Properties of surfatron acceleration of electrons

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The process of surfatron acceleration of electrons trapped by a potential wave moving transversely to a uniform magnetic field is investigated. Two feasible versions of the problem are analyzed. In one an electron beam is injected in the direction of motion of the wave, and in the other the electrons are trapped in a longitudinal wave in a plasma during its creation. Solutions of the relativistic equations of motion of the particles are found analytically using a simple model of the spatial structure of the fields. The conditions for the trapping of electrons in the wave are determined. The influence of the intrinsic space charge and radiation of the electrons on their acceleration is considered, and the damping of the wave as a consequence of the energy it loses to accelerate the particles is evaluated. © *1995 American Institute of Physics.*

1. When particles are accelerated by powerful electromagnetic waves, the presence of synchronism between the wave and the particles takes on great importance,[1] and one of the possibilities for sustaining such synchronism is based on preventing the particles from outrunning the wave by deflecting them along the front using a constant magnetic field. A machine based on this concept is called a surfatron.[2]

Actually, the mechanism of particle acceleration operating in a surfatron was first considered by Sagdeev[3] in connection with an analysis of the motion of ions in a magnetosonic shock-wave front. The energy of the accelerated ions obtained for the parameters of the shock wave considered in Ref. 3 is limited, and this is attributed to the fact that the amplitude of the electric field E_0 in the wave front is smaller than the amplitude of the magnetic field B_0. If the condition $E_0>B_0$ is satisfied in a potential wave moving transversely to a magnetic field, arbitrarily large energies can be obtained, in principle, as a consequence of the temporally unlimited ("perpetual") acceleration of the particles in the wave. Just this occurs in a surfatron.

It has been reported[2,4] that a particle acquires energy during surfatron acceleration in an electric field, which is stronger, the greater is the amplitude of the electric field of the wave; therefore, there have been various practical proposals for realizing the idea of the surfatron acceleration of particles to high energies within a short time, generally by using powerful potential waves traveling transversely to a constant magnetic field. They are based on either a longitudinal plasma wave excited by a laser or an electron beam,[2,5,6] an isomagnetic discontinuity formed at high Mach numbers in a perpendicular magnetosonic shock wave,[7] or acceleration of an electron beam in a vacuum by an electromagnetic wave (a TM mode).[8] Although, as was noted in Ref. 8, the last case has some advantage over an ordinary linear accelerator, it is impossible to obtain large fields, since the amplitude of the electric field in the wave, as in a linear accelerator, is restricted by breakdown on the waveguide walls ($E_{\rm dis}\sim 10^7$ V/m). At the present time, there is apparently only one possibility for obtaining large fields for the purpose of practically realizing the idea of the surfatron acceleration of electrons, viz., a wave in a plasma, in which the electric field can reach values of $\sim 10^{10}$ V/m or more.

2. Let us analyze the surfatron acceleration of electrons in detail using a simple model. A uniform magnetic field with a given magnitude B_0 points in the negative z direction. We consider a one-dimensional wave traveling strictly transversely to the magnetic field in the negative x direction. All the characteristics of this wave depend on the position and the time in the expression $k_0x-\omega t$, where k_0 is the wave number, ω is the frequency, and the electric field has a saw-tooth form (see Fig. 1). A saw-tooth electric field was chosen for two reasons. First, a field with a nearly saw-tooth pattern was obtained in the calculations in Refs. 5 and 6, and, second, as will be seen below, such a choice somewhat simplifies the problem, making it possible to obtain an analytical solution for the equations of motion of electrons trapped in the wave. In addition, the properties of a saw-tooth field are similar to those of a traditional sinusoidal field.

For the steady-state wave under consideration, it is convenient to analyze the motion of particles in the wave reference frame. We assume that the velocity of the wave in the laboratory reference frame $u=\omega/k_0$ does not exceed the velocity of light c, and we introduce the characteristic multiplier for the transformation to the wave reference frame in the form $\gamma_f=1/\sqrt{1-\beta^2}$, where $\beta=u/c$. In the wave reference frame the components of the electromagnetic fields do not depend on the time and take the forms $B_z=-\gamma_f B_0=-B$ and $E_y=-\gamma_f\beta B_0=-\beta B$ (the amplitude of the x component of the electric field does not vary, and the remaining components are equal to zero). The transformation formula for the wave vector is $k=k_0/\gamma_f$. Thus, according to our assumptions, in the wave reference frame the components of the fields E_y and B_z are spatially uniform, the x component of the electric field and the potential depend only on x, and in the range $-d<x<d$ which interests us, these functions have the form (see Fig. 1)

$$E(x)=E_A x/d,\quad \varphi(x)=\varphi_A(1-x^2/d^2),$$

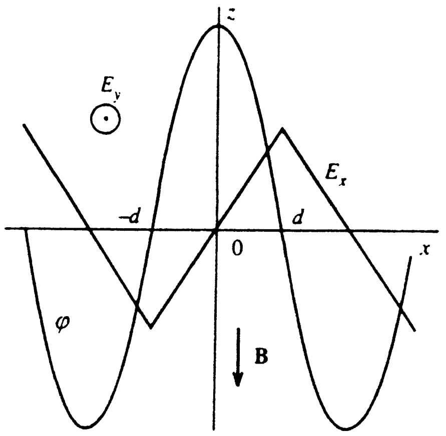

FIG. 1. Schematic drawing of the distribution of the fields in a wave.

