The collective excitation spectrum and resistance behavior of layered superconductors

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We use the mean-field approximation to describe a phase transition in layered superconductors with Josephson coupling between the layers. The transition results in the emergence of a finite collective excitation concentration and proceeds as a first-order phase transition. We study the effect of transport current on the nature of the transition. At high currents, $I>I_{c1}$, the transformation of the excitation spectrum proceeds as a second-order phase transition and affects the concentration of free vortices determining the resistance behavior of the superconductor. The exponentially weak interaction of the screened vortices in a dipole leads to a current-dependence of the free-vortex concentration above the phase transition temperature. This manifests itself in the broadening of the universal jump in the temperature dependence of the current–voltage characteristics exponent $a(T)$. We show that the Nelson–Kosterlitz jump in superconductors with Josephson coupling is present only in the temperature dependence of $\partial \ln E/\partial \ln[I-I_c(T)]$. © *1996 American Institute of Physics.* [S1063-7761(96)01901-4]

1. INTRODUCTION

High-T_c superconductors are known for their layered structure, which ensures a specific behavior of the current–voltage characteristics. In experiments they exhibit a resistance transition, which takes place at a temperature T_R below the superconductivity transition, or critical, temperature T_{c0}. A distinctive feature of this transition is the temperature dependence of the exponent $a(T)$ in the current–voltage characteristics of the form $V\sim I^{a(T)}$. This dependence demonstrates a linear decrease in $a(T)$ with temperature at $T < T_R$ up to $a(T_R)\approx 3$. A further increase in temperature causes a sudden drop in $a(T)$ to unity, which corresponds to a transition to the ohmic behavior of resistance, with the value remaining unchanged at higher temperatures.[1,2] The jump in voltage is associated with the Nelson–Kosterlitz jump,[3,4] which makes it possible to interpret the resistance transition as a Berezinskiĭ–Kosterlitz–Thouless (BKT) transition.[5,6]

A BKT transition is possible in two-dimensional systems where topological defects with a finite topological charge, or vortices, can exist. An isolated vortex cannot emerge as a result of thermal fluctuations, since its self-energy is logarithmically divergent and in a real sample is cut off only by the sample's size. In layered superconductors defects of this type are the magnetic vortices in isolated planes.[7–9] The two signs of topological charges, ± 1, are related to the two directions of the vortex magnetic flux.

In two-dimensional systems, the vapors of vortices of different signs are the thermal excitations. The topological charges of such dipoles are zero. Dipoles whose energy is proportional to the logarithm of the distance between the charges (the arm length of the dipole) are present in a two-dimensional system at any finite temperature. The BKT transition is related to the transformation of the excitation spectrum taking place at a certain temperature T_{KT} as a result of the interaction between the dipoles. Here the elementary excitations become unstable against the emergence of new collective formations. Screening of the vortex interaction similar to the Debye screening of electric charges occurs in the process. The energy of a screened vortex becomes finite, and the vortex coupling in a dipole whose length is greater than the screening length δ becomes exponentially small. A gas of free vortices is formed with a net topological charge equal to zero. The free-vortex concentration $\nu_0\sim\delta^{-2}$. At the transition point T_{KT} the screening length δ becomes infinitely large, $n_0\rightarrow 0$, and the energy of collective excitations becomes equal to that of elementary excitations. The transition between the high-temperature ($T>T_{KT}$) and low-temperature ($T<T_{KT}$) states of the system occurs continuously, so that BKT transitions are second-order.

Two-dimensional systems include layered superconductors with no Josephson coupling between the layers. The temperature at which a BKT transition takes place in such a structure is[7–9]

$$T_{KT}=\frac{\phi_0^2}{16\pi^2\Lambda(T_{KT})}, \tag{1}$$

where ϕ_0 is the magnetic flux quantum, and $\Lambda=2\lambda_{ab}^2/s$, with λ the London depth of penetration of the magnetic flux into the superconducting layers. When $T<T_{KT}$, thermally excited dipoles consist of elementary vortices whose attraction force is inversely proportional to the dipole's arm length. Only vortices whose mutual attraction is weaker than the Lorentz force exerted by the transport current will move under the action of that current and, therefore, will lead to a voltage drop. The concentration of such dipoles increases with the current, with the result that the current–voltage characteristics for $T<T_{KT}$ are nonlinear. When $T>T_{KT}$, a free-vortex gas emerges, which ensures ohmic (linear) behavior of the current–voltage characteristics.

In real layered superconductors, between the layers there may flow a superconducting tunnel current caused by the Josephson effect. Such superconductors cannot be considered two-dimensional. The simplest elementary excitation in

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such superconductors is a dipole whose magnetic field is curved into a ring by means of two Josephson vortices situated between superconducting planes.[9] There can be no BKT transition in such a system. Even if the magnetic interaction is completely screened, there is still the energy of the Josephson attraction of the vortices proportional to the dipole's arm length. The Josephson interaction can also be renormalized as a result of dipole–dipole interaction, and this must manifest itself in the nature of the phase transition.

