Fluctuations and anisotropy of cosmic rays in the Galaxy

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The problem of calculating the fluctuations in the particle number and anisotropy of the cosmic rays in the Galaxy in a model with randomly distributed sources is considered. It is shown that the distributions with respect to the particle number and the modulus of the flux are stable laws with exponents $\alpha=5/3$ and $\alpha=5/4$, respectively. The expression obtained for the anisotropy reproduces the available experimental data at energies $E<10^{16}$ eV if it is assumed that the energy dependence of the time of confinement of the particles in the Galaxy has an abrupt change of slope at $E\sim 3\cdot 10^{15}$ eV. © *1995 American Institute of Physics.*

1. INTRODUCTION

One of the central problems of the astrophysics of cosmic rays is that of their origin. There are by now weighty arguments for assuming that cosmic rays with energy $E<10^{17}$ eV have a mainly galactic origin.[1] It is assumed that the most probable sources of cosmic rays that meet the energy requirements are supernova explosions.

To prove this and determine the spacetime distribution $S(t,\mathbf{r})$ of the sources, it is necessary, strictly speaking, to solve an inverse problem—from the properties of the primary cosmic rays observed on the Earth and other radio and gamma astronomical data to obtain information about the location of the sources and the duration of their active phase. However, information about the sources has been traditionally obtained by comparing the experimental data with the results of the solution of some direct problems obtained under different assumptions concerning $S(t,\mathbf{r})$.[1]

The dependence of the theoretical results on the distribution $S(t,\mathbf{r})$ of the sources and the obvious impossibility of their complete determination, especially in the situation in which the observed cosmic rays have come from presently invisible ("dead") sources, lead to the problem of predicting the expected properties of the cosmic rays when information about the sources is not complete. In the calculations, one could, of course, use some particular source distribution, but the choice of one system of sources from the complete set of possible realizations cannot be sufficiently well grounded. As in Ref. 2, we see the way out of this difficulty in the introduction of a statistical ensemble of sources in which one considers the complete set of possible realizations of a system of sources with a probability defined on this set. In such a formulation of the problem, the experimental results are to be compared with the complete distribution of the theoretical results generated by the introduction of the ensemble of sources. This means that to predict the expected properties of the cosmic rays observed near the Earth and solve the problem of the sources of the cosmic rays a statistical approach must be used.

A statistical approach in the problem of estimating the properties of cosmic rays has been considered in several studies.[3–6] The most detailed exposition of this approach is given in Ref. 6, which develops and generalizes previous investigations in this field. In Ref. 6, the statistical characteristics of the investigated quantities (the particle number density, flux, etc.) are expressed in terms of a Green's function that describes the propagation of particles from a discrete monoenergetic galactic source, and a formal expression is given for the probability density of the particle density N of the primary cosmic rays at a given point of space. Analysis of the asymptotic behavior of the distribution at a high density N established an appreciable asymmetry of the distribution and divergence of the dispersion (variance). The divergence of the dispersion is discussed in Ref. 6, and it is noted that in the case of uniformly and randomly distributed discrete sources the dispersion cannot be a good measure of the possible fluctuations of the cosmic rays near the Earth.

However, Lee did not solve the problem of estimating the fluctuations of the cosmic rays. In the estimates made in Ref. 6, and then in Ref. 1, a cutoff parameter τ_0 was introduced to eliminate the divergence. This parameter has the meaning of a time and excludes from consideration very young and close sources. To obtain agreement with the experimental data in the estimates of the fluctuations and of the anisotropy, it is necessary to choose different values of τ_0.

In our view, the introduction of the cutoff parameter τ_0 is not physically justified. The reason for the divergence that arises is the special form of the distributions that are implicitly used in the problem and for which there is no finite second moment. In the case of such distributions, the variance is not a measure of the fluctuations, as is correctly noted in Ref. 6. To estimate the fluctuations, it is necessary to use a certain characteristic width of the distribution, and this requires knowledge of the distribution.

The aim of this paper is to develop a method of calculation of the distributions with respect to the particle number and the modulus of the flux and to estimate the fluctuations and anisotropy of the cosmic rays in a model with randomly distributed galactic sources.

