Stimulated bremsstrahlung effect in an ultrastrong laser field

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The Kramers–Henneberger method is used to investigate the stimulated bremsstrahlung absorption (emission) of photons accompanying the scattering of an electron by a one-dimensional quantum system in an ultrastrong electromagnetic field. The properties of the bremsstrahlung spectra are discussed for different intensities of the radiation. The results are compared with calculations made by direct numerical integration of the one-dimensional time-dependent Schrödinger equation. The limits of applicability of the Kramers–Henneberger approximation for the model are established. © *1996 American Institute of Physics.* [S1063-7761(96)00701-2]

1. INTRODUCTION

Bremsstrahlung absorption (emission) of photons accompanying the scattering of electrons by atoms in a strong electromagnetic field is one of the most important elementary processes that determine the dynamics of the interaction of rarefied gases and plasmas with intense laser radiation. Stimulated bremsstrahlung was first considered outside the framework of perturbation theory with respect to the interaction of a quantum system with the field of an electromagnetic wave in Refs. 1 and 2. The following connection between the amplitude of n-photon stimulated bremsstrahlung and the elastic scattering amplitude f_{pp_0} was obtained in the Born approximation for the scattering potential, with Volkov functions describing the state of a free electron in the field of an electromagnetic wave:

$$f_{SBE}^{(n)}=f_{pp_0}J_n(N_{pp_0}). \tag{1}$$

Here p_0 and p are the initial and final values of the electron momentum, J_n is a cylindrical Bessel function of argument

$$N_{pp_0}=\frac{e\varepsilon\,\delta\mathbf{p}}{m\hbar\omega^2},$$

ε is the amplitude of the electric field of the wave with frequency ω, and $\delta\,\mathbf{p}=\mathbf{p}-\mathbf{p}_0$ is the change in the momentum during the scattering process.

In subsequent years, numerous attempts were made (see, for example, Refs. 3–10) to generalized the expression (1) beyond the Born approximation. A general theory of stimulated bremsstrahlung in the case of potential scattering in an electromagnetic field has yet to be created. In ultrastrong optical fields, in which the field strength of the wave exceeds the strength of the atomic field, the dynamics of the quantum system in the electromagnetic field can be treated by the Kramers–Henneberger method.[11] Interest in the investigation of the elementary processes in a strong electromagnetic field in the Kramers–Henneberger approximation is due to the successful use of this approach to describe the stabilization of atoms in an ultrastrong field[12–14] and continues to be stimulated by recent discussion of the physical utility of Kramers coordinates.[15] For example, it is emphasized in Ref. 15 that the elementary concepts of the Kramers–Henneberger formalism, such as the Kramers–Henneberger potential, the Kramers–Henneberger stationary states, and the wave functions that correspond to them are real characteristics of elementary processes in strong fields, and in this sense the Kramers coordinate system is distinguished among other possible coordinate systems. In fact, given that the limits of applicability of the Kramers–Henneberger approximation has yet to be adequately investigated, the question raised above is perhaps largely a methodological one.

This paper is devoted to an investigation of electron scattering by a one-dimensional quantum system with short-range potential in the Kramers–Henneberger formalism. The results are compared with calculations based on direct numerical integration of the time-dependent Schrödinger equation for a quantum system in the field of an electromagnetic wave.

2. THE KRAMERS COORDINATE SYSTEM AND THE KRAMERS–HENNEBERGER APPROXIMATION

By definition, the Kramers coordinate system is attached to a free electron oscillating in the field of an electromagnetic wave. The coordinates x of the laboratory system and the coordinates x' of the Kramers coordinate system are related by

$$x'=x-a_e\cos\omega t,$$

where $a_e=e\varepsilon_0/m\omega^2$ is the amplitude of the oscillatory motion of the electron, and ε_0 and ω are the amplitude and frequency of the wave field. The transition to this coordinate system is made by means of a time-dependent unitary transformation

$$\psi_{\mathrm{KH}}(x',t)=S(t)\psi(x,t), \tag{2}$$

where

$$S(t)=\exp\left\{\frac{i}{\hbar}\int_{-\infty}^{t}H_{\mathrm{int}}(t)dt\right\},$$

in which $H_{\mathrm{int}}(t)$ is the operator of the interaction of the electron with the field of the electromagnetic wave.