For various high-T_c superconductors, measurements of the current–voltage characteristics reveal the presence of an internal critical current $I_c(T)$ related to the Josephson coupling between layers.[10–12] The current–voltage characteristics of such layered materials reflect the decoupling of vortex dipoles and have a threshold nature, $E \sim (I-I_c)^{a(T)}$ (see Ref. 13). The current I_c is needed to overcome the linear tension of the Josephson vortices in a dipole.

Theoretical investigations of layered superconductors have taken two paths. One seeks to establish the nature of the transformation of Josephson coupling between layers caused by critical fluctuations near the phase transition. The early works,[14,15] in which the renormalization group approach was used, discovered the three-dimensional behavior of layered systems. On the other hand, Weber and Jensen[16] employed the Monte Carlo method to study the critical behavior of the XY-model of two coupled layers and found that the coupling between the layers disappears at temperatures above the phase transition point. Later Pierson[17] modified the renormalization-group equations and showed that the critical behavior of a system of Josephson-coupled layers can also be two-dimensional. Results of experiments do not provide a final solution of this problem either. Martin *et al.*[18] and Wan *et al.*[19] report the results of measurements of the temperature dependence of the resistance of Bi-based single crystals. They also point out that the temperature at which the resistance in the planes vanishes, T_c^{ab}, is much lower than the temperature T_c^c at which the resistance between the layers vanishes. Wan *et al.*[19] found that $T_c^c - T_c^{ab} \approx 2$ K. This implies that after the superconducting layers have transformed into the resistive state at T_c^{ab}, Josephson coupling between the layers remains up to T_c^c. It is still unknown whether T_c^c coincides with the critical temperature T_{c0} or $T_c^c < T_{c0}$. Yeh and Tsuei,[20] on the other hand, found that in YBCO crystals $T_c^c \simeq T_c^{ab}$ and $T_{c0} - T_c^c \approx 0.2$ K.

The second path taken by theorists is related to building the current–voltage characteristics of layered superconductors with Josephson coupling. In one of the first papers, Jensen and Minhegen[21] built for coupled superconductors the nonlinear current–voltage characteristics reflecting the rupture of vortex dipoles by weak currents. These current–voltage characteristics have the power-law form $E \sim I(I/I_c - 1)^{a(T)}$, where the exponent a depends solely on temperature. Pierson[22] analyzed current–voltage characteristics of the same type. Finally, Gupta *et al.*[23] obtained the current–voltage high-current asymptotics of the type $E \sim I^{a(T)}$ for coupled layers at currents exceeding I_c.

In the present paper we discuss the phase transition of the BKT type and the related resistance to the transport current in layered superconductors with Josephson coupling. This transition leads to the onset of a finite concentration of collective excitations, vortex dipoles whose magnetic interaction is screened as a result of their coupling. The difference between this transition and a BKT transition lies in the presence of Josephson coupling between the layers. It is a first-order transition and begins at a temperature $T_c > T_{\mathrm{KT}}$ when a nonzero concentration of the collective excitations emerges. The upper limit of stability of the metastable state coincides with the temperature at which coupling between the layers disappears.

We studied the effect of the transport current on the nature of the transition and the transformation of the excitation spectrum. The appearance of collective excitations at low currents in a system of Josephson-coupled layers occurs as a first-order phase transition, whose upper limit of stability of the metastable state is monotonically shifted downward as the current grows. There exists a critical value of current I_{c1} at which the type of the phase transition changes from first-order to second-order. Under a further increase of current the temperature of the second-order phase transition decreases monotonically to zero.

The fact that there is a transport current leads to the formation of free vortices whose attraction to each other is overcome by the Lorentz force. The motion of these vortices determines the resistive behavior of the system. The free-vortex concentration n_f strongly depends on the excitation spectrum. Transformation of the excitation spectrum as a result of the phase transition leads to the universal Nelson–Kosterlitz jump. The current curves for n_f obtained here can be divided into two groups. The low-temperature ($T < T_c$) n_f vs I curves, where the concentration of collective excitations is extremely low, are determined by the elementary excitation spectrum, which agrees with the results of Jensen and Minhegen.[21] The high-temperature n_f vs I curves are determined by the collective excitation spectrum. The exponentially weak coupling of screened vortices in the dipoles leads to a weak dependence of the concentration n_f on current. This results in a smearing of the universal jump by the current. In Josephson-coupled superconductors this jump is present in the temperature dependence of $\partial \ln E / \partial \ln[I - I_c(T)]$.