2. MOMENTS OF PARTICLE NUMBER AND FLUX MODULUS DISTRIBUTIONS

Suppose cosmic rays are generated at random instants of time by point sources distributed randomly in the volume V_R of the Galaxy. We shall denote the finite region with volume

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$V_R \times T$ in the four-dimensional spacetime $R^3 \times R^1$ by U. The source situated at point $\mathbf{r}_i$ emits N_i particles at the time t_i. For simplicity, we shall assume that all the sources have the same power, i.e., $N_i = N_0$. It is convenient to describe the propagation of the particles in the interstellar medium by the Green's function $G(t,\mathbf{r};t_i,\mathbf{r}_i)$, which gives the contribution to the particle number at the point of observation $y=(t,\mathbf{r})$ from a particle generated in the source with coordinates $x_i=(t_i,\mathbf{r}_i)$. The total concentration $N_R(t,\mathbf{r})$ of particles at the observation point produced by the radiation of all sources in the volume V_R is given by

$$N_R(t,\mathbf{r}) = \sum_i N_i = N_0 \sum_i G(t,\mathbf{r};t_i,\mathbf{r}_i). \tag{1}$$

We make the following assumptions concerning the system of sources.

1) The system of sources is a Poisson ensemble,[7] i.e., it possesses the following properties:

a) the numbers of sources n_1 and n_2 in the regions U_1 and U_2, respectively, are independent random variables;

b) the probability $\mathsf{P}(n=k)$ for any $k>0$ depends on k and the volume u of the region U but not on its shape;

c) for small values of the volume u,

$$\mathsf{P}(n=1) = qu + o(u),$$

$$\mathsf{P}(n \geqslant 2) = o(u),$$

where q is a constant that gives the mean density of the sources in the region U.

If these properties hold, the distribution of n satisfies Poisson's law with parameter $\lambda = qu$:

$$\mathsf{P}(n=k) = \frac{\lambda^k e^{-\lambda}}{k!};$$

d) the distribution of the positions of the source in U is uniform, i.e., the distribution density is $\Psi = 1/u$.

2. For any region U of finite volume, the number n of sources in U and their positions $x_1, x_2, \ldots$ are independent random variables.

Traditionally, the fluctuations of the cosmic rays associated with the random nature of the sources are calculated by means of the moments of the corresponding distributions.

The mean particle concentration at the point $(t,\mathbf{r})$ can be calculated in accordance with the expression

$$\langle N(t,r) \rangle = \bar{n} N_0 \iint_{U_R} dt' d^3r' \Psi(t',r') G(t,\mathbf{r};t',\mathbf{r}'), \tag{2}$$

where $\bar{n}$ is the mean number of sources in the Galaxy.

In the case of a Poisson ensemble, the expression for the dispersion of the particle concentration at the point of observation has the form

$$\mathsf{D}N(t,\mathbf{r}) = \bar{n} N_0^2 \iint_{U_R} dt' d^3r' \Psi(t',\mathbf{r}') G(t,\mathbf{r};t',\mathbf{r}')^2. \tag{3}$$

In the case of diffuse motion, the particle flux is

$$\mathbf{j} = -D \nabla N.$$

The mean flux and its dispersion are, respectively,

$$\langle \mathbf{j}(t,\mathbf{r}) \rangle = -\bar{n} N_0 D \times \iint_{U_R} dt' d^3r' \Psi(t',\mathbf{r}') \nabla_{\mathbf{r}} G(t,\mathbf{r};t',\mathbf{r}'), \tag{4}$$

$$\mathsf{D}j(t,\mathbf{r}) = -\bar{n} N_0^2 D^2 \iint_{U_R} dt' d^3r' \Psi(t',\mathbf{r}') \times [\nabla G(t,\mathbf{r};t',\mathbf{r}')]^2. \tag{5}$$

We shall consider the propagation of the cosmic rays in the Galaxy in the framework of the diffusion approximation. We shall assume that there are no fluctuations during the passage of the particles from the source to the point of observation. The concentration of the cosmic rays at a given point y will be a random variable due to the random nature of the sources. We write the equation that describes this process in the form[1]

$$\frac{\partial N}{\partial t} - D\Delta N + \Gamma N = N_0 \sum_i \delta(t-t_i)\delta(\mathbf{r}-\mathbf{r}_i), \tag{6}$$

where the term ΓN describes approximately the loss of particles due to their escape from the Galaxy.