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In the dipole approximation in the dE gauge, this operator has the form

$$H_{\text{int}}(t) = -ex\varepsilon_0 \cos \omega t. \tag{3}$$

Choosing as basis $\{\psi(x,t)\}$ in the laboratory system the Volkov functions

$$\psi_V(x,t) = \exp\left(\frac{i}{\hbar}\left[\left(p - \frac{e\varepsilon_0}{\omega}\sin \omega t\right)x - \int_{-\infty}^{t} \frac{\left(p - \frac{e\varepsilon_0}{\omega}\sin \omega t'\right)}{2m} dt'\right]\right)$$

and applying the transformation (2), we obtain in the Kramers coordinate system the plane-wave basis

$$\psi_{\text{KH}}(x',t) = \exp\left(i\frac{p}{\hbar}x'\right)\exp\left(-\frac{ip^2t}{2m\hbar}\right)e^{iF}$$

(e^{iF} is a phase factor that does not depend on the coordinate and momentum and therefore vanishes in the calculation of the matrix elements between the states $\psi_{\text{KH}}(x,t)$).

In the Kramers coordinate system, the Schrödinger equation for the electron wave function $\psi_{\text{KH}}(x',t)$ can be written in the form

$$i\hbar \frac{\partial \psi_{\text{KH}}}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi_{\text{KH}}}{\partial x^2} + V(x' + a_e \cos \omega t) \times \psi_{\text{KH}}(x',t), \tag{4}$$

where $V(x)$ is the atomic potential. Thus, the atom–field interaction has been reduced to a time-dependent shift in the argument of the potential.

Expanding the potential $V(x' + a_e \cos \omega t)$ in a Fourier series, we obtain

$$V(x' + a_e \cos \omega t) = \sum_{n=-\infty}^{\infty} V_n(x',a_e)e^{in\omega t}, \tag{5}$$

where

$$V_n(x',a_e) = \frac{1}{2\pi}\int_0^{2\pi} V(x' + a_e \cos \omega t)e^{-in\omega t}d(\omega t). \tag{6}$$

The Kramers–Henneberger approximation consists of ignoring in (4) all the terms of the series except the zeroth: $n=0$. As a result, the motion of the electron in the atomic potential in the presence of the alternating wave field is reduced to the problem for a Schrödinger equation with time-independent potential $V_0(x,a_e)$ (Kramers–Henneberger potential):

$$i\hbar \frac{\partial \psi_{\text{KH}}}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi_{\text{KH}}}{\partial x^2} + V_0(x,a_e)\psi_{\text{KH}}(x,t). \tag{7}$$

Here and in what follows, we omit the prime.

All the terms of the series (5) with $n \neq 0$ are assumed to be small; they can then be regarded as a perturbation:

$$\delta V(x,a_e,t) = V(x + a_e \cos \omega t) - V_0(x,a_e). \tag{8}$$

A key concern in this process is to establish that the modes V_n are small and to find the regime in which the Kramers–Henneberger approximation works well, i.e., to find the constraints that must be imposed on the permissible parameters of the laser radiation: the frequency, intensity, duration, and shape of the pulse. It is obvious that in contrast to the ionization problem, in the case of the stimulated bremsstrahlung we need not discuss the constraints on the duration of the wavefront and on the pulse shape, i.e., questions associated with turning the field on and off and the occupation of the Kramers–Henneberger states.[16]

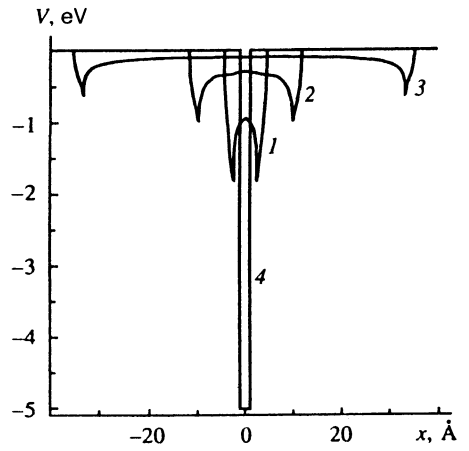

FIG. 1. Kramers–Henneberger potential for the following intensities of the laser radiation (W/cm^2): 1) 10^{14}, 2) 10^{15}, 3) 10^{16}; 4) the original atomic potential.

3. ONE-DIMENSIONAL MODEL OF THE QUANTUM SYSTEM

In this paper, we consider a rectangular potential well of depth $V_0 = 5$ eV and width $d = 2$ Å, which was used previously in Ref. 16. In such a well, there is a single bound state with energy $\varepsilon_1 = -2.55$ eV and wave function $\varphi_1(x)$, the eigenvalue and eigenfunction of the model potential $V(x)$. The atomic intensity for this well is $P^* \simeq 8 \cdot 10^{13}$ W/cm^2. The Kramers–Henneberger potential for this atom for different intensities and $\hbar\omega = 2.5$ eV is shown in Fig. 1. At high intensities ($P > 10^{14}$ W/cm^2), it has a characteristic double-well shape (the distance between the wells is $2a_e$). Such a shape of the Kramers–Henneberger potential is typical and does not depend on the particular details of the original potential.

