_\mu g_n=g_n[-(n-1)\varepsilon/2-\gamma_{g_n}], \tag{10}$$

with $F=\nu,m,g_n$; and $\tilde{\mathscr{D}}_\mu\equiv\mu\partial_\mu$ for fixed e_0. Note that $\mathscr{D}_{\mathrm{RG}}$ in (9) is the operation $\tilde{\mathscr{D}}_\mu$ in the variables e and μ. We note also that because there is no field renormalization the initial Green's functions $G_n(e_0)$ coincide with the renormalized Green's functions $G_n(e,m)$ (the difference is only in the choice of variables), with the result that both can be employed on an equal basis when critical behavior is analyzed.

We calculate the constants Z in (5) in the one-loop approximation. In the expansion of the generating functional $\Gamma_R(\Phi)$ of the 1-irreducible Green's functions of model (5) in the number p of loops,

$$\Gamma_R(\Phi)=\sum_{p=0}^{\infty}\Gamma_p(\Phi), \quad \Gamma_0(\Phi)=S_R(\Phi), \tag{11}$$

the loopless ("tree-like") contribution is simply the action (5), and the one-loop contribution is given by the following relation (see, e.g., Ref. 23):

$$\Gamma_1(\Phi)=\left(-\frac{1}{2}\right)\mathrm{Tr}\,\ln(W/W_0), \tag{12}$$

where W is a linear operation with the kernel

$$W(x,y)=-\frac{\delta^2 S_R(\Phi)}{\delta\Phi(x)\,\delta\Phi(y)}, \tag{13}$$

and W_0 is a similar expression for the free (quadratic in the fields) parts of the action (5). Both W and W_0 are 2-by-2 matrices in the pair $\Phi=h,h'$; the matrix that is the inverse of matrix W_0 represents the lines in the diagrams of model (5).

The constants Z can be found from the requirement that the UV divergences (poles in ε) in Eq. (11) are removed, and if the additional condition that they are of the type $Z=1+$only poles in ε (the minimal subtraction scheme) holds, they are determined uniquely. When the one-loop calculation is performed, in (12) we must put $Z=1$, and in the loopless contribution (5) we must allow for the contributions of order g in the constants Z. We introduce the notation

$$V(h)\equiv\sum_{n=2}^{\infty}\lambda_n h^n(x)/n!\ ,\quad V_R(h)\equiv\sum_{n=2}^{\infty}Z_n\lambda_n h^n(x)/n!\ , \tag{14}$$

interpreting these objects as functions of a single variable $h(x)$, and V', V'', etc., as the corresponding derivatives with respect to this variable. In this notation the matrix (13) with $Z=1$ can be symbolically represented as

$$W=\begin{pmatrix} -\partial^2 h' V'' & L^{\mathrm{T}} \\ L & -2 \end{pmatrix}, \tag{15}$$

where $L\equiv\partial_t-\nu(\partial^2-m^2)-\partial^2 V'$, and $L^{\mathrm{T}}=-\partial_t-\nu\times(\partial^2-m^2)-V'\partial^2$ is the transposed operation.

To determine the constants Z we do not need the exact expression (11) but only its divergent part, which, as we already know (see Sec. 2), has the form

$$\int dx\ \partial^2 h'(x)R(h(x))$$

with a function $R(h)$ similar to (14). This implies that the Tr ln of matrix (15) is needed only to first order in its hh-element $-\partial^2 h' V''$. Using the well-known formula $\delta(\mathrm{Tr}\ln K)=\mathrm{Tr}[K^{-1}\delta K]$, from (15) we obtain (with the required accuracy)

$$\mathrm{Tr}\ln(W/W_0)\cong-\mathrm{Tr}[D_{hh}V''\partial^2 h']$$

$$\equiv-\int dx\ D_{hh}(x,x)V''(h(x))\partial^2 h'(x), \tag{16}$$

where $D_{hh}\equiv(W^{-1})_{hh}$ at $h'=0$. By its very meaning, D_{hh} is the ordinary propagator $\langle hh\rangle$ of model (5) with $Z=1$ and with $\nu\partial^2+\partial^2 V'$ substituted for $\nu\partial^2$.

