Emission of scalar photons by an accelerated mirror in 1+1 space and its relation to the radiation from an electrical charge in classical electrodynamics

A. I. Nikishov and V. I. Ritus

P. N. Lebedev Institute of Physics, Russian Academy of Sciences, 117924 Moscow, Russia
(Submitted 29 March 1995)
Zh. Éksp. Teor. Fiz. **108**, 1121–1140 (October 1995)

It is shown that the spectrum of scalar photons emitted by an accelerated mirror in 1+1 space–time is the same as the photon spectrum emitted by an electrical charge in the analogous motion in 3+1 space–time. There is a deeper functional identity between the tensor energy-momentum of the scalar field produced by the mirror and the force of the Abragam–Lorentz–Dirac radiation reaction in classical electrodynamics. The numerical values of the functionally identical physical variables differ by the quantum factor $4\pi\alpha$, where α is the fine-structure constant. The negative energy-momentum fluxes discussed in the literature correspond to the change in the Schott term in the radiation reaction. They participate in the formation of emitted real particles, but are hard to observe because the uncertainty relation for the energy is not satisfied. The explanation for the constant energy flux with a thermal spectrum that proceeds from an exponentially moving mirror is that the exponentially increasing proper acceleration of the mirror exactly balances the radiation red shift, which is increasing in time. This does not hold for the mirror moving away with constant proper acceleration, for which an event horizon also exists, but for which the total emitted energy is finite. Thus, the presence of the horizon and the associated information loss about part of the system nevertheless does not lead to thermal emission. © *1995 American Institute of Physics.*

1. INTRODUCTION

As is well known, the mechanism proposed by Hawking[1] for the formation of particles by the gravitational field of a collapsing mass is analogous in many ways to the emission from an ideal mirror accelerated in vacuum.[2,3] Even the simplest model—the evolution of a mastless scalar quantum field in planar 1+1 space–time due to the acceleration of a reflecting boundary—has the main features of Hawking radiation. This model is treated in the present work in order to exhibit the spectral lines of the emission from scalar photons. It is found that not only the spectral lines, but also the spectra themselves, are functionally identical (when the appropriate covariant variables have been identified) with the spectra of photons emitted by an accelerated electric charge in ordinary 3+1 space–time. This assertion is proved in Sec. 5 for an arbitrary mirror or charge world line (in the x, t plane). Furthermore, as shown in Sec. 6, the space–time distribution of the energy–momentum for scalar and electromagnetic fields in these two problems are identical. Only the scales of the corresponding quantities differ; their ratio is equal to $4\pi\alpha$, (where α is the fine-structure constant). In Sec. 7 it is shown for the special but important case of quasihyperbolic motion that the negative energy fluxes which exist where true radiation forms do not satisfy the uncertainty relation for energy. This means that it is difficult to distinguish effects associated with the field accompanying an accelerated source in a region where true radiation develops.

In problems with moving mirrors two sets of solutions of the wave equation are ordinarily used:

$$\phi_{\text{in }\omega'}(u,v)=\frac{1}{\sqrt{2\omega'}}[e^{-i\omega' v}-e^{-i\omega' f(u)}],\tag{1}$$

$$\phi_{\text{out }\omega}(u,v)=\frac{1}{\sqrt{2\omega}}[e^{-i\omega g(v)}-e^{-i\omega u}],\tag{2}$$

given the $(x,\,t)$ plane as functions of the characteristic variables

$$u=t-x,\quad v=t+x.\tag{3}$$

These solutions satisfy the zero boundary condition $\phi(u,v)=0$ on the mirror whose v and u coordinates, respectively, are described by the functions $f(u)$ and $g(v)$:

$$v^{\text{mir}}=f(u),\quad u^{\text{mir}}=g(v).\tag{4}$$

It is evident that the functions f and g are inverses of one another.

An arbitrary solution of the wave equation with zero boundary condition on the mirror can be expanded in either the in or out system. The Bogolyubov coefficients α and β appear when one of these systems is expanded with respect to the other:

$$\phi_{\text{out }\omega}=\int_0^\infty\frac{d\omega'}{2\pi}(\alpha_{\omega'\omega}\phi_{\text{in }\omega'}+\beta_{\omega'\omega}\phi^*_{\text{in }\omega'}),\tag{5}$$

$$\phi_{\text{in }\omega'}=\int_0^\infty\frac{d\omega}{2\pi}(\alpha^*_{\omega'\omega}\phi_{\text{out }\omega}-\beta_{\omega'\omega}\phi^*_{\text{out }\omega}),\tag{6}$$

$$\alpha_{\omega'\omega}=i\int\phi^*_{\text{in }\omega'}\overleftrightarrow{\frac{\partial}{\partial t}}\phi_{\text{out }\omega}dx,\tag{7}$$

$$\beta^*_{\omega'\omega}=i\int\phi^*_{\text{in }\omega'}\overleftrightarrow{\frac{\partial}{\partial t}}\phi^*_{\text{out }\omega}dx.\tag{8}$$

From the conditions of orthogonality and normalization for these sets it follows that

$$\int_0^\infty \frac{d\omega''}{2\pi}(\alpha_{\omega''\omega}\alpha^*_{\omega''\omega'}-\beta_{\omega''\omega}\beta^*_{\omega''\omega'})=2\pi\delta(\omega-\omega'),$$
$$\int_0^\infty \frac{d\omega''}{2\pi}(\alpha_{\omega''\omega}\beta_{\omega''\omega'}-\beta_{\omega''\omega}\alpha_{\omega''\omega'})=0. \tag{9}$$

If the sets coincide, then we have $\beta_{\omega'\omega}=0$, and $\alpha_{\omega'\omega}=2\pi\delta(\omega'-\omega)$. Consequently, if we write

$$\alpha_{\omega'\omega}=2\pi\delta(\omega'-\omega)+\frac{A(\omega',\omega)}{\sqrt{2\omega 2\omega'}},$$
$$\beta^*_{\omega'\omega}=\frac{A(\omega',-\omega)}{\sqrt{2\omega 2\omega'}}, \tag{10}$$

then Eq. (9) implies, in particular, the optical theorem

$$-\mathrm{Re}\,A(\omega,\omega)=\int_0^\infty \frac{d\omega'}{8\pi\omega'}[|A(\omega',\omega)|^2-|A\times(\omega',-\omega)|^2]=\int_{-\infty}^\infty \frac{d\omega'}{8\pi\omega'}|A(\omega',\omega)|^2. \tag{11}$$

In the last identity we have used the relation $A(\omega',-\omega)=A^*(-\omega',\omega)$.

The explicit form of the in- and out-sets enables us to write the Bogolyubov coefficients in terms of Fourier transforms:

$$\alpha_{\omega'\omega},\beta^*_{\omega'\omega}=\sqrt{\frac{\omega'}{\omega}}\int_{-\infty}^\infty dv\,\exp\{i\omega' v\mp i\omega g(v)\} \tag{12}$$
$$=\sqrt{\frac{\omega}{\omega'}}\int_{-\infty}^\infty du\,\exp\{\mp i\omega u+i\omega' f(u)\}. \tag{13}$$

The spectrum $d\bar{n}_\omega$ of the average number of scalar protons produced by a moving mirror is given by

$$d\bar{n}_\omega=\left(\int_0^\infty |\beta_{\omega'\omega}|^2\frac{d\omega'}{2\pi}\right)\frac{d\omega}{2\pi}. \tag{14}$$

This physical meaning of $d\bar{n}_\omega$ is a consequence of the second-quantization field ϕ, in which the expansion coefficients of the field in terms of plane waves with positive and negative frequencies have the meaning of operators for the absorption of particles and the creation of antiparticles.