In this model, the harmonics $V_n(x,a_e)$ can be calculated analytically, and the perturbation $\delta V(x,a_e,t)$ in the region $x \in (-a_e - d/2, a_e + d/2)$ has the form

$$\delta V(x,a_e,t) = -\frac{|V_0|}{\pi n}\sum_{n=1}^{\infty}(-1)^n\left[\sqrt{1-\xi_1^2}\,U_{n-1}(\xi_1)\delta(\xi_1) - \sqrt{1-\xi_2^2}\,U_{n-1}(-\xi_2)\delta(\xi_2)\right]\begin{Bmatrix}\cos n\omega t \\ \sin n\omega t\end{Bmatrix}. \tag{9}$$

Here

$$U_n(\xi) = \frac{\sin((n+1)\cos^{-1}\xi)}{\sin \xi} \tag{10}$$

are the Chebyshev polynomials of the second kind,[17] with arguments $\xi_{1,2} = (a \mp x)/a_e$, $a = d/2$,

$$\delta(\xi)=\begin{cases}1, & \xi^2<1,\\ 0, & \xi^2>1.\end{cases}$$

In the expression (9), $\cos n\omega t$ and $\sin n\omega t$ correspond to even and odd n, respectively.

A similar model was considered in Ref. 16, an investigation of ionization in the Kramers–Henneberger approximation. The existence of stabilization in such a system enables to draw some *a priori* conclusions about the spectrum of stimulated bremsstrahlung. Indeed, in the general Kramers–Henneberger formalism above, what happens in the Kramers system is electron scattering by a corresponding potential in the presence of the field of the harmonics V_n. In lowest-order perturbation theory, each harmonic V_n gives rise to transitions between continuum states separated in energy by $n\hbar\omega$. Then the main results of Ref. 16 enable us to conclude that the intensity is increased, the number of multiphoton peaks in the stimulated bremsstrahlung spectrum must increase, while their absolute magnitude must decrease.

It should also be noted here that our model, which enables us to implement the Kramers–Henneberger transformation analytically, is not at all well suited for demonstrating the applicability of this approximation in a strong field for either ionization or stimulated bremsstrahlung. This is due to the fact that for a piecewise continuous potential, the harmonics V_n decrease more slowly than for a smooth function $V(x)$.

4. INVESTIGATION OF THE STIMULATED BREMSSTRAHLUNG SPECTRUM

4.1. Analytic estimates in the Kramers–Henneberger method

The basic idea of the proposed solution is that by using the Kramers–Henneberger method we can reduce scattering by the model potential in the ultrastrong field to scattering by an effective potential in a weak field, i.e., to a problem that can be solved with perturbation theory. This possibility arises because in the presence of electric field strengths greater than the atomic field strength, there is a Kramers–Henneberger regime in which the harmonics V_n are small in the sense that they excite only single-photon processes with photon energy $n\hbar\omega$.

It is easy to estimate the intensity at which this regime commences for the model considered. As is shown in Ref. 1, multiphoton processes can be ignored if

$$N\simeq\frac{e\varepsilon p}{m\hbar\omega^2}\ll 1,$$

and this can be rewritten in the form

$$N\simeq\frac{mv_e v_d}{\hbar\omega}\ll 1, \tag{11}$$

where $v_e=e\varepsilon/m\omega$ is the oscillatory velocity of the electron in the field of the electromagnetic wave, and $v_d=p/m$ is the electron drift velocity. In our case, the condition (11) has the form