where $d=\pi/2k$, and E_A and $\varphi_A=E_A d/2$ are the amplitudes of the electric field and the potential, respectively. The transformation formulas yield

$$E_A=E_0, \quad \varphi_A=\gamma_f\varphi_0,$$

where E_0 and φ_0 are the amplitudes of the electric field and the potential in the laboratory reference frame.

Let a small group of electrons be injected initially at $t=0$ at the bottom of the potential well of the wave, where $x=0$, $E(0)=0$, and $\varphi(0)=\varphi_A$. Such a formulation of the problem is justified for an electron beam injected with a velocity close to the wave velocity in the direction of its motion and at a definite phase (a case similar to that described in Ref. 8). We are interested only in the electrons whose motion is confined to the interval $-d\leqslant x\leqslant d$. The justification for this restriction will be made clear below. We consider the behavior of electrons trapped in this manner in the wave reference frame, which will move in the xy plane in the prescribed electromagnetic fields under the assumptions adopted in accordance with the equations of motion

$$\frac{dP_x(t)}{dt}=-\frac{eE_Ax(t)}{d}+\frac{eV_y(t)B}{c},$$

$$\frac{dP_y(t)}{dt}=\frac{euB}{c}-\frac{eV_x(t)B}{c},$$

where V_x, $P_x=\gamma mV_x$, V_y, and $P_y=\gamma mV_y$ are, respectively, the x and y components of the velocity and the momentum, e and m are the electron rest charge and mass, and $\gamma(t)=1/\sqrt{1-V_x^2/c^2-V_y^2/c^2}$. With no loss of generality, we assume that $V_z=dz/dt=0$ and $P_z=0$. We introduce the dimensionless variables:

$$\tau=\omega_Bt, \quad v=V_x/c, \quad w=V_y/c, \quad p_x=P_x/mc,$$

$$p_y=P_y/mc, \quad \chi=\omega_Bx/c, \quad \nu=\omega_By/c,$$

where $\omega_B=eB/mc$ is the nonrelativistic cyclotron frequency. The equations of motion written in these variations have the form

$$dp_x(\tau)/d\tau=w(\tau)-D^2\chi(\tau), \qquad (1)$$

$$dp_y(\tau)/d\tau=\beta-v(\tau). \qquad (2)$$

Here and below the following notation is used for the dimensionless parameters:

$$R=E_0/B=E_0/\gamma_fB_0, \quad D^2=R^2/2\psi_A, \quad \psi_A=e\varphi_A/mc^2.$$

The potential energy of an electron is assumed to equal zero when $\chi=0$:

$$\psi(\chi)=-e[\varphi(\chi)-\varphi_A]/mc^2,$$

therefore,

$$\psi(\chi)=D^2\chi^2/2=R\chi^2/2\chi_d,$$

where $\chi_d=2\psi_A/R=R/D^2$ (we recall that $-\chi_d\leqslant\chi\leqslant\chi_d$). Equation 2 can be integrated once to obtain

$$p_y(\tau)=p_{y0}+\beta\tau-\chi(\tau). \qquad (3)$$

Relation 3 takes into account the initial conditions adopted:

$$\chi(0)=\nu(0)=0, \quad v(0)=v_0, \quad w(0)=w_0,$$

$$p_x(0)=p_{x0}, \quad p_y(0)=p_{y0},$$

from which it follows that we restrict ourselves to consideration of the behavior of electrons found initially at the bottom of the well.

Using 3, we can write the y component of the dimensionless velocity and the total energy of an electron, respectively, as

$$w(\tau)=\frac{p_y}{\gamma_v(\tau)\sqrt{1+p_y^2}}, \qquad (4)$$

$$\gamma(\tau)=\gamma_v(\tau)\sqrt{1+p_y^2}, \qquad (5)$$

where $\gamma_v(\tau)=1/\sqrt{1-v^2}$, and the conservation law of the total energy under the assigned initial conditions has the form

$$\gamma(\tau)+\psi(\chi)-\beta\nu(\tau)=\gamma_0, \qquad (6)$$

where $\gamma_0=1/\sqrt{1-v_0^2-w_0^2}$. Since we assume that the ensemble of initially trapped electrons is nonrelativistic, we take $\gamma_0\simeq1$. The equations obtained above completely describe the behavior of the trapped electrons at all times.

The solution of the equations of motion will be sought separately in two limiting cases, $\psi_A\ll1$ and $\psi_A\gg1$.