Note that if we replace the boundary condition $\phi=0$ on the mirror with the condition that the derivative of the field ϕ normal to the mirror world line equals zero,

$$n_\alpha\frac{\partial\phi}{\partial x_\alpha}\equiv\sqrt{f'(u)}\frac{\partial\phi}{\partial v}-\sqrt{g'(v)}\frac{\partial\phi}{\partial u}=0, \tag{15}$$

then in the *in*- and *out*-solutions (1) and (2) the signs in front of the second terms, i.e., the waves leaving to the right, are changed. However, the Bogolyubov coefficients remain the same, so that the radiation spectrum is insensitive to a similar alteration in the boundary condition on the mirror.

We will now consider the Bogolyubov coefficients and spectra for some special cases of the mirror motion.

2. INSTANTANEOUS CHANGE FROM A STATE OF REST TO UNIFORM MOTION WITH VELOCITY β

In this case we have $g(v)=v$ for $v\leqslant 0$ and $g(v)=(1-\beta/1+\beta)$ for $v\geqslant 0$. From (12) it follows that

$$\alpha_{\omega'\omega},\beta^*_{\omega'\omega}=\pi\sqrt{\frac{\omega'}{\omega}}[\delta_-(\omega'\mp\omega)+\delta_+(\omega'\mp\tilde{\omega})],$$
$$\tilde{\omega}=\frac{1-\beta}{1+\beta}\omega. \tag{16}$$

The singular $\delta_\pm$ functions are defined as in Ref. 4.

The calculation of the $d\bar{n}_\omega$ spectrum is conveniently performed by using for the $\delta_\pm$ functions the so-called ε representation:

$$\delta_\pm(x)=\frac{\pm i}{\pi(x\pm i\varepsilon)},\quad \varepsilon\to 0. \tag{17}$$

Then we obtain

$$d\bar{n}_\omega=\left(\frac{\tanh^{-1}\beta}{\beta}-1\right)\frac{d\omega}{2\pi^2\omega}. \tag{18}$$

This expression, multiplied by e^2, the square of the electrical charge in Heaviside units, is identical with the spectrum of the average number of photons emitted by an electric charge in similar motion (cf. Sec. 69 in Ref. 5). This similarity is no accident; it will be explained below. However, we can already say that the power-law behavior $\propto\omega^{-1}$ of the spectrum in the ultraviolet region is a consequence of the arbitrarily small portion of the trajectory with the velocity discontinuity, while that in the infrared region is due to the infinite portion with a nonzero velocity variation.

Note that the amplitude

$$A(\omega',\omega)=2\pi\omega'[\delta_+(\omega'-\tilde{\omega})-\delta_+(\omega'-\omega)] \tag{19}$$

in the positive range of frequencies ω', ω, determined by Eqs. (10) and (16), has a singularity. Hence the proof of the optical theorem encounters a problem associated with the integration of the square of a singular function. Thus, the usual rule for integrating the square of a δ function,

$$\int_{-\infty}^\infty \delta^2(x)dx=\delta(0) \tag{20}$$

is actually carried out by using for the δ function the K representation $\delta_K(x)=\sin Kx/\pi x$, in the limit $K\to\infty$. In this case $\delta_K(0)=K/\pi$ appears on the right-hand side of (20). However, it is not obtained when we use the ε representation $\delta_\varepsilon(x)=\varepsilon/\pi(x^2+\varepsilon^2)$, $\varepsilon\to 0$, in which after integration instead of $\delta_\varepsilon(0)=1/\pi\varepsilon$ we find half this quantity. Consequently, we find $-\mathrm{Re}\,A(\omega,\omega)=2\omega/\varepsilon$ when we use the ε representation (17) for the amplitude A in the left-hand side of optical theorem (11) and a quantity half as large on the right-hand side. The nonsingular contributions to the right-hand side corresponding to $\omega'>0$ and $\omega'<0$ do not depend on the representation; they are equal to

$$\pm\left(\frac{\tanh^{-1}\beta}{\beta}-1\right)\frac{1}{\pi} \tag{21}$$

and cancel out.

3. FINITE MOTION WITH TWO VELOCITY DISCONTINUITIES

Now we consider a mirror which moves during a finite time interval $0 \leqslant t \leqslant \pi/\Omega$, so that the variation of its u coordinate is given by $g(v) = v + a \sin \Omega v$, $0 \leqslant \Omega v \leqslant \pi$, while outside this interval it is at rest at the point $x=0$, i.e., $g(v) = v$ for $v \leqslant 0$ and $v \geqslant \pi/\Omega$. If $\xi(t)$ is the x coordinate of the mirror as a function of time, then its velocity is

$$\dot{\xi}(t) = \frac{1 - g'(v)}{1 + g'(v)}. \tag{22}$$

Hence the initial and final velocities β_0 and β_1 of this motion,

$$\beta_0 \equiv \dot{\xi}(0) = -\frac{\beta}{2+\beta}, \quad \beta_1 \equiv \dot{\xi}\left(\frac{\pi}{\Omega}\right) = \frac{\beta}{2-\beta}, \quad \beta = a\Omega, \tag{23}$$

have opposite signs, and their absolute values are different and are less than $|\beta| < 1$. We will assume $\Omega > 0$, so that the sign of β is the same as that of the amplitude a.

It is convenient to start by considering the amplitude $A(\omega', -\omega)$, which can be expressed in terms of Anger functions $\mathbf{J}_\nu(z)$ and Weber functions $\mathbf{E}_\nu(z)$:

$$A(\omega', -\omega) = 2\omega' \int_0^{\pi/\Omega} dv e^{i(\omega'+\omega)v}[e^{i\omega(g(v)-v)} - 1]$$

$$= \frac{2\pi\omega'}{\Omega} e^{i\pi\nu}\left[\mathbf{J}_\nu(z) - i\mathbf{E}_\nu(z) - \frac{1 - e^{-i\pi\nu}}{i\pi\nu}\right]. \tag{24}$$

Here we have written $\nu = (\omega' + \omega)/\Omega$ and $z = \omega a$. The expression in square brackets can be written according to Refs. 6 and 7 in the form

$$\mathbf{J}_\nu(z) - i\mathbf{E}_\nu(z) - \frac{1 - e^{-i\pi\nu}}{i\pi\nu} = e^{-i\pi\nu/2}\left[\sin\frac{\pi\nu}{2} R_\nu(z) + i \cos\frac{\pi\nu}{2} S_\nu(z)\right], \tag{25}$$

where R and S are even and odd functions of z, which can be represented in the form of convergent series

$$R_\nu(z) = -\frac{2}{\pi\nu}\left[\frac{z^2}{2^2 - \nu^2} - \frac{z^4}{(2^2-\nu^2)(4^2-\nu^2)} + \ldots\right], \tag{26}$$

$$S_\nu(z) = \frac{2}{\pi}\left[\frac{z}{1^2 - \nu^2} - \frac{z^3}{(1^2-\nu^2)(3^2-\nu^2)} + \ldots\right]. \tag{27}$$

Similarly, for the amplitude $A(\omega', \omega)$ we find

$$A(\omega', \omega) = \frac{2\pi\omega'}{\Omega}\left[\mathbf{J}_\nu(z) + i\mathbf{E}_\nu(z) + \frac{1 - e^{i\pi\nu}}{i\pi\nu}\right]$$

$$= \frac{2\pi\omega'}{\Omega} e^{i\pi\nu/2}\left[\sin\frac{\pi\nu}{2} R_\nu(z) - i \cos\frac{\pi\nu}{2} S_\nu(z)\right], \tag{28}$$

where we have written $z = \omega a$ as before, but now $\nu = (\omega' - \omega)/\Omega$. Then the integrals on the right-hand side of (11) take the form

$$\int_0^\infty \frac{d\omega'}{8\pi\omega'} |A(\omega', \pm\omega)|^2 = \frac{\pi}{2}\int_{\mp\mu}^\infty d\nu (\nu \pm \mu) Q(\nu, z), \tag{29}$$

where $\mu = \omega/\Omega$ and

$$Q(\nu, z) = \left|\mathbf{J}_\nu(z) - i\mathbf{E}_\nu(z) - \frac{1 - e^{-i\pi\nu}}{i\pi\nu}\right|^2$$

$$= \sin^2\frac{\pi\nu}{2} R_\nu^2 + \cos^2\frac{\pi\nu}{2} S_\nu^2, \tag{30}$$

which is even in ν and z.

