Resonant tunneling relaxation of electrons generated by light in coupled quantum wells

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The relaxation kinetics of electrons generated by light in double quantum wells has been analyzed in quantum-mechanically under the conditions when the splitting energy Δ_T between tunneling-coupled states is comparable to their scattering broadening energy $\hbar/\tau$. Scattering from both short-range and long-range inhomogeneities of a heterostructure has been considered. Within a unified approach, the coherent oscillations of the electron density and slow tunneling relaxation of the electron population in a well have been described. The rate of this relaxation has been calculated for both high tunneling-electron energy ($\bar{\varepsilon} \gg \hbar/\tau$) and the case $\bar{\varepsilon} \leqslant \hbar/\tau$. In the latter case, the difference between scattering rates affects the relaxation rate in the resonant region $\Delta_T \leqslant \hbar/\tau$. The calculations of the relaxation rate versus Δ_T, which describe the shape of the resonant relaxation peak, are compared to available experimental data.

1. INTRODUCTION

The tunneling relaxation of electrons generated by light in double quantum wells has been studied by time-resolved luminescence spectroscopy.[1–8] This technique yields decay times of photoluminescence signals due to electrons in states of quantum wells coupled through tunneling annihilating with holes localized in either left (l) or right (r) well without tunnelling between them. If the electron-hole recombination time is much longer than the typical time of tunneling, decay curves of luminescence lines due to different electron states yield information about the population kinetics in coupled quantum wells. Such processes have been studied theoretically[9–12] taking into account various scattering processes for the case in which the splitting energy Δ_T between coupled levels ($\Delta_T = \sqrt{\Delta^2 + 4T^2}$, where Δ is the nonperturbed energy difference controlled by the transverse electric field, and T is the tunneling matrix element, which determines the minimum splitting $2T$) is much larger than the typical level width due to scattering, $\hbar/\bar{\tau}$. The kinetic equations for the electron density in this case are derived from the kinetic equation system for the function of electron distribution among tunnel-coupled states (the nondiagonal components of the density matrix are small in $\hbar/\bar{\tau}\Delta_T$). In this paper we also consider the case $\hbar/\bar{\tau} \geqslant \Delta_T$, quantum interference between coupled states is partly or fully destroyed by scattering, and classical approach cannot be applied to the kinetic equation.

A consistent quantum approach based on the nonequilibrium Feynman diagram technique should be applied to the resonant tunneling relaxation in the case when $\hbar/\bar{\tau} \geqslant \Delta_T$. The difference from the conventional approach[13] is that electron Green's functions in the problem are considered in the isospin (two-level) formalism. It is convenient to select the ground-state functions of the left and right quantum wells as basis functions describing the motion along z. The technique developed for scattering from steady-state heterostructure inhomogeneities was presented in our previous publication[14] describing in-plane electron transport. In this work we have derived kinetic equations for electron concentration under the conditions of scattering from short-range and long-range inhomogeneities of a heterostructure using nonequilibrium diagram techniques. The resulting equations describe both the slow tunneling relaxation of electron density and coherent oscillations of the dipole moment between two quantum wells, which were previously studied in experiment[-17] and theoretical work.[18–20] We shall discuss the qualitative difference between the oscillations and exponential decay of electron density due to tunneling between quantum wells. Features of the tunneling relaxation related to the mixing of quantum states due to scattering from barrier nonuniformities[12] and tunneling of low-energy electrons (when the average kinetic energy $\bar{\varepsilon}$ is less than $\hbar/\bar{\tau}$) will be discussed. The calculated relaxation rate versus splitting energy Δ, controlled by the external electric field, and versus the tunneling matrix element T is compared to experimental data measured in the resonant relaxation mode.

In Sec. 2 kinetic equations describing the three-dimensional electron distribution under resonant tunneling conditions are derived. In Sec. 3 the case when the average electron kinetic energy $\bar{\varepsilon}$ is much larger than both the level broadening due to scattering and the splitting between levels, and the scattering potential is short-range is considered. The solutions of these equations which describe the evolution of the population over a time shorter than $\bar{\tau}$ (in this case the conditions of the oscillating regime are analyzed) and over a time much longer than $\bar{\tau}$ when the slow tunnelling relaxation is essential are given. In Sec. 4 we derive the expression for the slow relaxation rate not limited to the case of the short-range scattering potential for all splitting energies. In Sec. 5 we shall discuss the population equation for slow relaxation when $\bar{\varepsilon} \leqslant \hbar/\bar{\tau}$, calculate the relaxation rate, and consider the effect of difference between scattering rates on the tunneling. The final section discusses available experimental data and compares them to calculations.

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2. GENERALIZED KINETIC EQUATION

Electron states in coupled quantum wells are described by a delayed Green's function, which is a 2×2 matrix in the basis formed by states in the right and left wells:[14]

$$\hat{G}^R_{\mathbf{p}}(\varepsilon)=(\varepsilon-\varepsilon_p-\hat{h}-\hat{\Sigma}^R_{\mathbf{p}}(\varepsilon))^{-1}, \tag{1}$$

where $\varepsilon_p=p^2/2m$ is the kinetic energy of lateral motion in the case of equal effective masses in the left and right wells, $\hat{h}=(\Delta/2)\hat{\sigma}_z+T\hat{\sigma}_x$ is the matrix of potential energy expressed in terms of Pauli matrices $\hat{\sigma}_i$. The nonequilibrium distribution of electrons among the states described by Eq. (1) is derived from the equation for the function $\hat{F}_{\mathbf{p}}(\varepsilon,t)$[13] (in this section $\hbar=1$):

$$\begin{aligned}
&i\frac{\partial\hat{F}_{\mathbf{p}}(\varepsilon,t)}{\partial t}+[\hat{F}_{\mathbf{p}}(\varepsilon,t),\hat{h}]\\
&=\frac{1}{\pi}\int dt'\int d\varepsilon'\{\exp[-2i(\varepsilon-\varepsilon')(t-t')]\\
&\times[\hat{\Omega}_{\mathbf{p}}(\varepsilon,t')\hat{G}^A_{\mathbf{p}}(\varepsilon')-\hat{F}_{\mathbf{p}}(\varepsilon,t')\hat{\Sigma}^A_{\mathbf{p}}(\varepsilon')]\\
&+\exp[2i(\varepsilon-\varepsilon')(t-t')][\hat{\Sigma}^R_{\mathbf{p}}(\varepsilon')\hat{F}_{\mathbf{p}}(\varepsilon,t')\\
&-\hat{G}^R_{\mathbf{p}}(\varepsilon')\hat{\Omega}_{\mathbf{p}}(\varepsilon,t')]\},
\end{aligned} \tag{2}$$

where $\hat{G}^A_{\mathbf{p}}=G^{R^+}_{\mathbf{p}}(\varepsilon)$. In the Born approximation the self-energy functions $\hat{\Sigma}_{\mathbf{p}}(\varepsilon,t)$ and $\hat{\Sigma}^{R,A}_{\mathbf{p}}(\varepsilon')$ are

$$\hat{\Sigma}^{R,A}_{\mathbf{p}}(\varepsilon)=\sum_{jj'}\sum_{\mathbf{q}} w_{jj'}\exp(-q^2l_c^2/4)\hat{P}_j\hat{G}^{R,A}_{\mathbf{p}-\mathbf{q}}(\varepsilon)\hat{P}_{j'}, \tag{3}$$

$$\hat{\Omega}_{\mathbf{p}}(\varepsilon,t)=\sum_{jj'}\sum_{\mathbf{q}} w_{jj'}\exp(-q^2l_c^2/4)\hat{P}_j\hat{F}_{\mathbf{p}-\mathbf{q}}(\varepsilon,t)\hat{P}_{j'}, \tag{4}$$

where the projection operators $\hat{P}_j$ $(j=l,r,t)$ are defined as

$$\hat{P}_l=(1+\hat{\sigma}_z)/2,\quad \hat{P}_r=(1-\hat{\sigma}_z)/2,\quad \hat{P}_t=\hat{\sigma}_x \tag{5}$$

and scattering due to the random potential in the left and right wells is described by the Gaussian correlator with correlation lengths equal to l_c for all the potentials. Explicit expressions for the correlator amplitudes $w_{jj'}$ in terms of the electron quantization energy in both wells, ε_l and ε_r, widths of the wells, d_l and d_r, and the reciprocal ranges of the wave functions under the barrier between wells, κ, for the model of scattering due to interface roughness[12-14] in the approximation of rectangular potential wells were given in Ref. 14.