3. We first seek solutions for $\psi_A\ll1$. In this case, setting $R\geqslant1$, we obtain a large value for D:

$$D=R/\sqrt{2\psi_A}\gg1.$$

The well is narrow here:

$$\chi_d=R/D^2\ll1.$$

In the nonrelativistic stage ($\beta\tau\leqslant1, \gamma\geqslant1$) the equations of motion have analytical solutions:

$$\chi(\tau)=\frac{v_0-v_{d0}}{\Omega}\sin\Omega\tau+\frac{w_0(1-\cos\Omega\tau)}{\Omega^2}+v_{d0}\tau,$$

$$v(\tau)=(v_0-v_{d0})\cos\Omega\tau+\frac{w_0}{\Omega}\sin\Omega\tau+v_{d0},$$

$$w(\tau)=w_0+\beta\tau-\chi(\tau), \quad (7)$$

$$\nu(\tau)=\frac{w_0\tau D^2}{\Omega^2}+\frac{\beta\tau^2 D^2}{2\Omega^2}+\frac{w_0}{\Omega^3}\sin\Omega\tau$$

$$-(v_0-v_{d0})(1-\cos\Omega\tau)/\Omega^2,$$

where $\Omega=\sqrt{1+D^2}\simeq D$ and $v_{d0}=\beta/\Omega^2$. The energy conservation law has the form

$$(D^2\chi-w)^2/\Omega^2+(v-v_{d0})^2=w_0^2/\Omega^2+(v_0-v_{d0})^2. \quad (8)$$

It follows from the solutions (7) that for $D\gg 1$ an electron trapped in the well performs a large number of oscillations during a time $\tau\leqslant 1$, the amplitude of the oscillations remaining constant. The absolute value of the velocity component v does not vary, and the velocity component w increases as $\beta\tau$ with time. The motion of the particles takes the form of drift along the x axis with a velocity v_{d0} and continuous acceleration along the y axis, on which oscillations with a frequency Ω are superimposed. In particular, we note that the electrons having the velocity $v_0=v_{d0}$ and $w_0=0$ at $\tau=0$ and $\chi=0$ subsequently move toward the wave (along the x axis) with a strictly constant velocity v_{d0}, while all the other particles also move with the same velocity, but only on the average (the averaging is over the oscillation period). We also note that the value of the drift velocity v_{d0} can be obtained from Eq. (1) by setting the right-hand side equal to zero.

As we shall show below, the conditions for prolonged retention of the particles initially trapped in the well are strongly dependent on the value of R: for $R<1$ all the particles leave the well after a certain time; for $R\geqslant 1$ a very small part is retained; and for $R\gg 1$ almost all the particles are retained. However, it is not convenient to choose an excessively large value of R, since the accelerating field in the surfatron for an assigned value of E_0 would then decrease. In fact, during surfatron acceleration, a particle acquires energy in the electric field $E_y=\beta\gamma_f B_0=\beta E_0/R$; therefore, when E_0 is given, to obtain large values of E_y the value of R must be close to unity, and the velocity of the wave must be close to the velocity of light. The condition $\beta\rightarrow 1$ is possible, if the particles are trapped in the wave during its creation, as, for example, in the case of excitation of a wave in a plasma by a laser or a beam.[2,5,6] However, if the particles are injected into the wave, as was done in Ref. 8, we must restrict ourselves to $\gamma_f<10$ and $\beta\simeq 1$ (these values correspond to an energy of the injected electrons $\leqslant$ 10 MeV).

Thus, we take $R\geqslant 1$, at which a certain fraction of the particles is still retained in the region $|\chi|<\chi_d$, and, consequently, we shall not consider the large values of R needed to retain particles outside this region. The conditions for retaining a particle in the spatial region $|\chi|<\chi_d$ can be determined from Eq. (8). An electron which performs more than one oscillation in the well drifts to the right (see Fig. 1), and all the particles departing from this region detrap and move toward the wave. This means that all the electrons leaving the trap impart energy to the wave. The coordinates of the turning points of an oscillating electron are denoted by $\chi_-^+(\tau)=\pm\chi_0+\rho(\tau)$, where $\rho(\tau)$ is the displacement of the particle as a consequence of drift and χ_0, which is the amplitude of the oscillations of the particle in the well in the absence of a magnetic field, is found from the relation

$$\psi(\chi_0)=D^2\chi_0^2/2=\gamma_0-\gamma(v=0)\simeq\gamma_0-1\simeq v_0^2/2.$$

Thus, $\chi_0=v_0/D$. Substituting the coordinates of the turning points into (8) and setting $\chi^+(\tau^+)=\chi_d$, $\beta\tau^+\simeq 1$, and $w_0\ll 1$, we obtain the condition for trapping electrons in the nonrelativistic state in the desired region:

$$R=\frac{1}{1-v_0/\sqrt{2\psi_A}}. \quad (9)$$

Similarly, it can be shown that the condition for trapping of particles in the well, i.e., in the range $-2d\leqslant x\leqslant 2d$, has the form

$$R=\frac{1}{1-v_0/2\sqrt{\psi_A}}.$$

Hence it follows that particles initially trapped in the interval $|\chi|<\chi_d$ will be retained in the well for $R\leqslant 2$ and that when the optimal values of R are slightly greater than unity and with initial velocity spread $\Delta v_0\sim\sqrt{2\psi_A}$, almost all the electrons initially trapped in the well will leave it. As the numerical calculations show, for values of R very close to unity, only particles with initial velocity components $w_0\simeq 0$ and $0<v_0<2v_d$ will remain in the group of "perpetually" accelerated electrons.