As a result of this the optical theorem takes the form

$$-\mathrm{Re}\, A(\omega, \omega) \equiv 2\pi\mu(1 - J_0(z)) = \pi\mu \int_0^\infty d\nu Q(\nu, z). \tag{31}$$

The expression for the nontrivial integral

$$\int_0^\infty d\nu \left|\mathbf{J}_\nu(z) - i\mathbf{E}_\nu(z) - \frac{1 - e^{-i\pi\nu}}{i\pi\nu}\right|^2 = 2(1 - J_0(z)) \tag{32}$$

does not appear available in the standard tables, and so we checked it by integrating with respect to ν the first terms in the z expansion of the integrand.

The spectrum of the average number of emitted photons is given by Eq. (29) with the lower sign,

$$d\bar{n}_\omega = \frac{d\omega}{4\omega}\int_\mu^\infty d\nu (\nu - \mu) Q(\nu, z), \tag{33}$$

and depends on frequency through the two independent dimensionless parameters $z = \omega a$ and $\mu = \omega/\Omega$. Let us focus our attention on the fact that the spectrum does not depend on the sign of a, a change in which corresponds to time reversal $t \to -t$ and reflection $x \to -x$.

At high frequencies where $\mu \sim z \gg 1$ holds we find

$$d\bar{n}_\omega = \left\{2\left(\frac{\tanh^{-1}\beta}{\beta} - 1\right) - \ln\frac{1}{\sqrt{1-\beta^2}} + \ldots\right\}\frac{d\omega}{2\pi^2\omega}. \tag{34}$$

The ellipsis stands for small terms $\sim \mu^{-2} \sim z^{-2}$. Using (23) it is not hard to see that the expressions in curly brackets are equal to

$$\frac{\tanh^{-1}\beta_0}{\beta_0} - 1 + \frac{\tanh^{-1}\beta_1}{\beta_1} - 1, \tag{35}$$

that is, the high-frequency part of the spectrum consists of a sum of contributions from the two small portions of the trajectory in which the velocity changes instantaneously [compare the spectrum (18)]. For low frequencies, when $\mu \sim z \ll 1$ holds, we find

$$d\bar{n}_\omega = \left(\frac{\pi}{4}\mathrm{Si}(\pi) - \frac{1}{2} - \frac{\pi^2}{8}\mu + \ldots\right)\frac{z^2 d\omega}{\pi^2\omega}. \tag{36}$$

Here the ellipsis stands for terms $\sim\mu^2$, z^2. The infrared singularity in the spectrum is gone, since the trajectory no longer has an arbitrarily large portion with a nonzero change in velocity.

4. SEMIHYPERBOLIC TRAJECTORY WITH A DISCONTINUITY IN ACCELERATION

We will call the mirror trajectory semihyperbolic if prior to the time $t=0$ it is at rest at the point $x=0$, and then begins to move along the hyperbolic trajectory $\xi(t) = B - \sqrt{B^2+t^2}$. It is not difficult to show that the mirror velocity changes continuously, while the proper acceleration at $t=0$ jumps from 0 to $-B^{-1}<0$ and thereafter remains constant. In u, v language such a trajectory can be written in the form $g(v)=v$ for $v\leqslant 0$ and $g(v)=Bv/B-v$ for $0\leqslant v<B$. Then, using Eq. (12) with the upper limit replaced by B, we find

$$\beta^*_{\omega'\omega}=B\sqrt{\frac{z'}{z}}\left[e^{i(z'-z)}\int_0^1 dte^{iz/t-iz't}-\frac{i}{z+z'-i\varepsilon}\right], \tag{37}$$

where $z=\omega B$ and $z'=\omega' B$. The coefficient $\alpha_{\omega'\omega}$ is obtained by replacing z with $-z$ in the square brackets of (37). Integration with respect to ω' of the square of the absolute value of the amplitude $\beta_{\omega'\omega}$ leads to the following expression for the spectrum of the average number of emitted quanta:

$$d\bar{n}_\omega=-\mathrm{Re}\left\{1+e^{iz}\,\mathrm{Ei}(-iz)+ize^{2iz}\int_0^1 dte^{-iz(t+1/t)}\right.$$
$$\left.\times\mathrm{Ei}(-iz(1-t))\right\}\frac{d\omega}{2\pi^2\omega}. \tag{38}$$

At low frequencies, when $z\ll 1$ holds, we have

$$d\bar{n}_\omega=\left(\ln\frac{1}{\gamma z}-1+\pi z+\dots\right)\frac{d\omega}{2\pi^2\omega},\quad \ln\gamma=0.577\dots, \tag{39}$$

that is, the spectrum has an infrared singularity enhanced by a logarithm. This is associated with the existence of an arbitrarily large portion of the trajectory with a velocity which is not just finite but increasing. In the ultraviolet region, i.e., for $z\gg 1$, we have

$$d\bar{n}_\omega=\left(\frac{1}{10z^2}-\frac{1}{21z^4}+\dots\right)\frac{d\omega}{2\pi^2\omega}. \tag{40}$$

The more rapid decrease in the spectrum as a function of frequency then in Eqs. (18) and (34) ($\propto\omega^{-3}$) is due to the discontinuity in acceleration rather than to the velocity. Since this trajectory has a horizon $v=B$, the function $g(v)$ is defined only in the interval $-\infty<v<B$. Consequently, in calculating $\alpha_{\omega'\omega}$, $\beta^*_{\omega'\omega}$ we have actually used for $\phi_{\mathrm{out}\ \omega}$ the solutions

$$\phi_{\mathrm{out}\ \omega}(u,v)=\frac{1}{\sqrt{2\omega}}\left[e^{-i\omega g(v)}\theta(B-v)-e^{-i\omega u}\right] \tag{41}$$

with a Heaviside θ-function in the first term. Even though there are no waves in these solutions beyond the horizon moving to the left, they have the usual orthonormality properties. However, the representation of the coefficient $\alpha_{\omega'\omega}$ in the form (10) leaves the amplitude $A(\omega',\omega)$ singular at the point $\omega'=\omega$. Hence verification of the optical theorem (11) requires some subtlety, but the proof of the more general relations (9) for the Bogolyubov coefficients in the form (12) with the upper limit B encounters no difficulty if we use the following approach. In carrying out the integrals with respect to v of functions containing $\exp[-i\omega g(v)]$ near the upper limit $v=B$ we must shift the integration contour into the complex v plane so as to make $-\mathrm{Re}\,i\omega g(v)\to-\infty$ in the limit $v\to B$. This condition produces uniform damping of the leftward-moving wave ϕ_{out} on an arbitrarily small interval before the horizon. It corresponds to the usual rule for evaluating integrals with plane waves $e^{-i\omega v}$ in the limit $v\to\infty$.