The equation for the Green's function that corresponds to (6) has the form

$$\frac{\partial G(t,\mathbf{r};t',\mathbf{r}')}{\partial t} - D\Delta G(t,\mathbf{r};t',\mathbf{r}') + \Gamma G(t,\mathbf{r};t',\mathbf{r}') = \delta(t-t_i)\delta(\mathbf{r}-\mathbf{r}_i),$$

from which we readily obtain

$$G(t,\mathbf{r};t',\mathbf{r}') = \frac{\Theta(t-t')}{[4\pi D(t-t')]^{3/2}} \exp\left(-\Gamma(t-t') - \frac{|\mathbf{r}-\mathbf{r}'|^2}{4D(t-t')} \right), \tag{7}$$

where

$$\Theta(t) = \begin{cases} 1, & t>0, \\ 0, & t<0. \end{cases}$$

Using the Green's function (7) and the expressions (2)–(5), we can find the fluctuations in the particle number and the anisotropy of the cosmic rays. Since the mean number of sources in the Galaxy is $\bar{n} = qV_R T$, where q is the frequency of occurrence of a source in unit volume in unit time, in the limit $V_R \to \infty$, $T \to \infty$

$$\langle N(t,\mathbf{r}) \rangle = qN_0 \Gamma^{-1}, \quad \langle \mathbf{j}(t,\mathbf{r}) \rangle = 0,$$

$$\mathsf{D}N = qN_0 (4\pi D)^{-3/2} 2^{-3/2} \int_0^{+\infty} d\tau \tau^{-3/2} e^{-2\Gamma\tau} = \infty, \tag{8}$$

$$\mathsf{D}j = qN_0^2 (16\pi)^{-1} (8\pi D)^{-1/2} \frac{3}{2} \int_0^{+\infty} d\tau \tau^{-5/2} e^{-2\Gamma\tau} = \infty. \tag{9}$$

Because the integrals in (8) and (9) diverge, estimates are obtained by introducing a cutoff parameter $\tau_0(\Gamma\tau_0 \ll 1)$; this is a time and excludes very young sources from consideration. In this case,

$$\mathrm{D}N = qN_0(4\pi D)^{-3/2}(2\tau_0)^{-1/2},$$

$$\mathrm{D}j = qN_0^2(16\pi)^{-1}(8\pi D)^{-1/2}\tau_0^{-3/2}.$$

Then for the relative fluctuations of the concentration and of the anisotropy, we obtain

$$\delta N = \frac{\sqrt{\mathrm{D}N}}{\langle N\rangle} = q^{-1/2}\Gamma(4\pi D)^{-3/4}(2\tau_0)^{-1/4},$$

$$\delta j = \frac{3\sqrt{\mathrm{D}j}}{\langle N\rangle c} = \frac{3}{4c}\, q^{-1/2}\Gamma\pi^{-1/2}(8\pi D)^{-1/4}\tau_0^{-3/4}.$$

We estimate the fluctuation of the particle number and the anisotropy for values of the parameters of the Galaxy typical of a model with a large halo (Ref. 1): $q=3\cdot 10^{-78}$ $\mathrm{cm}^{-3}\cdot\mathrm{s}^{-1}$, corresponding to the explosion of one supernova every 30 yr in the complete Galaxy, $\Gamma^{-1}=10^8$ yr, $D=5\cdot 10^{28}$ $\mathrm{cm}^2\cdot\mathrm{s}^{-1}$.

An estimate of τ_0 made under the assumption that almost the entire concentration of cosmic rays at the Earth is determined by a single nearby source that exploded at the time $t-\tau_0$ gives $\tau_0 \approx 10^4$ yr. In this case, the fluctuations of the concentration are of order 1% and the anisotropy is $\delta_j \approx 0.1\%$, exceeding the required value by an order of magnitude.[1] To obtain an anisotropy $\delta_j \approx 0.01\%$ corresponding to the given values of Γ and D, it is necessary to set $\tau_0 \approx 10^5$ yr.

