Quantum kinetics of the formation of a one-dimensional nucleating center

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A model of the tunneling formation of a nucleating center of a stable phase on an Ising chain is considered. In the model, the states of the nucleating center are described by Bessel functions $J_\nu(z)$, the argument z characterizing the strength of the weak perpendicular field, while the number ν is related to the size of the nucleating center. An expression is obtained for the reciprocal of the lifetime of the metastable phase at zero temperature under the assumption that the spin–lattice interaction is fairly weak. In this case, the main channel of the irreversible transition is tunneling from the state of an elementary nucleating center to the state of a nucleating center of critical size with emission of one phonon. © *1995 American Institute of Physics.*

1. At zero temperature, a phase transition from a metastable to a stable state occurs by the tunneling formation of a nucleating center of the stable phase. In Refs. 1 and 2, a theory of this process was developed for the case of large nucleating centers, for which the methods of macroscopic physics can be effectively used. It was shown in Ref. 3 that for the description of a quantum phase transition in discrete crystalline structures it is necessary to take into account the fact that the number of particles in a nucleating center is discrete even if this number is large.

The present paper is devoted to the development of a theory of the tunneling formation of nucleating centers of any size in an exactly solvable one-dimensional model in which the static part of the Hamiltonian has the form of the ($s=1/2$) Ising Hamiltonian, while the dynamical part is formulated in such a way that growth of the stable phase occurs only on the surface of a nucleating center. In this model, there is no formation of noncompact nucleating centers, which make the description of the process much harder.

The stationary states of a nucleating center are described by a discrete Schrödinger equation, which has the form of a functional Bessel equation. An irreversible process of growth of a supercritical nucleating center occurs only if the spin subsystem interacts with a thermal bath, for example, with acoustic phonons. Allowance for the discreteness has the consequence that the logarithm of the lifetime of the metastable state at zero temperature increases quasilinearly with increasing size N_c of the critical nucleating center:

$$\ln \tau \propto N_c \ln N_c . \tag{1.1}$$

If we assume formally that the number of particles in a nucleating center is a continuous variable and apply the theory of Ref. 1, then we obtain the stronger dependence

$$\ln \tau \propto N_c^{3/2} . \tag{1.2}$$

Thus, neglect of the discreteness severely underestimates the rate of tunneling formation of supercritical nucleating centers (see Refs. 3 and 4).

2. We consider a linear chain of $N_0 \to \infty$ two-level sites described by the Hamiltonian

$$\hat{H}_0 = -h\sum_r \left(\hat{S}_r^z + \frac{1}{2}\right) - \sum_{rg} I_g\left(\hat{S}_r^z \hat{S}_{r=g}^z - \frac{1}{4}\right)$$
$$= -h\sum_r \hat{n}_r - \sum_{rg} I_g\left(\hat{n}_r \hat{n}_{r+g} - \frac{1}{2}\hat{n}_r - \frac{1}{2}\hat{n}_{r+g}\right). \tag{2.1}$$

Here $h>0$ is a magnetic field or the energy of a two-level system, $I_g>0$ is the energy of the spin interaction between sites r and $r+g$ (r and g are integers), and $S_r^z = n_r - 1/2$, where $n_r = 0$ or 1. The energy of the metastable state when all the two-level systems are in excited states (all $S_r^z = -1/2$, $n_r = 0$) is chosen equal to zero.

We shall call the state of the system in which the spins at the sites $r, r+1, \ldots, r+n-1$ are directed upward and the remainder downward a compact nucleating center of n "particles." In the model (2.1), the energy of this nucleating center is

$$E_0(n) = -hn + \sum_{g \geqslant 1} I_g[n-(n-g)\theta(n-g)]. \tag{2.2}$$

The first term on the right is the internal energy of the nucleating center, while the second is a "surface" energy. A nucleating center with number of particles n_0 determined by the following inequalities has the maximum energy