$$N\simeq\frac{mv_e^{\mathrm{KH}}v_d}{n\hbar\omega}\ll 1 \tag{12}$$

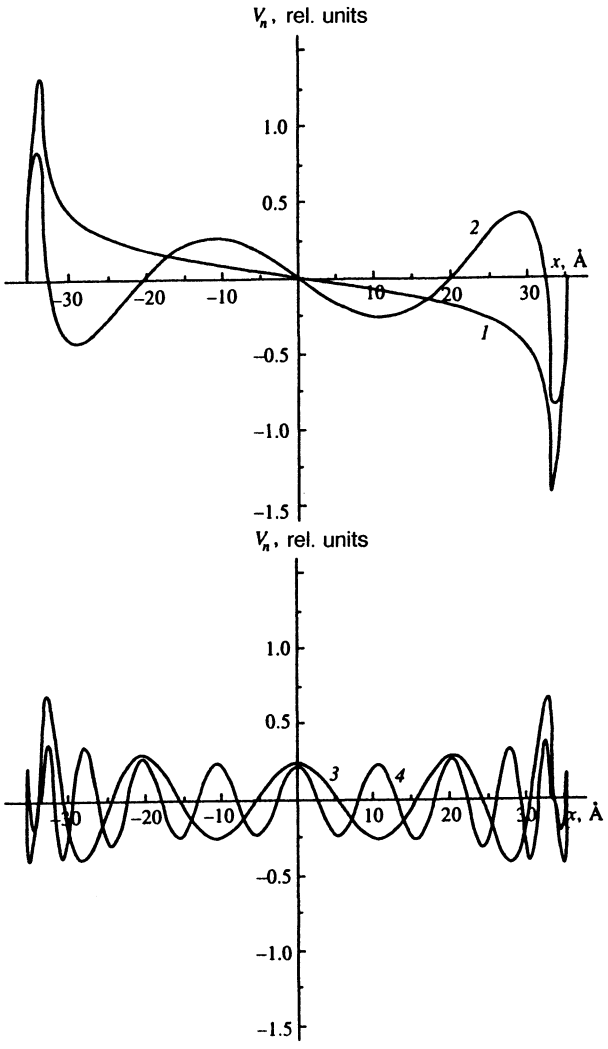

FIG. 2. Harmonics of the potential: 1) $V_1(x)$, 2) $V_5(x)$, 3) $V_{10}(x)$, 4) $V_{20}(x)$ for the Kramers–Henneberger potential and $\hbar\omega=2.5$ eV.

and must hold for all n. Here v_e^{KH} is the characteristic oscillatory velocity of the electron in the field of the harmonic V_n. Bearing in mind that the harmonics V_n are nonzero in a region of order a_e and have a highly nonuniform coordinate dependence (Fig. 2), we estimate the mean field created by the harmonic V_n to be $\bar{\varepsilon}_n\sim V_n/ea_e$. Then the oscillatory velocity v_e^{KH} of the electron in the field $\bar{\varepsilon}_n$ in (12) is $v_e^{\mathrm{KH}}\sim e\bar{\varepsilon}_n/mn\omega$. Bearing in mind that the characteristic depth of the Kramers–Henneberger potential is related to the depth of the original potential by

$$V_{\mathrm{KH}}\simeq V_0(a/a_e),$$

and $V_n\simeq V_{\mathrm{KH}}$, we find that

$$v_e^{\mathrm{KH}}\simeq\frac{V_0\omega}{e\varepsilon_0}\frac{a}{a_e}.$$

Then for a photon multiplicity N we have

$$N\simeq\frac{v_d}{v_e}\frac{V_0}{\hbar\omega}\frac{a}{a_e}\ll n^2. \tag{13}$$

Estimating N for the energy $\hbar\omega=2.5$ eV of the photon of the electromagnetic field and for $v_d\cong 10^8$ cm/s, we find that $N\simeq 1$ for an intensity $P^*\simeq 10^{14}$ W/cm^2.

In this paper, the stimulated bremsstrahlung spectrum is investigated in first-order perturbation theory in the Kramers–Henneberger potential and the field of the harmonics V_n. There is no doubt that the Born approximation holds in a Kramers–Henneberger scattering potential over a wide range of initial velocities v_d in the Kramers–Henneberger regime. Then for the probability $W^{\pm n}$ of absorption or emission of n photons by the electron we obtain

$$W^{\pm n}=2\pi\left[\sum_{p_m}\frac{V_{\mathrm{KH}}(p_p-p_m)V_n(p_m-p_k)}{E_m-E_k\pm n\hbar\omega}+\frac{V_n(p_p-p_m)V_{\mathrm{KH}}(p_m-p_k)}{E_m-E_k}\right]^2\times\delta(E_k\pm n\hbar\omega-E_p),\tag{14}$$

where $E_k, E_m, E_p=E_k\pm n\hbar\omega$ are, respectively, the initial, intermediate, and final energies of the electron, and the matrix elements

$$V_{\mathrm{KH}}(p_p-p_m)=\int\psi^{\mathrm{KH}}_{p_m}(x)V_{\mathrm{KH}}(x)\psi^{\mathrm{KH}}_{p_p}(x)dx,$$

$$V_n(p_p-p_m)=\int\psi^{\mathrm{KH}}_{p_m}(x)V_n(x)\psi^{\mathrm{KH}}_{p_p}(x)dx$$

have the form

$$V_{\mathrm{KH}}(\Delta p)=2V_0a\ \mathrm{sinc}\left(\frac{\Delta pa}{\hbar}\right)J_0\left(\frac{\Delta p}{\hbar}a_e\right),$$

$$V_n(\Delta p)=\frac{\delta(n)}{n}2V_0a\ \mathrm{sinc}\left(\frac{\Delta pa}{\hbar}\right)J_n\left(\frac{\Delta p}{\hbar}a_e\right).$$

Here Δp denotes either p_p-p_m or p_m-p_k (p_p, p_m, p_k are, respectively, the final, intermediate, and initial electron momenta):

$$\delta(n)=\begin{cases}1, & n\text{—even},\\ -i, & n\text{—odd}.\end{cases}$$

Further, following the standard scheme, we obtain

$$W=\frac{2\pi}{\hbar}|V^{(2)}|^2\rho(E_p)n_a.\tag{15}$$

where $V^{(2)}$ is the two-photon matrix element in (14), and n_a is the number density (the number of particles per unit length).

