Quantum coherence in heavy-flavor production on nuclei

N. N. Nikolaev

Institut für Kernphysik, Forschungszentrum Jülich, D-52425, Jülich, Germany; L. D. Landau Institute for Theoretical Physics, GSP-1, 117940, ul. Kosygina 2, 117334 Moscow, Russia

G. Piller

Department of Physics and Mathematical Physics, University of Adelaide, S.A. 5005, Australia; Physik Department, Technische Universität, München, D-85747, Garching, Germany

B. G. Zakharov

L. D. Landau Institute for Theoretical Physics, GSP-1, 117940, ul. Kosygina 2, 117334 Moscow, Russia

(Submitted 27 December 1994)
Zh. Éksp. Teor. Fiz. **108**, 1554–1568 (November 1995)

We describe a light-cone wave-function formalism for hadroproduction of heavy flavors via gluon–gluon fusion. The approach is well suited to address coherence effects in heavy-quark production on nuclei at small $x_2 \lesssim 0.1A^{-1/3}$. Nuclear attenuation in hadroproduction of heavy-flavors is found to be similar to shadowing effects for heavy-flavor structure functions in deep inelastic scattering from nuclei. Nevertheless, the remaining differences in the theoretical formulation of both processes imply corrections to the factorization theorems.

1. INTRODUCTION

Heavy-flavor production in hadronic interactions at high energy is one of the traditional applications of perturbative quantum chromodynamics (pQCD). Although perturbative calculations are available now in next-to-leading order pQCD, and seem to be in reasonable agreement with experimental data (see Ref. 1 and references therein), the production of heavy quark–antiquark ($Q\bar{Q}$) pairs has certainly not exhausted its potential in revealing precious details of strong interaction physics. One of the missing ingredients is the quantitative understanding of nuclear attenuation effects, which offer a chance to learn about the propagation of colored $Q\bar{Q}$ pairs through the nuclear environment. This matter is closely connected to the effects of "color-transparency" and "color-opacity", which are widely discussed issues in a large variety of processes, ranging from quasielastic electron scattering to heavy quarkonium production (see, e.g., Refs. 2 and 3 and references therein).

In the leading order of pQCD, the cross-section for heavy quark–antiquark production via the collision of hadrons h_1 and h_2 at a squared center-of-mass energy s can be written as

$$\sigma_{Q\bar{Q}}(s) = \sum_{i,j} \int dx_1 dx_2 f^i_{h_1}(x_1,\mu^2) f^j_{h_2}(x_2,\mu^2) \sigma_{ij}(x_1 x_2 s,\mu^2). \tag{1}$$

Here $f^i_{h_1}(x_1,\mu^2)$ and $f^j_{h_2}(x_2,\mu^2)$ are the densities of partons i and j, carrying fractions x_1 and x_2 of the light-cone momenta of the colliding projectile and target respectively. The partonic subprocess $i+j \to Q+\bar{Q}$ is described by the cross-section $\sigma_{ij}(x_1x_2s,\mu^2)$ and requires a squared center-of-mass energy $x_1x_2s > 4m_Q^2$, where m_Q is the mass of the heavy quark. Factorization in parton densities and hard partonic subprocesses is carried out on a typical scale $\mu^2 \sim 4m_Q^2$.

In this work we will be concerned with nuclear effects in inclusive hadroproduction of $Q\bar{Q}$ pairs which carry a high energy in the laboratory frame, where the target nucleus is at rest. In particular, we consider processes with small target light-cone momentum fractions $x_2 \lesssim 0.1A^{-1/3}$. Furthermore, we will restrict ourselves to moderate x_F, which is defined as the fraction of the projectile momentum carried by the $Q\bar{Q}$ pair as seen in the center-of-mass frame. Thus we concentrate on the kinematic domain where $Q\bar{Q}$ production is dominated by the gluon fusion subprocess $g+g \to Q+\bar{Q}$ and neither annihilation of light quarks nor excitations of higher-twist intrinsic heavy-quark components of the projectile wave function[4] are of importance. The pQCD cross-section (1) is then proportional to the density of gluons in the beam and target. In the case of open-charm production at typical Tevatron energies ($s=1600$ GeV^2) this implies that we focus on the region $0<x_F<0.5$.

We develop a description of heavy-flavor hadroproduction in the laboratory frame in terms of light-cone wave functions (LCWF).[5] We find, that the underlying QCD mechanism of $Q\bar{Q}$ production can be viewed as diffractive dissociation of projectile gluons into $Q\bar{Q}$ pairs. The resulting formalism closely resembles the description of coherence effects in diffractive scattering on nuclei. Although there is much similarity between nuclear shadowing effects in hadroproduction of heavy flavors and shadowing of heavy-flavor contributions in deep inelastic scattering (DIS), they are nevertheless different and cannot be cast in the form of modified nuclear gluon distributions which are unique for all hard scattering processes.

We apply our formalism to open-charm production and find only small nuclear attenuation effects in the kinematic region considered. For the conventional parametrization we have $\sigma_{c\bar{c}} \propto A^\alpha$, where A is the nuclear mass number, our re-

 1063-7761/95/110851-09$10.00

sults correspond to an exponent $\alpha \approx 0.99$. The corresponding experimental situation is not absolutely conclusive yet (for a comprehensive review on open-charm production see Refs. 4 and 6). In early experiments[7,8] open-charm production was estimated indirectly from the observed yield of prompt leptons, rather than by a direct observation of the charmed particles produced. The strong nuclear effects found in these experiments, $\alpha \sim 2/3$, led to the suggestion that large intrinsic charm components are present in the respective hadron projectile (see Ref. 4 and references therein). More recent experiments directly observe D-mesons and obtain $\alpha \approx 1$, which is in good agreement with our results ($\alpha(\langle x_F \rangle = 0.24) = 0.92 \pm 0.06$ in the WA82 experiment with a 340-GeV π^- beam,[9] $\alpha(0 < x_F < 0.5) = 1.00 \pm 0.05$ in the E769 experiment with a 250-GeV $\pi^\pm$-beam[10]).

Our presentation is organized as follows: In Sec. 2 we analyze the space-time picture of heavy-quark production at small x_2. In Sec. 3 we develop a LCWF description for hadroproduction of heavy flavors on free nucleons. The central result in this section is the representation of the inclusive $Q\bar{Q}$ production cross-section in terms of the cross-section for the scattering of a three-particle, color-singlet, $Q-\bar{Q}$-gluon state from the nucleon target. We also comment on the cancellation of co-mover interactions. We extend our discussion to nuclear targets in Sec. 4 and apply our formalism explicitly to open charm. Here we make a special emphasis on breaking of the so-called factorization theorems by nuclear shadowing. Finally we summarize and conclude in Sec. 5 and indicate the difference between nuclear attenuation in inclusive open charm and exclusive charmonium production.