The density operator $\hat{\rho}_{\mathbf{p}}(t)$ for electrons is expressed in the momentum representation in terms of Green's functions:

$$\begin{aligned}
\hat{\rho}_{\mathbf{p}}(t)&=-\frac{i}{4\pi}\int d\varepsilon[\hat{F}_{\mathbf{p}}(\varepsilon,t)+\hat{G}^A_{\mathbf{p}}(\varepsilon)-\hat{G}^R_{\mathbf{p}}(\varepsilon)]\equiv\\
&-\frac{i}{4\pi}\int d\varepsilon\hat{\mathscr{F}}_{\mathbf{p}}(\varepsilon,t),
\end{aligned} \tag{6}$$

and the function $\hat{\mathscr{F}}_{\mathbf{p}}(\varepsilon,t)$ satisfies the equation

$$\begin{aligned}
&i\frac{\partial\hat{\mathscr{F}}_{\mathbf{p}}(\varepsilon,t)}{\partial t}+[\hat{\mathscr{F}}_{\mathbf{p}}(\varepsilon,t),\hat{h}]\\
&=\frac{1}{\pi}\int dt'\int d\varepsilon'\sum_{jj'}\sum_{\mathbf{q}} w_{jj'}\exp\left(-\frac{q^2l_c^2}{4}\right)\\
&\times\{\exp[-2i(\varepsilon-\varepsilon')(t-t')]\\
&\times[\hat{P}_j\hat{\mathscr{F}}_{\mathbf{p}-\mathbf{q}}(\varepsilon,t')\hat{P}_{j'}\hat{G}^A_{\mathbf{p}}(\varepsilon')\\
&-\hat{\mathscr{F}}_{\mathbf{p}}(\varepsilon,t')\hat{P}_j\hat{G}^A_{\mathbf{p}-\mathbf{q}}(\varepsilon')\hat{P}_{j'}]\\
&+\exp[2i(\varepsilon-\varepsilon')(t-t')][\hat{P}_j\hat{G}^R_{\mathbf{p}-\mathbf{q}}(\varepsilon')\hat{P}_{j'}\hat{\mathscr{F}}_{\mathbf{p}}(\varepsilon,t')\\
&-\hat{G}^R_{\mathbf{p}}(\varepsilon')\hat{P}_j\hat{\mathscr{F}}_{\mathbf{p}-\mathbf{q}}(\varepsilon,t')\hat{P}_{j'}]\}.
\end{aligned} \tag{7}$$

In deriving this equation, we used the identity

$$\begin{aligned}
&[\hat{G}^R_{\mathbf{p}}(\varepsilon)-\hat{G}^A_{\mathbf{p}}(\varepsilon),\hat{h}]\\
&=\frac{1}{\pi}\int dt'\int d\varepsilon'\exp[-2i(\varepsilon-\varepsilon')(t-t')]\\
&\times\{[\hat{\Sigma}^R_{\mathbf{p}}(\varepsilon)-\hat{\Sigma}^A_{\mathbf{p}}(\varepsilon)]\hat{G}^A_{\mathbf{p}}(\varepsilon')-[\hat{G}^R_{\mathbf{p}}(\varepsilon)\\
&-\hat{G}^A_{\mathbf{p}}(\varepsilon)]\hat{\Sigma}^A_{\mathbf{p}}(\varepsilon')+\text{H.c.}\},
\end{aligned} \tag{8}$$

which can be verified by integrating with respect to ε' and t', taking into account Eq. (1). In order to derive the equation for the density operator from Eq. (7), it is integrated with respect to ε using the expression for $\hat{\mathscr{F}}_{\mathbf{p}}(\varepsilon,t)$ in terms of $\hat{\rho}_{\mathbf{p}}(t)$,

$$\hat{\mathscr{F}}_{\mathbf{p}}(\varepsilon,t)=2[\hat{\rho}_{\mathbf{p}}(t)\hat{G}^A_{\mathbf{p}}(\varepsilon)-\hat{G}^R_{\mathbf{p}}(\varepsilon)\hat{\rho}_{\mathbf{p}}(t)], \tag{9}$$

which is a generalization of the respective equation for scalar Green's functions[13] and is proved using the identity

$$\int d\varepsilon\hat{G}^A_{\mathbf{p}}(\varepsilon)=-\int d\varepsilon\hat{G}^R_{\mathbf{p}}(\varepsilon)=i\pi. \tag{10}$$

The result is the generalized kinetic equation

$$\begin{aligned}
&\frac{\partial\hat{\rho}_{\mathbf{p}}(t)}{\partial t}+i[\hat{h},\hat{\rho}_{\mathbf{p}}(t)]\\
&=\sum_{jj'}\sum_{\mathbf{q}}\int_{-\infty}^{0}d\tau w_{jj'}\exp\left(-\frac{q^2l_c^2}{4}\right)\\
&\times\left[\hat{P}_j\hat{G}^R_{\mathbf{p}-\mathbf{q}}(-\tau)\hat{\rho}_{\mathbf{p}-\mathbf{q}}\left(t+\frac{\tau}{2}\right)\hat{P}_{j'}\hat{G}^A_{\mathbf{p}}(\tau)\right.\\
&+\hat{G}^R_{\mathbf{p}}(-\tau)\hat{P}_j\hat{\rho}_{\mathbf{p}-\mathbf{q}}\left(t+\frac{\tau}{2}\right)\hat{G}^A_{\mathbf{p}-\mathbf{q}}(\tau)\hat{P}_{j'}\\
&-\hat{G}^R_{\mathbf{p}}(-\tau)\hat{\rho}_{\mathbf{p}}\left(t+\frac{\tau}{2}\right)\hat{P}_j\hat{G}^A_{\mathbf{p}-\mathbf{q}}(\tau)\hat{P}_{j'}-\hat{P}_j\\
&\left.\times\hat{G}^R_{p-q}(-\tau)\hat{P}_{j'}\hat{\rho}_{\mathbf{p}}\left(t+\frac{\tau}{2}\right)\hat{G}^A_{\mathbf{p}}(\tau)\right],
\end{aligned} \tag{11}$$

in which the right-hand side (collision integral) is expressed in terms of time representation of the Green's functions introduced above:

$$\hat{G}^{R,A}_{\mathbf{p}}(\tau)=\int\frac{d\varepsilon}{2\pi}e^{-i\varepsilon\tau}\hat{G}^{R,A}_{\mathbf{p}}(\varepsilon). \tag{12}$$

The expression for the collision integral at $\Delta_T \gg \hbar/\bar{\tau}$ derived for this scattering mechanism in Ref. 12 is obtained by substituting the Green's function of free motion into Eq. (11) and diagonalizing the density operator [this can be done by the unitary transformation $\exp(i\hat{\sigma}_y\varphi)$, where $2\varphi=\tan^{-1}(2T/\Delta)$, which transforms the hamiltonian $\hat{h}$ to the diagonal form $(\Delta_T/2)\hat{\sigma}_z$]. In the diagonal representation, the matrix equation (11) is separated into two Boltzmann kinetic equations, which describes the evolution of the electron distribution function over well-defined tunnel-coupled states. If the resonant tunneling relaxation is considered under the condition when the coherent interference of tunnel-coupled states is disrupted by scattering, this separation of the kinetic equation is impossible, and the matrix nature of the collision integral is essential.

In what follows, we shall not search for a general solution of the matrix integral equation (7) or (11), but investigate several limiting cases, when these equations are reduced to equations for components of the density matrix $\hat{n}(t)$ defined as

$$\hat{n}(t)=-\frac{i}{2\pi S}\int d\varepsilon \sum_{\mathbf{p}} \hat{\mathscr{F}}_{\mathbf{p}}(\varepsilon,t)$$

$$=\frac{2}{S}\sum_{\mathbf{p}} \hat{\rho}_{\mathbf{p}}(t)=n_0\hat{I}+\sum_i \hat{\sigma}_i n_i(t), \tag{13}$$

where S is the heterostructure area, $\hat{I}$ is the unit matrix, $n_i(t)$ $(i=x,y,z)$ are the components of the introduced "vector" of the isospin density, $\mathbf{n}(t)=(n_x,n_y,n_z)$. The scalar part n_0 of the density operator is time independent because scattering does not change the total number of electrons in the system (their annihilation is ignored in this approximation). The operator $\hat{n}(t)$ determines the distribution of electrons along z in the system,

$$N(z,t)=\sum_{jj'} [\hat{n}(t)]_{jj'}\varphi_j(z)\varphi_{j'}(z), \tag{14}$$

through the wave functions of the left and right wells, $\varphi_j(z)$, $j=l,r$. Hence the electron density in the left and right well, $n_l(t)$ and $n_r(t)$, and the electric dipole moment of the structure

$$D(t)=e\int dz\ zN(z,t)$$

are expressed in terms of the z-component of the isospin density, $n_z(t)$:

$$n_l(t)=n_0+n_z(t), \quad n_r(t)=n_0-n_z(t),$$

$$D(t)=e(\Delta z)n_z(t)+D_0, \tag{15}$$

where $(\Delta z)=\int dz\ z[\varphi_l^2(z)-\varphi_r^2(z)]$ is the parameter whose order of magnitude equals to the distance between quantum well centers, and D_0 is a time-independent part of the dipole moment.

3. POPULATION DYNAMICS FOR THE MODEL OF SHORT-RANGE SCATTERING POTENTIAL

In this and subsequent sections, we limit our consideration to the case when the average kinetic energy $\bar{\varepsilon}$ of electrons is much larger than the collision broadening energy:

$$\bar{\varepsilon} \gg \hbar/\bar{\tau}. \tag{16}$$

This is the case when electron states can be described in terms of the momentum $\mathbf{p}$, and their kinetics can be analyzed using Eq. (11) for the density operator in the momentum representation. In our consideration of the resonant relaxation, we also assume $\bar{\varepsilon} \gg \Delta_T$, keeping in mind that in the case $\bar{\varepsilon} \leqslant \Delta_T$ the relation $\Delta_T \gg \hbar/\bar{\tau}$ would hold and the classical approach would be applicable. If Eq. (16) is valid and $\bar{\varepsilon} \gg \Delta_T$ holds, the matrix Green's functions can be approximated by scalar expressions

$$\hat{G}^A_{\mathbf{p}}(\tau)\cong\hat{G}^{R*}_{\mathbf{p}}(-\tau)\cong\exp(-i\varepsilon_p\tau/\hbar+\delta\tau), \quad \delta\to+0.$$

Since the typical time over which the density operator changes is determined either by the relaxation time $\bar{\tau}$ or by the period of "Bloch" oscillations, $\hbar/\Delta_T$ (both these times are much longer than $\hbar/\bar{\varepsilon}$), we may also take $\hat{\rho}_{\mathbf{p}}(t+\tau/2)=\hat{\rho}_{\mathbf{p}}(t)$, i.e., the non-Markovian nature of the collision integral is neglected. Besides, in this section we presume that $\bar{\varepsilon} \ll \hbar^2/(ml_c^2)$ so that the scattering potential can be treated as short-range, i.e., the dependence of the scattering amplitude on the imparted momentum $\hbar\mathbf{q}$ on the right of Eq. (11) can be ignored.