We move on to a search for solutions of Eqs. (1)–(2) in the relativistic stage ($\beta\tau\gg 1$). We first find the conditions for departure of the particles from the well. Using (6), we obtain

$$\gamma_+-\gamma_--\beta\int_{\tau_-}^{\tau_+}w\,d\tau=\psi_a(\chi_-^2-\chi_+^2)/\chi_d^2,$$

where $\gamma_-^+=\gamma(\tau_-^+)$, and τ_-^+ denotes the times when an electron reaches the turning points with the coordinates χ_-^+. Now setting $\gamma\simeq p_y=p_{y0}+\beta\tau-\chi$ and $w\simeq 1$, we obtain the trapping condition exactly in the form (9). Thus, the condition for departure from the well remains unchanged; therefore, out of the total group of particles leaving the well after an infinite time, the vast majority leave the well in the nonrelativistic stage.

Next, taking into account that the absolute values of the velocity component v do not vary in the nonrelativistic stage, i.e., remain the same as at the onset, assuming $v\ll 1$ when $\beta\tau\gg 1$ holds, and setting $\gamma_v\simeq 1$, from Eqs. (3)–(5) we obtain the following solutions:

$$\gamma(\tau)\simeq\sqrt{1+\beta^2\tau^2},\quad w(\tau)\simeq\beta\tau/\gamma(\tau),\quad p_y\simeq\beta\tau,$$

$$\nu(\tau)\simeq\gamma(\tau). \quad (10)$$

Assuming that the character of the solutions of Eqs. (1)–(2) remains unchanged in the relativistic stage, we seek the solution for the remaining unknown $\chi(\tau)$ from Eq. (1) in the form of a sum of two terms: $\chi(\tau)=\chi_D(\tau)+\xi(\tau)$, where χ_D is the coordinate of the point of equilibrium between the forces acting along the χ axis, which moves with the drift velocity $v_d(\tau)=d\chi_D/d\tau$, and $\xi(\tau)$ is the oscillating part of the solution. Equating the right-hand side of Eq. (1) to zero, we obtain an expression for the coordinate of the point of local equilibrium in the zeroth approximation in the form $\chi_D(\tau)=\beta\tau/\Omega^2\gamma(\tau)$. Therefore, the drift velocity is $v_d(\tau)=\beta/\Omega^2\gamma^3(\tau)$. Restricting ourselves to this approximation, we obtain the solutions for $\chi_D(\tau)$ and $v_d(\tau)$ in the form

$$\chi_D(\tau)=v_{d0}\tau/\gamma(\tau),\quad v_d(\tau)=v_{d0}/\gamma^3(\tau).$$

The equation for the oscillating part

$$\tau\frac{d^2\xi}{d\tau^2}+\frac{d\xi}{d\tau}+\frac{D^2}{\beta}\xi=0$$

has the solution $\xi(\tau)=J_0(2D\sqrt{\tau/\beta})$, where J_0 is a zeroth-order Bessel function of the first kind. Thus, we obtain the general solutions for $\chi(\tau)$ and $v(\tau)$ in the form

$$\chi(\tau)\simeq\frac{v_{d0}\tau}{\gamma}+\frac{\beta^{1/4}}{\sqrt{\pi D}\tau^{1/4}}\sin(2D\sqrt{\tau/\beta}),\tag{11}$$

$$v(\tau)\simeq\frac{v_{d0}}{\gamma^3}+\frac{\sqrt{D/\pi}}{\beta^{1/4}\tau^{3/4}}\cos(2D\sqrt{\tau/\beta}).\tag{12}$$

Here the expressions for $\chi(\tau)$ and $v(\tau)$ were written using the representation of a Bessel function at large values of the arguments. Thus, Eqs. (7)–(12) comprise a complete set of solutions describing the behavior of an electron in a surfatron when $\psi_A\ll 1$.

An analysis of the results obtained leads to a simple and very important conclusion: in all stages a particle which is being accelerated continuously along the y axis tends to move along the x axis in the vicinity of the point at which the sum of the x components of all the forces [i.e., the right-hand side of Eq. (1)] is equal to zero. Apparently, this conclusion is quite general for the surfatron mechanism of particle acceleration. As a consequence of this remarkable fact, a special regime for accelerating an electron in the equilibrium state from the very beginning was discovered in the numerical calculations. If an electron has the initial velocity components $v_0=v_{d0}$ and $w_0=0$, it moves with a strictly constant velocity v_{d0} in the nonrelativistic stage. At a certain given value of $R\simeq 1$ an electron having an initial velocity $v_0=v_{d0}$ is confined in the well longer than an electron having an initial velocity $v_0=0$. Hence it can be concluded that it is better to inject particles with a velocity which is smaller than that of the wave by v_{d0}.