2. THE ELEMENTARY EXCITATION SPECTRUM

We consider a superconductor to be a periodic system of parallel conducting planes separated by insulating layers of thickness s. We assume the conducting planes to be London superconductors, $\lambda_{ab} \gg \xi_{ab}$, with Josephson coupling between the layers. Here λ is the depth of penetration by the magnetic field and ξ is the coherence length. We select a Cartesian system of coordinates with the axes directed along the crystallographic axes, with the $\hat{z}$ axis, as well as the anisotropy axis **c**, being perpendicular to the planes. Then the superconductor can be described by the Lawrence–Doniach model,[24] and its free energy

$$F=\frac{\phi_0^2}{16\pi^3\Lambda}\sum_n \int d^3r\left[\left(\nabla\theta_n-\frac{2\pi}{\phi_0}\mathbf{A}\right)^2+\frac{2}{\lambda_J^2}(1-\cos\Omega_n)\right]\delta(z-ns)+\int d^3r\frac{(\nabla\times\mathbf{A})\cdot(\nabla\times\mathbf{A})}{8\pi} \quad (2)$$

can be expressed in terms of the vector potential $\mathbf{A}(\mathbf{r})$ and the phase $\theta_n(\mathbf{x})$ of the order parameter in the nth layer. Here $\mathbf{r}=(\mathbf{x},z)$, $\Lambda=2\lambda_{ab}^2/s$, $\lambda_J=\gamma s$, $\gamma=\xi_{ab}/\xi_c=\lambda_c/\lambda_{ab}$ is the anisotropy parameter of the system, and

$$\Omega_n=\theta_{n+1}-\theta_n-\frac{2\pi}{\phi_0}\int_{ns}^{(n+1)s}dz\,A_z.$$

Varying the potential (2) in the independent variables $\mathbf{A}$ and θ leads to the following system of equations:[25]

$$\frac{\partial}{\partial\mathbf{x}}\left(\frac{\partial\mathbf{A}}{\partial\mathbf{x}}\right)-\Delta\mathbf{A}(\mathbf{r})=\sum_n\frac{4\pi}{c}I_n(\mathbf{x})\,\delta(z-ns), \quad (3)$$

$$\frac{4\pi}{c}I_n(\mathbf{x})=\frac{2}{\Lambda}\left[\frac{\phi_0}{2\pi}\nabla\theta_n(\mathbf{x})-\mathbf{A}_n(\mathbf{x})\right],$$

$$\frac{\partial}{\partial z}\left(\frac{\partial\mathbf{A}}{\partial\mathbf{x}}\right)=\frac{\phi_0}{2\pi\lambda_c^2 s}\sum_n f_n\sin\Omega_n, \quad (4)$$

$$\left(\frac{\partial}{\partial\mathbf{x}},\frac{\phi_0}{2\pi}\nabla\theta_n(\mathbf{x})-\mathbf{A}(\mathbf{x})\right)=\frac{\phi_0}{2\pi\lambda_J^2}[\sin\Omega_{n-1}(\mathbf{x})-\sin\Omega_n(\mathbf{x})], \quad (5)$$

where $(\cdot,\cdot)$ stands for a scalar product.

The step function f_n is equal to unity inside the interval $ns<z<(n+1)s$ and is zero outside. So we select the gauge of the vector potential in the form[25]

$$\mathbf{A}_z=0. \quad (6)$$

The system of equations (3)–(6) is completely defined if we fix in each layer the coordinates of two-dimensional vortices ($\mathbf{x}_\alpha^k$) and antivortices ($\mathbf{x}_\beta^k$), the magnetic-field sources:

$$\left[\frac{\partial}{\partial\mathbf{x}},\nabla\theta^k(\mathbf{x})\right]=2\pi\hat{z}\sum_\alpha\delta(\mathbf{x}-\mathbf{x}_\alpha^k)-2\pi\hat{z}\sum_\beta\delta(\mathbf{x}-\mathbf{x}_\beta^k),$$

where $[\cdot,\cdot]$ stands for a vector product.

In this paper we solve only the linearized system of equations. Such an approach to layered high-T_c materials was developed by Bulaevskiĭ *et al.*[25,26] The linear approximation makes it possible to calculate the distribution of the phase of the order parameter of a single dipole and to use a linear combination of the solutions obtained for a gas of dipoles. In Sec. 2 we find the expression for the energy of interaction of two vortex dipoles and the self-energy of a single dipole.

Combining Eqs. (2) and (3), we find that in the linear approximation the interaction energy for two dipoles with arms $\mathbf{l}$ and $\mathbf{l}'$ situated in the layers m and k at a distance $\mathbf{x}$ from each other is

$$\mathcal{U}^{mk}(\mathbf{x},\mathbf{l},\mathbf{l}')=\frac{1}{4\pi}\sum_n\int d^2x'\left[\frac{\phi_0}{c}(\mathbf{I}_n^m(\mathbf{x}',\mathbf{l}),\nabla\theta_n^k(\mathbf{x}'-\mathbf{x},\mathbf{l}'))+\frac{\phi_0^2}{(2\pi)^2\lambda_J^2\Lambda}\Omega_n^m(\mathbf{x}',\mathbf{l})\Omega_n^k(\mathbf{x}'-\mathbf{x},\mathbf{l}')\right]. \quad (7)$$

By the lower index we designate the number of the layer for which we are calculating the given function induced by the vortex whose source is in the layer designated by the upper index. As the structure of the above expression shows, the interaction energy of two dipoles positioned in the layers n and m are expressed in terms of the current density and the gradients of the phase and phase differences between neighboring phases induced by these vortices in all the layers.