$$\sum_{g \geqslant n_0+1} I_g < h < \sum_{g \geqslant n_0} I_g . \tag{2.3}$$

In what follows, we shall measure the energy in units of h, i.e., we set $h=1$, and we assume that only I_1 and $I_2 = I > 1$ are nonzero. In this case we have $n_0 = 2$, and the energy (2.2) for $n \geqslant 2$ depends linearly on n:

$$U(n) = E_0(n) - E_0(1) = I + 1 - n. \tag{2.4}$$

At zero temperature, the rate of the phase transition will be determined by the amplitude for tunneling from the state of the elementary nucleating center ($n=1$) to the state n_c with a critical nucleating center, which is the smallest nucleating center whose energy is less than $E_0(1)$:

$$N_c = [I] + 2 \tag{2.5}$$

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(here $[x]$ is the integer part of the number x). If δ is the fractional part of the number l, then the function (2.4) can be rewritten as

$$U(n)=U_c+N_c-n,\quad U_c=\delta-1. \tag{2.6}$$

We choose the dynamical part of the Hamiltonian so that it admits transitions only at the interface of the two phases:

$$\hat{H}_1=\xi\sum_r \hat{P}_r\hat{S}_r^x. \tag{2.7}$$

The operator $\hat{S}_r^x$ realizes transitions between the states of the two-level system at site r, and

$$\hat{P}_r=(\hat{S}^z_{r-1}-\hat{S}^z_{r+1})^2=(\hat{n}_{r-1}-\hat{n}_{r+1})^2 \tag{2.8}$$

is the operator of projection onto the boundary of the two phases, and it ensures compactness of the nucleating center during its growth. The Hamiltonian (2.7) can neither create nor annihilate an elementary nucleating center. Therefore, the number of nucleating centers in the system is conserved.

Any wave function of the system with one nucleating center can be represented as an expansion in a complete set of functions with fixed position of the left-hand boundary r and number n of sites in the nucleating center:

$$\Psi=\frac{1}{\sqrt{N_0}}\sum_{n,r}\psi(r,n)|r,r+1,\dots,r+n-1\rangle. \tag{2.9}$$

We shall assume that the state of the system is homogeneous and the amplitudes $\psi(r,n)$ do not depend on the coordinate r. In this case,

$$\Psi=\sum_n \psi(n)|n\rangle,$$

$$|n\rangle=\frac{1}{\sqrt{N_0}}\sum_r |r,r+1,\dots,r+n-1\rangle, \tag{2.10}$$

$$\sum |\psi(n)|^2=1.$$

Then the stationary equation

$$(\hat{H}_0+\hat{H}_1)\Psi=E\Psi \tag{2.11}$$

reduces to the simple system of recursion relations

$$E\psi(1)=\xi\psi(2), \tag{2.12}$$

$$[E-U(n)]\psi(n)=\xi[\psi(n-1)+\psi(n+1)],\quad n\geqslant 2. \tag{2.13}$$

We introduce the notation

$$z=2\xi,\quad \nu=E-U(n),\quad J_\nu(z)=\psi(n), \tag{2.14}$$

in which Eq. (2.13) takes the standard form of the functional Bessel equation

$$J_{\nu-1}(z)+J_{\nu+1}(z)=\frac{2\nu}{z}J_\nu(z),\quad \nu\geqslant E-U(2). \tag{2.15}$$

Thus, the stationary states of the nucleating center are described by the Bessel functions

$$\begin{gathered}|N\rangle=\sum_{n\geqslant 1}\psi_N(n)|n\rangle,\\ \psi_N(n)=\mathrm{const}\cdot J_\nu(2\xi),\quad \nu=E_N-U(n).\end{gathered} \tag{2.16}$$

At the same time, Eq. (2.12) plays the role of a boundary condition that fixes the eigenvalues E_N of the nondegenerate discrete spectrum.