2. THE SPACE-TIME PICTURE OF $Q\bar{Q}$ PRODUCTION AT SMALL x_2

In the laboratory frame the quantum-mechanical mechanism of nuclear attenuation in hadroproduction of heavy $Q\bar{Q}$ pairs at small x_2 and moderate x_F is similar to the mechanism of nuclear shadowing in deep inelastic lepton scattering at small values of the Bjorken variable $x = Q^2/2M\nu$ (with $Q^2 = -\nu^2 + \mathbf{q}^2$ the squared momentum transfer and M the nucleon mass). In DIS the absorption of the exchanged virtual photon on the target proceeds, for $x < 0.1$, via the interaction of quark–antiquark fluctuations present in the photon wave function. These $q\bar{q}$ pairs are formed at a typical distance $l_{q\bar{q}}$ from the target,

$$l_{q\bar{q}} \sim \frac{2\nu}{Q^2} = \frac{1}{Mx}. \tag{2}$$

For $x < 0.1$ the propagation length of the $q\bar{q}$ pairs exceeds the average nucleon–nucleon separation in nuclei, $l_{q\bar{q}} > R_{NN}$. Consequently, the quark pairs will start to interact coherently with several nucleons inside the target. This leads to shadowing (see, e.g., Refs. 11 and 12). If x decreases further,

$$x \lesssim \frac{1}{R_A M} \sim 0.1 A^{-1/3}, \tag{3}$$

the propagation length of the quark pairs becomes larger than the size of the nuclear target itself, i.e.,

$$\lambda_{q\bar{q}} > R_A. \tag{4}$$

Hence, the quark–antiquark pairs will interact coherently with the whole target nucleus while their transverse size $|\mathbf{r}|$ is frozen.[11] Shadowing will be fully developed then.

Similarly, at small values of x_2 and moderate x_F hadroproduction of heavy quarks can also be treated in the laboratory frame in terms of $Q\bar{Q}$ fluctuations of gluons interacting with the hadron projectile. Their typical propagation length is

$$l_{Q\bar{Q}} \sim \frac{2\nu_G}{4m_Q^2} \approx \frac{1}{Mx_2}, \tag{5}$$

where $\nu_G = x_1 E_{\rm lab}$ denotes the laboratory energy of the respective parent gluon and $E_{\rm lab}$ stands for the beam energy. Evidently, if the propagation length of the $Q\bar{Q}$ pairs is smaller than the average internucleon distance in the target nucleus, $l_{Q\bar{Q}} \lesssim R_{NN}$, then $Q\bar{Q}$ production on nuclei will be equal to the incoherent sum of $Q\bar{Q}$ production on individual nucleons within the target. In this regime the intranuclear interactions of the beam gluons reduce to rotations in color space and the flux of gluons is preserved.[1] However, if the energy of the interacting gluons in the beam is large or equivalently if $x_2 < 0.1A^{-1/3}$ is small, one obtains as before

$$l_{Q\bar{Q}} > R_A, \tag{6}$$

which means that the formation of $Q\bar{Q}$ pairs takes place coherently on the whole nucleus, very much like DIS at small x.

Once produced, a $Q\bar{Q}$ pair evolves into charmed hadrons after a typical formation (recombination) length

$$l_f \sim \frac{2\nu_G}{4m_D^2 - 4m_Q^2} \gg l_{Q\bar{Q}}, \tag{7}$$

where m_D is the D-meson mass. The inequality between l_f and $l_{Q\bar{Q}}$ greatly simplifies our further consideration, as it implies that we may neglect the intrinsic evolution of a produced $Q\bar{Q}$ pair as it propagates through the nucleus.

At large $x_F \sim 1$, the momentum of the projectile is transferred to a heavy $Q\bar{Q}$ state collectively by several partons of the hadron projectile.[4] Such a $Q\bar{Q}$ pair can therefore not be assigned to a particular projectile gluon. Consequently, the excitation of heavy-quark components of the projectile at $x_F \sim 1$ is quite different from the $Q\bar{Q}$ production mechanism at moderate x_F. In this work we will focus on moderate x_F only, where the above mechanism is not important.

From a diagrammatic point of view an accurate analysis of coherence effects requires the calculation of higher order diagrams, in which the incoming projectile gluon interacts with more than one nucleon. In standard pQCD this becomes a nearly impossible task. As for DIS at $x < 0.1A^{-1/3}$, an analysis of coherence effects is greatly simplified if one uses a LCWF formalism. This allows one to make explicit use of the fact that at high energies, or small x_2, the transverse size of partonic fluctuations is conserved due to Lorentz time dilatation. A very important difference between hadroproduction of heavy flavors and DIS is that in DIS one starts out with a color-singlet photon, whereas in the former case the incident gluon carries color charge. Therefore, one might have expected to encounter an infrared divergent $Q\bar{Q}$ production cross-section, unless the interactions with comoving spectator partons of the incident color-singlet hadron were fully included. We shall demonstrate, however, that the LCWF formalism of Ref. 5 can be generalized to hadroproduction of heavy flavors in an infrared-stable manner.

3. HADROPRODUCTION OF $Q\bar{Q}$ PAIRS ON FREE NUCLEONS

In this section we develop a formalism for hadroproduction of heavy flavors in terms of LCWF valid at small $x_2<0.1$ and moderate x_F. Applied to hadron–hadron collisions our results can be shown to be equivalent to the conventional pQCD parton model. However our new formalism becomes truly valuable, when it is applied to coherence effects in hadron–nucleus interactions.

First we will consider the production of heavy $Q\bar{Q}$ pairs through the interaction of a gluon from the projectile with the nucleon target. To finally obtain the total production cross-section we have to convolute our result with the gluon distribution of the beam hadron.

In the LCWF technique one must carefully distinguish between "bare" and "dressed" (physical) partons. In our specific process the incident gluon will evolve after the interaction with the target into a state which contains a $Q\bar{Q}$ Fock component. If this component is projected onto a final $Q\bar{Q}$ state to calculate the production amplitude of heavy-quark pairs, one has to bear in mind that the physical gluon in the LCWF formalism has a non trivial Fock decomposition which also contains a heavy $Q\bar{Q}$ state.

To leading order in the QCD coupling constant α_S the incoming gluon G has a rather simple Fock decomposition. It contains bare gluons, two-gluon states, and quark–antiquark components. As discussed in Sec. 2, at small $x_2<0.1$ the propagation length of the leading order Fock states exceeds the target size. Therefore, the S-matrix describing the interaction of the beam gluon with the target will not mix states containing a different number of partons. Furthermore and most important, S conserves the transverse separations and the longitudinal momenta of all partons present in a particular Fock state. Consequently, it is natural to work with light-cone wave functions in a "mixed" representation, given by these conserved quantities.