These solutions imply that as $\tau\to\infty$ the electron oscillation period ($\propto\sqrt{\tau}$) increases, and both the drift velocity ($\propto\tau^{-3}$) and the amplitude of the oscillations ($\propto\tau^{-1/4}$) decrease, tending to zero. These factors provide a striking illustration of the phenomenon mentioned in Ref. 1 of phase focusing or phase stability when particles are accelerated in a surfatron.

4. We move on to the case of the motion of an electron in the well in the limit $\psi_A\gg 1$. In this case the dimensionless parameters for the optimal values $R\gg 1$ have the values $D\ll 1$, $\Omega\simeq 1$, and $v_{d0}\simeq 1$, and the width of the well becomes very large, $\chi_d\gg 1$. This means that the second term on the right-hand side of Eq. (1) is negligibly small in the initial stage of motion. Physically, this means that the motion of an electron actually begins here in the constant uniform fields B and $E_y=\beta B$. Setting $\beta\simeq 1$, $D^2\chi\simeq 0$, $p_{x0}\ll 1$, $p_{y0}\ll 1$, and $\gamma_0\simeq 1$, we can write the solutions of Eqs. (1)–(2) as functions of the variable $p_y=p_y(\tau)$, which can be determined from the equation $p_y+p_y^3/6=\tau$:

$$p_x(\tau)=\nu(\tau)=p_y^2/2,\quad \gamma(\tau)=1+p_y^2/2,$$

$$w(\tau)=2p_y/(2+p_y^2),\quad v(\tau)=p_y^2/(2+p_y^2),\quad \chi(\tau)=p_y^3/6.$$

In the nonrelativistic stage we have $p_y(\tau)\simeq\tau<1$, and these solutions have a simple form:

$$p_x(\tau)=v(\tau)=\nu(\tau)=\tau^2/2,$$
$$w(\tau)=\tau,\quad \gamma(\tau)=1+\tau^2/2,\tag{13}$$
$$\chi(\tau)=\tau^3/6.$$

In the relativistic stage we have $p_y=(6\tau)^{1/3}>1$, and the solutions are

$$p_x(\tau)=\gamma(\tau)=\nu(\tau)=(6\tau)^{2/3}/2,$$
$$\chi(\tau)=\tau,\quad w(\tau)=2(6\tau)^{-1/3},\tag{14}$$
$$v(\tau)=(6\tau)^{2/3}/[2+(6\tau)^{2/3}].$$

It follows from these solutions that at the time $\tau\simeq 2$, at which $p_y=\sqrt{2}$, the velocity component w reaches its maximum value $w=1/\sqrt{2}$, and $v=1/2$. Subsequently, $v\to 1$, and w decreases with time as $w\propto\tau^{-1/3}$ law. The solutions (14) are valid for $1\ll\tau<\tau_q$, where τ_q is the time determined from the condition $w(\tau_q)=D^2\chi(\tau_q)$: $\tau_q\propto D^{-3/2}\gg 1$. At this time the particle reaches a distance $\chi\simeq\tau_q\simeq D^{-3/2}\ll\chi_q=D^{-2}$, and the condition of equilibrium between the forces acting along the x axis will be achieved for it. As we have already noted, the motion of the particle in x will subsequently follow the law of surfatron acceleration, so that the equilibrium condition would be maintained. Thus, at $\tau>\tau_q$, solutions (10)–(12), which have $\Omega\simeq 1$ and $\beta\simeq 1$, become possible, and, consequently, the velocity component v begins to decrease, while w increases. At some instant, when the particle has the velocity components $w\simeq 1$ and $v\ll 1$, it reaches the point χ_q and begins to oscillate in the vicinity of that point. Therefore, the character of the motion along the x axis remains unchanged even for $\psi_A\gg 1$: at first the particle moves toward the asymptotic equilibrium point $\chi_q=1/D^2$ with the drift velocity (in the time interval $1<\tau<\tau_q$ the velocity is $v_{d0}=\beta/\Omega^2\simeq 1$), and then it begins to oscillate about this point with a time-decaying amplitude.