We define the coefficients of reflection and transmission accompanied by the absorption (emission) of n photons, $R^{\pm n}$ and $T^{\pm n}$, respectively, by

$$R^{\pm n}=\frac{\dot W_R^{\pm n}}{n_a v},\quad T^{\pm n}=\frac{\dot W_T^{\pm n}}{n_a v},\tag{16}$$

where $W_R^{\pm n}$ and $W_T^{\pm n}$ are the probabilities of reflection and transmission accompanied by the absorption (emission) of n photons.

Going over in (15) from summation to integration and using the pole approximation, we obtain the following expression for $V_R^{(2)}\equiv V^{(2)}$ for $p_p, p_k<0$ and $V_T^{(2)}\equiv V^{(2)}$ for $p_p, p_k>0$:

$$V_T^{(2)}=i\frac{\delta(n)}{n}\frac{B}{ka}\left[\frac{V_0a}{L}\mathrm{sinc}[(\lambda-1)ka]J_n[(\lambda-1)ka_e]\times[1+1/\lambda]-\frac{V_0a}{L}\mathrm{sinc}[(\lambda+1)ka]J_n[(\lambda+1)ka_e]\times\left[\mathrm{sinc}(2ka)J_0(2ka_e)+\frac{(-1)^n}{\lambda}\mathrm{sinc}(2ka_e)J_0(2ka_e)\right]\right],\tag{17}$$

$$V_R^{(2)}=i\frac{\delta(n)}{n}\frac{B}{ka}\left[(-1)^n\frac{V_0a}{L}\mathrm{sinc}[(\lambda+1)ka]\times J_n[(\lambda+1)ka_e][1-1/\lambda]+\frac{V_0a}{L}\mathrm{sinc}[(\lambda-1)ka]\times J_n[(\lambda-1)ka_e]\left[\frac{1}{\lambda}\mathrm{sinc}(2\lambda ka)J_0(2\lambda ka_e)-(-1)^n\mathrm{sinc}(2ka)J_0(2ka_e)\right]\right].\tag{18}$$

Here $\lambda=p_p/p_k=\sqrt{1\pm n\hbar\omega/E_k}$, $B=2mV_0a^2/\hbar^2$, $k=p_k/\hbar$, and L is the normalization volume.

In the case $\lambda\gg 1$, (17) and (18) take the form

$$V_T^{(2)}=i\frac{\delta(n)}{n}\frac{B}{ka}\frac{V_0a}{L}\mathrm{sinc}(\lambda ka)J_n(\lambda ka_e)\times[1-\mathrm{sinc}(2ka)J_0(2ka_e)],\tag{19}$$

$$V_R^{(2)}=(-1)^n i\frac{\delta(n)}{n}\frac{B}{ka}\frac{V_0a}{L}\mathrm{sinc}(\lambda ka)J_n(\lambda ka_e)\times[1-\mathrm{sinc}(2ka)J_0(2ka_e)],\tag{20}$$

from which it can be seen that $|V_T^{(2)}|=|V_R^{(2)}|^2$.

In all that follows, we shall consider the case $\lambda\gg 1$, corresponding to the absorption of a large number of photons and $\hbar\omega\geqslant E_k$. Then for the coefficient $R^{\pm n}$ we obtain, following the scheme described above,

$$R^{\pm n}=\frac{1}{n^2}\left(\frac{B}{ka}\right)^2R^{\mathrm{el}}\lambda J_n^2(\lambda ka_e)\times[1-\mathrm{sinc}(2ka)J_0(2ka_e)]^2,\tag{21}$$

where

$$R^{\mathrm{el}}=\left(\frac{m}{\lambda p_k\hbar}\right)^2(V_0a)^2\ \mathrm{sinc}^2(\lambda ka)$$

is the reflection coefficient calculated in the Born approximation for the scattering of an electron with energy $E_p=E_k\pm n\hbar\omega$ by the rectangular potential well.

For $T^{\pm n}$, we have a similar result.