In this representation the LCWF of the physical incident gluon can be written as (wherever it shall not lead to confusion, we suppress the virtuality of the incident gluon k_1^2)

$$|G(k_1^2)\rangle=\sqrt{1-n_{Q\bar{Q}}-n_\xi}|g\rangle+\sum_{z,\mathbf{r}}\Psi_G(z,\mathbf{r})|Q\bar{Q};z,\mathbf{r}\rangle+\sum_{\xi}\Psi(\xi)|\xi\rangle. \tag{8}$$

Here $\Psi_G(z,\mathbf{r})$ is the projection of the physical gluon wave function onto a $Q\bar{Q}$ state in the $(z,\mathbf{r})$-representation, where the light-cone variable z represents the fraction of the gluon momentum carried by the quark and $\mathbf{r}$ is the transverse separation of the $Q-\bar{Q}$ in impact-parameter space; $\Sigma_\xi\Psi(\xi)|\xi\rangle$ represents the light quark-antiquark $q\bar{q}$ and gluon–gluon gg Fock components. The normalization of the bare-gluon state in the presence of the $Q\bar{Q}$, $q\bar{q}$, and gg components is given by

$$\langle g|G\rangle=\sqrt{1-n_{Q\bar{Q}}-n_\xi}, \tag{9}$$

where

$$n_{Q\bar{Q}}=\int_0^1 dz\int d^2\mathbf{r}|\Psi_G(z,\mathbf{r})|^2, \tag{10}$$

determines the weight of $Q\bar{Q}$ states in the physical gluon. Similar, n_ξ denotes the normalization of the light-flavor $q\bar{q}$ and gg components. The $Q\bar{Q}$ wave function Ψ_G of the gluon can be obtained immediately from the $Q\bar{Q}$ wave function of a photon Ψ_{γ^*} as derived in Ref. 11. The only difference is a color factor and the substitution of the strong-coupling constant for the electromagnetic one:

$$|\Psi_G(z,\mathbf{r})|^2=\frac{\alpha_S}{6\alpha_{em}}|\Psi_{\gamma^*}(z,\mathbf{r})|^2=\frac{\alpha_S}{(2\pi)^2}\{[z^2+(1-z)^2]\epsilon^2K_1^2(\epsilon r)+m_Q^2K_0^2(\epsilon r)\}, \tag{11}$$

where $K_{0,1}$ are modified Bessel functions, $r=|\mathbf{r}|$, and

$$\epsilon^2=z(1-z)k_1^2+m_Q^2. \tag{12}$$

After the interaction with the target nucleon at an impact parameter $\mathbf{b}$, the incident-gluon state is transformed into

$$|G;\mathbf{b}\rangle\to\hat{S}(\mathbf{b})|G;\mathbf{b}\rangle. \tag{13}$$

Here $\hat{S}(\mathbf{b})$ is the scattering matrix in the impact-parameter representation. The familiar eikonal form[14] of the $\hat{S}$-matrix for the scattering of two partonic systems a and b is

$$\hat{S}(\mathbf{b})=\exp\left[-i\sum_{i,j}V(\mathbf{b}+\mathbf{b}_i-\mathbf{b}_j)\hat{T}_i^\alpha\hat{T}_j^\alpha\right], \tag{14}$$

with the one-gluon exchange potential (eikonal function)

$$V(\mathbf{b})=\frac{\alpha_S}{\pi}\int\frac{d^2\mathbf{k}}{\mathbf{k}^2+\mu_g^2}\exp(i\mathbf{k}\cdot\mathbf{b}), \tag{15}$$

$\hat{T}_{i,j}^\alpha$ are color $SU(3)$ generators acting on the individual partons of a and b at transverse coordinates $\mathbf{b}_i$ and $\mathbf{b}_j$, respectively. The effective gluon mass μ_g is introduced as an infrared regulator. In heavy-flavor production we have $m_Q\gg\mu_g$, which ensures that our final results will not depend on the exact choice of μ_g. Furthermore, as we will show shortly, μ_g can be absorbed into the definition of the target gluon density. To lowest nontrivial order, we have to keep terms up to second order in $V(\mathbf{b})$ only.[2)]

Unitarity of the $\hat{S}$-matrix at a fixed impact parameter $\mathbf{b}$ simplifies the calculation of the inclusive $Q\bar{Q}$ production cross-section. For every value of $\mathbf{b}$ it relates the probability $P_{Q\bar{Q}}$ to detect a $Q\bar{Q}$ pair in the final state with the probabilities P_G and P_ξ, to find a physical gluon or light-flavor $q\bar{q}$ or gg states:

$$P_G+P_{Q\bar{Q}}+P_\xi=1. \tag{16}$$

Let us first consider P_G, which is defined through

$$P_G=\frac{1}{8}\sum_{f,i,N'}\langle G^fN'|\hat{S}(\mathbf{b})|G^iN\rangle\langle G^fN'|\hat{S}(\mathbf{b})|G^iN\rangle^*, \tag{17}$$

here N and N' stand for the initial target nucleon and the final nucleonic three-quark states; the indices f and i characterize the color charges of the final and incident gluon, where we average over the latter. Using the explicit form of the $\hat{S}$-matrix and applying closure for the final nucleonic states N', we may simplify Eq. (17) in order α_S^2 to

$$P_G=\frac{1}{8}\sum_{i,f}\langle G^f\bar{G}^f N|\hat{S}(\mathbf{b})|G^i\bar{G}^i N\rangle. \tag{18}$$

We explicitly have used the fact that $\{-(\hat{T}^\alpha)^*\}$ are the antiparticle color generators and regrouped the gluon wave functions in Eq. (17), accordingly. Carrying out the color sums leaves us with the elastic scattering amplitude for the interaction of a color-singlet two-gluon state $(GG)_1$ with the target nucleon:

$$P_G=\langle (GG)_1 N|\hat{S}(\mathbf{b})|(GG)_1 N\rangle. \tag{19}$$

Making use of the Fock-state decomposition (8) of the dressed gluon, we obtain

$$\begin{aligned}P_G=&(1-n_{Q\bar{Q}}-n_\xi)\langle (gg)_1 N|\hat{S}(\mathbf{b})|(gg)_1 N\rangle\\&+2\int dz d^2\mathbf{r}|\Psi_G(z,\mathbf{r})|^2\langle (Q\bar{Q}g)_1 N|\hat{S}(\mathbf{b})\\&\times|(Q\bar{Q}g)_1 N\rangle\\&+2\int d\xi|\Psi(\xi)|^2\langle (\xi g)_1 N|\hat{S}(\mathbf{b})|(\xi g)_1 N\rangle.\end{aligned} \tag{20}$$

To this order in pQCD the contributions of different flavors in (16) and (20) do not interfere. We therefore suppress $q\bar{q}$ and gg terms in the following. In the color-singlet $(gg)_1$ state in Eq. (20) both gluons enter at the same impact parameter. Consequently this state has a vanishing color-dipole moment and

$$\langle (gg)_1 N|\hat{S}(\mathbf{b})|(gg)_1 N\rangle=1. \tag{21}$$

From Eqs. (20) and (21) and the explicit form (10) for $n_{Q\bar{Q}}$, we obtain

$$\begin{aligned}P_{Q\bar{Q}}=&2\int dz d^2\mathbf{r}|\Psi_G(z,\mathbf{r})|^2\langle (Q\bar{Q}g)_1 N|[1\\&-\hat{S}(\mathbf{b})]|(Q\bar{Q}g)_1 N\rangle.\end{aligned} \tag{22}$$