3. The form of the Nth state of the nucleating center depends on the values of two parameters, ξ and l. We consider the simplest case

$$\xi\ll 1+\xi<l<2-\xi,\quad N_c=3. \tag{3.1}$$

The eigenfunctions and corresponding eigenvalues of Eq. (2.13) are approximately

$$\begin{gathered}\psi_1(1),\quad \psi_1(2)=-\frac{\xi}{U(2)},\quad \psi_1(3)=0,\\ E_1=-\frac{\xi^2}{U(2)},\end{gathered} \tag{3.2}$$

$$\psi_2(1)=\frac{\xi}{U(2)},\quad \psi_2(2)=1,\quad \psi_2(3)=\xi,\quad E_2=U(2), \tag{3.3}$$

$$\begin{gathered}\psi_N(N-1)=-\xi,\quad \psi_N(N)=1,\quad \psi_N(N+1)=\xi,\\ \psi_N(N\pm 2)=0,\quad E_N=U(N),\quad N\geqslant 3.\end{gathered} \tag{3.4}$$

Let $l=2$. Then we have $U(3)=0$, and the states $N=1$ and $N=3$ are in resonance. In such a case, the even superposition of these states has energy $E_3=-2\xi^2$, while the odd one has $E_1=0$.

If at the initial time there is one elementary nucleating center in the system, i.e.,

$$\Psi|_{t=0}=|n=1\rangle, \tag{3.5}$$

then after a time of order $|E_1-E_3|^{-1}$ the following oscillating state is established:

$$\Psi(t)=\sum_N \exp(-iE_Nt)|N\rangle\psi_N(1). \tag{3.6}$$

Here, if there is no resonance, the nucleating center has the minimum size 1 with a probability close to unity and the critical size 3 with probability $|\xi^2/U(3)|^2$. In the case of resonance, the two sizes have the same probability.

The metastable state $|N=1\rangle$ is destroyed and the system goes over irreversibly to supercritical states $|N\geqslant 3\rangle$ only if the interaction between the spin system and the thermal bath is included.

The kinetics of the growth of the nucleating center will be considered in Sec. 6.

4. Now suppose

$$\xi\ll 1\ll l,\quad N_c=[l]+2\gg 1. \tag{4.1}$$

For $z=2\xi\ll 1$, the Bessel function $J_\nu(z)$ for integer values of the parameter ν decreases rapidly as a function of $|\nu|$:

$$J_0(z)\cong 1,\quad J_\nu(z)=(-1)^{-\nu}J_{-\nu}(z)=\frac{1}{\nu!}\left(\frac{z}{2}\right)^\nu,\quad \nu\geqslant 1. \tag{4.2}$$

The function

$$\psi_N(n)=J_{n-N}(z)\,\theta(n-1) \tag{4.3}$$

at large integer values of N is a good approximation to the exact solution of the system of equations (2.12)–(2.13), since for small n this function is very small, and its truncation at the point $n=1$ plays no role. The requirement that N be an integer gives the energy eigenvalues of the states (4.3) [see (2.6)]:

$$E_N=U(N)=U(N_c)+N_c-N,\quad N\geqslant 3. \tag{4.4}$$

Among the functions (4.3), the function that describes the critical nucleating center,

$$\psi_R(n)=\psi_{N_c}(n)\cong\frac{(-\xi)^{N_c-n}}{(N_c-n)!}\,\theta(n-1),\quad E_R=\delta-1, \tag{4.5}$$

has the largest value.

The functions $\psi_1(n)$ and $\psi_2(n)$, localized around the points $n=1$ and 2, must be described by Bessel functions of fractional order. We do not require the highly excited state $\psi_2(n)$. Under the condition (4.1), the energy E_1 and the values of the function $\psi_1(n)$ at small n are, as before, described by the expressions (3.2). The following amplitudes of this function can be readily obtained directly from Eq. (2.13) if in it we ignore $\psi(n+1)$, or they can be obtained from the form of the Bessel function at small values of the argument z:

$$\psi_L(n)=\psi_1(n)=(-\xi)^{n-1}\,\frac{\Gamma[U(n)]}{\Gamma[U(1)]}\,\theta(N_c-n-0{,}1),$$
$$E_L=-\frac{\xi^2}{U(2)}. \tag{4.6}$$

Here $\Gamma(x)$ is the gamma function. The theta function cuts off the state $\psi_L(n)$ at $n\geqslant N_c$, where the argument of the gamma function in the numerator is negative.