We assume that the reason for the divergence that has arisen is the special form of the employed distributions, which do not have a finite second moment. In the case of such distributions, the dispersion is not a measure of the fluctuations. To estimate the fluctuations, it is necessary to use a certain characteristic width of the distribution, and this requires knowledge of the distribution.

3. DISTRIBUTION WITH RESPECT TO THE PARTICLE NUMBER

To find the distribution with respect to the particle number, we calculate the characteristic function $f_R(t)$ of the random variable (1) on the basis of assumptions 1a) and 1b) about the distribution of the sources. It can be seen from the expression (1) that $N_R(y)$ is a sum of the random number n of identically distributed random variables $N_j = N(y,x_j)$ that are independent of n and each other. If $v_R(t)$ is the characteristic function of each random variable N_j, we can write the characteristic function $f_R(t)$ in the form

$$f_R(t) = \mathrm{M}\exp\{itN_R\} = \sum_{k=0}^{\infty} \mathrm{P}(n=k)\mathrm{M}\exp\{it(N_1+N_2+\cdots+N_k)\} = \sum_{k=0}^{\infty} \exp(-qu_R)\frac{(qu_R)^k}{k!}\varphi_R^k(t) = \exp\{qu_R(\varphi_R(t)-1)\},$$

where $u_R = V_R T$. The actual form of the particle number distribution density is determined by the form of the Green's function. We calculate it for the case when the propagation of the cosmic rays is described by Eq. (6).

We transform the expression in the exponential of the last equation:

$$qu_R[\varphi_R(t)-1] = qu_R\int_{U_R}[\exp(itN(x,y))-1]\frac{dx}{u_R} = q\int_{U_R}[\exp(itN(x,y))-1]dx.$$

Then for the function $\ln f_R(t)$ we obtain

$$\ln f_R(t) = q\int_{U_R}[\exp(itN(x,y))-1]dx.$$

Since the distribution in which we are interested has a finite mean value, this last expression can be written in the form

$$\ln f_R(t) = iqt\langle n\rangle + q\int_{U_R}[\exp(itN(x,y))-1-itN]dx,$$

or, going over to $\tilde{N} = N - \langle N\rangle$,

$$\ln \tilde{f}_R(t) = q\int_{U_R}[\exp(itN(x,y))-1-itN]dx. \qquad (10)$$

By the concentration of particles $N(t,\mathbf{r})$ produced by the complete system of sources, we shall understand the limit $\lim_{V_R,T\to\infty} N_R$. To elucidate the conditions of existence of such a limit, it is sufficient to consider the limiting value as $V_R\to\infty$, $T\to\infty$ of the characteristic function of the quantity $N_R(y)$. It is shown in Ref. 7 that under the conditions that hold in our problem a limit distribution exists, by virtue of which (10) can be written in the form

$$\ln \tilde{f}(t) = q\int_U[\exp(itN(x,y))-1-itN]dx. \qquad (11)$$

We note that this result is a special case of the result of the model of point influence sources proposed in Ref. 7. It is shown there that the integral (11) can be transformed to

$$\ln \tilde{f}(t) = q\int_V[\exp(itN)-1-itN]\mu(dN), \qquad (12)$$

where $V=\{N(x):x\in U\}$, and

$$\mu(\omega) = q\int_{\{x:N(x)\in\omega\}V} dx. \qquad (13)$$

An analytic calculation of the integral (12) can be made only for a definite form of the function $N(y,x)$ (in the terms of Ref. 7, the function N should be called the "influence function"). Our obtained expression

$$N(x,y) = \frac{N_0}{[4\pi D(t-t')]^{3/2}}\exp\left(-\Gamma(t-t') - \frac{|\mathbf{r}-\mathbf{r}'|^2}{4D(t-t')}\right) \qquad (14)$$

has a form that does not permit analytic calculation of (12). The expression (14) differs from the form required in Ref. 7 by the factor $\exp(-\Gamma(t-t'))$. If this factor is ignored, then the integral (13) takes the form of the integral of Ref. 8 and can be calculated analytically:

$$\mu(\omega)=d^3r \int\int_{N(x)>\omega} dt=4\pi\int\int_{N(x)>\omega} r^2 dr dt$$