The introduction of damping on the arbitrarily small interval $0<B-v<\delta\to 0$ before the horizon will lead to no misunderstanding if we recall the boundary conditions in the solution of the wave equation. As is well known, in order to obtain a unique solution of the wave equation in the $(t,\,x)$ plane to the right of a timelike curve Γ it is necessary to specify the value of the function on this curve and of its derivative normal to the curve. Rather than specifying the derivative on Γ we can specify the values of the function on the characteristic $u=u_R^-$ intersecting Γ in the distant past ($u_R^-\to-\infty$) or on the characteristic $v=v_R^+$ intersection Γ in the distant future ($v_R^+\to+\infty$). If in one of these two cases, e.g., in the past, the curve Γ and the characteristic $v=v_R^+$ do not intersect at any point because in the future the curve Γ has an asymptote (horizon) $v=B<v_R^+$, then the boundary values of the function on the curve Γ and the characteristic $v=v_R^+$ do not suffice to uniquely determine the solution in the region between them. If is necessary to prescribe other values of the function, e.g., on a portion of the characteristic $u=u_L^+$ as $u_L^+\to\infty$, connecting the curves Γ and $v=v_R^+$ in the far future. Only after this can the same solution be obtained both from the boundary conditions on Γ and $u=u_R^-$ and the boundary conditions on Γ, $u=u_L^+$ and $v=v_R^+$. In other words, this solution for prescribed values on Γ connects its values in the past on $u=u_R^-$ with its values in the future on $v=v_R^+$ and $u=u_L^+$.

Since the out system of functions (41) is not complete, the expansion with respect to it of an arbitrary solution $\phi(u,v)\equiv\phi_1(u)+\phi_2(v)$ of the wave equation with zero boundary condition on the trajectory Γ with horizon $v=B$ has the following form:

$$\phi(u,v)=\int_0^\infty\frac{d\omega}{2\pi}\left(\alpha_\omega\phi_{\mathrm{out}\ \omega}+\beta_\omega\phi^*_{\mathrm{out}\ \omega}\right)$$
$$+\phi_2(v)\theta(v-B). \tag{42}$$

The coefficients α_ω, β_ω are defined by Eqs. (7) and (8) with the usual replacements $\phi_{\mathrm{in}\ \omega'}\to\phi_{\mathrm{out}\ \omega}$, and $\phi_{\mathrm{out}\ \omega}\to\phi$. They contain all the information about $\phi_1(u)$ and about $\phi_2(v)$ for $v<B$, ensuring that the boundary condition on Γ and on $v=v_R^+$ is satisfied. The auxiliary term is determined by the boundary condition on $u=u_L^+$.

Note that the in system of solutions (1) for a trajectory with a horizon in the future is complete. Hence its expansion

(6) in terms of the incomplete out system in accordance with Eq. (42) should be supplemented by the term $(1/\sqrt{2\omega'})e^{-i\omega' v}\theta(v-B)$.

5. IDENTIFICATION OF THE EMISSION SPECTRA OF A SCALAR MIRROR IN 1+1 SPACE AND OF AN ELECTRIC CHARGE IN 3+1 SPACE

As is well known, the spectral distribution of the average number of photons emitted by an electrical charge moving along the trajectory $x_\alpha = x_\alpha(\tau)$ is given by the expression

$$d\bar{n}_{\mathbf{k}}^{el} = |j_\alpha(k)|^2 \frac{d^3k}{16\pi^3 k_0},$$

$$j_\alpha(k) = e\int_{-\infty}^{\infty} d\tau \dot{x}_\alpha(\tau) e^{-ik_\alpha x_\alpha(\tau)}, \tag{43}$$

where $j_\alpha(k)$ is the Fourier component of the current density of the charge, $\mathbf{k}$ and $k_0 = |\mathbf{k}|$ are the wave vector and frequency of the radiation, and τ is the proper time. For a charge which, like the mirror, executes rectilinear motion along axis 1, it is convenient to choose the variable $u \equiv x_-(\tau)$ in place of τ. Then the v coordinate of the charge, i.e., $x_+(\tau)$, will be a function of u. As before, we denote it by $f(u) \equiv x_+(\tau)|_{\tau=\tau(u)}$. As a result, the two nonzero components of the current density can be written in the form

$$(j_1, j_0) = -e\int_{-\infty}^{\infty} du \exp\left[\frac{i}{2}(k_+ u + k_- f(u))\right]\left(\frac{k_0}{k_-}, \frac{k_1}{k_-}\right). \tag{44}$$

Here and in what follows we write $k_\pm = k_0 \pm k_1$, $0 \leqslant k_\pm < \infty$. It can be seen that the current satisfies the transversality condition $k_\alpha j_\alpha = 0$. Using this expression for the current we find directly that

$$d\bar{n}_{\mathbf{k}}^{el} = \left| e\int_{-\infty}^{\infty} du \exp\left[\frac{i}{2}(k_+ u + k_- f(u))\right]\right|^2 \times \frac{k_+}{k_-}\frac{dk_+ dk_-}{16\pi^2}. \tag{45}$$

Here we have used the formula

$$\frac{d^3k}{k_0} = \frac{1}{2} dk_+ dk_- d\varphi \to \pi dk_+ dk_- \tag{46}$$

and have integrated with respect to the azimuthal angle φ, on which the spectrum does not depend.

The spectrum $d\bar{n}_{\mathbf{k}}^{el}$ differs from the spectrum

$$|\beta_{\omega'\omega}|^2 \frac{d\omega d\omega'}{(2\pi)^2} = \left|\int_{-\infty}^{\infty} du \times \exp[i(\omega u + \omega' f(u))]\right|^2 \frac{\omega d\omega d\omega'}{(2\pi)^2 \omega'} \tag{47}$$

of the radiation from the scalar mirror in 1+1 space–time only in the factor e^2, the square of the electrical charge in Heaviside units. Here the components $(1/2)k_+$, $(1/2)k_-$ of the wave vector k_α are identical with the frequencies ω, ω', respectively.

This similarity is not an accident, since in 1+1 space the parameter ω (ω') is at the same time both the frequency and the wave number of the wave propagating in the positive (negative) x direction. In a Lorentz transformation with velocity V in the x direction the quantities ω, ω' are therefore transformed according to the same rules,

$$\omega = D\tilde{\omega}, \quad \omega' = D^{-1}\tilde{\omega}', \quad D = \sqrt{\frac{1-V}{1+V}}, \tag{48}$$

as the components k_+, k_- and the characteristic coordinates v, u. The tilde indicates quantities in the transformed coordinate system. After this remark it is easy to see that the spectra (45) and (47), like the volume element (46), are invariant under this transformation.

It is useful to introduce an expression for the spectrum of the average number of quanta emitted by a scalar mirror (the source of the scalar field) moving along the same trajectory $x_\alpha(\tau)$ in 3+1 space. It differs from (43) in replacing the current density $j_\alpha(k)$ with the scalar charge density $\rho(k)$:

$$d\bar{n}_{\mathbf{k}}^{sc} = |\rho(k)^2| \frac{d^3k}{16\pi^3 k_0},$$
$$\rho(k) = e\int_{-\infty}^{\infty} d\tau \exp[-ik_\alpha x_\alpha(\tau)]. \tag{49}$$

For rectilinear motion parallel to axis 1, we find in u, v variables

$$d\bar{n}_{\mathbf{k}}^{sc} = \left| e\int_{-\infty}^{\infty} du \sqrt{f'(u)} \times \exp\left[\frac{i}{2}(k_+ u + k_- f(u))\right]\right|^2 \frac{dk_+ dk_-}{16\pi^2}. \tag{50}$$

This expression differs from $d\bar{n}_k^{el}$ in having the function $\sqrt{f'(u)}$ in the integrand rather than the factor $\sqrt{k_+/k_-}$. Thus, the spectra of scalar and electrical charges differ significantly. However, for a sufficiently smooth trajectory and large values of $k_\pm$, when the integrals in (45) and (50) can be evaluated by the method of steepest descent, these spectra are the same. Specifically, in this case the saddle point $u = u_0$ satisfies the equation

$$f'(u_0) = -\frac{k_+}{k_-} \tag{51}$$

and lies in the complex u plane, since $f'(u) > 0$ holds on the real axis. In this range of $k_\pm$ the spectra therefore have the exponential asymptotic form

$$d\bar{n}_{\mathbf{k}}^{el} \approx d\bar{n}_{\mathbf{k}}^{sc} \approx \frac{e^2 k_+}{k_-^2 |f''(u_0)|} \times \exp[-\mathrm{Im}(k_+ u_0 + k_- f(u_0))] \frac{dk_+ dk_-}{4\pi}. \tag{52}$$

This asymptotic behavior agrees with the theorem according to which the Fourier component of a smooth function in the high-frequency region falls off faster than any power of the frequency.[8] Compare Ref. 9 regarding the high-frequency asymptotic spectrum in electrodynamics.