If the distance from the level E_L to the level E_R is greater than ξ^{N_c-1}, there is no resonance between the states L and R. Then during a time $|E_L-E_R|^{-1}$ the elementary nucleating center is transformed into an oscillating state in which the "left-hand" state (4.6) is predominant and there is a very small ($\sim\xi^{2N_c^2}$) admixture of the "right-hand" state (4.5).

If without allowance for resonance $E_L-E_R=0$, then the strength of the repulsion of the resonance levels can be found by using the expression for the Bessel function that is valid for small value of the argument z and small deviation of the negative parameter of the order from an integer value:

$$J_{-\nu}(2\xi)=\frac{\Gamma(\nu)}{\xi^\nu}\frac{\sin(\pi\nu)}{\pi}+\frac{\xi^\nu}{\Gamma(\nu+1)}\cos(\pi\nu),\quad \nu>0. \tag{4.7}$$

Here the first term on the right describes a state that decreases rapidly with decreasing ν. Setting $\nu=-U(n)$, we can show that up to a factor it is equal to (4.6). The second term, which is localized around $\nu\approx 0$, can compete with the first only if $|\sin(\pi\nu)|\ll 0$. For $\nu=N_c-n$, this term goes over into (4.5).

We set

$$E=E_0-\varepsilon,\quad E_0=\delta-1=-\frac{\xi^2}{U(2)},\quad |\varepsilon|\ll 1. \tag{4.8}$$

Bearing in mind that the number $\nu=N_c-n+\varepsilon$ in the expression (4.7) is very close to its nearest integer value, we obtain

$$\psi(n)=(-1)^m\,\frac{\Gamma(m)}{\xi^m}\,[\varepsilon+\Delta(m)],$$
$$\Delta(m)=\frac{\xi^{2m}}{\Gamma(m)\Gamma(m+1)}, \tag{4.9}$$

where $m=N_c-n$ is an integer. We substitute this function in Eq. (2.12):

$$\left(\frac{\xi^2}{m_2}+\varepsilon\right)\frac{m_2}{\xi}\left[\varepsilon+\frac{\xi^2\Delta_2}{(m_2+1)m_2}\right]=\xi(\varepsilon+\Delta_2),$$
$$m_2=N_c-2=U(2)=I-1. \tag{4.10}$$

From this we find the half-separation between the resonance levels:

$$\varepsilon^2=\frac{\xi^2}{m_2}\,\Delta(m_2),\quad \varepsilon=\pm\frac{\xi^I}{\Gamma(I)}. \tag{4.11}$$

As was to be expected, the wave functions (4.9) with these values of ε are smallest in absolute value near $n=N_c/2$.

Usually, resonant states are sought as superpositions of states localized to the left, (4.6), and to the right, (4.5), of the potential barrier:

$$\psi=A\,\psi_L(n)+B\,\psi_R(n). \tag{4.12}$$

The nonorthogonality of the states L and R plays no role. The distance between the resonance levels is determined by

$$H_{LR}=\langle L|H|R\rangle\cong\xi\psi_L(N_c-1)$$
$$\cong\xi\psi_R(2)=(-1)^{N_c}\frac{\xi^I}{\Gamma(I)}, \tag{4.13}$$

which, as it must, is equal to ε (4.11).