$$=\frac{4\pi N_0^{5/3}}{D\omega^{5/3}}\int\int_{N(x)>1} r^2 dr dt=\frac{N_0^{5/3}}{D} k\omega^{-5/3},$$

where

$$k=4\pi\int\int_{N(x)>1} r^2 dr\ dt=4\pi\int_0^{(4\pi)^{-1}}\int_0^{[-6t\ \ln(4\pi t)]^{-1/2}} r^2 dr$$

$$=\frac{1}{10\pi}\left(\frac{3}{5}\right)^{3/2}.$$

Hence

$$\mu(d\omega)=\frac{5}{3}\frac{N_0^{5/3}}{D} k\omega^{-8/3} d\omega$$

and as a result we obtain for the characteristic function $\tilde{f}(t)$

$$\ln \tilde{f}(t)=q\int_V [\exp(itN(x,y))-1$$

$$-itN]\frac{5}{3}\frac{N_0^{5/3}}{D} k\omega^{-8/3} d\omega$$

$$=-\frac{3}{2}\Gamma\left(\frac{1}{3}\right)k\frac{qN_0^{5/3}}{D}\left[|t|^{5/3}\exp\left(i\frac{\pi}{6}\text{ sign } t\right)\right]. \tag{15}$$

In accordance with Ref. 7, the obtained distribution belongs to the class of stable laws with parameters

$$\alpha=\frac{5}{3},\quad \beta=1,\quad \gamma=0,\quad \lambda=\frac{3}{2}\Gamma\left(\frac{1}{3}\right)k\frac{qN_0^{5/3}}{D}.$$

The distribution of the original random variable N differs from $\tilde{N}$ only by the value of the parameter γ, which is a mathematical expectation and equal to $\langle N\rangle$.

The small extra complexity of the form of the function N associated with the presence of the factor $\exp[-\Gamma(t-t')]$ deprives us of the possibility of finding the characteristic function of the distribution analytically. Therefore, to find the characteristic function, we have used the numerical method. After the exponent α of the stable law and the factor k have been found, the remaining parameters of the distribution can be calculated analytically.

To test the program, we used the previously solved problem of temperature bursts in a nuclear reactor.[8] As a result of the test, it was established that the parameter α can be calculated with an error less than 1%.

The calculation for the function (14) showed that for the values of Γ and D that we employed the value of the parameter α hardly differs from 5/3, while $k=0.0147$.

4. DISTRIBUTION WITH RESPECT TO THE FLUX MODULUS

One can similarly obtain an expression for the characteristic function of the distribution with respect to the modulus of the particle flux in the Galaxy. In accordance with (12), this characteristic function has the form

$$\ln \tilde{f}(t)=q\int_V [\exp(itj)-1-itj]\mu(dj),$$

where

$$j=|-DN_0\nabla G(t,\mathbf{r};t',\mathbf{r}')|=\frac{2\pi DN_0|\mathbf{r}-\mathbf{r}'|}{[4\pi D(t-t')]^{5/2}}$$

$$\times\exp\left(-\Gamma(t-t')-\frac{|\mathbf{r}-\mathbf{r}'|^2}{4D(t-t')}\right). \tag{16}$$

The form of this function is appreciably more complicated than the form (14), and the calculations needed to find the characteristic function cannot be done analytically even if the factor $\exp[-\Gamma(t-t')]$ is ignored. By virtue of this, the parameter α was found numerically. According to the results of the calculations, the parameter α of the stable distribution for the flux modulus was taken equal to (1.25), since this number is the simple rational fraction 5/4. The choice of this number can also be justified by dimensional arguments.

The remaining parameters of the stable law can be found analytically. The values that we obtained are

$$\beta=1,\quad \gamma=0,\quad \lambda=4\Gamma\left(\frac{3}{4}\right)kqN_0^{5/4}D^{1/4},\quad k=0.0484.$$

5. FLUCTUATIONS OF THE PARTICLE NUMBER AND ANISOTROPY OF COSMIC RAYS IN THE GALAXY

Figure 1 shows the densities of the distributions with respect to the particle number and the flux modulus for the values of the parameters of the stable laws given above and a special choice of N_0 convenient for estimating the fluctuations of the particle number and the anisotropy. We obtained these distributions from standard stable distributions. Since for these distributions there are no second moments, to make the estimates we use a certain characteristic width of the distributions. For example, as such width we choose the interval of values that corresponds to the confidence level 68%.