In this approximation, the operator equation (11) is reduced after integration with respect to the momentum $\mathbf{p}$ to the system of Bloch equations for the components of the isospin density "vector"

$$\begin{vmatrix} \frac{\partial}{\partial t}+\frac{1}{\tau} & \frac{\Delta}{\hbar} & \nu \\ -\frac{\Delta}{\hbar} & \frac{\partial}{\partial t}+\frac{1}{\tau} & \frac{2T}{\hbar} \\ \nu & -\frac{2T}{\hbar} & \frac{\partial}{\partial t}+\frac{1}{\tau_z} \end{vmatrix} \begin{pmatrix} n_x \\ n_y \\ n_z \end{pmatrix} = 0 \tag{17}$$

and to the obvious equation $\partial n_0/\partial t=0$ due to the conservation of the total number of electrons. The three characteristic frequencies in Eq. (17) are

$$\tau^{-1}=\frac{m(w_{ll}+w_{rr})}{2\hbar^3}, \quad \tau_z^{-1}=\frac{2mw_{lt}}{\hbar^3},$$

$$\nu=\frac{m(w_{rt}-w_{lt})}{\hbar^3}. \tag{18}$$

Since the correlators w_{lt} and w_{rt} are proportional to the tunnelling matrix element T and $w_{tt} \propto T^2$,[14] the following relations hold: $w_{ll,rr} \gg w_{lt,rt} \gg w_{tt}$. Nonetheless, we retain the terms with ν and $1/\tau_z$ in Eq. (17) if they are responsible for radically new effects. For example, the term $1/\tau_z$ in the lower diagonal component of the matrix leads to a direct (i.e., not caused by quantum mixing of states) relaxation of the component n_z, and the parameter ν in xz and zx components leads to an asymmetry in the relaxation rate with respect to

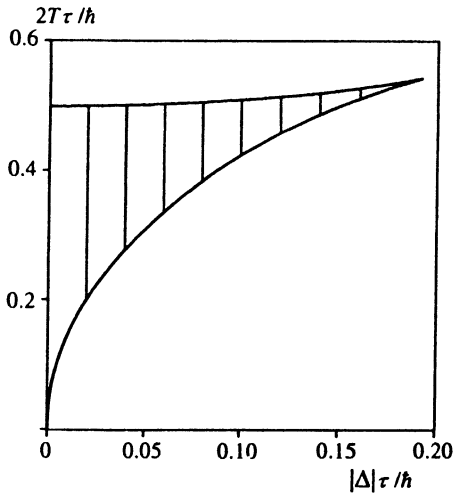


FIG. 1. Diagram of regions with different regimes of electron density relaxation in double quantum wells. In the hatched area the function $n_i(t)$ contains only three decaying components, and beyond this region only one purely decaying and two oscillating components with decaying amplitudes.

the sign reversal of Δ. These effects are more pronounced for $|\Delta| \sim \varepsilon_{l,r}/(\kappa d_{l,r})$,[12] which is in the range of the assumed two-level approximation in the case of "deep" states ($\kappa d_{l,r} \gg 1$). Around the resonance ($|\Delta| \sim T$), the terms proportional to ν and $1/\tau_z$ may be omitted in most cases. One can see in Eq. (17) that the effective relaxation time $\bar{\tau}$ is the averaged scattering time τ within one well. If the analogy between Eq. (17) and the Bloch equations for magnetic moment evolution theory[21] is considered, τ is similar to the "transverse relaxation time" of the magnetization vector. In Eq. (17) we have omitted in the xy and yx components the terms due to the inequality between scattering rates in different wells ($w_{ll} \neq w_{rr}$). These terms emerge when real parts of the Green functions are taken into account and make nonsymmetrical contributions to these components, so their contribution can be considered as a renormalization of Δ, and their presence in Eq. (17) is not essential. Similar terms leading to renormalization of the tunnelling matrix element T emerge in yz and zy components due to the correlators w_{lt} and w_{rt}, but their contribution is negligible by virtue of the inequality

$$\frac{\hbar \kappa d_{l,r}}{\varepsilon_{l,r}\tau} \ll 1, \tag{19}$$

which we assume to hold throughout this paper.

Analysis of the cubic equation, which determines eigenvalues of the equation system (17), reveals that each component n_i contains either three terms which decay with different time constants or two terms one of which decays and the other one oscillates with a decaying amplitude, the decay times always being of the order of τ, and in the range where three decaying components coexist, at least two of them decay with time constants of the order of τ. Figure 1 shows the regions of dimensionless parameters $2T\tau/\hbar$ and $|\Delta|\tau/\hbar$ corresponding to these two cases (in our calculation $\nu = 1/\tau_z = 0$). In principle, the oscillating regime takes place in a wide range of parameters, but the amplitude of oscillating components drops more sharply with the ratio $|\Delta|/T$ in comparison to the nonoscillating components.

The ratios of these terms are determined by initial conditions, which are not included in the discussed approximation. They derive from initial stages of the process, namely generation of electrons by light and their fast relaxation due to emission of optical phonons (the latter is essential if the typical energy of generated electrons is larger than that of an optical phonon). In order to demonstrate features of the oscillating relaxation, let us consider electron generation by an ultrashort laser pulse with a width τ_p of the order of 100 femtoseconds.[16,17] In this case a time-dependent parameter $\mathbf{G}(t)$, which describes the generation of carriers[30] and determines the initial conditions, is added to the right-hand side of Eq. (17). The function $\mathbf{G}(t)$ is most simple when the width of the laser pulse is much less than the times $\hbar/\Delta_T$ and τ. In this case the electrons, which are generated either in the left or right quantum well owing to their asymmetry,[16]–20 cannot penetrate into another well during τ_p, so $\mathbf{G}(t)$ can be approximated by a delta-function

$$\mathbf{G}(t) = (0, 0, \pm n_0)\delta(t), \tag{20}$$

where the plus sign corresponds to the "instantaneous" generation of electrons in the left well, and the minus sign in the right well. We stress that the characteristic energy of generated electrons in such experiments is controlled by the laser pulse spectral width $\hbar/\tau_p$, which is larger than $\hbar/\tau$ and Δ_T in the case of ultrashort pulses, hence this approximation is applicable.

Figure 2a,b shows the dipole moment of the structure derived from Eq. (17) with the function from Eq. (20) on the right versus time at different ratios among the parameters Δ, T, and $\hbar/\tau$ (ν and $1/\tau_z$ are neglected because $|\Delta| \sim T$). At $T \gg \hbar/\tau$ the oscillation period is determined by the period $2\pi\hbar/\Delta_T$ of Bloch oscillations, but for $T \sim \hbar/\tau$ also depends on τ. The coherence is lost, i.e., oscillations of the dipole moment decay in a time of the order of τ. For $\Delta \neq 0$ there is a nonoscillating component in the dipole moment. The oscillation amplitude decreases with Δ. Note that at $\Delta = 0$ the solution of Eq. (17) with respect to $n_z(t)$ can be expressed in a simple analytical form:

$$n_z(t) = n_0 \exp\left(-\frac{t}{2\tau}\right)\left[\cos \Omega t + \frac{1}{2\Omega\tau}\sin \Omega t\right],$$

$$\Omega^2 = \left(\frac{2T}{\hbar}\right)^2 - \frac{1}{(2\tau)^2}, \tag{21}$$

which describes an oscillating process for $\Omega^2 > 0$ and an exponential decay for $\Omega^2 < 0$, the decay time constant at $(2T\tau/\hbar)^2 \ll 1$ being $(2T/\hbar)^{-2}\tau^{-1}$.

A solution of the Bloch equations (17) can be derived without any initial conditions if the electron density relaxation is considered at $t \gg \tau$. This solution is interesting if there is a slow relaxation of the isospin density components characterized by a time constant $\tau_0 \gg \tau$. As follows from the analysis of the eigenvalue equation (see above), this relaxation may be due to one of the three terms which determine

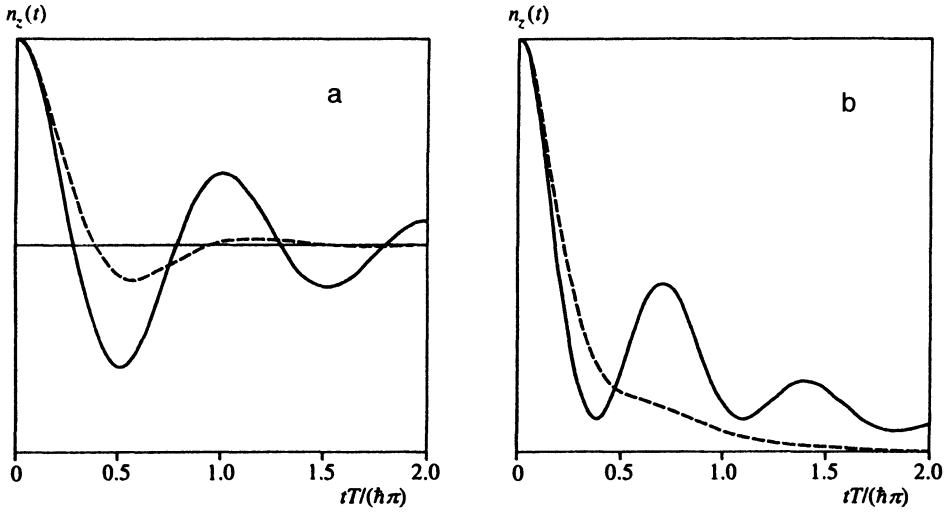


FIG. 2. Evolution of electric dipole moment of double quantum wells after generation of electrons in one well by an ultrashort laser pulse at $2T\tau/\hbar=3$ (solid lines) and $2T\tau/\hbar=1$ (dashed lines): a) $\Delta=0$; b) $\Delta=2T$.