5. Now suppose

$$1\ll\xi\ll I. \tag{5.1}$$

In this case, the potential energy (2.4) changes little compared with the transition amplitude when the size of the nucleating center changes by unity, and it would appear that the size can be regarded as a continuous parameter. However, for $z=2\xi\gg|\nu|$ the Bessel function behaves as

$$J_\nu(z)=\sqrt{\frac{2}{\pi z}}\cos\left(z-\frac{1}{2}\,\nu\pi-\frac{1}{4}\,\pi\right), \tag{5.2}$$

and for $z\ll|\nu|$

$$J_\nu(z)=\sqrt{\frac{1}{2\pi}}\left(\frac{ez}{2\nu}\right)^\nu. \tag{5.3}$$

Therefore the recursion relation (2.13) or (2.15) cannot be replaced for any values of the parameters by a differential equation of Schrödinger type.

It follows from (5.2) and (5.3) that the stationary states of the nucleating center have a localization radius $\Delta n \approx z$. Under the condition (5.1), this localization radius is small compared with the size N_c of the critical nucleating center. This means that, as in the previous cases with $\xi \ll 1$, the wave functions can be divided into an isolated "left-hand" state with energy $E_1 = -\xi^2/U(2)$ and a set of equidistant ($\delta E = 1$) "right-hand" states with centers at the integer points N.

Finally, in the case

$$1 \ll I \ll \xi \tag{5.4}$$

the localization of the stationary wave functions of the nucleating center is large compared with N_c, and the potential barrier plays an insignificant role. In this case the initial state with one elementary nucleating center is effectively absolutely unstable. The rate of decay of this state is limited solely by the interaction of the spin system with the thermal bath.

6. We now consider the growth kinetics of a nucleating center in the presence of a thermal bath, the role of which can be played, for example, by the phonon subsystem. At zero temperature, transition of the nucleating center to supercritical states through the potential barrier $U(n)$, $2 \leq n \leq N_c - 1$, is impossible. The system relaxes via a tunneling transition from the state $|N=1\rangle$ to the critical state $|N_c\rangle$ with emission of a phonon.

The irreversible process of tunneling of a nucleating center in the discrete space of sizes of the center is considered in detail in Ref. 3.

Since a complicated three-dimensional model was investigated in that study, it is worth calculating the lifetime of the metastable phase for the one-dimensional system from the beginning.

It is well-known that the relaxation of a quantum subsystem that interacts with a thermal bath is described by a kinetic equation for the density matrix f_{ab}:

$$\frac{\partial \hat{f}}{\partial t} + i[\hat{H}, \hat{f}] = -\hat{J}. \tag{6.1}$$

Here $\hat{H} = \hat{H}_0 + \hat{H}_1$ is the Hamiltonian (2.1), (2.7) of the quantum subsystem, and the "collision integral" in the Born approximation in the interaction V with the thermal bath is

$$\hat{J} = \mathrm{Tr}[\hat{V}, [\hat{\underline{V}}, \hat{f}\hat{\rho}]], \tag{6.2}$$

where Tr denotes summation over the states $\{\alpha\}$ of the thermal bath, and $\hat{\rho}$ is the equilibrium density matrix of the thermal bath. The underlining of an operator expresses the energy conservation law:

$$\underline{V}^{\alpha\beta}_{ab} = \pi \delta(E_a + E_\alpha - E_b - E_\beta) V^{\alpha\beta}_{ab}. \tag{6.3}$$

For definiteness, we choose the interaction in the multiplicative form

$$\hat{V} = \hat{A}\hat{B}, \quad \hat{A} = \sum_r \hat{S}^x_r, \quad \hat{B} = \sum_k (B_k a_k + B_k^* a_k^+). \tag{6.4}$$

Here $a_k(a_k^+)$ is the operator of absorption (respectively, creation) of a phonon.