Let us attempt to find the conditions for particle trapping when $\psi_A\gg 1$. In general, knowing practically everything about the character of the motion of the particles from the solutions obtained, we can find the condition for their retention in the well in a rough approximation from the following qualitative arguments. As follows from the calculations, particles leave the well in the initial stage of motion, during which the amplitude of their oscillations $\chi_0\simeq v_0/D$ remains practically constant. In addition, the position about which the particles oscillate ($\chi=\chi_D$) shifts with the drift velocity to the asymptotic equilibrium point ($\chi\to 1/D^2$). Assuming that a trapped particle reaches the vicinity of the point

$\chi_q = 1/D^2$ at some moment in time and oscillates about it with an amplitude $\chi_0 = v_0/D$, we obtain the condition for its trapping in the range $-\chi_d < \chi < \chi_d$ in the form

$$\chi_0 \leq \chi_d - 1/D^2 = (R-1)/D^2.$$

Hence we obtain an estimate of the value of R:

$$R \geq \frac{1}{1 - v_0/\sqrt{2\psi_A}},$$

which coincides with (9). Thus, Eq. (9) can be used for both $\psi_A \ll 1$ and $\psi_A \gg 1$.

We assume that the values of the initial velocity of a particle are bounded by the thermal velocity, which is much smaller than the velocity of light in all practical cases; therefore, $v_0/\sqrt{2\psi_A} \ll 1$ holds for $\psi_A \gg 1$. According to (9), under these conditions practically all the initially trapped particles become part of the group of "perpetually" accelerated particles when the value of R is slightly greater than unity. This conclusion is confirmed by the numerical calculations.

We note that in the limiting cases $\psi_A \ll 1$ and $\psi_A \gg 1$ considered here the time dependence of all the parameters obtained numerically and analytically is in good agreement.

5. We have considered the problem of the acceleration of electrons in a traveling wave after being injected in a special manner at the bottom of the potential well of the wave. Let us use the same formulation of the problem as above (see Fig. 1) to discuss the case of the surfatron acceleration of an electron by a longitudinal plasma wave traveling transversely to a constant uniform magnetic field, which is of interest from the standpoint of practical applications.

We assume that a plasma wave with an oscillation frequency $\omega^2 = \omega_p^2 + \omega_{B0}^2$, where $\omega_p = \sqrt{4\pi n_0 e^2/m}$ is the electronic plasma frequency and $\omega_{B0} = eB_0/mc$, is created by a laser or an electron beam in a uniform plasma with a density n_0 and an electron temperature $T_e \ll mc^2$. To eliminate the influence of the magnetic field on the dispersion properties of the plasma, the condition $\omega_p^2 \gg \omega_{B0}^2$ must be satisfied; therefore, we shall henceforth assume $\omega \simeq \omega_p$ and, consequently, that the phase velocity of the plasma wave is $u = \omega_p/k_0$. The highest theoretically possible amplitude of the electric field of a plasma wave is[2,5]

$$E_{m0} \sim n_0 e/k_0 \sim m u \omega_p/e.$$

We pass to the wave reference frame, in which the plasma as a whole moves with a velocity u. The amplitude of the potential in the plasma wave must be smaller than the kinetic energy of the particles moving with the velocity of the wave, since, otherwise, all the particles would be trapped by the wave and it would rapidly damp (Landau damping). This requirement places a definite restriction on the amplitude of the wave: $\varphi_A \leq (\gamma_f - 1)mc^2/e$. On the other hand, this inequality can be regarded as a condition which relates the amplitude of the potential and the velocity or the amplitude and the wave vector to one another. Next, since $\varphi_A \sim E_0 \gamma_f/k_0$, holds, we can write down a condition which restricts the amplitude of the electric field in the wave:

$$E_0 < \frac{mc^2 \omega_p(\gamma_f - 1)}{e\gamma_f u}.$$

Assuming that the amplitude of the electric field is equal to only a part ϵ of the theoretically possible amplitude, i.e., $E_0 = \epsilon E_{m0}$, we obtain a bound on ϵ:

$$\epsilon < \frac{\gamma_f}{\gamma_f + 1} < 1.$$

Thus, for a longitudinal wave in a plasma with an assigned density, the amplitude of the electric field depends on the velocity of the wave according to the relation $E_0 = \epsilon m u \omega_p/e$ and is largest in the limit $u \to c$ (in that case we have $\epsilon \leq 1$).

The limiting cases for a plasma wave with respect to ψ_A take on definite meanings. For example, the condition $\psi_A \ll 1$ can be written in the form $\psi_A < \gamma_f - 1 \ll 1$, whence it follows that this case is characteristic of a nonrelativistic wave: $\beta \ll 1$, $\gamma_f \geq 1$, and $\epsilon \leq 1/2$. If $\psi_A \gg 1$ holds in the wave, the velocity of the wave can be close to the velocity of light; therefore we have $\beta \simeq 1$, $\gamma_f \gg 1$, and $\epsilon \leq 1$. Since the field accelerating the particles in a surfatron is $E_y = \beta E_0/R \sim \beta^2 \omega_p/R$, at an assigned value of n_0 and $R \leq 2$ the rate of acceleration of the particles is significantly higher in a relativistic wave than in a nonrelativistic wave. Thus, the case $\beta \simeq 1$ is most interesting for practical applications, and we shall therefore consider it in greater detail, assuming that $\psi_A \gg 1$.