Note that $P_{Q\bar{Q}}$ takes the form of a profile function for the elastic scattering of a $(Q\bar{Q}g)_1$ state on a nucleon. After integrating over the impact parameter, we finally obtain the cross-section for the inclusive production of heavy-flavor quark–antiquark pairs in gluon-nucleon collisions:

$$\begin{aligned}&\sigma(GN\to Q\bar{Q}X)\\&=2\int dz d^2\mathbf{r}|\Psi_G(z,\mathbf{r})|^2\int d^2\mathbf{b}\langle (Q\bar{Q}g)_1 N|[1\\&\quad-\hat{S}(\mathbf{b})]|(Q\bar{Q}g)_1 N\rangle\\&=\int dz d^2\mathbf{r}|\Psi_G(z,\mathbf{r})|^2\sigma_3(\mathbf{r}_1,\mathbf{r}_2)=\langle\sigma_{3N}\rangle_{Q\bar{Q}g},\end{aligned} \tag{23}$$

where

$$\begin{aligned}\sigma_{3N}=\sigma_3(\mathbf{r}_1,\mathbf{r}_2)=&2\int d^2\mathbf{b}\langle (Q\bar{Q}g)_1 N|[1\\&-\hat{S}(\mathbf{b})]|(Q\bar{Q}g)_1 N\rangle=\frac{9}{8}\left[\sigma(r_1)+\sigma(r_2)-\frac{1}{9}\sigma(r)\right].\end{aligned} \tag{24}$$

As indicated, σ_{3N} is the total interaction cross-section for the scattering of a color-singlet three-parton state $(Q\bar{Q}g)_1$ from a nucleon.[5] The $g-Q$ and $g-\bar{Q}$ separations in the impact parameter space are labeled $\mathbf{r}_1$ and $\mathbf{r}_2$, respectively. Their difference $\mathbf{r}=\mathbf{r}_1-\mathbf{r}_2$ is identical to the $Q-\bar{Q}$ separation. The distances $\mathbf{r}_{1,2}$ are related to the light-cone momentum fractions carried by the Q and $\bar{Q}$, respectively:

$$\mathbf{r}_2=-z\mathbf{r},\quad \mathbf{r}_1=(1-z)\mathbf{r}. \tag{25}$$

The cross-section $\sigma(r)$ for the interaction of a $Q\bar{Q}$ color dipole of size $r=|\mathbf{r}|$ with a nucleon reads in the Born approximation[11]

$$\begin{aligned}\sigma_B(r)=&\frac{16}{3}\int\frac{d^2\mathbf{k}}{(\mathbf{k}^2+\mu_g^2)^2}\alpha_S^2(1-\mathcal{F}_2(\mathbf{k}))\\&\times(1-\exp(i\mathbf{k}\cdot\mathbf{r})),\end{aligned} \tag{26}$$

where $\mathcal{F}_2(\mathbf{k})=\langle N|\exp(i\mathbf{k}\cdot(\rho_1-\rho_2))|N\rangle$ is the two-quark form-factor of the nucleon. In general, the dipole cross-section is related in the leading-log (Q^2) approximation (LLQA) to the gluon distribution of the target.[5,16,17]

$$\sigma(r)\to\sigma(x_2,r)=\frac{\pi^2}{3}r^2\alpha_S(r)\left[x_2 g\left(x_2,k_2^2\approx\frac{\mathcal{A}}{r^2}\right)\right], \tag{27}$$

where $\mathcal{A}\approx 10$.[18] The explicit x_2-dependence in (27) results from higher order Fock components of the incident physical gluon, e.g., $Q\bar{Q}g$ states, and is determined through the BFKL evolution equation (for a detailed BFKL phenomenology of the dipole cross-section see Ref. 15). The gluon distribution in (27) absorbs the dependence on the infrared regularization μ_g in the same manner as in the conventional QCD analysis of hard scattering processes. The most important point for our further discussion is the color transparency property, i.e., the r^2-dependence of the dipole cross-section in (27).

When cast in the form (23), the heavy-flavor production cross-section resembles heavy-flavor contributions to real and virtual photoproduction. While the three-particle cross-section (24) enters in the former, the dipole cross-section $\sigma(r)$ is present in the latter. (See also Eq. (11) for the relationship between the corresponding wave functions.) Both processes have a similar dipole-size structure, and we briefly recapitulate the results of Refs. 5 and 11: The $Q\bar{Q}$ wave function (11) decreases exponentially for $r\gtrsim 1/\epsilon$. We may therefore conclude that for $k_1^2\lesssim 4m_Q^2$ small transverse sizes are relevant for the $Q\bar{Q}$ production process:

$$r^2,\ r_1^2,\ r_2^2\lesssim\frac{1}{m_Q^2}. \tag{28}$$

For highly virtual gluons, $k_1^2\gg 4m_Q^2$, the $Q\bar{Q}$ production cross-section (23) vanishes as $1/k_1^2$ due to the color transparency property of the dipole cross-section. (In view of the analogy to high-energy photoproduction, this behavior can be understood as the counterpart of Bjorken scaling.) Combining equations (23), (27), (28), we therefore find

$$\sigma(GN\to Q\bar{Q}X)\propto x_2 g(x_2,\mu^2\sim 4m_Q^2). \tag{29}$$

The color transparency property (27) of the dipole cross-section has important implications. First note that $n_{Q\bar{Q}}$, the probability that a physical gluon is realized in a $Q\bar{Q}$ Fock component, has a familiar logarithmic ultraviolet divergence at $r\to 0$. Nevertheless, in the LCWF formalism the heavy-quark production cross-section $\sigma(GN\to Q\bar{Q}X)$ is ultraviolet finite, since the color transparency property of the dipole cross-section (27) leads to a rapid convergence of the r-integration in the cross-section (23). The physical reason color transparency plays a role in this process is that the small-size $Q\bar{Q}$ pairs cannot be resolved by interacting t-channel gluons with wavelengths $\lambda\gtrsim r$. They are therefore indistinguishable from bare gluons and cannot be excited into $Q\bar{Q}$ states.

The second remarkable feature of the LCWF formalism is that $\sigma(GN\to Q\bar{Q}X)$ is also infrared-stable. This is not the case for the total gluon–nucleon cross-section, which is (logarithmically) infrared-divergent for $\mu_g\to 0$. Only the interaction of two color-singlet hadrons yields an infrared-finite total cross-section. This fact is due to the cancellation of the incident gluon color charge with the color charge of comoving spectator partons belonging to the color-singlet hadron projectile. However, soft t-channel gluons with $\mathbf{k}\to 0$ cannot resolve the $Q\bar{Q}$ component of the physical beam gluon and therefore cannot contribute to heavy-flavor production. We consequently obtain an infrared-stable $Q\bar{Q}$ production cross-section (23) even for an isolated colored gluon projectile.