It is instructive to compare the spectra of (45) and (50) for charges executing exponential motion,

$$f(u)=\frac{1}{\kappa}-\frac{1}{\kappa}e^{-\kappa u}.$$

In this case Eqs. (45) and (50) yield the Bose–Einstein and Fermi–Dirac distributions with respect to the component k_+:

$$d\bar{n}_{\mathbf{k}}^{el,sc}=\frac{e^2}{\exp(\pi k_+/\kappa)\mp 1}\frac{dk_+dk_-}{4\pi\kappa k_-}, \tag{53}$$

although the quanta of the two fields have integral spins 1 and 0. It is clear that the form of the spectrum is determined not by the statistics of the quanta but by the trajectory of the source and its interaction with the radiation field. For $k_+\gg\kappa$ the spectra become identical and are given by Eq. (52).

Note that the parameter κ determines the proper acceleration of a charge at the turning point, $a_0\equiv a(u=0)=-\kappa/2$. The spectra develop in a region of space–time near the turning point with dimensions of order κ^{-1}. If we transform to another Lorentz frame moving with velocity V in the x direction relative to the original one, the proper acceleration of the charge at the turning point of the new system will be $\tilde{a}_0=D^{-1}a_0$, i.e., the parameter $\varkappa$ transforms like k_+ [cf. Eq. (48)].

When electrical and scalar charges move along the hyperbolic trajectory $f(u)=u/(1+\kappa u)$ the radiation spectra are described by the square of a modified Bessel function of the second kind:

$$d\bar{n}_{\mathbf{k}}^{el,sc}=e^2K_{1,0}^2\left(\frac{\sqrt{k_+k_-}}{\kappa}\right)\frac{dk_+dk_-}{4\pi^2\kappa^2}. \tag{54}$$

Their order is the same as the spin of the emitted quanta. The spectra are symmetric with respect to the interchange $k_+\leftrightarrow k_-$ because of the t-invariance of the motion. The proper acceleration of a charge is constant along the entire trajectory, $a=-\kappa$, so under a Lorentz transformation the parameter κ remains unchanged. For $k_+k_-\gg\kappa^2$ spectra become identical, assuming the exponential form (52).

The fact that the radiation spectra of an electrical charge in 3+1 space and a scalar mirror in 1+1 space are identical raises the question, for what mirror in 1+1 space is the radiation spectra analogous to that of a scalar charge in 3+1 space? Such a mirror cannot be a scalar mirror with the modified boundary condition (15), since the Bogolyubov coefficients with such a modification remain unchanged. From Eq. (50) for $d\bar{n}_{\mathbf{k}}^{sc}$ it follows that in order to reproduce this spectrum in 1+1 space–time it is necessary that the Bogolyubov coefficients assume the form

$$\alpha_{\omega'\omega},\beta^*_{\omega'\omega}=\int_{-\infty}^{\infty}dv\,\sqrt{g'(v)}\exp[i\omega' v\mp i\omega g(v)] \tag{55}$$

$$=\int_{-\infty}^{\infty}du\,\sqrt{f'(u)}\exp[\mp i\omega u+i\omega' f(u)]. \tag{56}$$

It is not difficult to show that these coefficients satisfy conditions (9) with the plus sign in front of the second terms on the left-hand side. This means that in 1+1 space–time we are dealing with a spinor mirror interacting with a spinor field.[3]

6. RELATION BETWEEN THE ENERGY-MOMENTUM TENSOR OF A SCALAR FIELD AND THE ABRAGAM–LORENTZ–DIRAC RADIATION REACTION FORCE

The total energy of photons emitted by a scalar charge to infinity is obviously equal to

$$\mathscr{E}^R=\int_0^\infty\int_0^\infty\omega|\beta_{\omega'\omega}|^2\frac{d\omega d\omega'}{(2\pi)^2}. \tag{57}$$

From the fact established in the previous section, that the spectra and the transformation properties of the quantities ω, ω' are identical [cf. Eq. (48)], it follows that the classical electrodynamic quantity which is the same as $\mathscr{E}^R$ is

$$\frac{1}{2}G_+^R=\int\frac{1}{2}k_+d\bar{n}_{\mathbf{k}}^{el}. \tag{58}$$

Here we have written $G_+^R=G_0^R+G_1^R$, where G_0^R and G_1^R are the energy and the x component of the momentum of the radiation.

On the other hand, the energy-momentum density of the scalar field produced by an accelerated scalar mirror is determined by the energy-momentum tensor. According to Ref. 10 the only nonzero component $T_{uu}\equiv T_{++}$ in (u, v coordinates) is equal to

$$T_{++}(u)=\frac{1}{12\pi}\left[\left(\frac{f''}{2f'}\right)^2-\left(\frac{f''}{2f'}\right)'\right], \tag{59}$$

where the prime denotes a derivative with respect to u. We emphasize that the quantities (57) and (59) are quantum-mechanical; they are proportional to Planck's constant $\hbar$. Note that the additional singular term in (59) has dropped out as a result of "renormalization." The energy-momentum tensor, which is the average value of the product of two field operators at the same point, is ill-defined. It becomes a completely defined function of the coordinates of these points if they are separated by a timelike or spacelike interval. The singular term omitted from Eq. (59) appears when this function is expanded in powers of the small proper distance 2ε between the points, and in the limit $\varepsilon\to 0$ it behaves as ε^{-2} (Ref. 10). Its T_{++}^ε component is equal to

$$T_{++}^\varepsilon=-\frac{1}{16\pi\varepsilon^2}, \tag{60}$$

if the points are displaced in time or along the spatial axis. The next term in the expansion is $\sim\varepsilon_0$ and is given in (59). It is just this which is the physically observable variable. Such a procedure for eliminating the divergence can only be justified if the above assertion agrees with reality.

The integral

$$\mathscr{E}=\int_{-\infty}^{\infty}du\;T_{++}(u) \tag{61}$$

signifies the total energy of the field produced by an accelerated mirror, and it has the same transformation properties as does $\mathscr{E}^R$.

In classical electrodynamics the rate of change of the 4-momentum G_α of an accelerated charge is equal to

$$\frac{dG_\alpha}{d\tau}=\frac{e^2}{8\pi\varepsilon}\,a_\alpha(\tau)-g_\alpha(\tau),\quad g_\alpha=\frac{e^2}{6\pi}\left(\frac{d^2u_\alpha}{d\tau^2}-u_\alpha a^2\right),\tag{62}$$

where g_α is the 4-force of radiative reaction, u_α is the 4-velocity, and a_α is the 4-acceleration of the charge. The physical meaning of the two terms in the radiative reaction force are the self-radiation term

$$g_\alpha^R=-\frac{e^2}{6\pi}\,a^2u_\alpha\tag{63}$$

and the Schott curve

$$g_\alpha^S=\frac{e^2}{6\pi}\frac{d^2u_\alpha}{d\tau^2},\tag{64}$$

was discussed by Dirac,[11] Thirring,[12] Rohrlich,[13] and Teitelboim.[14] The singular term equal to the space–time derivative of the 4-vector $e^2u_\alpha/8\pi\varepsilon$ is usually called the Coulomb term. In fact, the 4-vector $e^2u_\alpha/8\pi\varepsilon$ in the rest frame of the charge reduces to the energy of its Coulomb field beyond the radius ε, and in a general system it is the energy and momentum of the field of a uniformly moving charge. In the Abragam–Lorentz–Dirac equation this term changes the inertial properties of the charge, causing its mass to be redefined (renormalized), $m_0\to m_0+e^2/8\pi\varepsilon\equiv m$. This also raises the question of the imprecise nature of a charge with dimensions ε greater than or equal to the classical radius $r_0=e^2/4\pi m$.