One more circumstance must be taken into account. After ∂^2 is shifted to the outer factor h', only a logarithmically divergent expression remains in the counterterm, with the result that in calculating the divergent part of a given diagram we can set all its external momenta to zero (IR regularization is ensured by the mass m). This means that in selecting the pole part (in ε) in (16) we can ignore the inhomogeneity of $\partial^2 h'(x)$ and $h(x)$, i.e., both quantities can be assumed constant. Then $D_{hh}(x,x)$ can easily be calculated by going over to the momentum–frequency representation:

$$D_{hh}(x,x)=\int\int\frac{d\omega\,dk}{(2\pi)^{d+1}}\frac{2}{\omega^2+[(\nu+V')k^2+\nu m^2]^2}$$

$$=\frac{\mu^{-\varepsilon}}{2\pi\varepsilon(\nu+V')}+\cdots, \tag{17}$$

where the dots stand for the UV-finite part. To avoid a misunderstanding we note that according to the rules of dimensional regularization[19] in an expression with low-order deviations (of order ε) of the dimensionality from an integral value these deviations are always represented by fractional powers of μ only, rather than any other variables. Substitution of (16) and (17) into (12) yields the following expression for the divergent part of $\Gamma_1(\Phi)$ with the necessary accuracy:

$$\tilde{\Gamma}_1(\Phi)=\frac{\mu^{-\varepsilon}}{4\pi\varepsilon}\int dx\ \frac{V''(h(x))}{\nu+V'(h(x))}\partial^2 h'(x). \tag{18}$$

In the sum of (18) and the loopless contribution in (11) the pole in ε cancels out, which makes it possible to find the one-loop contributions of order $1/\varepsilon$ in all constants Z. We introduce the notation

$$V''(h)(\nu+V'(h))=\sum_{n=0}^{\infty}\mu^{\varepsilon(n+1)/2}\nu^{(n+1)/2}r_n h^n/n!\ , \tag{19}$$

where r_n are completely dimensionless coefficients (polynomials in the charges g_n). If the above condition for the canceling out of poles in ε is combined with (6), we get

$$Z=1-r_1/4\pi\varepsilon+\cdots,\quad Z_n=1-r_n/4\pi\varepsilon g_n+\cdots. \tag{20}$$

It should be kept in mind that when the renormalization constants are used to calculate the RG-functions (10), in terms of the functions of the form (20) depending only on the charges g, the operation $\tilde{\mathscr{D}}_\mu$ in (10) assumes the form

$$\tilde{\mathscr{D}}_\mu=\sum_n(\tilde{\mathscr{D}}_\mu g_n)\partial_{g_n}=\sum_n\beta_n\partial_{g_n}.$$

To this accuracy it is sufficient to use only the first terms in the β-functions (10). This yields

$$\tilde{\mathscr{D}}_\mu\cong-(\varepsilon/2)\,\mathscr{D}_g,\quad \mathscr{D}_g\equiv\sum_{n=2}^{\infty}(n-1)g_n\partial_{g_n}. \tag{21}$$

Bearing this in mind and employing the constants (20) together with (8) and (10), we can easily arrive at the following expressions for the one-loop RG-functions:

$$\gamma_\nu=-(\gamma_m/2)=a\mathscr{D}_g r_1,\quad a\equiv\frac{1}{8\pi}, \tag{22a}$$

$$\beta_n=-\varepsilon(n-1)g_n/2+(n+1)g_n\gamma_\nu/2$$

$$-a(\mathscr{D}_g-n+1)r_n. \tag{22b}$$

From the definitions (19), (14), and (6) we find the explicit expressions for the first four coefficients r_n [the first term with r_0 in (19) contributes nothing to (18)],

$$r_1=g_3-g_2^2,\quad r_2=g_4-3g_2g_3+2g_2^3,$$

$$r_3=g_5-4g_2g_4-3g_3^2+12g_2^2g_3-6g_2^4,$$

$$r_4=g_6-5g_2g_5+20g_2^2g_4-10g_3g_4+30g_2g_3^2-60g_2^2g_3+24g_2^4,$$

which when substituted into (22) yield

$$\gamma_n=-\gamma_m/2=a(2g_3-2g_2^2), \tag{23a}$$

$$\beta_2=\varepsilon g_2/2+a(-2g_4+9g_2g_3-7g_2^3),$$

$$\beta_3=-2\varepsilon g_3/2+a(-2g_5+8g_2g_4+10g_3^2-28g_3g_2^2+12g_2^4), \tag{23b}$$

etc. ($a\equiv1/8\pi$).