the time dependence of the components $n_i(t)$, and this term does not oscillate. Let us seek the solution of Eq. (17) by iterations with small parameters

$$\frac{\tau}{n_x}\frac{\partial n_x}{\partial t} \quad \text{and} \quad \frac{\tau}{n_y}\frac{\partial n_y}{\partial t}.$$

In the lowest order approximation, the equation system is reduced to a single equation, which describes the relaxation of the component $n_z(t)$ (and the related components $n_l(t)$ and $n_r(t)$):

$$\frac{\partial n_z(t)}{\partial t}=\frac{\partial n_l(t)}{\partial t}=-\frac{\partial n_r(t)}{\partial t}$$

$$=-2\frac{\partial n_z(t)}{\tau_0}=-\frac{n_l(t)-n_r(t)}{\tau_0}. \tag{22}$$

The characteristic time τ_0 is given by the Breit–Wigner expression

$$\frac{1}{\tau_0}=\frac{2/\tau}{\Delta^2+\hbar^2/\tau^2}\left\{T^2+2T\Delta\,\frac{w_{rt}-w_{lt}}{w_{ll}+w_{rr}}+\Delta^2\frac{w_{tt}}{w_{ll}+w_{rr}}\right\}, \tag{23}$$

which has been derived by omitting terms with small parameters τ/τ_z and $(\nu\tau)^2$. Equation (23) also determines the range of parameters in which the slow relaxation is possible: $\tau_0\gg\tau$ when

$$4T^2\ll\Delta^2+\hbar^2/\tau^2, \tag{24}$$

i.e., when the tunneling coupling is weakened either by the nonresonant condition (large Δ) or by scattering.

The time τ_0 describes the tunneling relaxation between quantum wells, which is much slower in these conditions than the electron scattering within one well characterized by the time τ. The second and third terms on the right of Eq. (23) are not essential around the resonance ($|\Delta|\leqslant\hbar/\tau$), but they may be important in the classical region $|\Delta|\gg\hbar/\tau$, where they affect the dependence of the relaxation time τ_0 on Δ (note the asymmetry of the relaxation time with respect to the sign of Δ).

The description of the slow tunneling relaxation of the population and the range of parameters of this regime are naturally derived from the Bloch equation system (17) obtained in the approximation of short-range scattering potential and small difference between energies of levels in coupled quantum wells, $\Delta_T\ll\bar{\varepsilon}$ (otherwise the generalized kinetic equation (11) cannot be separated into the kinetic equations for populations and remains in operator form). But an analysis of the slow tunneling regime based on the smallness of the tunneling matrix element (Eq.(24)) is also possible beyond this approximation. This analysis in the case of large average electron energy in Eq. (16) is discussed in the next section

4. SLOW RELAXATION OF THE POPULATION: THE CASE OF LARGE ENERGY

Another approach not based on the assumption about short-range scattering potential can be applied to the slow relaxation of the system over a time longer than τ. This approach implies that after a sufficiently long time electrons are thermalized owing to nonelastic scattering (for example, due to acoustic phonons) which determines their energy distribution but contributes little to transitions between quantum wells (for this reason, a collision integral including this scattering is not added to the right of Eq. (11)). We shall seek the electron density matrix in the form

$$\hat{\rho}_{\mathbf{p}}(t)=\left(n_0+\sum_i\hat{\sigma}_i n_i(t)\right)\frac{1}{N}\exp\left(-\frac{\varepsilon_p}{T_e}\right),$$

$$N=\frac{2}{S}\sum_{\mathbf{p}}\exp\left(-\frac{\varepsilon_p}{T_e}\right). \tag{25}$$

In this case the average energy $\bar{\varepsilon}$ equals the electron temperature T_e. Substituting the density matrix from Eq. (25) into Eq. (11), summing over the momentum $\mathbf{p}$, and neglecting the non-Markovian nature of the collision integral owing to the slow relaxation rate, we obtain the kinetic equation for the components $n_i(t)$ in the form

$$\frac{\partial}{\partial t}n_i(t)+[\mathbf{L}n(t)]_i-M_{ii'}n_{i'}(t)=K_i n_0, \tag{26}$$

where the vector $\mathbf{L}$ is by definition $\mathbf{L}=(2T/\hbar,0,\Delta/\hbar)$, and the real matrix $M_{ii'}$ and column K_i are defined as follows:

$$M_{ii'}=\frac{1}{NS^2}\sum_{jj'}\sum_{\mathbf{pp'}} w_{jj'}\exp\left(-\frac{|\mathbf{p}-\mathbf{p}'|^2 l_c^2}{4\hbar^2}\right)$$
$$\times\int_{-\infty}^{0} d\tau\ \mathrm{Sp}\left\{\hat{G}^A_{\mathbf{p}'}(\tau)[\hat{\sigma}_i\hat{P}_j-\hat{P}_j\hat{\sigma}_i]\right.$$
$$\times\hat{G}^R_{\mathbf{p}}(-\tau)\left[\hat{\sigma}_{i'}\hat{P}_{j'}\exp\left(-\frac{\varepsilon_p}{T_e}\right)\right.$$
$$\left.\left.-\hat{P}_{j'}\hat{\sigma}_{i'}\exp\left(-\frac{\varepsilon_{p'}}{T_e}\right)\right]\right\}, \tag{27}$$

$$K_i=\frac{1}{NS^2}\sum_{jj'}\sum_{\mathbf{pp'}} w_{jj'}\exp\left(-\frac{|\mathbf{p}-\mathbf{p}'|^2 l_c^2}{4\hbar^2}\right)$$
$$\times\left[\exp\left(-\frac{\varepsilon_p}{T_e}\right)-\exp\left(-\frac{\varepsilon_{p'}}{T_e}\right)\right]$$
$$\times\int_{-\infty}^{0} d\tau\ \mathrm{Sp}\{\hat{G}^A_{\mathbf{p}'}(\tau)[\hat{\sigma}_i\hat{P}_j-\hat{P}_j\hat{\sigma}_i]\hat{G}^R_{\mathbf{p}}(-\tau)\hat{P}_{j'}\}. \tag{28}$$

Note that Eq. (26) has the form of the Bloch equation system. As in the previous section, we omit the derivatives $\partial n_x/\partial t$ and $\partial n_y/\partial t$ in solving the equation for the case of slow relaxation, then we express the components n_x and n_y in terms of n_z and n_0 and obtain a single equation

$$\frac{\partial n_z(t)}{\partial t}=A_z n_z(t)+A_0 n_0,$$

whose coefficients are functions of $M_{ii'}$, K_i, Δ, and T. In calculation of these coefficients, we retain only terms of the lowest order in T^2 and make the integration over τ in Eqs. (27) and (28) easier by taking into account Eq. (16), which allows us to use the expressions

$$\hat{G}^A_{\mathbf{p}}(\tau)\cong\hat{G}^{R*}_{\mathbf{p}}(-\tau)\cong\hat{P}_l\exp\left[-i\left(\varepsilon_p+\frac{\Delta}{2}\right)\frac{\tau}{\hbar}+\delta\tau\right]$$
$$+\hat{P}_r\exp\left[-i\left(\varepsilon_p-\frac{\Delta}{2}\right)\frac{\tau}{\hbar}+\delta\tau\right].$$

After simple transformations we obtain

$$\frac{\partial n_z(t)}{\partial t}=\frac{\partial n_l(t)}{\partial t}=-\frac{\partial n_r(t)}{\partial t}$$
$$=-\frac{1}{\tau_0}\left[n_l(t)-n_r(t)\exp\left(-\frac{\Delta}{T_e}\right)\right], \tag{29}$$

with the time τ_0 defined by Eq. (23), in which

$$\tau^{-1}=\frac{2\pi}{NS^2\hbar}(w_{ll}+w_{rr})\sum_{\mathbf{pp'}}\delta(\varepsilon_p-\varepsilon_{p'}+\Delta)$$
$$\times\exp\left(-\frac{\varepsilon_p}{T_e}\right)\exp\left(-\frac{|\mathbf{p}-\mathbf{p}'|^2 l_c^2}{4\hbar^2}\right)$$
$$=\frac{m(w_{ll}+w_{rr})}{2N\pi\hbar^5}\int_0^\infty p\,dp$$
$$\times\exp\left(-\frac{\varepsilon_p}{T_e}\right)\exp\left(-\frac{ml_c^2\Delta}{2\hbar^2}\right)$$
$$\times\exp\left(-\frac{p^2 l_c^2}{2\hbar^2}\right)I_0\left(\frac{l_c^2 p\sqrt{2m\Delta+p^2}}{\hbar^2}\right), \tag{30}$$

where $I_0(x)$ is the first-kind Bessel function of an imaginary argument. For $l_c\to 0$ Eq. (30) is identical to the expression for τ^{-1} in Eq. (18). In deriving Eqs. (29) and (30) we assumed the condition $\hbar^2/(ml_c^2)\gg\hbar/\tau$, keeping in mind that the scattering angle from smooth potential is so small that the electron momentum $\mathbf{p}$ is no longer a good quantum parameter. Note that in the classical region, $|\Delta|\gg\hbar/\tau$, the expression (23) for τ_0 with τ taken from Eq. (30) is identical to the result of Ref. 12, in which the problem was solved using the classical approach and the same scattering mechanism was assumed. It is also natural that at $\hbar^2/(ml_c^2)\gg T_e$ and $T_e\gg\Delta$ all the results of this section are equivalent to those given at the end of the previous section.