Moreover, for an accelerated charge the above covariant expression for the singular term, which does not depend on the motion of the charge in the past, occurs only for a particular way of integrating the energy—momentum tensor of the field of the charge, assuming also that in the distant past the world line of the charge was straight.[14] Consequently, the classical description of the effect of the proper field of the charge on its motion becomes internally inconsistent for very large proper accelerations, when the characteristic length c^2/a over which the radiation is produced and on which the charge changes its energy by an amount $\sim mc^2$ becomes comparable to the smallest classical dimension r_0 of the charge, i.e., $a\sim 4\pi mc^4/e^2$.

Because of quantum effects, however, the classical theory ceases to be valid for relatively low accelerations of the charge, when the length c^2/a becomes comparable to the Compton wavelength $\hbar/mc$, i.e., for $a\sim mc^3/\hbar$. The effective dimensions of the charge in self-energy effects also turns out to be on the order of the Compton wavelength, $\varepsilon\sim\hbar/mc$, which gives rise to a field part of the mass of order αm, where $\alpha=e^2/4\pi\hbar c$ is the fine-structure constant. We can therefore anticipate that when the charge has a sufficiently small proper acceleration $a\ll mc^3/\hbar$, the classical expression for the radiation reaction force will be valid. In this connection we recall the quantum-mechanical calculation of self-energy effects when an electron moves in a constant electric field,[15,16] which for small proper accelerations $a\ll mc^3/\hbar$ predicts the classical shift

$$\mathrm{Re}\,\Delta m=-e^2a/8\pi\tag{65}$$

in the electron mass and an average number of photons

$$\frac{d\bar n}{d\tau}\equiv-2\,\mathrm{Im}\,\Delta m=\frac{e^2a^2}{4\pi^2}\left(2\ln\frac{a}{\gamma k_{\perp\,\min}}-1\right),\tag{66}$$

emitter per unit proper time, exactly agreeing with the classical spectrum (54) and the classical radiation intensity determined by g_α^R.

We will be interested in the component $G_+=G_0+G_1$, or more precisely

$$\frac{1}{2}G_+=-\frac{1}{2}\int g_+d\tau=\frac{e^2}{12\pi}\int_{-\infty}^{\infty}d\tau\left(u_+a^2-\frac{d^2u_+}{d\tau^2}\right).\tag{67}$$

It has the same Lorentz—covariant property as does $\mathscr{E}$, and for an accelerated charge moving along the rectilinear trajectory of the mirror these quantities and their integrands are identical to within the factor $e^2=4\pi\alpha$:

$$e^2T_{++}du=-\tfrac{1}{2}\,g_+d\tau.\tag{68}$$

In fact, if $\xi(t)$ is the x coordinate of the charge as a function of the time t and $f(u)$ is its v coordinate as a function of the u coordinate, then for the charge velocity $\dot\xi$, the u_+ component of its 4-velocity, and its proper acceleration a we have the formulas

$$\dot\xi=\frac{f'-1}{f'+1},\quad u_+=\frac{1+\dot\xi}{\sqrt{1-\dot\xi^2}}=\sqrt{f'(u)},$$
$$a=\frac{\ddot\xi}{(1-\dot\xi^2)^{3/2}}=\frac{f''}{2f'^{3/2}}.\tag{69}$$

Using them we find three equivalent expressions for the right-hand side of (68):

$$-\frac{1}{2}g_+d\tau=\frac{e^2}{12\pi}\left(u_+a^2-\frac{d^2u_+}{d\tau^2}\right)d\tau\tag{70}$$

$$=\frac{e^2}{12\pi}\left[\frac{(1+\dot\xi)\ddot\xi^2}{(1-\dot\xi^2)^3}-\frac{d}{dt}\left(\frac{(1+\dot\xi)\ddot\xi}{(1-\dot\xi^2)^2}\right)\right]dt\tag{71}$$

$$=\frac{e^2}{12\pi}\left[\left(\frac{f''}{2f'}\right)^2-\frac{d}{du}\left(\frac{f''}{2f'}\right)\right]du,\tag{72}$$

confirming Eq. (68). The first, positive-definite, term in each of these expressions yields $(1/2)G_+^R$ when integrated with respect to the corresponding variable; this is the + component of the momentum of the radiation proceeding to infinity. For its spectral representation see Eq. (58). The second term in Eqs. (70)–(72) is the total differential of the quantity

$$-\frac{e^2}{12\pi}\frac{du_+}{d\tau}=-\frac{e^2}{12\pi}\frac{(1+\dot\xi)\ddot\xi}{(1-\dot\xi^2)^2}=-\frac{e^2}{12\pi}\left(\frac{f''}{2f'}\right),\tag{73}$$

which can be termed the Schott energy—momentum or the acceleration energy–momentum. Its changes constitute a reversible form of emission and absorption of the field energy–momentum in the region in which the radiation is formed, irreversibly proceedig to infinity. As a result of these variations the energy density in this region can be negative. However, we will show that the amount of this energy is inconsistent with the uncertainty principle.

The Schott energy–momentum does not contribute to G_+ if the charge acceleration vanishes at the ends of the trajectory or if the velocity and acceleration of the charge return to their original values. But if this does not happen, that the energy–momentum of the genuine emission cannot be distinguished from the energy–momentum accompanying the charge.

We assume that the Schott term in the energy–momentum tensor of the scalar field bounding the moving mirror describes the reversible exchange of energy–momentum between the mirror and the field, just as in electrodynamics. For a mirror with acceleration that does not vanish asymptotically (i.e., for a trajectory with a horizon) the region in which genuine radiation forms is infinitely extended and this emission cannot readily be distinguished from the reversible transmission of energy–momentum. As long as this is happening, the fields at two points separated by a spacelike interval (e.g., simultaneous) are causally related, correlated on account of that portion of the mirror trajectory which is between the past light cones of these points.

Now let us compare the singular terms in expressions (62) for $dG_\alpha/d\tau$ and in the energy–momentum tensor of a scalar field [cf. Eq. (60)]. The former becomes physically meaningful in the proper frame of the charge, where it represents the electromagnetic mass $e^2/8\pi\varepsilon$ of the charge. In the rest frame of the segment joining the separated points of the energy–momentum tensor, the second term represents the linear energy–momentum density on this interval. Since the proper length of the interval equals 2ε, the energy–momentum concentrated on it is equal to $-1/8\pi\varepsilon$, where the minus sign means that this energy is associated with binding and attraction. The fluctuations of the scalar field energy after time 2ε have this value.

Thus, the invariant quantities determined by the singular terms differ through the factor e^2. The dimensionless quantum factor $e^2=4\pi\alpha$ is the natural scale through which quantum-mechanical quantities for a scalar field differ from the classical variables of electrodynamics. We remark further that just as quantum electrodynamics introduces a lower bound for the length ε equal to the Compton wavelength $\hbar/mc$, the quantum theory of a scalar mirror should give a lower bound to the analogous length parameter equal to some characteristic quantity ε_0. It can be seen that ε_0 plays the role of the minimum dimensions of the mirror, which are unimportant as long as the proper acceleration of the mirror is not excessively large, namely, for $c^2/a \gg \varepsilon_0$. This restriction on the magnitude of the proper acceleration should be kept in mind, especially in treating exponential motion, when the proper acceleration

$$a=-\frac{\kappa}{2-\kappa\tau} \qquad (74)$$

increases from the characteristic value $\kappa/2$ to infinity in a finite proper time $\tau=2\kappa^{-1}$.