The width of the distribution for the concentration will be

$$\Delta N\propto\lambda^{1/\alpha}\propto q^{3/5}D^{-3/5}N_0,$$

and

$$\Delta j\propto\lambda^{1/\alpha}\propto q^{4/5}D^{1/5}N_0.$$

Defining the fluctuations of the concentration and the anisotropy as follows:

$$\delta N=\frac{\Delta N}{\langle N\rangle},\quad \delta j=\frac{3\Delta j}{\langle N\rangle c},$$

we find

$$\delta N\propto q^{-2/5}D^{-3/5}\Gamma, \tag{17}$$

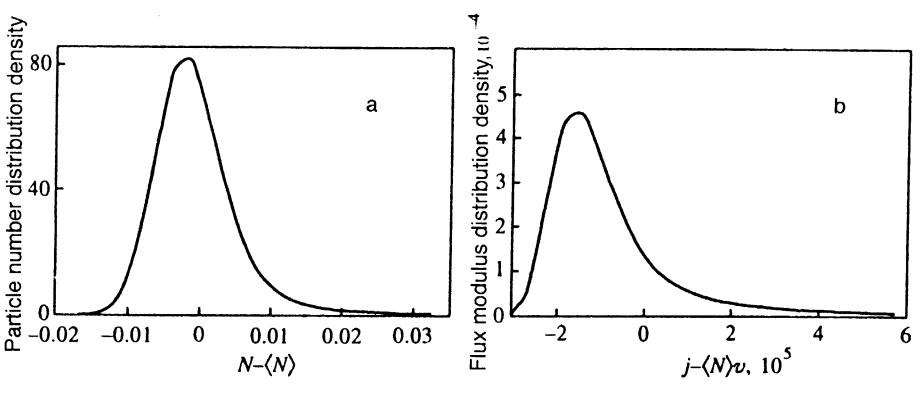

FIG. 1. a) Particle number distribution density; b) flux modulus distribution density.

$$\delta j \propto q^{-1/5} D^{1/5} \Gamma. \tag{18}$$

For example, for $D=5\cdot 10^{28}$ cm$^2\cdot$s^{-1}, which corresponds to the range of energies 10^9-10^{10} eV, $\Gamma^{-1}=10^8$ yr, we obtain $\delta N\sim 1\%$, $\delta j\sim 0.01\%$. These values agree well with the estimates of Refs. 1, 9, and 10.

We note that our solution to the problem of the fluctuations of the particle number and the anisotropy of the cosmic rays, which belongs to the class of stable laws, confirms at the rigorous mathematical level the point of view expressed in Ref. 1, namely, that the decisive causes of the fluctuations are the nearby sources. This follows from one of the properties of the stable laws, in accordance with which the distribution of the investigated quantity in the case of a Poisson ensemble of sources is determined by the contribution of the sources in the immediate vicinity of the considered point (see, for example, Refs. 11–13).

We have solved the problem for monoenergetic sources. In view of the fact that the energy losses of the particles as they pass through the Galaxy are small for $E<10^{17}$ eV, the particles do not go over from one energy interval to another, and the form of Eq. (6) will be the same for all energies. Therefore, we can go over from monoenergetic sources to sources with continuous spectrum and estimate the dependence of the anisotropy on the energy. For each energy interval, we shall obtain relations that are identical to (18) with corresponding values of D and Γ, i.e., the dependence of the anisotropy on the energy will have the form