The system of equations for the phase of the dipole with arm $\mathbf{l}$ in layer k has the form

$$\left(\frac{\partial}{\partial\mathbf{x}},\nabla\theta_n^k(\mathbf{x})\right)=\frac{1}{\lambda_c^2}\Omega_n^k-\frac{1}{\lambda_J^2}(\Omega_{n-1}^k+\Omega_{n+1}^k-2\Omega_n^k),$$

$$\left[\frac{\partial}{\partial\mathbf{x}},\nabla\theta_n^k(\mathbf{x})\right]=2\pi\hat{z}\left[\delta\left(\mathbf{x}-\frac{\mathbf{l}}{2}\right)-\delta\left(\mathbf{x}+\frac{\mathbf{l}}{2}\right)\right](\delta_{n+1}^k-\delta_n^k),$$

where δ_n^k is the Kronecker delta.

Linearization of the equations leads to the appearance of nonphysical discontinuities in the expression for the current in the direction of the $\hat{z}$ axis. To remove the discontinuities we introduce a cut in the form of a straight line along vector $\mathbf{l}$, a cut that connects the coordinates of the vortex and antivortex in the dipole. We identify this cut with a Josephson vortex. Performing the Fourier transformation

$$\Omega(\mathbf{q})=\int d^2x\,\Omega(\mathbf{x})e^{-i\mathbf{q}\mathbf{x}}$$

and allowing for the fact that the derivative of the phase, which is different in the direction of $\mathbf{l}$, along which there is no jump in phase, is

$$(\mathbf{l},\nabla\Omega)=i(\mathbf{q}\cdot\mathbf{l})\Omega,$$

we obtain the following system of equations:

$$\nabla\Omega_n^k(\mathbf{q})=i\mathbf{q}\Omega_n^k(\mathbf{q})+4\pi\frac{[\mathbf{l}\hat{\mathbf{z}}]}{(\mathbf{q}\cdot\mathbf{l})}\sin\left(\frac{\mathbf{q}\cdot\mathbf{l}}{2}\right)(\delta_{n+1}^k-\delta_n^k),$$

$$i(\mathbf{q},\nabla\Omega_n^k(\mathbf{q}))=\frac{1}{\lambda_c^2}\Omega_n^k-\frac{1}{\lambda_J^2}(\Omega_{n+1}^k-2\Omega_n^k+\Omega_{n-1}^k).$$

The solution of this system is

$$\Omega_n^k(\mathbf{q})=2\pi i\lambda_J^2\frac{(\hat{\mathbf{z}}[\mathbf{q}\cdot\mathbf{l}])}{(\mathbf{q}\cdot\mathbf{l})}\sin\left(\frac{\mathbf{q}\cdot\mathbf{l}}{2}\right)\frac{1}{a-1}T(V_{n+1}^k-V_n^k), \quad (8)$$

$$V_n^k=\sqrt{\frac{a-1}{a+1}}(a-\sqrt{a^2-1})^{|n-k|},$$

$$a=1+\frac{\lambda_J^2}{2}\left(q^2+\frac{1}{\lambda_c^2}\right),$$

and the phase gradient is given by the following expression:

$$\nabla \theta_n^k(\mathbf{q},\mathbf{l}) = \frac{4\pi}{q^2}\sin\left(\frac{\mathbf{q}\cdot\mathbf{l}}{2}\right)\left[[\mathbf{q}\hat{\mathbf{z}}]\delta_n^k + \mathbf{q}\,\frac{(\hat{\mathbf{z}}[\mathbf{q}\cdot\mathbf{l}])}{(\mathbf{q}\cdot\mathbf{l})}\right.$$
$$\left.\times\left(\delta_n^k - \frac{\lambda_J^2 q^2}{2}\frac{V_n^k}{\sqrt{a^2-1}}\right)\right].$$

The dipole current $\mathbf{I}_n^k(\mathbf{q})$ in the Fourier representation can be separated into two components, the potential component and the vortex component:

$$\mathbf{I}_n^k(\mathbf{q}) = \frac{\mathbf{q}}{q}(\mathbf{q},\mathbf{I}_n^k) + \frac{[\mathbf{q}\hat{\mathbf{z}}]}{q}([\mathbf{q}\hat{\mathbf{z}}],\mathbf{I}_n^k) \equiv \mathbf{I}_{\parallel n}^k(\mathbf{q}) + \mathbf{I}_{\perp n}^k(\mathbf{q}).$$

The vortex component of the current can be found by solving the equation

$$\frac{4\pi}{c}I_{\perp n}^k = \frac{2}{\Lambda}\left[\frac{\phi_0}{2\pi}\nabla\theta_\perp - \frac{4\pi}{c}\sum_m I_{\perp m}^k\,\frac{e^{-q|n-m|}}{2q}\right] \tag{9}$$

and does not contain Josephson coupling, while the potential component of the current,

$$\frac{4\pi}{c}I_{\parallel n}^k = -i\,\frac{\phi_0}{\pi\Lambda\lambda_J^2}\,\frac{\Omega_{n-1}^k(\mathbf{q}) - \Omega_n^k(\mathbf{q})}{q}$$
$$= \frac{4\phi_0}{\Lambda q}\frac{(\hat{\mathbf{z}}[\mathbf{q}\cdot\mathbf{l}])}{(\mathbf{q}\cdot\mathbf{l})}\sin\left(\frac{\mathbf{q}\cdot\mathbf{l}}{2}\right)(\delta_n^k - V_n^k), \tag{10}$$

depends only on the phase difference Ω_n^k between the layers defined in (8).