7. ENERGY–MOMENTUM DISTRIBUTION FOR QUASIHYPERBOLIC MOTION OF THE SOURCE AND THE UNCERTAINTY RELATION FOR ENERGY

Consider a mirror (or an electrical charge) moving along the trajectory

$$x=\xi(t), \quad \xi(t)=\frac{v_0^2}{\kappa}-v_0\sqrt{\left(\frac{v_0}{\kappa}\right)^2+t^2}, \qquad (75)$$

where $\pm v_0$ is the velocity of the mirror at $t\to\mp\infty$ and $-\kappa$ is its acceleration at the turning point at $t=0$. We will call this trajectory quasihyperbolic, since in the limit $v_0\to 1$ it approaches hyperbolic on an increasing portion of space–time. We obtained the radiation spectrum $d\mathscr{E}_{\mathbf{k}}\equiv\omega d\bar{n}_{\mathbf{k}}$ and the total emitted energy $\mathscr{E}$ of the charge on this trajectory previously:[17]

$$d\bar{n}_{\mathbf{k}}=\frac{e^2v_0^6z^2}{z_1^2}K_1^2(z_1)\frac{dk_+dk_-}{4\pi^2\kappa^2}, \quad \mathscr{E}=\frac{e^2\kappa v_0}{16\sqrt{1-v_0^2}},$$
$$z=\sqrt{k_+k_-}/\kappa, \quad z_1=(v_0/\kappa)\sqrt{k_+k_-+k_1^2(1-v_0^2)}, \qquad (76)$$
$$2k_1=k_+-k_-.$$

Since for arbitrary $v_-\neq 1$ the proper acceleration vanishes at infinity, the energy spectrum of the radiation has no infrared divergence and the total energy and effective emission time are finite. However, the spectrum of the average number of emitted quanta contains an infrared divergence due to the infinite part of the trajectory with a finite velocity variation.

Here we will not concern ourselves with the question regarding which part of the quasihyperbolic mirror trajectory has $T_{++}<0$ and what the effective size of this region is. Since for the quasihyperbolic trajectory we have

$$\ddot{\xi}=-\frac{\kappa}{v_0^3}(v_0^2-\dot{\xi}^2)^{3/2}, \qquad (77)$$

we find using Eqs. (68) and (71) that

$$T_{++}du=G(\dot{\xi})dt, \quad G(z)=\frac{\kappa^2(1-v_0^2)}{4\pi v_0^6}\frac{z(1+z)(v_0^2-z^2)^2}{(1-z^2)^3}. \qquad (78)$$

It can be seen that $T_{++}>0$ holds for $\dot{\xi}>0$ and $T_{++}<0$ holds for $\dot{\xi}<0$, i.e., in the portions $t<0$ and $t>0$, respectively. The function $G(z)$ vanishes at the turning point $z=0$ and at the ends $z=\pm v_0$ of the physical range of velocities $|z|\leqslant v_0$, while at the points z_1 and z_2 satisfying the equation

$$(1+z)(1-z^2)-(1-v_0^2)(1+z+4z^2)=0 \qquad (79)$$

and lying to the left and to the right of $z=0$ it attains a negative minimum and positive maximum, respectively.

The minimum and maximum values of the function $G(z)$ are equal, respectively, to

$$G(z_{1,2})=\begin{cases}\dfrac{\kappa^2(1-v_0^2)}{8\pi},\ \dfrac{2\kappa^2}{27\pi} & \text{for } 1-v_0^2\ll 1, \quad (80)\\[2ex] \mp\dfrac{4\kappa^2}{25\sqrt{5}\pi v_0} & \text{for } v_0\ll 1. \quad (81)\end{cases}$$

In order to find the effect of time over which in the region of formation the energy density acquires values on the order of those in Eqs. (80) and (81), we integrate (78) for $t>0$ and $t<0$, respectively. Here it is convenient to use the relation (77) and to transform to velocity as the integration variable. Then we obtain

$$\mathscr{E}_{1,2}\equiv\int_{t\gtrless 0}T_{++}du=\frac{\kappa v_0}{64\sqrt{1-v_0^2}}\times\left\{1\mp\frac{2}{\pi v_0^3}\left[\frac{\sin^{-1}v_0}{v_0}-(1-2v_0^2)\sqrt{1-v_0^2}\right]\right\}. \quad (82)$$

It can readily be seen that the first and second terms on the right are the contributions from the components g_0 and g_1 which enter into the makeup of T_{++}, as follows from Eq. (68). These components have positive and negative parity under time reversal and are related by

$$g_0=\dot{\xi}g_1. \quad (83)$$

The second term in (82) is therefore larger in absolute value than the first, since we have $\mathscr{E}_1<0$ and $0<-\mathscr{E}_1<\mathscr{E}_2$. In particular, in the ultrarelativistic and nonrelativistic limits we have

$$\mathscr{E}_{1,2}=\begin{cases}-\dfrac{\kappa\sqrt{1-v_0^2}}{32},\ \dfrac{\kappa}{32\sqrt{1-v_0^2}} & \text{for } 1-v_0^2\ll 1, \quad (84)\\[2ex] \mp\dfrac{\kappa}{12\pi}\left(1\mp\dfrac{3\pi}{16}v_0\right) & \text{for } v_0\ll 1. \quad (85)\end{cases}$$

The effective time intervals Δt_1 and Δt_2 during which the energy density assumes negative and positive values on the order of the extremal values is naturally determined by

$$G(z_{1,2})\Delta t_{1,2}=\mathscr{E}_{1,2}. \quad (86)$$

Then for $1-v_0^2\ll 1$ we have

$$\Delta t_{1,2}=\frac{\pi}{4\kappa\sqrt{1-v_0^2}},\ \frac{27\pi}{64\kappa\sqrt{1-v_0^2}}, \quad (87)$$

while for $v_0\ll 1$ we have

$$\Delta t_{1,2}=\frac{25\sqrt{5}v_0}{48\kappa}. \quad (88)$$

It is noteworthy that even in the ultrarelativistic case the times $\Delta t_{1,2}$ are comparable. Thus, for the magnitudes of the action on the intervals $\Delta t_{1,2}$ we find

$$|\mathscr{E}_1|\Delta t_1=\begin{cases}\dfrac{\pi}{128}, & \text{for } 1-v_0^2\ll 1, \quad (89)\\[2ex] \dfrac{25\sqrt{5}}{576\pi}v_0, & \text{for } v_0\ll 1, \quad (90)\end{cases}$$

$$\mathscr{E}_2\Delta t_2=\begin{cases}\dfrac{27\pi}{2048(1-v_0^2)}, & \text{for } 1-v_0^2\ll 1, \quad (91)\\[2ex] \dfrac{25\sqrt{5}}{576\pi}v_0, & \text{for } v_0\ll 1. \quad (92)\end{cases}$$

Although in the ultrarelativistic case the effective time interval Δt_1 can be very large, the energy $\mathscr{E}_1$ accumulated over this interval is too small to be observable and does not contradict the uncertainty relation for energy [cf. Eqs. (87) and (89)]. But increasing the observation time Δt_1 by a considerable amount causes the total radiative energy $\mathscr{E}=\mathscr{E}_1+\mathscr{E}_2$, which is always positive, to fall into the extended interval. Ford,[18] who showed that the second law of thermodynamics was not violated, confirmed a similar assertion about the negative energy.