$$\delta j(E) \propto q^{-1/5} D(E)^{1/5} \Gamma(E). \tag{19}$$

Similarly, the expression (17) can be rewritten in the form

$$\delta N(E) \propto q^{-2/5} D(E)^{-3/5} \Gamma(E). \tag{20}$$

According to the currently available data, $D\propto E^{\beta}$, $\beta\sim 0.2-0.7$ (Ref. 1) and $\Gamma\propto E^{\gamma}$, $\gamma\sim\beta$ (Refs. 1 and 14). For these values of β and γ, the nature of the variation of the anisotropy with increasing E is determined by the value of Γ. If it is assumed that γ varies from ~ 0.15 at $E\sim 10^{10}$ eV to ~ 0.54 at $E\sim 10^{15}$ eV, then our obtained expression (19) describes the existing experimental data[1,9,14] on the anisotropy (see Fig. 2). Figure 2 shows our estimates of the time of confinement of the cosmic rays in the Galaxy. The obtained energy dependence of this time makes it possible to explain the behavior of the cosmic ray spectrum. Indeed, because

$$\langle N\rangle = qN_0\Gamma^{-1} \propto E^{-\gamma_{\rm source}} E^{-\gamma(E)} = E^{-\gamma_{\rm source}-\gamma(E)},$$

an abrupt change in the dependence of the time of confinement (i.e., of γ) on the energy leads to a change in the exponent of the cosmic rays ($\gamma_{\rm source}+\gamma$). According to Refs. 1 and 15, the exponent of the cosmic ray spectrum changes at $E\sim 3\cdot 10^{15}$ eV from 1.7 to 2.1, in good agreement with the value we obtained for γ.

Thus, the abrupt change in the cosmic ray spectrum observed at $E\sim 3\cdot 10^{15}$ eV can be explained by the abrupt change in the dependence of the time of confinement of the particles in the Galaxy on the energy. The presence of the abrupt change in the time of confinement at $E\sim 10^{15}$ eV was also indicated by solution of a nonlinear kinetic equation for the propagation of cosmic rays.[16]

6. CONCLUSIONS

In this paper, we have considered the problem of calculating the fluctuations and anisotropy of the cosmic rays in a model with randomly distributed sources. The calculations have shown the following.

1. The distributions with respect to the particle number and the modulus of the flux are stable distributions with exponents 5/3 and 5/4, respectively, and this confirms the view

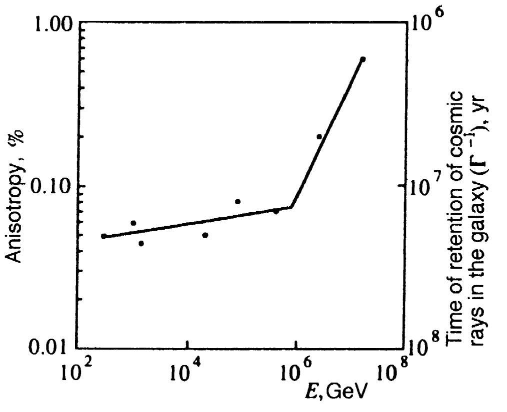

FIG. 2. Anisotropy and time of confinement of cosmic rays in the Galaxy; the points are experimental data on the anisotropy,[1,9,11] and the solid continuous lines are our estimates of the time of confinement of cosmic rays in the Galaxy.

that the nearby sources make the decisive contribution to the fluctuations of the particle number and to the anisotropy of the cosmic rays.

2. For the fluctuations of the particle number and the anisotropy in a model with large halo with parameters $\Gamma^{-1}=10^8$ yr, $D=5\cdot 10^{28}$ $cm^2\cdot s^{-1}$, $q=3\cdot 10^{-78}$ $cm^{-3}\cdot s^{-1}$, we have obtained as a result of our calculations the values $\delta N\sim 1\%$ and $\delta j\sim 0.01\%$, which do not contradict the estimates of Refs. 1, 9, and 10.

3. The relative fluctuations of the particle number and the anisotropy depend on the parameters of the problem as follows: $\delta N\propto D^{-3/5}\Gamma$ and $\delta j\propto D^{1/5}\Gamma$. The obtained dependence of the anisotropy on the energy makes it possible to explain the experimental data under the condition that $\Gamma(E)\propto E^{\gamma}$, where the exponent γ changes from ~ 0.15 to ~ 0.54 in the considered range of energies.

4. The abrupt change observed in the cosmic ray spectrum at $E\sim 3\cdot 10^{15}$ eV can be attributed to an abrupt change in the energy dependence of the time of confinement of particles in the Galaxy.

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