In this model, the matrix elements of the collision integral are

$$\begin{gathered} J_{ab} = A_{ac} A_{\overleftarrow{cd}} f_{db} + f_{ac} A_{\overrightarrow{cd}} A_{db} - A_{ac} f_{cd} A_{\overrightarrow{db}} - A_{\overleftarrow{ac}} f_{cd} A_{db}, \\ A_{ab} = \sum_n [\psi_a(n)\psi_b(n+1) + \psi_a(n+1)\psi_b(n)], \\ A_{\overrightarrow{ab}} = A_{\overleftarrow{ba}} = A_{ab}\Omega_{a\to b}, \\ \Omega_{a\to b} = \pi \sum_{\alpha,\beta} \rho_\alpha |B^{\alpha\beta}|^2 \delta(E_a + E_\alpha - E_b - E_\beta). \end{gathered} \tag{6.5}$$

At zero temperature, $\Omega_{a\to b}$ is nonzero only if $E_{ab} = E_a - E_b > 0$ holds and phonon emission is possible. We then have

$$\Omega_{a\to b} = \omega_0 \left(\frac{E_{ab}}{\omega_D} \right)^d, \tag{6.6}$$

where ω_D is the Debye frequency, and ω_0 is a quantity that depends on the amplitude $B^{\alpha\beta}$. The index d is equal to unity if a vibration of the chain is excited, and $d=3$ if a three-dimensional acoustic phonon is emitted.

The form of the kinetic equation (6.1) depends on the choice of the representation. To obtain expressions that are valid simultaneously for both the resonant and nonresonant cases, it is most convenient to describe the irreversible tunneling of the nucleating center in the representation (3.2)–(3.4) or (4.3), (4.6). In this representation, there is coherent tunneling, which is characterized by the amplitude of transition from the "left-hand" state $L(N=1)$ to the "right-hand" state $R(N=N_c)$, i.e., by a matrix element of the Hamiltonian H_{LR}, and incoherent tunneling, which is accompanied by phonon emission and is characterized by the probability $\gamma_{LR} = A_{\overrightarrow{LR}} A_{RL}$. In the models (3.1), (4.1), the corresponding quantities are

$$H_{13} = -\frac{\xi^2}{U(2)}, \quad \gamma_{13} = \xi^2 \left[1 + \frac{1}{U(2)} \right]^2 \Omega_{1\to 3}, \tag{6.7}$$

$$H_{LR} = (-1)^{N_c} \frac{\xi^I}{\Gamma(I)}, \quad \gamma_{LR} = \alpha^2 \left(\frac{\xi^{N_c - 2}}{\Gamma(I)} \right)^2 \Omega_{L\to R}. \tag{6.8}$$

The coefficient α in the final expression is approximately equal to

$$\alpha = \sum_{n=1}^{N_c - 1} (\delta - 1 + n)^{\delta - 1} \cong N_c - 1. \tag{6.9}$$

The kinetics of the process is described by the elements f_{LL}, f_{RR}, and f_{LR} of the density matrix. The equation for the nondiagonal element f_{LR} of the density matrix has the form

$$\begin{aligned} f_{LR} &+ iE_{LR} f_{LR} + iH_{LR}(f_{RR} - f_{LL}) \\ &= -(\gamma_{LR} + \Gamma) f_{LR} + \gamma_{LR} f_{RL} + \tilde{\gamma} f_{LL}, \end{aligned} \tag{6.10}$$

where

$$\Gamma = A_{\overrightarrow{N_c N_c+1}} A_{N_c+1 N_c} = \Omega_{N_c \to N_c+1} \gg \gamma_{LR}, \tag{6.11}$$

$$\tilde{\gamma} = A_{LL} A_{\overrightarrow{LR}} \sim \xi^{N_c - 1} \Omega_{L\to R}. \tag{6.12}$$

Ignoring in (6.10) the time derivative, we obtain

$$2\ \mathrm{Im}\, f_{LR}=\frac{1}{\Gamma^2+E_{LR}^2}\,[2H_{LR}\Gamma(f_{LL}-f_{RR})-2\tilde{\gamma}E_{LR}f_{LL}]. \tag{6.13}$$

This quantity must be substituted into the equations for the diagonal elements of the density matrix:

$$\dot{f}_{LL}+2H_{LR}\ \mathrm{Im}\, f_{LR}=-2\gamma_{LR}f_{LL}, \tag{6.14}$$

$$\dot{f}_{RR}-2H_{LR}\ \mathrm{Im}\, f_{LR}=-2\Gamma f_{RR}+2\gamma_{LR}f_{LL}. \tag{6.15}$$