If we leave only the radiative part in the energy–momentum tensor T_{++} [which according to Eq. (68) is identical with the first term of any of Eqs. (70)–(72) without the factor e^2], then in place of (82) we find

$$\mathscr{E}^R_{1,2}\equiv\int_{t\gtrless 0}T^R_{++}du=\frac{\kappa v_0}{64\sqrt{1-v_0^2}}\left\{1\mp\frac{2}{\pi v_0^3}\left[\frac{\sin^{-1}v_0}{v_0}-\left(1+\frac{2}{3}v_0^2\right)\sqrt{1-v_0^2}\right]\right\}. \quad (93)$$

Here the first and second terms on the right are contributions from the components g_0^R and g_1^R, which enter into T^R_{++}. In contrast to g_0 and g_1, they are related by

$$g_1^R=\dot{\xi}g_0^R, \quad (94)$$

the opposite of (83). Hence the second term in (93) is smaller in absolute value than the first, so that $0<\mathscr{E}_1^R<\mathscr{E}_2^R$ holds. We further note that the first terms in (82) and (93) are the same, since the contribution from g_0^S vanishes because of the vanishing of the Schott energy at both $t=0$ and $t=\pm\infty$. The second terms in (82) and (93) differ as a result of the contribution from the Schott component g_1^S, which is independent of v_0:

$$\mathscr{E}^S_{1,2}=\mathscr{E}_{1,2}-\mathscr{E}^R_{1,2}=\mp\frac{\kappa}{12\pi}. \quad (95)$$

Now we consider the radiation from a mirror on a semihyperbolic trajectory, for which prior to $t=-\varepsilon$, $\varepsilon\to 0$ the mirror was at rest. Then it underwent an acceleration $-\kappa$ over a vanishingly small interval $-\varepsilon<t<0$; then it moved along the quasihyperbolic trajectory. In this case the result reduces to the upper form of Eq. (93):

$$\int_{t>-\varepsilon}T_{++}du=\frac{\kappa v_0}{64\sqrt{1-v_0^2}}\left\{1-\frac{2}{\pi v_0^3}\left[\frac{\sin^{-1}v_0}{v_0}-\left(1+\frac{2}{3}v_0^2\right)\sqrt{1-v_0^2}\right]\right\}, \quad (96)$$

since the contributions from the Schott component g_+^S, which are equal to $\mathscr{E}_1^S$ and $\mathscr{E}_2^S$ on the segment $0<t<\infty$ and $-\varepsilon<t<0$, cancel out and the contribution from g_+^R in the segment $-\varepsilon<t<0$ vanishes in the limit $\varepsilon\to 0$.

A notable property of Eq. (96) is that in the limit $v_0 \to 1$, i.e., for a semihyperbolic trajectory, it approaches the finite value

$$\int_{t>-\varepsilon} T_{++} du = \frac{\kappa}{12\pi}. \tag{97}$$

This means that the total radiated energy from a scalar mirror for semihyperbolic motion is finite and equal to

$$\int_0^\infty \omega d\bar{n}_\omega = \frac{\kappa}{12\pi}, \tag{98}$$

whereas for hyperbolic motion it is infinite. The finiteness of the radiated energy essentially follows from the spectrum $d\bar{n}_\omega$ of the average number of radiated quanta given in Eq. (38) and its behavior for small and large frequencies. However, calculation of the total radiated energy by direct integration of $\omega d\bar{n}_\omega$ with respect to frequency is difficult.

Note that for the exponential and hyperbolic motions of the radiation source treated in Sec. 5 $T_{++} \geqslant 0$ holds everywhere. In the former case we have $f''/2f' = \text{const}$, i.e., the Schott energy is constant and only the first (positive) term in the power of the real emission remains in Eq. (59). In the latter case the power of the real emission is exactly equal to the rate of change of the Schott energy, so that $T_{++}=0$ holds.

8. CONCLUSION

Thus, there is a close analogy between the radiation of scalar quanta from an accelerated mirror in 1+1 space and the radiation of photons by an accelerated electrical charge in 3+1 space. As a result, the radiated spectra and the space–time distributions of the energy–momentum of the radiated field are the same. Since the electrodynamic processes have been studied quite thoroughly, this analogy should lead to a better understanding of processes induced in vacuum by an accelerated mirror, and also to an understanding of the difference between them and Hawking radiation. In particular, it was found that for any prescribed motion of the mirror, where one can ignore recoil, the probability of emitting a certain number of quanta is distributed according to a geometrical progression, as in the production of electron pairs by a field.[19,20] Furthermore, the field produced by a classical current is in a pure coherent state,[21] while the probability of radiating a certain number of photons is given by a Poisson distribution (cf. Sec. 9 in Ref. 12). Accordingly we feel that the Bose–Einstein and Fermi–Dirac spectra in Eqs. (53) with a "temperature" proportional to the acceleration at the turning point are not related to the statistics of the emitted quanta, but are determined by the exponential shape of the source trajectory and the spin of the field.

The referee has drawn our attention to Refs. 22 and 23, in which the effects of null oscillations of the electromagnetic field are considered. The relation between these effects and the production of particles by an accelerated mirror, treated in Refs. 1–3, 10, and 18 and the present work has not been established.

We express our gratitude to the Soros Fund and the Russian Fund for Fundamental Research for financial support.

[1] S. W. Hawking, Commun. Math. Phys. **43**, 199 (1975).
[2] N. D. Birrell and P. C. W. Davies, *Quantum Fields in Curved Space*, Cambridge Univ. Press, Cambridge (1982).
[3] B. S. DeWitt, Physics Reports **19C**, 295 (1975).
[4] A. I. Akhiezer and V. B. Berestetskii, *Quantum Electrodynamics*, Wiley, New York (1965).
[5] L. D. Landau and E. M. Lifshitz, *The Classical Theory of Fields*, Pergamon, Oxford (1975).
[6] E. Jahnke, F. Emde, and F. Lösch, *Tables of Higher Functions*, McGraw-Hill, New York (1960).
[7] A. Erdély (ed.), *Higher Transcendental Functions*, McGraw-Hill, New York (1953).
[8] E. C. Titchmarsh, *Introduction to the Theory of Fourier Integrals*, Clarendon Press, Oxford (1937).
[9] I. I. Abbasov, B. M. Bolotovskii, and V. A. Davydov, Usp. Fiz. Nauk **149**, 709 (1986) [Sov. Phys. Usp. **29**, 788 (1986)].
[10] S. A. Fulling and P. C. W. Davies, Proc. R. Soc. Lond. **A348**, 393 (1976).
[11] P. A. M. Dirac, Proc. R. Soc. Lond. **A167**, 148 (1938).
[12] W. E. Thirring, *Principles of Quantum Electrodynamics*, Academic, New York (1958).
[13] F. Rohrlich, in *The Physicist's Conception of Nature*, J. Mehra (ed.), Reidel PC, Dordrecht (1973).
[14] C. Teitelboim, Phys. Rev. **D1**, 1572 (1970).
[15] V. I. Ritus, Zh. Éksp. Teor. Fiz. **75**, 1560 (1978) [Sov. Phys. JETP **48**, 783 (1978)].
[16] V. I. Ritus, Trudy FIAN **168**, 52 (1986).
[17] A. I. Nikishov and V. I. Ritus, Zh. Éksp. Teor. Fiz. **56**, 2035 (1969) [Sov. Phys. JETP **29**, 1093 (1964)].
[18] L. H. Ford, Proc. R. Soc. Lond. **A364**, 227 (1978).
[19] A. I. Nikishov, Nucl. Phys. **B21**, 346 (1970).
[20] R. M. Wald, Commun. Math. Phys. **45**, 9 (1975).
[21] A. I. Akhiezer and V. B. Berestetskiĭ, *Quantum Electrodynamics*, Interscience, New York (1965).
[22] G. Barton, J. Phys. A **24**, 991 (1991).
[23] V. B. Braginsky and F. Ya. Khalili, Phys. Lett. A **161**, 197 (1991).

Translated by David L. Book