A simple consideration shows, that the $Q\bar{Q}$ production cross-section (23) is not affected by the presence of spectator partons in the beam. Consider for example the case of one spectator "parton" s, which forms with the incident gluon a color-singlet state $(Gs)_1$. The spectator parton is assumed to scatter elastically only, i.e., without dissociating into a $Q\bar{Q}$ pair. Generalizing the above analysis, we finally end up with

$$P_G\propto\sum_{f,d}\langle (Gs)_1(\bar{G}\bar{s})_1 N|\hat{S}(\mathbf{b})|(G^f s^d)(\bar{G}^f\bar{s}^d)N\rangle. \tag{30}$$

To obtain the inclusive production cross-section we have to sum over the colors f and d of the final gluon and spectator states respectively. This yields a color-singlet two-gluon state, as well as a color-singlet spectator $s\bar{s}$ state. Since both partons, s and $\bar{s}$, of this singlet state enter at the same impact parameter, their color-dipole moment vanishes and they decouple from the interacting gluons. This leaves us with our previous result given in Eq. (23). A similar cancellation takes place for an arbitrary number of spectators, as long as we neglect interference effects between $Q\bar{Q}$ pairs belonging to different gluons of the beam hadron. However, such a restriction to elastic rescatterings of spectators is a standard ingredient of pQCD calculations in the framework of the conventional LLQA. In this respect the cancellation of interactions with spectator partons can be traced back to the LLQA factorization of the LCWF, containing the heavy-quark component. Consequently, in LLQA the production cross-section of $Q\bar{Q}$ pairs in the kinematic region under consideration is determined by the gluonic subprocess $G+N\to Q\bar{Q}+X$, i.e., the underlying process is diffractive dissociation of gluons into $Q\bar{Q}$ pairs.

We are now in the position to write down the heavy-quark production cross-section for hadron–nucleon collisions $\sigma_{Q\bar{Q}}(h,N)$. For this purpose we have to multiply $\sigma(GN\to Q\bar{Q}X)$ from Eq. (23) with the gluon density of the incoming hadron projectile and integrate over the virtuality k_1^2 of the projectile gluon:

$$\frac{d\sigma_{Q\bar{Q}}(h,N)}{dx_1}=\int\frac{dk_1^2}{k_1^2}\frac{\partial[g_1(x_1,k_1^2)]}{\partial\log k_1^2}\sigma(GN\to Q\bar{Q}X;x_2,k_1^2). \tag{31}$$

As mentioned above, the leading contributions to $\sigma(GN\to Q\bar{Q}X)$ result from the region $k_1^2\lesssim 4m_Q^2$, where we approximate

$$\sigma(GN\to Q\bar{Q}X;x_2,k_1^2)\approx\sigma(GN\to Q\bar{Q}X;x_2,k_1^2=0).$$

We therefore obtain

$$\frac{d\sigma_{Q\bar{Q}}(h,N)}{dx_1}\approx g_1(x_1,\mu^2=4m_Q^2)\times\sigma(GN\to Q\bar{Q}X;x_2,k_1^2=0). \tag{32}$$

Note, that we have recovered the dependence on the gluon density of the beam as in the parton model ansatz (1), whereas (29) yields the gluon density of the target. This shows the equivalence of the LCWF formulation of heavy-flavor production and the conventional parton model approach.

Let us explore the above result for charm production in nucleon–nucleon collisions. To be specific we choose a beam energy $E_{\rm lab}=800$ GeV as used in the Fermilab E743 experiment.[19] We employ the dipole cross-section of Ref. 15, which has been shown to yield a good quantitative description of the closely related real photoproduction of open charm, and the small x proton structure function, as measured at HERA. According to the analyses in Refs. 18 and 20, one obtains for transverse sizes $r^2\sim 1/m_c^2$ a precocious asymptotic BFKL behavior, i.e.,

$$\sigma(x_2,r)\approx\sigma_B(r)\left(\frac{x_0}{x_2}\right)^{\Delta}, \tag{33}$$

valid at $x_2\lesssim x_0$. The Born cross-section $\sigma_B(r)$ from Eq. (26) is evaluated for $\mu_g=0.75$ GeV; $\Delta=0.4$ is the intercept of the BFKL pomeron, and $x_0=0.03$. The charm-quark mass is fixed at $m_c=1.5$ GeV. The gluon distribution of the nucleon projectile enters at moderate to large values of x_1. We may therefore use the parametrization of Ref. 21. Because the $g+g\to Q+\bar{Q}$ cross-section decreases rapidly with the invariant mass of the heavy-quark pair, one finds $x_1x_2s\sim 4m_Q^2$. We may now calculate $d\sigma_{c\bar{c}}(N,N)/dx_F$ from Eq. (32). In Fig. 1 we compare our results with the data from the E743 proton–proton experiment.[19] In the kinematic region $0<x_F<0.5$, where our approach is well justified, we obtain reasonable agreement with the experimental data.

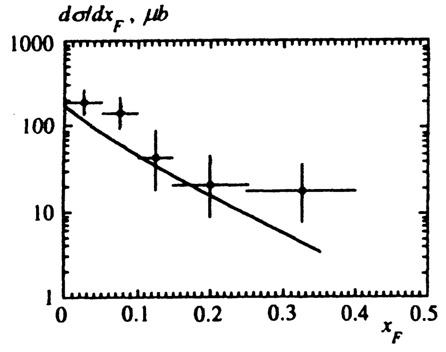

FIG. 1. Differential cross-section of open-charm production in nucleon-nucleon collisions at $E_{\rm lab}=800$ GeV. The experimental data are from Ref. 14.

4. HADROPRODUCTION OF $Q\bar{Q}$ PAIRS ON NUCLEAR TARGETS

Although the conventional pQCD result for heavy-flavor production and the LCWF formulation are equivalent, there is an important advantage in the latter. Namely, in Eq. (23) for $\sigma(GN\to Q\bar{Q}X)$ the LCWF of the $Q\bar{Q}$ state and its interaction cross-section factor. This feature is crucial for the now following generalization to nuclear targets.