Now by performing simple transformations we can write the expression for the interaction energy of two dipoles [Eq. (7)] in the form

$$\mathscr{U}^{nk}(\mathbf{x}_0,\mathbf{l},\mathbf{l}') = \frac{\phi_0}{c}\int\frac{d^2q}{(2\pi)^2}\frac{1}{q}\sin\left(\frac{\mathbf{q}\cdot\mathbf{l}'}{2}\right)\left[I_{\perp n}^k(\mathbf{q},\mathbf{l})\right.$$
$$\left. + \frac{(\hat{\mathbf{z}}[\mathbf{q}\cdot\mathbf{l}'])}{(\mathbf{q}\cdot\mathbf{l}')}I_{\parallel n}^k(\mathbf{q},\mathbf{l})\right]e^{i\mathbf{q}\mathbf{x}_0}, \tag{11}$$

and the dipole's self-energy is

$$\mathscr{F}(\mathbf{l}) = \mathscr{U}^{kk}(\xi_{\mathrm{ab}},\mathbf{l},\mathbf{l}) = \mathscr{F}_M + \mathscr{F}_J, \tag{12}$$

$$\mathscr{F}_M = \frac{\phi_0^2}{4\pi^2\Lambda}\ln\frac{l}{\xi_{ab}},$$

$$\mathscr{F}_J = \frac{\phi_0^2}{4\pi^2\Lambda}\begin{cases}\dfrac{l^2}{\lambda_J^2}\ln\left(1+\dfrac{4\lambda_c^2}{l^2}\right), & l \ll \lambda_J\\[2ex] \dfrac{2l}{\lambda_J}\left[tE\left(\dfrac{1}{t}\right) - \dfrac{t^2-1}{t}K\left(\dfrac{1}{t}\right)\right], & l \gg \lambda_J,\end{cases}$$

with E and K the complete elliptic integrals, and $t = \sqrt{1+(\lambda_J/2\lambda_c)^2}$.

3. THE COLLECTIVE EXCITATION SPECTRUM

The energy of a vortex dipole in a layered superconductor is always finite, with the result that for any nonzero temperature the layers contain a dipole gas caused by thermal fluctuations. This is not an ideal gas: the interaction of the dipoles with each other leads to local polarization and spatial inhomogeneity of the gas and, as a result, to renormalization of the dipole energy.

Let us calculate the energy of a dipole that has an arm $\mathbf{l}$, is positioned at the origin of coordinates of the zeroth layer, and interacts with dipole gas. We employ the Debye–Hückel method.[27] In view of the linearity of the problem, the current $\mathbf{I}_n(\mathbf{x})$ in the nth layer is the sum of the currents $\mathbf{I}_n^m(\mathbf{x}-\mathbf{x}_{\alpha_m},\mathbf{l}_{\alpha_m})$ induced in this layer by all the other dipoles at points $\mathbf{x}_{\alpha_m}$ of all the layers m:

$$\frac{4\pi}{c}\mathbf{I}_n(\mathbf{x}) = \sum_m\sum_{\alpha_m}\frac{4\pi}{c}\mathbf{I}_n^m(\mathbf{x}-\mathbf{x}_{\alpha_m},\mathbf{l}_{\alpha_m}) \approx \frac{4\pi}{c}\mathbf{I}_n^0(\mathbf{x},\mathbf{l})$$
$$+\sum_m\int d^2x'\int d^2l'\,n^m(\mathbf{x}',\mathbf{l}')$$
$$\times\frac{4\pi}{c}\mathbf{I}_n^m(\mathbf{x}-\mathbf{x}',\mathbf{l}'), \tag{13}$$

where

$$\frac{4\pi}{c}\mathbf{I}_n^m(\mathbf{x}-\mathbf{x}_{\alpha_m},\mathbf{l}_{\alpha_m}) = \frac{2}{\Lambda}\left[\frac{\phi_0}{2\pi}\nabla\theta_n^m(\mathbf{x}-\mathbf{x}_{\alpha_m},\mathbf{l}_{\alpha_m})\right.$$
$$\left. - \mathbf{A}_n^m(\mathbf{x}-\mathbf{x}_{\alpha_m},\mathbf{l}_{\alpha_m})\right].$$