We assume that in the wave reference frame the mean value of the velocity component v of the trapped particles at a point with potential $\varphi = 0$ is equal to zero, and that the velocity spread is $\Delta v \sim v_T = \sqrt{\theta}$, where $\theta = T_e/mc^2 \ll 1$. Thus, we assume that in a plasma wave each particle begins its motion at a point where the potential is equal to zero and the electric field strength peaks (the point $x = d$ in Fig. 1). Since we have $R > 1$, all the electrons with the initial velocity components $|v_0| < v_T \ll 1$ and $|w_0| < v_T \ll 1$ fall under the action of the peak force of the electric field into the well, in which the laws of motion of the particles are already known.

Let us find the character of the motion of an electron in this case, setting $v_0 \ll 1$, $w_0 \ll 1$, and $\chi(0) = \chi_d$. For the limiting values $\psi_A \gg 1$ in the nonrelativistic stage, we substitute $\chi(\tau) = \chi_d - \xi(\tau)$ into (1)–(3) and set $\Omega \simeq 1$, $\chi_d = R/D^2$, and $\xi(0) = 0$. This gives $w(\tau) = \tau + \xi(\tau)$ and an equation for finding $\xi(\tau)$:

$$\frac{d^2\xi}{d\tau^2} = R - \tau - \xi.$$

This equation is easily solved:

$$\xi(\tau) = R[1 - \cos\tau] - \tau + \sin\tau.$$

Setting $\xi \simeq R\tau^2/2$ at $\tau < 1$, we ultimately obtain solutions in the form

$$\chi(\tau) \simeq R/D^2 - R\tau^2/2, \quad v(\tau) \simeq -R\tau, \quad w(\tau) \simeq \tau + R\tau^2/2,$$

from which it follows that after a time $\tau < 1$ the value of the velocity component w for $R \geq 1$ becomes close to unity, and the particle closely approaches the asymptotic equilibrium point $\chi \simeq 1/D^2$. Subsequently, its motion will be confined to the vicinity of this point in accordance with Eqs. (10)–(12).

In the case of a plasma wave with an amplitude $\psi_A \ll 1$, a trapped electron falling into the well from the point $x=d$ reaches the bottom of the well after a time $\tau \sim \Omega^{-1} \ll 1$. Thereafter the problem of the motion of such an electron reduces to that considered above except that the initial value v_0 of the velocity component at the bottom of the well is greater than $\sqrt{2\psi_A}$. As we know, under these conditions retention of an appreciable portion of the originally trapped electrons in the well requires $R \geqslant 2$, and the motion of the particles will be described by Eqs. (7)–(12).

6. Let us discuss some possible restrictions which prevent the theoretically unlimited increase in the energy of the trapped electrons. The principal and most widely known limiting factors are, first, the loss of energy in the form of radiation and, second, the finite dimensions of the wave front in real situations. Since the case in which the energy of the particles is restricted by the finite value of the transverse dimension is simple and self-evident, we shall not dwell on it noting only the conclusion in Ref. 9 that the restriction on the transverse dimension can be relaxed to some extent when the front of the potential wave has some curvature.

As for radiation, according to the analysis performed, in the case of a surfatron we are dealing with the radiation of an electron accelerated in a constant and uniform electric field E_y. In fact, as follows from the solutions obtained in the wave reference frame, no other forces besides the force of the electric field E_y act on an electron in the relativistic and ultrarelativistic stages. Thus, in a surfatron the radiated power is constant:

$$W_I = \text{const } E_y^2 = \text{const } \beta^2 E_0^2 / R^2.$$

It is negligibly small compared with the rate of the buildup of energy by an electron [the ratio between these quantities is $\sim E_y/(e/r_0^2) \to 0$, where $r_0 = e^2/mc^2$ is the classical radius of an electron. Since the radiated power is a relativistic invariant, this conclusion is quite general. We note in this context that the claims of several authors (see, for example, Ref. 2) regarding a strong dependence of the radiated power on the energy of a particle in a surfatron seem very doubtful.

Let us consider in greater detail the restriction associated with the influence of the intrinsic space charge on the motion of particles trapped by a wave, which was not taken into account anywhere in the analysis of the surfatron acceleration of electrons. We shall attempt to evaluate this effect in the case of a uniform plasma wave. As follows from the numerical calculations performed, only a small portion n_T (n_T is the number of particles per unit volume) of the initial number of trapped particles remains at optimal values of R ($R \simeq 1$). As has already been noted above, the group of trapped electrons as a whole has a certain drift velocity v_{d0}, which signifies the presence of an uncompensated flux of particles along the x axis from the onset. (For example, in the case of a longitudinal wave in a plasma, the initially trapped ensemble of electrons, whose velocity distribution function in the wave reference frame has the form of a part of the exponentially decreasing tail of a Maxwell function, has a certain drift velocity toward the wave at the onset.) As a consequence of conservation of the flux

$$J = n_T v_{d0} = n(\tau) v_d(\tau)$$

during the acceleration process, in the final, relativistic stage, in which the drift velocity v_d tends to zero, the density of the trapped electrons increases without bound. Obviously, when this density reaches a certain value, the space charge of the electrons begins to influence the macroscopic structure of the wave.