The derivation of Eq. (23) was based upon the observation that at high energies, where the coherence length of the $Q\bar{Q}$ fluctuation of the incident gluon is larger than the target size, i.e., $l_{Q\bar{Q}}>R_{\rm target}$, and the transverse separations of the beam partons are frozen. This led to a diagonalization of the scattering $\hat{S}$-matrix in a mixed $(z,\mathbf{r})$-representation. In high-energy hadron–nucleus collisions with $l_{Q\bar{Q}}\gtrsim R_A$, the S-matrix can evidently be diagonalized the same way. Consequently we obtain the $Q\bar{Q}$ production cross-section for nuclear targets by substituting σ_{3A} for σ_{3N} in Eq. (23). The cross-section σ_{3A} for the scattering of a three-parton, color-singlet, $(Q\bar{Q}g)_1$ state from a nucleus with mass number A is given in the frozen-size approximation through the conventional Glauber formalism (we suppress the parton separations $\mathbf{r},\mathbf{r}_{1,2}$):

$$\sigma_{3A}=2\int d^2\mathbf{b}\left\{1-\left[1-\frac{1}{2A}\,\sigma_{3N}T(\mathbf{b})\right]^A\right\}$$
$$\approx 2\int d^2\mathbf{b}\left\{1-\exp\left[-\frac{1}{2}\,\sigma_{3N}T(\mathbf{b})\right]\right\}. \qquad (34)$$

Here $\mathbf{b}$ is the impact parameter of the $(Q\bar{Q}g)$-nucleus scattering process, which must not be confused with the impact parameter of the $(Q\bar{Q}g)$-nucleon interaction in Sec. 3; $T(\mathbf{b})$ stands for the optical thickness of the nucleus,

$$T(\mathbf{b})=\int_{-\infty}^{+\infty}dz\,n_A(\mathbf{b},z), \qquad (35)$$

with the nuclear density $n_A(\mathbf{b},z)$ normalized to $\int d^3\mathbf{r}\,n_A(\mathbf{r})=A$. In LLQA the inclusive cross-section for $Q\bar{Q}$ hadroproduction on nuclei can then be written as

$$\frac{d\sigma_{Q\bar{Q}}(h,A)}{dx_1}\approx g_1(x_1,\mu^2=4m_Q^2)$$
$$\times\sigma(GA\to Q\bar{Q}X;x_2,k_\perp^2=0), \qquad (36)$$

where

$$\sigma(GA\to Q\bar{Q}X;x_2,k_\perp^2)=2\int dz\,d^2\mathbf{r}|\Psi_G(z,r)|^2\int d^2\mathbf{b}$$
$$\times\left\{1-\exp\left[-\frac{1}{2}\,\sigma_{3N}T(\mathbf{b})\right]\right\}=\langle\sigma_{3A}\rangle_{Q\bar{Q}g}. \qquad (37)$$

In the multiple scattering series (34), (37) nuclear coherence effects are controlled by the expansion parameter

$$\tau_A=\sigma_{3N}T(\mathbf{b})\propto\sigma_{3N}A^{1/3}. \qquad (38)$$

Expanding the exponential in (34),(37) in powers of τ_A, one can identify terms $\propto\tau_A^\nu$. They describe contributions to the total production cross-section resulting from the coherent interaction of the $(Q\bar{Q}g)$ state with ν nucleons inside the target nucleus. In leading order, $\nu=1$, we are left with the incoherent sum over the nucleon production cross-sections:

$$\sigma(GA\to Q\bar{Q}X)=\int d^2\mathbf{b}T(\mathbf{b})\sigma(GN\to Q\bar{Q}X)$$
$$=A\sigma(GN\to Q\bar{Q}X). \qquad (39)$$

This yields the conventional impulse approximation component of the nuclear cross-section, which is proportional to the nuclear mass number. Effects of coherent higher order interactions are usually discussed in terms of the nuclear transparency

$$T_A=\frac{\sigma_{Q\bar{Q}}(h,A)}{A\sigma_{Q\bar{Q}}(h,N)}. \qquad (40)$$

In the impulse approximation we have $T_A=1$. The driving contribution to nuclear attenuation results from the coherent interaction of the three-parton $(Q\bar{Q}g)$ state with two nucleons inside the target:

$$T_A=1-\frac{1}{4}\frac{\langle\sigma_{3N}^2\rangle_{Q\bar{Q}g}}{\langle\sigma_{3N}\rangle_{Q\bar{Q}g}}\int d^2\mathbf{b}T^2(\mathbf{b}). \qquad (41)$$

For $k_\perp^2\ll 4m_Q^2$ shadowing effects can easily be estimated, using $|\Psi_G(z,r)|^2\propto\exp(-2m_Qr)$ and $\sigma(r)\approx Cr^2$. We further approximate $\sigma_3(\mathbf{r}_1,\mathbf{r}_2)\approx 1/2\sigma(r)$, and obtain

$$\frac{\langle\sigma_{3N}^2\rangle_{Q\bar{Q}g}}{\langle\sigma_{3N}\rangle_{Q\bar{Q}g}}\approx\frac{5C}{2m_Q^2}. \qquad (42)$$

For Gaussian nuclear densities with mean square charge radii $R_{\rm ch}=1.1A^{1/3}$ fm we find

$$\int d^2\mathbf{b}T^2(\mathbf{b})=\frac{3A}{4\pi R_{\rm ch}^2}. \qquad (43)$$

Altogether, we finally obtain the following estimate for nuclear attenuation effects:

$$1-T_A\approx\frac{15CA}{32\pi R_{\rm ch}^2m_Q^2}. \qquad (44)$$

At pre-asymptotic energies with coherence length $l_{Q\bar{Q}} \lesssim R_A$, the shadowing term in (41) will be proportional to the squared nuclear charge form factor $F_A^2(\kappa)$, where $\kappa = 1/l_{Q\bar{Q}} = Mx_2$ is the longitudinal momentum transfer in the $G+N \to Q\bar{Q}+N$ transition.[22] This form factor quantifies the onset of nuclear coherence effects.

Applied to open-charm production at current Tevatron energies ($s=1600$ GeV^2), we have to remember that x_2 is bounded below through the kinematic constraint

$$x_1 x_2 \gtrsim \frac{4m_c^2}{s} \approx (0.5-1)\cdot 10^{-2}. \tag{45}$$

In the accessible region of small $x_2 \sim 0.01$, we find at $r \sim 1/m_c$ typically $C \approx 2.5$,[15] which leads to

$$1-T_A \sim 6\cdot 10^{-3} A^{1/3}. \tag{46}$$

Approximating this by the ansatz $T_A \approx A^{\alpha-1}$, we get $1-\alpha \sim 7\cdot 10^{-3}$ in agreement with more recent experimental results.[9,10] However, this also shows that data with high accuracy are necessary to observe nuclear attenuation of open-charm production.

Although nuclear shadowing effects turn out to be small, it is interesting to consider their consequences for factorization theorems. Remember the close similarity between nuclear shadowing in heavy-flavor hadroproduction and heavy-flavor contributions to nuclear structure functions. It mainly involves an exchange of the three-parton cross-section σ_{3N} in (41) with the dipole cross-section $\sigma(r)$, which differs in this process by a factor ~ 2 [see the discussion following Eq. (41)]. Indeed, comparable ($\sim 1\%$) nuclear shadowing for the charm structure function of lead was found in Ref. 16. Following the analysis of Ref. 11, we note that the shadowing effect in Eq. (41) does not vanish at high virtualities $k_1^2 \gg 4m_Q^2$ of the projectile gluon and consequently is a leading twist effect. (The same is true for nuclear shadowing in DIS.) Our formalism correctly reproduces the proportionality of the $Q\bar{Q}$ hadroproduction cross-section to the gluon structure function of the target nucleon, in agreement with the conventional parton model formulation shown in Sec. 3. A similar proportionality to the gluon structure function of the target nucleon holds also for real (and weakly virtual $Q^2 \lesssim 4m_c^2$) photoproduction of open charm on nucleons. It is therefore tempting to assume such a factorization also in nuclear production processes. However, the three-parton cross-section and the dipole cross-section, which enter in hadro- and photoproduction, respectively, are not equal. Hence the A-dependence of both processes cannot be described in terms of modified nuclear gluon distributions, which would be unique for all hard processes. The fact that nuclear shadowing defies factorization, despite being a leading twist effect, was already encountered earlier in connection with DIS and Drell–Yan production on nuclei,[16] and also in nuclear production of vector mesons[23] and jets.[24]