Here we have separated the current of the test dipole and replaced the current of the other dipoles by a mean current determined by the concentration $n^m(\mathbf{x}',\mathbf{l}')$ of the dipoles in layer m. The spatial inhomogeneity of the distribution of dipoles is caused only by their interaction. We assume that the concentration is given by the Boltzmann relationship

$$n^m(\mathbf{x}',\mathbf{l}') = A\,\exp\left(-\frac{1}{T}[\mathscr{F}(\mathbf{l}') + \mathscr{U}^m(\mathbf{x}',\mathbf{l}')]\right)$$
$$= n_0(l')\exp\left(-\frac{1}{T}\mathscr{U}^m(\mathbf{x}',\mathbf{l}')\right), \tag{14}$$

where A is the normalization constant, and $\mathscr{U}^m(\mathbf{x}',\mathbf{l}')$ is the energy of the interaction of the dipole with arm $\mathbf{l}'$ at point $\mathbf{x}'$ of layer m with all the other dipoles:

$$\mathscr{U}^m(\mathbf{x}',\mathbf{l}') = \sum_k\sum_{\alpha_k}\mathscr{U}^{mk}(\mathbf{x}'-\mathbf{x}_{\alpha_k},\mathbf{l}',\mathbf{l}_{\alpha_k})$$
$$= \frac{\phi_0}{c}\int\frac{d^2q}{(2\pi)^2}\frac{1}{q}\sin\left(\frac{\mathbf{q}\cdot\mathbf{l}'}{2}\right)\left[I_{\perp m}(\mathbf{q})\right.$$
$$\left. + \frac{(\hat{\mathbf{z}}[\mathbf{q}\cdot\mathbf{l}'])}{(\mathbf{q}\cdot\mathbf{l}')}I_{\parallel m}(\mathbf{q})\right]\exp(i\mathbf{q}\mathbf{x}'). \tag{15}$$

Here the integrand contains the total current flowing in level m. Assuming the interaction energy is small, we expand the function (14) in a power series and keep only the term linear in $\mathscr{U}$:

$$n^m(\mathbf{x}',\mathbf{l}') \approx n_0(l')\left[1 - \frac{1}{T}\mathscr{U}^m(\mathbf{x}',\mathbf{l}')\right].$$

Carrying out a two-dimensional transformation of (13) and eliminating the vector potential, we arrive at the following equation for the components of the total current flowing in a layer:

$$\frac{4\pi}{c}I_{\perp n}(\mathbf{q})=\frac{2}{\Lambda}\left[\frac{\phi_0}{2\pi}\nabla\theta^0_{\perp m}(\mathbf{q})-\frac{4\pi}{c}\times\sum_m I_{\perp m}(\mathbf{q})\frac{e^{-q|n-m|}}{2q}-\frac{\phi_0}{\pi\Lambda T}\times\sum_m\int d^2l' n_0(l')\mathscr{U}^m(\mathbf{q},\mathbf{l}')\nabla\theta^m_{\perp n}(\mathbf{q},\mathbf{l}'),\right. \tag{16}$$

$$\frac{4\pi}{c}I_{\parallel n}(\mathbf{q})=-i\frac{\phi_0}{\pi\Lambda\lambda_J^2}\frac{\Omega^0_{n-1}(\mathbf{q})-\Omega^0_n(\mathbf{q})}{q}+i\frac{\phi_0}{\pi\Lambda\lambda_J^2 T}\sum_m\int d^2l' n_0(l')\mathscr{U}^m(\mathbf{q},\mathbf{l}')\times\frac{\Omega^m_{n-1}(\mathbf{q},\mathbf{l}')-\Omega^m_n(\mathbf{q},\mathbf{l}')}{q}. \tag{17}$$

The expressions for $I_{\perp n}$ and $I_{\parallel n}$ are separated as a result of integration over the angles $\widehat{\mathbf{q}\mathbf{l}'}$.

Let us start with the equation for the vortex component of the current (Eq. (16)). Substituting the explicit expressions for $\mathscr{U}^m$ and $\nabla\theta^m_{\perp n}$, we obtain

$$\frac{4\pi}{c}I_{\perp n}(\mathbf{q})\left[1+\frac{\phi_0^2}{\pi\Lambda q^2 T}\int d^2l' n_0(l')\sin^2\left(\frac{\mathbf{q}\cdot\mathbf{l}'}{2}\right)\right]=\frac{2}{\Lambda}\left(\frac{\phi_0}{2\pi}\nabla\theta^0_{\perp n}(\mathbf{q})-\frac{4\pi}{c}\sum_m I_{\perp m}(\mathbf{q})\frac{e^{-q|n-m|}}{2q}\right). \tag{18}$$