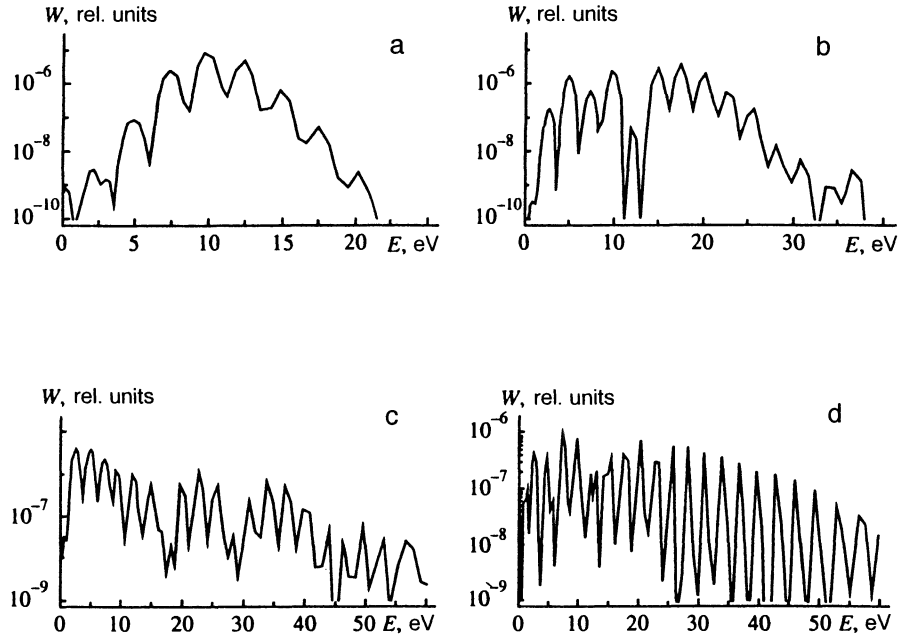

FIG. 3. Energy spectra of back-scattered electrons for the radiation intensities P (W/cm^2)$=10^{12}$ (a), 10^{13} (b), 10^{14} (c), 10^{15} (d); $E=10$ eV.

Analysis of the expression (21) enables us to make the following remarks concerning the stimulated bremsstrahlung spectrum.

1. Comparing the argument of the Bessel function with the index, we estimate the number of peaks in the stimulated bremsstrahlung spectrum:

$$n_{\max}=\lambda k a_e. \tag{22}$$

Bearing in mind that in the case we consider, corresponding to the absorption of a large number of photons, $\lambda\simeq(n\hbar\omega/E_k)^{1/2}$, we obtain

$$n_{\max}\simeq\frac{e^2\varepsilon^2}{m\omega^2}\frac{1}{\hbar\omega}. \tag{23}$$

Thus, as the intensity is increased the number of peaks in the stimulated bremsstrahlung spectrum increases.

2. Using the asymptotic representation for the Bessel function at large values of the argument,

$$J_n^2(\lambda k a_e)\cong\frac{2}{\pi\lambda k a_e}\cos\left(\lambda k a_e-\frac{\pi n}{2}-\pi/4\right). \tag{24}$$

we obtain from (21) the following intensity dependence of the probability of the n-photon process:

$$R^{\pm n}\propto P^{-1/2}.$$

3. Taking (22) and (24) into account, we obtain from (21) the following dependence of the height of the peaks in the stimulated bremsstrahlung spectrum on n:

$$R^{\pm n}\propto n^{-4}.$$

4.2. Numerical experiment

The main conclusions presented above are confirmed by a numerical experiment that we made for the model described in Sec. 3.

The initial-state wave function was specified in the form of a Gaussian packet:

$$\varphi_0(x)=\frac{1}{\Delta x\sqrt{\pi}}\exp\left\{\frac{i}{\hbar}p_0x\right\}\exp\left\{-\frac{1}{2}\left(\frac{x-x_0}{\Delta x}\right)^2\right\}, \tag{25}$$

where Δx is the half-width, and x_0 and p_0 are the initial mean values of the position and momentum. If $p_0\gg\hbar/\Delta x$, then p_0 determines the mean energy of the electron packet: $E\cong p_0^2/2m$. In our calculations, we assumed $\Delta x=20$ Å. The field of the electromagnetic wave was specified in the form

$$\varepsilon=\varepsilon_0\cos\omega t.$$

The numerical solution of Eqs. (4) and (7) with initial condition (25) was analyzed for times at which electron scattering by the oscillating potential (or the Kramers–Henneberger potential) had ended and two wave packets—one reflected by the scattering potential and one transmitted through it—had formed.

In what follows, we restrict ourselves to an analysis of the energy spectra of back-scattered electrons. These spectra were determined by means of the relation

$$W(E)=\left|\int\psi^*(x,t)e^{-ikx}dx\right|^2.$$

Here $E=\hbar^2k^2/2m$, $\psi(x,t)$ is the result of numerical solution of (4) with the initial condition (25), and the integral is taken over the region to the left of the potential.