A detailed discussion of the subtleties of factorization breaking goes beyond the scope of the present paper. Here we only wish to note that in DIS and heavy-flavor hadroproduction nonfactorizable effects result from the contributions of $Q\bar{Q}$ Fock states present in the projectile photon or gluon, respectively. Furthermore, shadowing of $Q\bar{Q}$ contributions in DIS must be reinterpreted as a modification of the nuclear sea quark distributions.[5] This, however, is not possible for heavy-flavor hadroproduction. Shadowing effects, which can be ascribed to nuclear modifications of gluon structure functions, emerge in DIS first through contributions from higher-order $Q\bar{Q}g$ Fock states of the photon. For the scattering from free nucleons these lead to the conventional QCD evolution of the target gluon distribution function[5] included in Eq. (27). However, in the case of nuclear targets the situation becomes different, as we will outline briefly. In DIS at large momentum transfers a $Q\bar{Q}g$ Fock state, which contains a soft gluon, is characterized by a small transverse separation of the quark–antiquark pair $r_{Q\bar{Q}} \sim 2/\sqrt{Q^2+4m_Q^2}$, but a large separation $r \sim R_g = 1/\mu_g$ of the gluon from both quarks.[5,25] Note that the latter is independent of the photon virtuality Q^2 and m_Q. A $Q\bar{Q}g$ Fock state of the virtual photon can therefore be treated as an octet–octet color dipole of size r, because the $Q\bar{Q}$ pair acts as a pointlike color charge by virtue of $r \gg r_{Q\bar{Q}}$. Furthermore, the squared wave function of the $Q\bar{Q}g$ Fock state factors into the square of the $Q\bar{Q}$ wave function $|\Psi_G(z,r_{Q\bar{Q}})|^2$, and the distribution of soft gluons around the nearly pointlike $Q\bar{Q}$ pairs:

$$|\Psi_3(z,r_{Q\bar{Q}},z_g,r)|^2 = \frac{1}{z_g}\frac{4}{3\pi}\,\alpha_S(r_{Q\bar{Q}})|\Psi_{\gamma^*}(z,r_{Q\bar{Q}})|^2 \times \frac{r_{Q\bar{Q}}^2}{r^4}\,\Phi(\mu_g r). \tag{47}$$

Here z_g is the momentum fraction carried by the soft gluon and

$$\Phi(x) = x^2\left[K_1^2(x) + xK_1(x)K_0(x) + \frac{1}{2}x^2K_0^2(x)\right].$$

Nuclear attenuation of $Q\bar{Q}g$ states will be controlled by the color octet dipole cross-section $\sigma_8(r) = (9/4)\sigma(r)$.[5] On the other hand, the contributions of $Q\bar{Q}g$ components to shadowing can be related to diffractive dissociation of virtual photons into large mass states $M^2 \gg Q^2$, which is successfully described in terms of the so-called triple-pomeron coupling $A_{3\mathbf{P}}$. For $x_2 \lesssim x_A = 0.1A^{1/3}$ one finds[5,22,26]

$$1-T_A \approx \frac{3A}{R_{\rm ch}^2}A_{3\mathbf{P}}\log\left(\frac{x_A}{x}\right) \sim 1.5\cdot 10^{-2}A^{1/3}\log\left(\frac{x_A}{x}\right). \tag{48}$$

We used $A_{3\mathbf{P}} \approx 0.16$ GeV^{-2}, as determined in the photoproduction experiment.[27]

Similar considerations are applicable to the excitation of $Q\bar{Q}g$ states in hadroproduction of heavy flavors. Postponing detailed numerical calculations, we merely cite the main ideas. The underlying process will be diffractive excitation of incident projectile gluons G_1 into quark–antiquark–gluon states, $G_1 \to Q\bar{Q}g_2$. Consequently, the generalization of the nucleon (23) and nuclear (37) production cross-section will contain the interaction cross-section of a four-parton color singlet state $Q\bar{Q}g_1g_2$, where the transverse size $r_{Q\bar{Q}} \sim 1/m_Q$ of the $Q\bar{Q}g_1$ component is much smaller than the separation $r \sim R_g$ of the soft gluon g_2. Furthermore, they will involve the factored three-parton wave function from Eq.

(47). Since the size of the three-parton $Q\bar{Q}g$ state in DIS, as well as the size of the four-parton $Q\bar{Q}g_1g_2$ state in hadroproduction, is determined by the soft-gluon separation $r\sim 1/R_g$, nuclear attenuation of $Q\bar{Q}g$ states will be similar in both processes. Only x_2 has to be substituted for x in Eq. (48).

For $R_g \gtrsim r_{Q\bar{Q}}$ the triple-pomeron coupling becomes approximately independent of Q^2 and quark masses.[5] We may therefore reinterpret the nuclear attenuation of $Q\bar{Q}g$ Fock states as nuclear modifications of target gluon distributions. These are then unique for all hard processes. However, in DIS the nonfactorizable contributions of light-flavor $q\bar{q}$ pairs dominate nuclear shadowing down to $x\lesssim 10^{-3}$. Higher-order Fock states are important at much smaller x only.[5] Hence the observed nuclear shadowing in DIS is not an appropriate place to determine nuclear modifications of gluon distributions. In open-charm hadroproduction the triple-pomeron contribution (48) will be relevant even at $x_2\sim 10^{-2}$. Nevertheless, possible attenuation effects will not exceed $\sim$(5–10)% even for heavy nuclei. This corresponds to an exponent $0.98\lesssim\alpha\lesssim 1$—a deviation from unity which is still below the current experimental accuracy. Of course, Eq. (48) gives a crude estimate of the x_2 dependence of nuclear attenuation only. A more detailed calculation using the pomeron structure function of Ref. 25 will be presented elsewhere.

Finally, the estimate (48) will be applicable to nuclear shadowing in Drell–Yan and exclusive vector-meson production on nuclei. In the latter case $q\bar{q}$ contributions to nuclear shadowing decrease asymptotically, $\propto 1/(Q^2+m_V^2)$, but are numerically very large. The influence of color transparency[28] demands an extremely large momentum transfer Q^2 for a substantial decrease of quark–antiquark $q\bar{q}$ contributions and the onset of the universal attenuation of Eq. (48).