To evaluate the expression in the square brackets, we note that the integrand has two asymptotes. For small lengths, $l'\ll 1/q$, the sine can be replaced with its argument, and for large lengths, $l'\gg 1/q$, the square of the sine can be replaced with its average value $\frac{1}{2}$. After this Eq. (18) assumes the form

$$\frac{4\pi}{c}I_{\perp n}(\mathbf{q})=\frac{2\mathscr{K}(q)}{\Lambda\epsilon_\perp}\left(\frac{\phi_0}{2\pi}\nabla\theta^0_{\perp n}(\mathbf{q})-\frac{4\pi}{c}\times\sum_m I_{\perp m}(\mathbf{q})\frac{e^{-q|n-m|}}{2q}\right), \tag{19}$$

where $\mathscr{K}(q)=\delta^2q^2/(1+\delta^2q^2)$, with

$$\delta=\sqrt{\frac{2\pi\Lambda\epsilon_\perp T}{\phi_0^2 n_0}} \tag{20}$$

the screening length,

$$n_0=\int_{l\geq\delta}d^2l n_0(l) \tag{21}$$

the collective excitation concentration, and

$$\epsilon_\perp=1+\frac{\phi_0^2}{4\Lambda T}\int_0^\delta dl l^3 n_0(l)$$

the dielectric constant in the direction of $[\mathbf{q}\cdot\hat{\mathbf{z}}]$. In evaluating the above integrals we allowed for the fact that the principal contribution to the dipole energy is provided by the wave vectors $q\leq 1/\delta$ and is restricted to asymptotic branches according to the parameter. Note that short dipoles have no effect on the screening length δ and contribute only to the dielectric constant. On the other hand, long dipoles ($l>\delta$) determine only the screening length.

The solution to Eq. (20) is found by a Fourier transformation along the $\hat{z}$ axis:

$$\frac{4\pi}{c}I_{\perp n}(\mathbf{q})=\frac{4\phi_0\mathscr{K}(q)}{q\Lambda\epsilon_\perp}\sin\left(\frac{\mathbf{q}\cdot\mathbf{l}}{2}\right)(\delta^0_n-W^0_n), \tag{22}$$

$$W^k_n(q)=\frac{\mathscr{K}(q)}{q\Lambda\epsilon_\perp}\sinh(qs)\frac{(G-\sqrt{G^2-1})^{|n-k|}}{\sqrt{G^2-1}},$$

$$G(q)=\cosh(qs)+\frac{\mathscr{K}(q)}{q\Lambda\epsilon_\perp}\sinh(qs).$$

Now let us study the screening of the potential component of the current $I_{\parallel n}(\mathbf{q})$. Solving Eq. (17) for $I_{\parallel n}$ as we did Eq. (16), we find that

$$\frac{4\pi}{c}I_{\parallel n}=-i\frac{\phi_0}{\pi\Lambda\lambda_J^2\epsilon_\parallel(q)}\frac{\Omega^0_{n-1}(\mathbf{q})-\Omega^0_n(\mathbf{q})}{q}=\frac{4\phi_0}{q\Lambda\epsilon_\parallel(q)}\left(\frac{\hat{\mathbf{z}}[\mathbf{q}\cdot\mathbf{l}]}{\mathbf{q}\cdot\mathbf{l}}\right)\sin\left(\frac{\mathbf{q}\cdot\mathbf{l}}{2}\right)(\delta^0_n-V^0_n). \tag{23}$$

Here we have employed the notation $V^0_n(q,a)$ introduced by (8),

$$a=1+\frac{\lambda_J^2}{2\epsilon_\parallel}\left(q^2+\frac{1}{\lambda_c^2}\right),$$

and

$$\epsilon_\parallel=1+\frac{\phi_0^2}{\pi\Lambda q^2 T}\int d^2l n_0(l)\frac{[\mathbf{q}\cdot\mathbf{l}]^2}{(\mathbf{q}\cdot\mathbf{l})^2}\sin^2\left(\frac{\mathbf{q}\cdot\mathbf{l}}{2}\right)$$

is the dielectric constant in the direction of $\mathbf{q}$.

In what follows we ignore spatial dispersion and assume δ, $\epsilon_\perp$, and $\epsilon_\parallel$ to be q-independent.

The self-energy of a dipole interacting with a gas of thermally excited dipoles can be found from Eqs. (12) and (15). Substituting (22) and (23) into (15) and integrating with respect to q, we get

$$\mathscr{F}(l)=\mathscr{F}_M+\mathscr{F}_J, \tag{24}$$

$$\mathscr{F}_M=\frac{\phi_0^2}{4\pi^2\epsilon_\perp}\left[\ln\frac{\delta}{\xi_{ab}}-K_0\left(\frac{l}{\delta}\right)\right],$$

$$\mathscr{F}_J=\frac{\phi_0^2}{4\pi^2\Lambda}\begin{cases}\dfrac{l^2}{\lambda_J^2}\ln\left(1+\dfrac{4\lambda_c^2}{l^2}\right), & l\ll\lambda_J\\ \dfrac{2l}{\lambda_J\sqrt{\epsilon_\parallel}}\left[tE\left(\dfrac{1}{t}\right)-\dfrac{t^2-1}{t}K\left(\dfrac{1}{t}\right)\right], & l\gg\lambda_J,\end{cases}$$

with

$$t=\sqrt{1+\left(\frac{\lambda_J}{2\lambda_c\epsilon_\parallel}\right)^2},$$

where K_0 is the modified Bessel function of the second kind.