When τ is calculated in the resonant region ($|\Delta|\leqslant\hbar/\tau$), parameters proportional to Δ/T_e and $ml_c^2\Delta/2\hbar^2$ may be omitted, after which we have instead of Eq. (30) the equation

$$\tau^{-1}=\frac{m(w_{ll}+w_{rr})}{2\hbar^3}\left(1+\frac{2ml_c^2T_e}{\hbar^2}\right)^{-1/2}, \tag{31}$$

which describes how the time of scattering within a single quantum well and the related time of resonant tunnelling relaxation change with the correlation length [$\hbar^2/(ml_c^2)\leqslant T_e$]. This expression should be also used in the energetic denominator of Eq. (23) at all Δ. Note also that Eq. (29) in the resonant region is identical to Eq. (22). Our analysis demonstrates that the general trend of the tunneling relaxation time as a function of Δ depends on whether the scattering potential is long-range or short-range only far from the resonance. Near the resonance only the value of τ depends on the shape of scattering potential [compare Eqs. (31) and (18)], and its dependence on Δ is not affected.

It follows from Eq. (29) that the populations in the left and right well under conditions of slow tunnelling relaxation change exponentially with a time constant

$$\tau_d=\tau_0/[1+\exp(-\Delta/T_e)]. \tag{32}$$

The time constant τ_d can be compared to measurements of the decay (or build-up) time of photoluminescence due to annihilation of electrons in one of coupled quantum wells because the drop of the electron population in one well leads to its increase in the other well.[3,5] Around the resonance we have $\tau_d=\tau_0/2$ since $\exp(-\Delta/T_e)\cong 1$. In the case of exact resonance ($\Delta=0$) we have a simple formula

$$\tau_d=(2T/\hbar)^{-2}\tau^{-1}.$$

Note that because of the temperature dependence of τ at $\hbar^2/(ml_c^2)\leqslant T_e$ this parameter drops with the temperature.

5. SLOW RELAXATION OF THE POPULATION: THE CASE OF LOW ENERGY

In this section we shall consider slow tunnelling relaxation of electrons in the case of a small average energy $\bar{\varepsilon} \leqslant \hbar/\tau$, when electron states cannot be classified according to the momentum $\mathbf{p}$. The technique based on the Keldysh diagram method and used in Sec. 2 can be applied in this situation. Moreover, in the short-range-potential approximation used below in this section, there is no need to solve integral equations.

Let us start with Eq. (7) for the function $\hat{\mathscr{F}}_{\mathbf{p}}(\varepsilon,t)$. Since we study the slow relaxation, we may neglect the non-Markovian nature of the collision integral in this equation, after which integration with respect to ε' and t' is easy:

$$
\begin{aligned}
&i\hbar\frac{\partial\hat{\mathscr{F}}_{\mathbf{p}}(\varepsilon,t)}{\partial t}+[\hat{\mathscr{F}}_{\mathbf{p}}(\varepsilon,t),\hat{h}]\\
&\quad=\frac{1}{\pi}\sum_{jj'}\sum_{\mathbf{q}} w_{jj'}\exp\left(-\frac{q^2l_c^2}{4}\right)\{\hat{P}_j\hat{\mathscr{F}}_{\mathbf{p}-\hbar\mathbf{q}}(\varepsilon,t)\hat{P}_{j'}\hat{G}^A_{\mathbf{p}}(\varepsilon)\\
&\quad-\hat{G}^R_{\mathbf{p}}(\varepsilon)\hat{P}_j\hat{\mathscr{F}}_{\mathbf{p}-\hbar\mathbf{q}}(\varepsilon,t)\hat{P}_{j'}+\hat{P}_j\hat{G}^R_{\mathbf{p}-\hbar\mathbf{q}}(\varepsilon)\hat{P}_{j'}\hat{\mathscr{F}}_{\mathbf{p}}(\varepsilon,t)\\
&\quad-\hat{\mathscr{F}}_{\mathbf{p}}(\varepsilon,t)\hat{P}_j\hat{G}^A_{\mathbf{p}-\hbar\mathbf{q}}(\varepsilon)\hat{P}_{j'}\}. \qquad (33)
\end{aligned}
$$

Introducing the components of the matrix $\hat{\mathscr{F}}_{\mathbf{p}}(\varepsilon,t)$ in the isospin representation:

$$
\hat{\mathscr{F}}_{\mathbf{p}}(\varepsilon,t)=\hat{I}\mathscr{F}_{0\mathbf{p}}(\varepsilon,t)+\sum_i \hat{\sigma}_i\mathscr{F}_{i\mathbf{p}}(\varepsilon,t), \qquad (34)
$$

we find that the operator equation (33) is reduced to a system of three integral equations for $\mathscr{F}_{i\mathbf{p}}(\varepsilon,t)$ $(i=x,y,x)$ and the equation $\partial\mathscr{F}_{0\mathbf{p}}(\varepsilon,t)\partial t=0$, from which follows $\mathscr{F}_{0\mathbf{p}}(\varepsilon,t)=\mathscr{F}_{0\mathbf{p}}(\varepsilon,)$. If the components $\mathscr{F}_{x\mathbf{p}}(\varepsilon,t)$ and $\mathscr{F}_{y\mathbf{p}}(\varepsilon,t)$ are expressed through $\mathscr{F}_{z\mathbf{p}}(\varepsilon,t)$, we obtain a single integral equation for $\mathscr{F}_{z\mathbf{p}}(\varepsilon,t)$:

$$
\begin{aligned}
&\hbar\frac{\partial\mathscr{F}_{z\mathbf{p}}(\varepsilon,t)}{\partial t}+2\Bigg\{\Sigma''_{0\mathbf{p}}(\varepsilon)\mathscr{F}_{z\mathbf{p}}(\varepsilon,t)\\
&\quad-\sum_{\mathbf{q}}\exp\left(-\frac{q^2l_c^2}{4}\right)\mathscr{F}_{z\mathbf{p}-\hbar\mathbf{q}}(\varepsilon,t)[w_+G''_{0\mathbf{p}}(\varepsilon)\\
&\quad+w_-G''_{z\mathbf{p}}(\varepsilon)]\Bigg\}+\frac{4T}{[\Delta+2\Sigma'_{z\mathbf{p}}(\varepsilon)]^2+[2\Sigma''_{0\mathbf{p}}(\varepsilon)]^2}\\
&\quad\times\Bigg\{2T\Sigma''_{0\mathbf{p}}(\varepsilon)\mathscr{F}_{z\mathbf{p}}(\varepsilon,t)-\sum_{\mathbf{q}}\exp\left(-\frac{q^2l_c^2}{4}\right)\\
&\quad\times\mathscr{F}_{z\mathbf{p}-\hbar\mathbf{q}}(\varepsilon,t)\Big[2w_+G''_{0\mathbf{p}}(\varepsilon)G'_{x\mathbf{p}}(\varepsilon)\\
&\quad+w_-[\Delta+2\Sigma'_{z\mathbf{p}}(\varepsilon)]G''_{x\mathbf{p}}(\varepsilon)-\sum_{\mathbf{q}}\exp\left(-\frac{q^2l_c^2}{4}\right)\\
&\quad\times\mathscr{F}_{0\mathbf{p}-\hbar\mathbf{q}}(\varepsilon)[2w_-G''_{0\mathbf{p}}(\varepsilon)G'_{x\mathbf{p}}(\varepsilon)+w_+\\
&\quad\times[\Delta+2\Sigma'_{z\mathbf{p}}(\varepsilon)]G''_{x\mathbf{p}}(\varepsilon)]\Bigg\}=0 \qquad (35)
\end{aligned}
$$

where $w_\pm=(w_{ll}\pm w_{rr})/2$, and the terms including correlators of higher orders w_{lt}, w_{rt}, and w_{tt} are omitted. Equation (35) includes the real and imaginary components (marked by one and two primes, respectively) of advanced Green's functions and self-energy functions in the isospin formalism. In calculating these components, terms of higher order in T should be omitted. The inclusion of these terms would not be appropriate to the actual accuracy of calculations of the slow tunneling relaxation under the conditions when the tunneling coupling is suppressed by electron scattering within a single quantum well. In the zero-order approximation in T, the states in quantum wells are independent of each other and described by the Green's functions for the left and right wells, $G_{l\mathbf{p}}(\varepsilon)$ and $G_{r\mathbf{p}}(\varepsilon)$:

$$
G_{j\mathbf{p}}(\varepsilon)=\left[\varepsilon-\varepsilon_p\pm\frac{\Delta}{2}-\Sigma_{j\mathbf{p}}(\varepsilon)\right]^{-1}, \qquad (36)
$$

$$
\Sigma_{j\mathbf{p}}(\varepsilon)=w_{jj}\sum_{\mathbf{q}}\exp\left(-\frac{q^2l_c^2}{4}\right)G_{j\mathbf{p}-\hbar\mathbf{q}}(\varepsilon), \qquad (37)
$$

where $j=l,r$, and plus and minus signs are taken for the indices l and r, respectively. The components of the Green's and self-energy functions in Eq. (35) are expressed as follows:

$$
\begin{aligned}
&G_{0\mathbf{p}}(\varepsilon)=[G_{l\mathbf{p}}(\varepsilon)+G_{r\mathbf{p}}(\varepsilon)]/2, G_{z\mathbf{p}}(\varepsilon)\\
&\quad=[G_{l\mathbf{p}}(\varepsilon)-G_{r\mathbf{p}}(\varepsilon)]/2, \qquad (38)
\end{aligned}
$$

$$
G_{x\mathbf{p}}(\varepsilon)=\frac{T}{\Delta+\Sigma_{l\mathbf{p}}(\varepsilon)-\Sigma_{r\mathbf{p}}(\varepsilon)}[G_{l\mathbf{p}}(\varepsilon)-G_{r\mathbf{p}}(\varepsilon)], \qquad (39)
$$

$$
\begin{aligned}
&\Sigma_{0\mathbf{p}}(\varepsilon)=[\Sigma_{l\mathbf{p}}(\varepsilon)+\Sigma_{r\mathbf{p}}(\varepsilon)]/2, \Sigma_{z\mathbf{p}}(\varepsilon)\\
&\quad=[\Sigma_{l\mathbf{p}}(\varepsilon)-\Sigma_{r\mathbf{p}}(\varepsilon)]/2. \qquad (40)
\end{aligned}
$$