The increase in the number of harmonics with increasing intensity is demonstrated in Fig. 3, in which we have plotted on a logarithmic scale the energy spectra of the back-scattered electrons with initial energy $E=10$ eV in a radiation field with $\hbar\omega=2.5$ eV in the intensity range 10^{12}–10^{15} W/cm^2. For the given set of parameters, the two-well Kramers–Henneberger potential begins to be formed at the

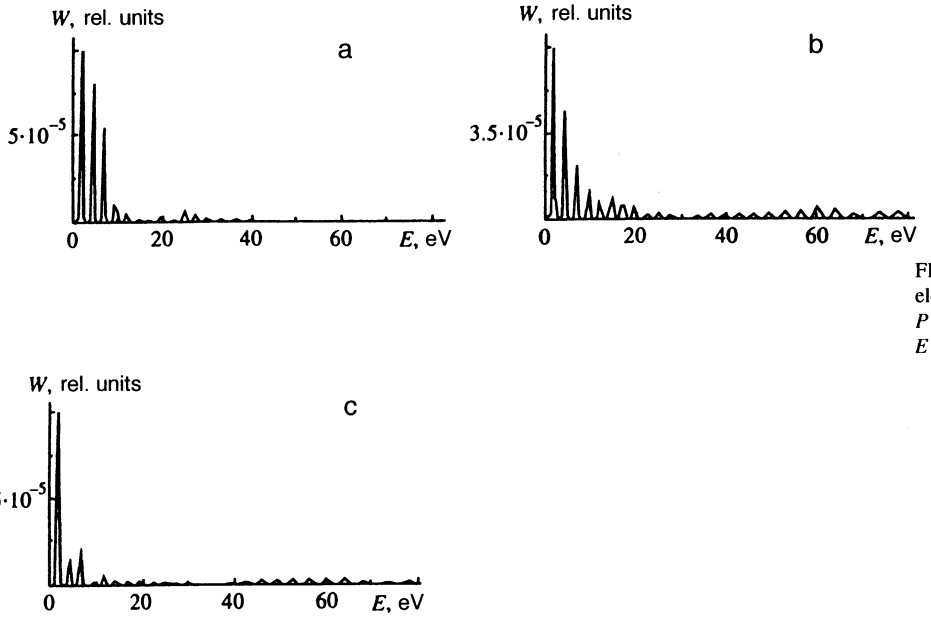

FIG. 4. Energy spectra of back-scattered electrons for the radiation intensities $P(\mathrm{W/cm^2})=10^{14}$ (a), $3\cdot10^{14}$ (b), 10^{15} (c); $E=2$ eV.

intensity $P\simeq10^{14}$ W/cm^2. Tracing the change in the spectra with increasing intensity, we can readily see that the approach to the Kramers–Henneberger regime is manifested by a significant increase in the number of harmonics in the spectra shown in Figs. 3c and 3d. The set of energy spectra of electrons with initial energy $E=2$ eV for intensities 10^{14}–10^{15} W/cm^2, shown on a linear scale in Fig. 4, also makes it possible to follow the onset of the Kramers–Henneberger regime. In the indicated intensity range, we observe a rapid increase in the number of multiphoton peaks of approximately equal height, in the stimulated bremsstrahlung spectrum with a simultaneous decrease in the intensity of the adjacent peaks ($n=1$–5) compared with the peak corresponding to elastic scattering by the Kramers–Henneberger potential.

The photoelectron spectra obtained during the solution of Eq. (7) with allowance for different numbers of harmonics $V_n(x,a_e)$ ($n=2$–10, $P=10^{15}$ W/cm^2), which are shown in Fig. 5, illustrate the validity of the estimate (13) and, with it, the scheme proposed for analytic treatment of the stimulated bremsstrahlung spectrum. Indeed, in the spectra presented in Fig. 5, the number of peaks of comparable height is determined by the number of harmonics V_n taken into account in the solution of the Schrödinger equation.