When we discard the g_n and β_n with $n\geq3$, Eq. (23b) yields an expression for the β-function of model (3) modi-

fied only by the introduction of the IR regularizer m. The expression differs qualitatively from Pavlik's result[1] since it has a different sign in the one-loop contribution (the charge g in Ref. 1 is related to g_2 in (23b) as follows: $g=g_2^2/4\pi$). But such an approach is, of course, not consistent, since even for zero leading charges g_n with $n\geqslant 3$ their β-functions do not disappear because of the presence of "generation terms" of the form g_2^4 in β_3.

We now return to the complete system (23b) of β-functions. The fixed points of these functions, $g_*\equiv\{g_{n*}\}$, can be found from the condition $\beta_n(g_*)$ for all $n=2,3,\cdots$. The explicit form of the β-functions (23b) implies that in determining g_* we can choose the quantities g_{2*} and g_{3*} arbitrarily, and all the other g_{n*} with $n\geqslant 4$ can then be uniquely determined from the equations $\beta_k(g_*)=0$ with $k\geqslant 3$. This means that in the infinite-dimensional space of the charges $g\equiv\{g_n\}$ the RG-equation (9) has a two-dimensional surface of fixed points g_* parametrized by the values of g_{2*} and g_{3*}.

Generally, studying the nature of these points is a difficult task. According to the general rule,[17] a point g_* is IR-stable if the real parts of all the eigennumbers of the matrix $\omega_{nm}=\partial\beta_n/\partial g_m|_{g*}$ are strictly positive. The necessary condition for IR-stability is the requirement that all the diagonal elements ω_{nn} be positive. Equation (2) can be used to calculate these elements for all values of n:

$$\omega_{22}=-\varepsilon/2+a[9g_{3*}-21g_{2*}^2],$$

$$\omega_{33}=-\varepsilon+a[20g_{3*}-28g_{2*}^2]$$

and for $n\geqslant 4$ we have

$$\omega_{nn}=-\varepsilon/2(n-1)+a(n+1)[(n+1)g_{3*}-(2n+1)g_{2*}^2].$$

In a certain region $g_{3*}\geqslant 7g_{2*}^2/3+\text{const}$ all these quantities are positive. Although this is only a necessary condition and not a sufficient condition, we can assume that the surface of fixed points g_* contains a region of IR-stability. If this is so, then in the model considered there can be IR-scaling with nonuniversal critical dimensionalities (i.e., depending on the choice of the parameters g_{2*} and g_{3*} in the stability region). In dynamic models of the type (3) the critical dimensionality Δ_F of an arbitrary quantity F is given by the following relationship:[22]

$$\Delta_F=d_F^k-\Delta_t d_F^\omega+\gamma_F^*, \qquad \Delta_t=-2+\gamma_\nu^*, \tag{24}$$

where d_F^k and d_F^ω are canonical dimensionalities, Δ_t is the critical time dimensionality, $\gamma_F^*\equiv\gamma_F(g_*)$ is the proper anomalous dimensionality of F if it is renormalized,

$$F=Z_F F_R, \qquad \gamma_F=\tilde{\mathscr{D}}\mu\ \ln Z_F.$$

In the given case for $F=h$ we have $\gamma_h^*=0$ since the fields are not renormalized. Then, combining (24) with the data of Table I, we arrive at the exact relationship $2\Delta_h=\Delta_t+d$, and for Δ_t in the one-loop approximation we find from (23a) that $\Delta_t=-2+(g_{3*}-g_{2*}^2)/4\pi$. Generally, these quantities do not satisfy the condition $\Delta_h+\Delta_t=-2$, which in the model of Ref. 6 is assumed to hold and which, apparently, agrees with the numerical experiments discussed in Refs. 4, 5, 10, and 11.

CONCLUSION

Even with proper treatment, the model of Ref. 1 can hardly claim to be a satisfactory "microtheory" for the problem of a growing phase boundary. In any case, it has no advantages over the usual model of Ref. 2 although the latter is unable to rigorously substantiate IR-scaling. We also note that the Pavlik model has a disadvantage compared to the model of Ref. 6 in that the dynamic equation (2) contains the height h itself, rather than only its derivatives.

The general conclusion is pessimistic: the problem of building a satisfactory microtheory of the given phenomenon remains unresolved.

One of the authors (N.V.A.) would like to express his gratitude to S. É. Derkachev for fruitful discussions. The work was done with financial support from the Russian Fund for Fundamental Research (Project No. 93-02-14515 and the International Science Foundation (Grant ISF R 63000). N.V. Antonov acknowledges financial support by Association INTAS in the framework of research program of the International Center for Fundamental Physics in Moscow (Grant INTAS N33-2492).

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Translated by Eugene Yankovsky