Substituting here $f_{LL}\sim f_{RR}\sim \exp(-t/\tau)$, we find the reciprocal of the time of formation of a supercritical nucleating center:

$$\frac{1}{\tau}\cong 2\gamma_{LR}+\frac{2H_{LR}^2\Gamma-2\tilde{\gamma}H_{LR}E_{LR}}{\Gamma^2+E_{LR}^2}. \tag{6.16}$$

This expression is analogous to the expression that determines the quantum diffusion of particles in crystals.[5]

We shall assume that

$$\Gamma \ll 1, \tag{6.17}$$

i.e., that Γ, which determines the width of the level E_{N_c}, is small compared with the distance $h=1$ between the levels. It is then possible to have the case in which $\Gamma<E_{LR}<1$ holds and the incoherent process dominates, since both terms of the numerator in (6.16) are ξ^2 times smaller than γ_{LR}. If $E_{LR}=0$ holds, then we have $\gamma_{LR}=\tilde{\gamma}=0$, and the relaxation takes place through the coherent channel:

$$\frac{1}{\tau}\cong\frac{2H_{LR}^2}{\Gamma^2}. \tag{6.18}$$

In (6.14) and, accordingly, in (6.16), we have omitted the terms that describe the incoherent nonresonant transitions $L\rightarrow N$ to the supercritical states $N\geqslant N_c+1$. These terms are certainly small as long as the inequality (6.17) holds.

In the case (5.1), the kinetics is, as before, described by the expressions of this section, but the accuracy of the estimates decreases, since they do not take into account the modifications (5.2) of the eigenfunctions in the region where $|U(n)|<\xi$ holds.

Finally, we discuss the dependence of the lifetime of the metastable state on the extent to which it is an equilibrium state. The closer it is to equilibrium, the greater is the size N_c of the critical nucleating center. All the terms of (6.16) have the same explicit dependence on $N_c\sim I/h$:

$$\ln \tau \propto N_c \ln(N_c h/e\xi). \tag{6.19}$$

Here we have gone over from measurement of the energy in units of h to the ordinary energy units, and we have used Stirling's formula $\ln \Gamma(x)\cong x\ln(x/e)$. If formally we replace the expression in the square brackets on the right-hand side of (2.13) by $2\xi\psi+2\xi\psi''$, then we obtain a differential Schrödinger equation with effective mass $m\sim\xi^{-1}$, from which there follows the semiclassical formula

$$\ln \tau_{QC}\propto\int_0^{N_c} dn\sqrt{2mU(n)}\propto N_c^{3/2}\sqrt{h/\xi}\,. \tag{6.20}$$

The dependences (6.19) and (6.20) are given in Sec. 1 [see (1.1) and (1.2)]. It may be more correct to characterize the degree of disequilibrium by the strength of the longitudinal magnetic field h for fixed values of I and ξ. In this notation, the expressions (6.19) and (6.20) take the form

$$\ln \tau \propto \frac{I}{h}\ln\left(\frac{I}{e\xi}\right), \quad \ln \tau_{QC} \propto \frac{I}{h}\sqrt{\frac{I}{\xi}}\,. \tag{6.21}$$

Thus, neglect of the discrete nature of the number of particles in a nucleating center gives a result that is substantially too large if the static part of the energy is large compared with the dynamic part, $I\gg\xi$.

In this paper, the interaction with the thermal bath was taken into account in the Born approximation, and only one phonon participated in the formation of the critical nucleating center. For three-dimensional phonons, this is valid if the separation between the levels is sufficiently small $(h\ll\omega_D)$. For one-dimensional phonons, the opposite inequality is required. The problem of multiphonon relaxation of a discrete nucleating center ("tunneling with dissipation") requires special consideration (see Ref. 4).

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