In the short-range-potential approximation, the integral equation (35) is reduced to an algebraic equation. By introducing the functions $N_l(\varepsilon,t)$ and $N_r(\varepsilon,t)$ defined as

$$
N_l(\varepsilon,t)=-\frac{i}{4\pi S}\sum_{\mathbf{p}}[\mathscr{F}_{0\mathbf{p}}(\varepsilon)+\mathscr{F}_{z\mathbf{p}}(\varepsilon,t)], \qquad (41)
$$

$$
N_r(\varepsilon,t)=-\frac{i}{4\pi S}\sum_{\mathbf{p}}[\mathscr{F}_{0\mathbf{p}}(\varepsilon)-\mathscr{F}_{z\mathbf{p}}(\varepsilon,t)]
$$

[integrals of these functions over energy equal the electron densities in the left and right quantum wells, $n_l(t)$ and $n_r(t)$], and integrating Eq. (35) with respect to the momentum $\mathbf{p}$, we obtain equations for $N_l(\varepsilon,t)$ and $N_r(\varepsilon,t)$ which describe the tunnelling relaxation:

$$
\begin{aligned}
&\frac{\partial N_l(\varepsilon,t)}{\partial t}=-\frac{\partial N_r(\varepsilon,t)}{\partial t}\\
&\quad=-\left[\frac{\Sigma''_r(\varepsilon)}{w_{rr}}N_l(\varepsilon,t)-\frac{\Sigma''_l(\varepsilon)}{w_{ll}}N_r(\varepsilon,t)\right]R(\varepsilon), \qquad (42)
\end{aligned}
$$

where the function $R(\varepsilon)$ symmetrical with respect to the change of the left well by the right one is expressed as

$$R(\varepsilon)=\frac{4T^2}{\hbar}\frac{w_+\Delta^2+\Delta[w_{ll}\Sigma_l'(\varepsilon)-w_{rr}\Sigma_r'(\varepsilon)]+w_-[|\Sigma_l(\varepsilon)|^2-|\Sigma_r(\varepsilon)|^2]}{([\Delta+2\Sigma_z'(\varepsilon)]^2+[2\Sigma_0''(\varepsilon)]^2)([\Delta+2\Sigma_z'(\varepsilon)]^2+[2\Sigma_z''(\varepsilon)]^2)}. \tag{43}$$

These equations also take into account that in the short-range-potential approximation the eigenfunctions are constant with the momentum. These functions are found by substituting Eq. (36) into Eq. (37) and integrating with respect to the momentum, as a result, we have complex equations for $\Sigma_j(\varepsilon)$:

$$\Sigma_j(\varepsilon)=B_j\left[\ln\left(-\varepsilon\mp\frac{\Delta}{2}+\Sigma_j(\varepsilon)\right)-\ln\varepsilon_m\right],$$

$$B_j=\frac{m}{2\pi\hbar^2}w_{jj}, \tag{44}$$

where ε_m is the cut-off energy introduced to get rid of divergence of integrals.[22] The nondefined logarithm $\ln\varepsilon_m$ is excluded from Eq. (44) by renormalizing real parts of $\Sigma_j(\varepsilon)$ concurrently with the renormalization of the energy ε and energy difference Δ.[23] In this procedure Eq. (42) does not change, as was expected (see below).

In order to derive from Eqs. (42) and (43) kinetic equations for electron densities, let us note that

$$N_j(\varepsilon,t)=\frac{n_j(t)}{N_j}D_j(\varepsilon)\exp\left(-\frac{\varepsilon}{T_e}\right),$$

$$N_j=\int d\varepsilon D_j(\varepsilon)\exp\left(-\frac{\varepsilon}{T_e}\right), \tag{45}$$

where

$$D_j(\varepsilon)=\frac{2}{\pi w_{jj}}\Sigma_j''(\varepsilon)$$

is the density of states of two-dimensional electrons in the nonhomogeneous potential of the j-th quantum well and T_e is the electron temperature. In deriving Eq. (45) we assumed the Boltzmann distribution of electrons after thermalization in their respective wells (at a higher electron density the Fermi distribution may be considered). By integrating Eq. (42) with respect to the energy, we obtain an equation similar to Eq. (29):

$$\frac{\partial n_l(t)}{\partial t}=-\frac{\partial n_r(t)}{\partial t}=-\frac{1}{\tau_0}\left[n_l(t)-\frac{N_l}{N_r}n_r(t)\right], \tag{46}$$

where the time τ_0, as in the previous section, is the time of transition from the left to right well due to the slow tunneling relaxation. In this case it is determined by the formula

$$\frac{1}{\tau_0}=\frac{\pi}{2}\frac{1}{N_l}\int d\varepsilon D_l(\varepsilon)D_r(\varepsilon)R(\varepsilon)\exp\left(-\frac{\varepsilon}{T_e}\right), \tag{47}$$

in which the integrand on the right contains the product of electron density of states in the left and right well by both the tunnelling probability between these wells described by the function $R(\varepsilon)$ and the energy distribution function. The second term on the right of Eq. (46) describes the reverse flow of electrons and is controlled by the factor N_l/N_r, which equals $\exp(-\Delta/T_e)$ in the case of symmetric scattering ($w_{ll}=w_{rr}$). Note that both the time τ_0 and the function $R(\varepsilon)$ are proportional to T^2. The time constant of the luminescence decay (build-up) is determined by the formula

$$\tau_d=\tau_0/[1+N_l/N_r] \tag{48}$$

(compare to Eq. (32)). This parameter is invariant with respect to the exchange of wells.

In calculating τ_0 and τ_d we use the following expressions for real parts of $\Sigma_j(\varepsilon)$ in terms of dimensionless imaginary parts $\sigma_j(\varepsilon)=\Sigma_j''(\varepsilon)/B_j$:

$$\Sigma_j'(\varepsilon)=-B_j\ln\left[\frac{\sin[\sigma_j(\varepsilon)]}{\sigma_j(\varepsilon)}\frac{T_e}{B_j}\right]. \tag{49}$$

The imaginary parts are derived from an algebraic equation system[28]

$$\varepsilon=B_l\left[1-\sigma_l(\varepsilon)\cot\sigma_l(\varepsilon)-\ln\frac{\sin[\sigma_l(\varepsilon)]}{\sigma_l(\varepsilon)}\right],$$

$$\varepsilon+\Delta_R=B_r\left[1-\sigma_r(\varepsilon)\cot\sigma_r(\varepsilon)-\ln\frac{\sin[\sigma_r(\varepsilon)]}{\sigma_r(\varepsilon)}\right], \tag{50}$$

where $\Delta_R=\Delta-B_l[1-\ln(B_l/T_e)]+B_r[1-\ln(B_r/T_e)]$. In deriving Eqs. (49) and (50) from Eq. (44) we used the renormalizations

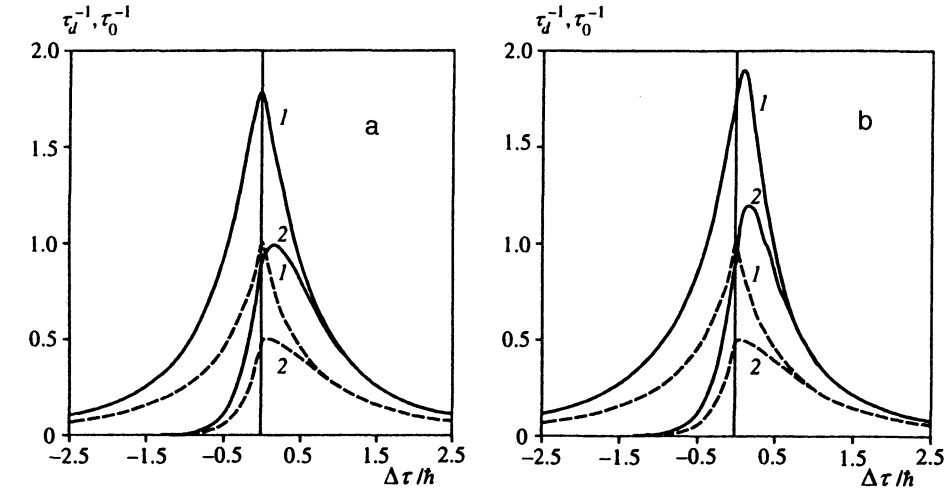
$$\Sigma_j\to\Sigma_j-B_j\ln\left(\frac{\varepsilon_m}{T_e}\right),$$

$$\varepsilon\to\varepsilon-\frac{1}{2}(B_l+B_r)\ln\left(\frac{\varepsilon_m}{T_e}\right), \tag{51}$$

$$\Delta\to\Delta+(B_l-B_r)\ln\left(\frac{\varepsilon_m}{T_e}\right),$$

which cancel out the cut-off energy, and the following shift of the energy zero: $\varepsilon\to\varepsilon+\Delta/2-B_l[1-\ln(B_l/T_e)]$, which shifts it to the band edge in the left well, where $\Sigma_l''(\varepsilon)$ drops to zero (the latter renormalization is not essential and is performed for convenience).