5. CONCLUSIONS

The purpose of this paper was the presentation of a light-cone wave function approach to hadroproduction of heavy quarks on nucleons and nuclei valid at small $x_2\lesssim 0.1A^{1/3}$ and moderate $x_F\not\to 1$. We found that in the laboratory frame the mechanism of heavy-flavor production in LLQA can be described as a diffractive dissociation of projectile gluons into $Q\bar{Q}$ pairs. The central result is the representation of the inclusive $Q\bar{Q}$ production cross-section in terms of the cross section for the scattering of a three-parton, color singlet, $Q-\bar{Q}$-gluon state from the target. Although the incident gluon carries color charge, the light-cone formalism leads to an infrared-stable heavy-flavor production cross-section, which is not affected by interactions with comoving spectators. Nuclear attenuation effects were found to be small, $\sim A/(m_QR_A)^2$. We applied our formalism to open-charm production and obtained good agreement with more recent experimental data. Comparing nuclear effects in photoproduction and hadroproduction of open charm we observed that, despite being a leading twist effect, leading shadowing effects defy factorization. However, nuclear attenuation of higher-order production processes can be cast in the form of nuclear modifications of gluon distributions.

Weak nuclear attenuation in inclusive hadroproduction of open charm does not imply weak nuclear effects for exclusive charmonium channels. It should, however, be possible to extend the present technique to the latter. As far as nuclear effects are concerned, the difference between them should be quite similar to the well established difference between inclusive photoproduction of open charm and exclusive photoproduction of charmonium bound states. In the first case nuclear attenuation is controlled by the $c\bar{c}$ dipole cross-section at small distances $r\sim 1/m_c$. This leads to a weak nuclear attenuation.[5,16] On the other hand, in exclusive production processes the typically transverse scale is given by the size of the charmonium bound states $R_{J/\Psi}\cdot R_\chi \gg 1/m_c$ with the consequence of much stronger nuclear effects.[28]

In finalizing this paper we learned that B. Z. Kopeliovich is working on a similar analysis of nuclear shadowing effects in Drell–Yan processes.[3)]

G.P. and B.G.Z. would like to thank J. Speth and the theoretical physics group at the Institut für Kernphysik, KFA Jülich, for their hospitality during several visits. B.G.Z. acknowledges discussions with B. Z. Kopeliovich. We also thank A. W. Thomas for a careful reading of the manuscript. This work was partially supported by the INTAS grant 93-239 and grant N9S000 from the International Science Foundation.

[1)] Effective nuclear attenuation is possible though via the tension of color strings formed by color exchanges (predominantly on the front surface of the target nucleus).[13]

[2)] The fact that the color generators in $\hat{S}$ do not commute is not important to this order in α_S, because our final results will contain the symmetric part of the product of two generators only.

[3)] B. Z. Kopeliovich, private communication.

[1] S. Fixione, M. L. Mangnao, G. Ridolfi, and P. Nason, Nucl. Phys. B **431**, 453 (1994).

[2] N. N. Nikolaev, Comm. Nucl. Part. Sci. **21**, 41 (1992).

[3] G. Piller, J. Mutzbauer, and W. Weise, Z. Phys. A **343**, 247 (1992); Nucl. Phys. A **560**, 437 (1993).

[4] R. Vogt, S. J. Brodsky, and P. Hoyer, Nucl. Phys. B **383**, 643 (1992).

[5] N. N. Nikolaev and B. G. Zakharov, Zh. Éksp. Teor. Fiz. **105**, 1117 (1994). [JETP **78**, 598 (1994)].

[6] J. A. Appel, Ann. Rev. Nucl. Part. Sci. **42**, 367 (1992).

[7] M. E. Duffy, G. K. Fanourakis, R. J. Loveless *et al.*, Phys. Rev. Lett. **55**, 1816 (1985).

[8] H. Cobbaert, R. Roosen, M. G. Catanesi *et al.*, Phys. Lett. B **191**, 456 (1987).

[9] M. Adamovich, Y. Alexandrov, F. Antinori *et al.*, Phys. Lett. B **284**, 453 (1992).

[10] G. A. Alves, S. Amato, J. C. Anjos *et al.*, Phys. Rev. Lett. **70**, 722 (1993).

[11] N. N. Nikolaev and B. G. Zakharov, Z. Phys. C **49**, 607 (1991).

[12] G. Piller and W. Weise, Phys. Rev. C **42**, 1834 (1990); S. A. Kulagin, G. Piller, and W. Weise, Phys. Rev. C **50**, 1154 (1994).

[13] B. Z. Kopeliovich, Phys. Lett. B **243**, 141 (1990).

[14] J. F. Gunion and D. E. Soper, Phys. Rev. D **15**, 2617 (1977).

[15] N. N. Nikolaev and B. G. Zakharov, Phys. Lett. B **327**, 149 (1994); J. Nemchik, N. N. Nikolaev, and B. G. Zakharov, Phys. Lett. B **341**, 228 (1994).

[16] V. Barone, M. Genovese, N. N. Nikolaev *et al.*, Z. Phys. C **58**, 541 (1993).

[17] V. Barone, M. Genovese, N. N. Nikolaev *et al.*, Int. J. Mod. Phys. A **8**, 2779 (1993).

[18] N. N. Nikolaev and B. G. Zakharov, Phys. Lett. B **332**, 184 (1994).

[19] R. Ammar, R. C. Ball, S. Banerjee *et al.*, Phys. Rev. Lett. **61**, 2185 (1988).

[20] N. N. Nikolaev, B. G. Zakharov, and V. R. Zoller, Zh. Éksp. Teor. Fiz. **105**, 1498 (1994), JETP **78**, 806 (1994) Phys. Lett. B **328**, 486 (1994).

[21] J. F. Owens, Phys. Lett. B **266**, 126 (1991).

[22] N. N. Nikolaev and B. G. Zakharov, Phys. Lett. B **260**, 414 (1991).

[23] J. Nemchik, N. N. Nikolaev, and B. G. Zakharov, in *Proceedings of Workshop on CEBAF at Higher Energies*, 14–16 March 1994, CEBAF, ed. by N. Isgur and P. Stoler, pp. 415–464.
[24] N. N. Nikolaev, B. G. Zakharov, and V. R. Zoller, Z. Phys. A **351**, 435 (1995).
[25] M. Genovese, N. N. Nikolaev, and B. G. Zakharov, Zh. Éksp. Teor. Fiz. **108**, 1141, 1155 (1995). [JETP **81**, 625, 633 (1995)].
[26] N. N. Nikolaev and B. G. Zakharov, Z. Phys. C **53**, 331 (1991).
[27] T. J. Chapin, R. L. Cool, K. Goulianos *et al.*, Phys. Rev. D **31**, 17 (1985).
[28] B. G. Kopeliovich and B. G. Zakharov, Phys. Rev. D **44**, 3466 (1991); B. G. Kopeliovich, J. Nemchik, N. N. Nikolaev, and B. G. Zakharov, Phys. Lett. B **309**, 179 (1993); **324**, 469 (1993).

Published in English in the original Russian journal. Reproduced here with stylistic changes by the Translation Editor.