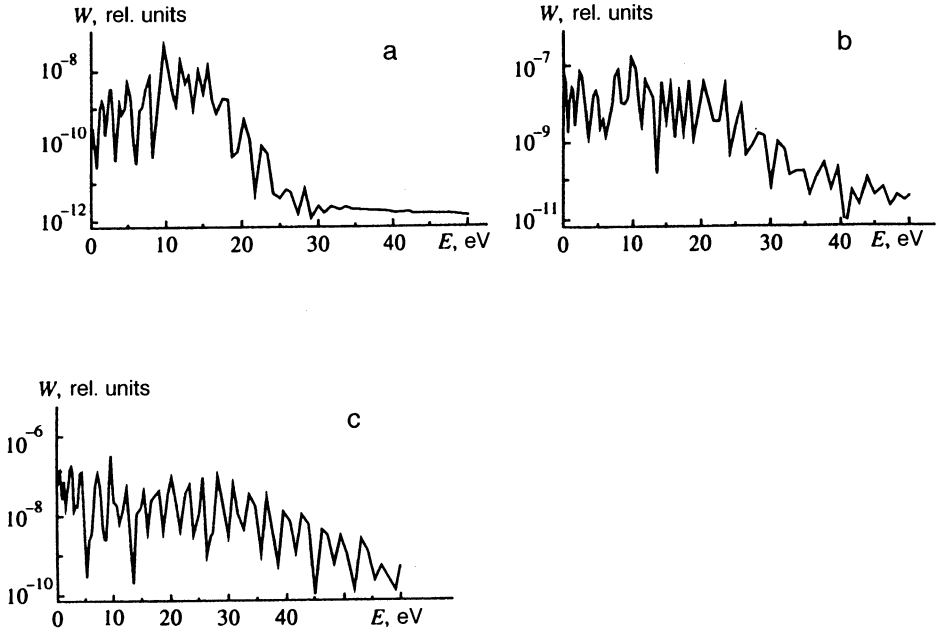

FIG. 5. Spectra of back-scattered electrons calculated in the Kramers–Henneberger approximation with allowance for different numbers of harmonics $V_n(x,a_e)$, [$n=2$ (a), 5 (b), 10(c)]. The radiation intensity is $P=10^{15}$ W/cm^2, and the electron energy is $E=10$ eV.

5. CONCLUSIONS

To summarize, we must indicate the position accorded the Kramers–Henneberger approach in the general theory of the stimulated bremsstrahlung effect. In particular, it is interesting to compare the regions of applicability of this approach and the approach developed in Ref. 1. The question of the applicability of the Born approximation with respect to the scattering potential in the presence of the strong field of an electromagnetic wave evidently requires separate study. However, if it is assumed that in the present case the Born approximation is valid when $E \gg V_0$ (E is the electron kinetic energy), we must also require that

$$v_d \gg v_e, \tag{26}$$

since otherwise the electron will be slow during certain intervals of time. The effect of the existence of classical turning points of the electron on the applicability of the Born approximation is at present not clear, and it is therefore also unclear how important (26) is for the calculation in the Born approximation.

On the other hand, following the general logic of the Kramers–Henneberger approach, averaging the potential $V(x+a_e \cos \omega t)$ over the period of the laser field is meaningful if

$$\omega \tau \gg 1, \tag{27}$$

where $\tau = \max\{\Delta x/v_d, a_e/v_d\}$ is the scattering time of a wave packet of width Δx by the Kramers–Henneberger potential.

Bearing in mind that to have $\Delta x \gg \hbar p_0$ it is sufficient that $\Delta x \sim a_e$ (in this range of parameters of the laser radiation, $p_0 a_e/\hbar = e\varepsilon p_0/m\hbar\omega^2$), from (27) we have $a_e\omega/v_d \gg 1$ or $v_e \gg v_d$, i.e., the condition opposite to (26).

Nevertheless, there is undoubted interest in carrying out in a comparative analysis of the results obtained in this paper with the results of calculations in the first Born approximation in the scattering potential using Volkov functions in the laboratory coordinate system. For electrons back-scattered by a rectangular potential well in the field of an electromagnetic wave, the Bunkin–Fedorov approach yields

$$R^{\pm n} = R^{\mathrm{el}} \lambda J_n^2(\lambda k a_e). \tag{28}$$

Comparison of (28) and (21) shows that the two methods obtain the same estimate for the number of peaks in the stimulated bremsstrahlung spectrum and that they exhibit the same dependence of the probability of n-photon absorption on the radiation intensity.

At the same time, it follows from (28) that the intensity of the different peaks in the stimulated bremsstrahlung spectrum decreases as

$$R^{\pm n} \propto n^{-2},$$

and this is different from the analogous dependence obtained via the Kramers–Henneberger approximation.

This behavior is obtained for large n, and this circumstance makes it difficult to obtain an unambiguous comparison of the analytic calculations with the numerical model. Nevertheless, analysis of the stimulated bremsstrahlung spectra in Figs. 3c and 3d suggests that the dependence $R^{\pm n} \propto n^{-4}$ gives a better description of the numerical estimates. However, detailed comparison of the Kramers–Henneberger approximation and the Bunkin–Fedorov approach to the description of stimulated bremsstrahlung in ultrastrong fields requires additional numerical calculations to be made in the region of stronger fields, this being due to the need to increase the number of peaks in the spectrum of the scattered electrons.

At the same time, it must be borne in mind that the upper limit of the Kramers–Henneberger regime is fixed by the choice of model. If the intensity is raised above 10^{16}W/cm^2 under our conditions, this must take us beyond the dipole approximation. A corresponding estimate can be obtained by comparing the magnetic component of the Lorentz force and the intra-atomic force $\simeq V_0/d$ acting on the electron.

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