The time dependence of the density of the trapped electrons can be evaluated using (12) and the flux conservation law in the one-dimensional problem under consideration, yielding

$$n(\tau) \simeq n_T \gamma^3(\tau).$$

At the time τ_L, when this density becomes comparable to the plasma density n_0, the surfatron acceleration mechanism apparently breaks down. Therefore, with consideration of the effect of the space charge, the limiting energy is approximately

$$\mathscr{E}_{LQ} = mc^2 \gamma(\tau_L) \simeq mc^2 (n_0/n_T)^{1/3}. \tag{15}$$

Let us compare the limitation due to the increase in the space charge with the limitation of the energy due to the damping of the wave as a consequence of the reciprocal influence of the trapped particles on it. Damping of the wave begins to be manifested to a significant degree when the energy density of the group of electrons being accelerated becomes comparable to that in the wave:

$$E_0^2 \simeq n(\tau_L) \gamma(\tau_L) mc^2.$$

Setting $E_0 \sim u\sqrt{mn_0}$ for a longitudinal plasma wave, we obtain a rough estimate of the limiting energy:

$$\mathscr{E}_{LD} = mc^2 \gamma(\tau_L) \simeq mc^2 \sqrt{\beta} (n_0/n_T)^{1/4}. \tag{16}$$

A comparison of (15) and (16) reveals that $\mathscr{E}_{LQ}$ is always greater than $\mathscr{E}_{LD}$, i.e., the limitation of the energy due to the damping of the wave is stronger.

From (15) and (16) we can evaluate the limiting energies of electrons accelerated in a longitudinal nonrelativistic ($\psi_A \simeq \gamma_f - 1 \ll 1$) wave. The number of trapped electrons in a longitudinal plasma wave is $n_T \propto n_0 \exp[-(\gamma_f - 1)/\theta]$, where we have, as a rule, $\theta \ll 1$ and $\gamma_f - 1 \gg \theta$. As we see, for a given value of θ the number of trapped electrons depends on the wave velocity and is infinitesimal for relativistic waves ($n_T = 0$ when $\beta = 1$). Substituting the expression for n_T into (15) and (16), we obtain

$$\mathscr{E}_{LQ} \sim mc^2 \exp[(\gamma_f - 1)/3\theta],$$

$$\mathscr{E}_{LD} \sim mc^2 \sqrt{\beta} \exp[(\gamma_f - 1)/4\theta].$$

Thus, for a plasma wave the limiting energies (15) and (16) increase with wave velocity. We take the following values for the parameters of the wave in the plasma as an example: $\psi_A \sim \gamma_f - 1 \sim \beta^2/2 \sim 10^{-3}$ ($u \simeq 10^9$ cm/s, $\varphi_A \simeq 5$ keV), $\theta \sim 10^{-5}$ ($T_e \simeq 5$ eV). Then for the limiting energies we obtain $\mathscr{E}_{LQ} \sim 10^{20}$ eV and $\mathscr{E}_{LD} \sim 10^{16}$ eV. Larger values of the limiting energy were actually obtained due to the extremely small number of trapped particles.

7. In conclusion, we summarize the main results of the investigations performed and what we have learned.

1. Solutions of the relativistic equations describing the surfatron acceleration of electrons in an electromagnetic wave have been obtained in the model adopted.

2. The results obtained are applicable to two practical setups for realizing the surfatron acceleration mechanism: electrons injected into an electromagnetic wave in the form of an electron beam are accelerated; some of the electrons of a plasma trapped by a longitudinal plasma wave during its creation undergo acceleration.

3. The conditions for the trapping of electrons in a wave have been found. These conditions are determined mainly by the ratio of the amplitude of the electric field of the wave to the amplitude of the magnetic field.

4. It follows from the calculations that electrons which are trapped in a wave and accelerated continuously along the wave front oscillate about a certain coordinate and move (drift) in the direction opposite to the motion of the wave to a point where the amplitude of the electric field is comparable to the amplitude of the magnetic field (in the wave reference frame). Subsequently, during the surfatron acceleration of electrons, the amplitude of their oscillations decreases, and each particle being accelerated actually moves together with the wave.

Thus, the phase stability or phase focusing associated with the surfatron acceleration of electrons follows explicitly from the solutions obtained.

5. It has been shown in the case of the surfatron acceleration of electrons by an electromagnetic wave traveling transversely to a constant uniform magnetic field that the energy lost in the form of radiation is negligibly small.

6. Two possible mechanisms which can disrupt the "perpetual" acceleration of particles in a surfatron, viz., the presence of the intrinsic space charge of the group of particles being accelerated and the loss of energy from the wave to accelerate the particles, have been considered. According to the evaluations, the latter restriction is stronger than the former.

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