It is expedient to compare τ_0 and τ_d as functions of the energy difference Δ to similar functions for the case of high electron temperature obtained in the previous section at $\hbar^2/(ml_c^2)\ll T_e,\Delta$ without terms including w_{lt}, w_{rt}, and w_{tt}:



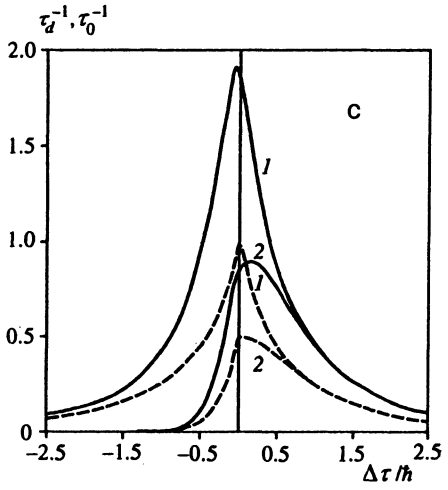


FIG. 3. 1) Relaxation rate of the electron population in double quantum wells, τ_d^{-1}, and 2) departure rate from the left well, τ_0^{-1}, at low temperatures ($T_e\tau/\hbar=0.25$) at different ratios of scattering frequencies in two wells: a) $w_{ll}=w_{rr}$; b) $w_{rr}/w_{ll}=1/3$; c) $w_{rr}/w_{ll}=3$. The solid lines are calculations using Eqs. (47) and (48), dashed lines are calculations using Eq. (52). All calculations are normalized to $(2T/\hbar)^2\tau$.

$$\tau_d^{-1}=\frac{2T^2\tau^{-1}}{\Delta^2+(\hbar/\tau)^2}\left[1+\exp\left(-\frac{|\Delta|}{T_e}\right)\right],$$
$$\tau_0^{-1}=\frac{2T^2\tau^{-1}}{\Delta^2+(\hbar/\tau)^2}\begin{cases}1, & \Delta>0,\\ \exp(\Delta/T_e), & \Delta<0,\end{cases} \tag{52}$$

where τ is determined by the first part of Eq. (18). Figures 3–5 show the decay rates τ_d^{-1} and τ_0^{-1} normalized to $(2T/\hbar)^2\tau$ (the resonant value from Eq. (52)) versus Δ at different ratios of the electron temperature to the collision broadening $\hbar/\tau$. Figures 3–5b,c demonstrate the effect of the difference between scattering rates in the two wells (i.e., difference between the correlators w_{ll} and w_{rr}) on these curves. This effect not only leads to the shift of the resonant peak (it can be eliminated by renormalizing Δ) but also to a considerable asymmetry of the peak. This asymmetry is more significant in the intermediate temperature range (Fig. 4) than at low temperature (Fig. 3). At high temperature the asymmetry is not so pronounced, and in the high-temperature limit, when Eq. (52) is valid, the peak should be symmetrical.

The calculations shown in Figs. 3–5 indicate that τ_d^{-1} and τ_0^{-1} increase with the decreasing temperature around the resonance. This increase is not too large: the ratio of the resonance relaxation rate at $T_e=0.25\hbar/\tau$ to that in the high-temperature limit (Eq. (52)) is about two and changes little with decreasing temperature. The widths of the resonance peaks on the curves of τ_d^{-1} and τ_0^{-1} versus Δ also increase with decreasing temperature, but this increase is even smaller. The differences between calculations of τ_d^{-1} and τ_0^{-1}, on one side, and their high-temperature limits, on the other side, are less significant at larger Δ. Thus we have found that although the relaxation mechanism at low temperature is radically different from that at high temperatures, the relaxation rates may be roughly estimated at all temperatures using the high-temperature limit given by Eq. (52).

Note that the formulas given in this section rely on the self-consistent Born approximation [Eqs. (3) and (4)] used to calculate the Green functions of low-energy electrons. This approximation, in particular, yields a sharp edge of the density of state of two-dimensional electrons, which is not a satisfactory result from the physical viewpoint. Other methods, such as functional integration,[24] must be used in a more detailed study.

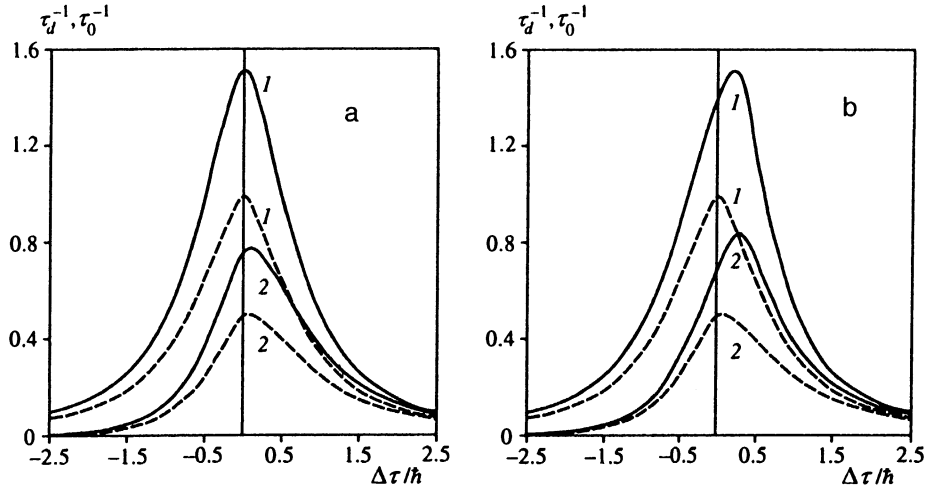
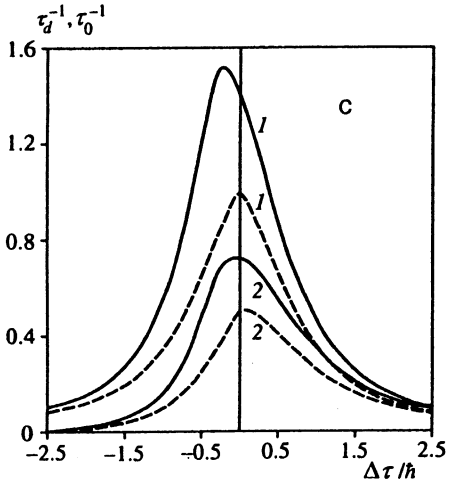


FIG. 4. Plots similar to those of Fig. 3 for an intermediate temperature: $T_e\tau/\hbar = 1$.



6. REVIEW OF EXPERIMENTAL DATA AND DISCUSSION

The kinetics of nonequilibruim electrons in tunnel-coupled quantum wells is interesting both from the viewpoint of fundamental physics (study of tunneling between two-dimensional electron systems) and with a view of developing new electronic devices.[25] Their kinetics has been studied extensively since the late 1980s, largely by optical spectroscopy. Most publications (Refs. 1–8 and others) were dedicated to the slow resonant and nonresonant tunneling relaxation of electrons whose rate was measured by the decay (or build-up) of luminescence due to electrons in the left or right quantum well. The time resolution in such experiments is usually about several picoseconds, but with state-of-the-art techniques it is down to 100 femtoseconds.[8] The energy difference Δ between levels in the well can be tuned by electric field aligned with the z-axis applied to the structure in order to investigate the region of resonant tunnelling around $\Delta = 0$. On the other hand, several publications in recent years reported on investigations of electron density oscillations due to coherent tunneling observed through oscillations of the probing pulse transmission[15] or detection of electromagnetic waves in the terahertz band.[16,17]

The existing theoretical models of processes observed in these experiments are not satisfactory for the following reasons. First, the resonant relaxation of electrons in coupled quantum wells was not studied in detail; only a simple phenomenological model, which yielded a formula similar to Eq. (52), was proposed by Leo *et al.*[26] (note also an earlier publication by Kazarinov and Suris,[27] in which a similar formula was derived from a microscopic theory for tunneling in superlattices). Second, the existing theories of coherent oscillations in coupled quantum wells either ignore electron scattering[20] or include it in the form of phenomenological decay times introduced into dynamic equations.[18,19] This model does not reveal any connection between coherent oscillations and slow tunneling relaxation, although these are two manifestations of one process, namely the evolution of a space distribution of nonequilibrium electrons.

In this work, we have investigated the evolution of electron distribution in the regime of both coherent electron den-

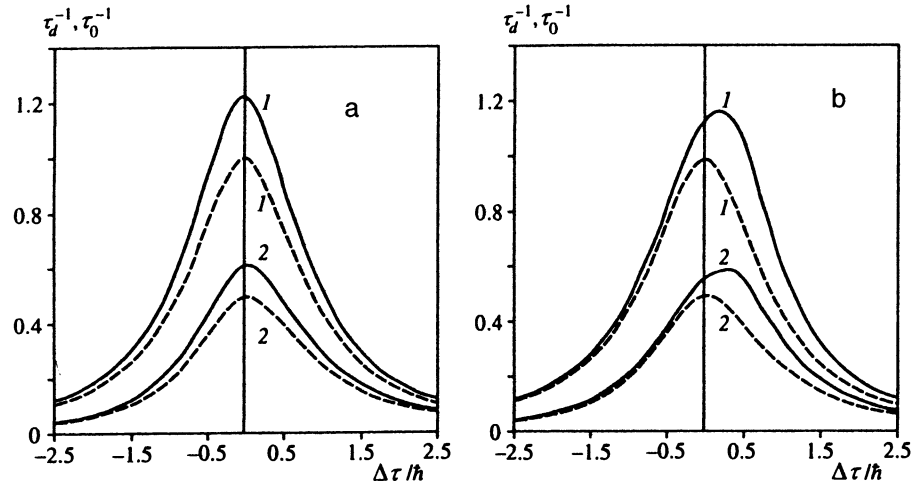
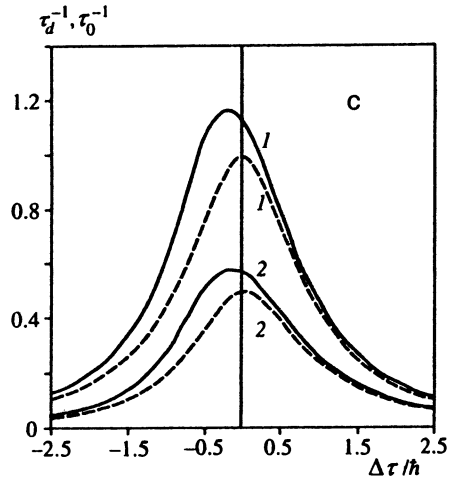


FIG. 5. Plots similar to those of Fig. 3 at a high temperature: $T_e\tau/\hbar = 4$.



sity oscillations and slow relaxation of the electron population using a unified approach based on the kinetic equation (7) or (11) and the model of scattering from time-independent structural nonuniformities (interface roughness). Let us reiterate the main features of the transition between these two regimes due to changes in parameters which control the electronic system evolution. Slowly decaying oscillations are observable when the energy difference $2T$ between tunnel-coupled levels is slightly larger than the collision broadening energy $\hbar/\tau$. This condition was realized in the experiment by Roskos *et al.*,[16] in which $2T = 6$ meV, and an estimate based on the measured decay time constant yielded $\hbar/\tau \approx 1$ meV. On the contrary, if $\hbar/\tau$ is larger than $2T$, there is a range of parameters in which oscillations are impossible (Fig. 1). If the energy difference Δ is increased, oscillations with smaller amplitude and period are seen against a nonoscillating decaying background (Fig. 2b), which makes their observation more difficult. After a time $t \sim \tau$ the oscillations decay and the system comes to equilibrium.

If the coherent tunneling is strongly suppressed by scattering or owing to a large energy difference Δ [Eq. (24)], the relaxation continues for $t \gg \tau$. This slow relaxation is due to the tunneling of electrons between quantum wells, which are well insulated from each other when the condition of Eq. (24) holds. This is the relaxation process observed in experiments on the photoluminescence decay, in which the relaxation time is 10–100 ps even around the resonance, and this time is much longer than typical values of τ. In Secs. 3–5 we attempted to give a detailed description of the slow tunneling relaxation taking into account its features deriving from the following conditions: (a) the scattering couples the quantum wells owing to the nonuniformity of the barrier thickness (the correlators w_{ll}, w_{rl}, and w_{tt} are included); (b) the scattering potential is long-range (the correlation length l_c is large); (c) electrons with energies less than $\hbar/\tau$ tunnel between the wells (the electron temperature, T_e is low). The conditions (a) and (b) have a notable effect on the curves of tunneling times τ_0 and τ_d plotted against Δ only at a large Δ. The condition (c) is essential around the resonance, when $|\Delta| \leqslant \hbar/\tau$. We have found that for $T_e \leqslant \hbar/\tau$ the tunneling relaxation rate is higher than for $T_e \gg \hbar/\tau$ and the resonant peak is asymmetrical due to the difference between scattering rates in the coupled wells. These conditions have little

effect on τ_0 and τ_d, and experimental data on the tunnelling relaxation around the resonance can be analyzed using simple formulas of Eq. (52), which are valid at $T_e \gg \hbar/\tau$.

It is noteworthy that in most experiments on the resonant tunneling relaxation,[1,2,6–8] electrons tunnel between the ground state in the left quantum well and the second (or even third or fourth) dimensional subband of the right well. Under this condition, the electrons arriving in the right well rapidly relax to lower subbands, and there is no back flow of electrons to the left well. This means that the tunneling relaxation is controlled by the time of transition from the left to the right well, τ_0. Note that the curve of this time plotted against Δ is asymmetrical because the factor $\exp(\Delta/T_e)$ is present for $\Delta<0$ and absent for $\Delta>0$ (Eq. (52) and Figs. 3–5). This asymmetry is clearly seen in experimental plots of τ_0 versus Δ.[1,2,6,8] Strictly speaking, our equations can be applied to this process only when the transition time between subbands is longer than τ (for example, owing to a large momentum transferred in such transitions). If these times are comparable, the two-level model of the tunnelling relaxation is not suitable, and our results can be considered only as rough estimates.

In comparing our calculations using Eq. (52) to experimental data on the tunneling relaxation, we estimated the broadening $\hbar/\tau$ from both the luminescence decay time at resonance (the tunnelling matrix element T was calculated using parameters of heterostructures in the flat band approximation) and from the relaxation time as a function of the energy difference Δ (i.e., from the resonant peak width). Estimates from the decay time at resonance yielded 4.5 meV;[6] 1.1 and 3.0 meV (structures with barriers 6 and 8 nm wide, respectively);[2] 0.7 and 0.7 meV (the structure with thicknesses 13.5/6.5/10.5 nm; tunnelling to the second and third level, respectively).[6] The estimates from the resonance peak width are 9 meV,[1] 4.5 and 8.0 meV,[2] 4.0 and 4.0 meV,[6] respectively. The differences among these estimates may be caused by errors in T, since the slope of band edges due to electric field was not taken into account; given large widths of wells and barriers, this could change the estimates of T by a factor of two to three (four to nine in the case of τ). Another reason is that the two-level approximation is not applicable to this case (see the previous section). These two factors are not essential for experimental data by Mantsutsue *et al.*,[4] concerning a GaAs/AlAs/GaAs structure with a high thin barrier and fairly narrow quantum wells, in which the level in the right well was depleted through electron tunneling across the thin barrier to continuum states. No asymmetry of the resonant peak, $\tau_0(\Delta)$, due to the temperature was observed, which may be accounted for assuming that $T_e \gg \Delta$ (the large width of luminescence lines supports this assertion). The experimental curve[4] of the relaxation time τ_0 versus Δ was approximated fairly well by Eq. (23) at $\hbar/\tau = 5.7$ meV, supposing that roughness amplitudes on all interfaces were equal (Fig. 6) (the assumed broadening, $\hbar/\tau$, may be caused by monolayer irregularities with a correlation length of 3 nm, which are fairly reasonable parameters). Since the condition $(\kappa d_{l,r})^2 \gg 1$ was satisfied in this structure, a weak asymmetry of the $\tau_0(\Delta)$ curve [due to the linear with Δ term in Eq. (23)] can be seen (see also Ref. 12).

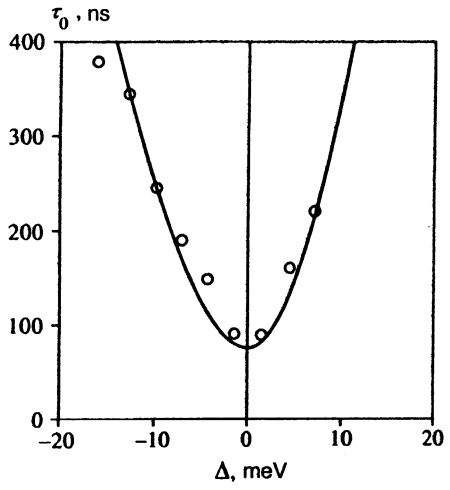


FIG. 6. Time τ_0 of tunneling relaxation from the left well of the GaAs/AlAs/GaAs structure with widths of 7.1/3.1/5.1 nm versus the energy difference Δ controlled by electric field. Experimental points are taken from Ref. 4.

At the resonance ($\Delta=0$) the rate of slow tunnelling relaxation is proportional to the square of the tunneling matrix element at all temperatures of tunneling electrons. This conclusion is in a qualitative agreement with experimental data by Heberle *et al.*[8] concerning tunneling relaxation of electrons between coupled quantum wells separated by barriers of different thicknesses when electrons from the first level of the left well went to the second level of the right well (Fig. 7) (in our estimates we used T calculated in Ref. 8). The fit of experimental data to a simple formula $\tau_0 = (T/\hbar)^{-2}\tau^{-1}/2$ yields $\hbar/\tau \approx 11$ meV. Relaxation times when the first level of the left well was exactly tuned to the n-th level ($n=2,3,4$) of the right well were also compared in

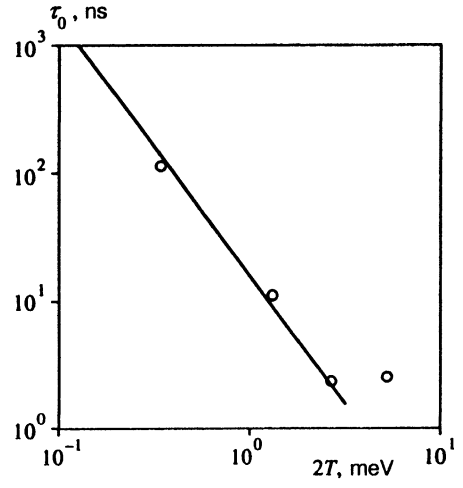


FIG. 7. Time τ_0 of tunneling relaxation from the left well in a $GaAs/Ga_{0.7}Al_{0.3}As/GaAs$ structure with thicknesses of $5/d/10$ nm at $\Delta=0$ versus the energy difference $2T$ controlled by the barrier width d. Experimental points are taken from Ref. 8.

Ref. 8. The experimental points did not fit the curve $\tau_0 \propto (2T)^{-2}$, which may be accounted for by an increase in the broadening energy $\hbar/\tau$ with the resonance number. This behavior is typical of scattering from interface inhomogeneities, but may be also interpreted in terms of other scattering mechanisms (such as transitions between subbands with emission of optical phonons).

In conclusion, note that the shape of resonance peaks may be affected by another factor not mentioned in this paper. This is an nonhomogeneous broadening due to large-scale nonuniformities of quantum well widths ("islet" nonuniformities) that leads to variations in Δ over the structure area. In this case the relaxation rate should be lower and the peak width larger. This effect should be more pronounced in structures with narrow wells, and our analysis of experimental data indeed indicates this tendency.

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