Comparing Eqs. (24) and (12) makes it possible to understand the nature of the transformation of the excitation spectrum, a transformation caused by the dipole interaction. If the collective excitation concentration (21) n_0 is zero, then the screening length δ is infinite, and the energy (24) of the dipoles differs from the energy of free dipoles only by a dielectric constant $\epsilon_\perp > 1$ that allows for the polarization of the gas of excitations. A finite concentration n_0 of collective excitations corresponds to a finite screening length δ. The screened vortices in dipoles with arm lengths l exceeding δ are characterized by an exponentially weak interaction. In a system of uncoupled planes we have $\mathscr{F}=0$, with the collective excitations consisting almost entirely of free vortices. In the case of coupled planes such excitations are the screened vortices united in dipoles by means of Josephson vortices. The energy of short dipoles with $l<\delta$ depends on their length logarithmically, and the dipoles continue being the elementary excitations.

Thus, transformation of the energy spectrum is related to the emergence of a nonzero concentration n_0 of collective excitations. The equation for $n_0(T)$ is derived and analyzed in Sec. 4.

4. THE PHASE TRANSITION SCENARIO

The definition of n_0 given in the process of calculating the dipole energy is not constructive, since it is expressed in terms of the known dipole concentration $n(\mathbf{l})$.

Here we define n_0 differently. The equilibrium concentration n_0 of collective excitations emerges as a result of dynamic equilibrium between creation and annihilation of screened dipoles. The variation of n_0 in this case is described by the kinetic equation[28]

$$\frac{dn_0}{dt}=\Gamma-\mu n_0^2. \tag{25}$$

The first term on the right-hand side of the equation describes the rate at which collective excitations appear as a result of thermal fluctuations. We assume that $\Gamma\approx B\exp\{-\mathscr{F}(\delta)/T\}$, where B is a constant. The second term describes the rate of disappearance of screened excitations as a result of annihilation of vortices and antivortices belonging to different dipoles.

The time-independent equilibrium concentration n_0 is determined by requiring that the right-hand side of Eq. (25) vanish:

$$n_0=\sqrt{\frac{B}{\mu}}\exp\left(-\frac{1}{2T}\mathscr{F}(\delta)\right). \tag{26}$$

Using the expression (24) for the dipole energy and introducing a dimensionless concentration $z=\pi\xi_{ab}^2 n_0$, we can write Eq. (26) as

$$z=qz^p\exp\left(-\alpha\sqrt{\frac{2p}{z}}\right), \tag{27}$$

where $\alpha=\sqrt{\epsilon_\perp/\epsilon_\parallel}(\xi_{ab}/\lambda_J)$ and $p=\phi_0^2/16\pi^2\Lambda(T)\epsilon_\perp T$. The specific temperature dependence of p is determined by the

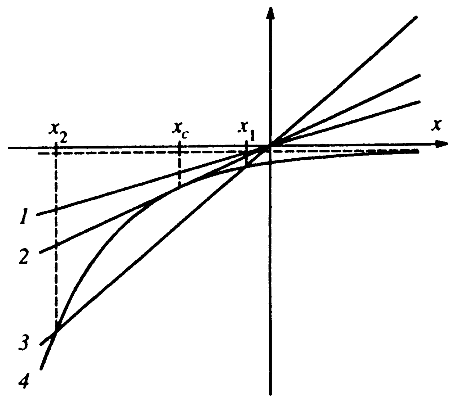

FIG. 1. Graphical solution of Eq. (27) for the collective excitation concentration n_0, with $x=\ln(\pi n_0\xi^2)$.

temperature dependence of ξ_{ab} and λ_{ab}; the quantity tends to infinity as $T\to 0$, $p(T_{\rm KT})=1$, and $p(T_{c0})=0$.

Obviously, for all $T<T_{c0}$ Eq. (27) has the trivial solution $z=0$, which corresponds to the absence of collective excitations. To facilitate subsequent analysis of Eq. (27), take the logarithms of both sides of this equation and introduce a new variable $x=\ln z$. The result is

$$(1-p)x=\ln q-\alpha\sqrt{2p}\exp\left(-\frac{x}{2}\right). \tag{28}$$

First we examine the system of uncoupled superconducting layers ($\alpha=0$). We analyze Eq. (28) with $\alpha=0$ graphically (Fig. 1). The left-hand side of the new equation is represented by a straight line through the origin, with the slope of the line determined by p. The right-hand side, $\ln q$, is represented by curve *4* in Fig. 1. If $q<1$, then for $p\geqslant 1$ the equation has no physically meaningful solutions corresponding to low values of the collective excitation concentration $n_0<1/\pi\xi^2$ ($x<0$). But for $p<1$ there is a nontrivial solution,

$$z=q^{1/[1-p(T)]}, \tag{29}$$

which is zero at $p=1$ and increases monotonically with temperature.

This means that at a temperature $T_{\rm KT}$ given by Eq. (1) and fixed by the condition that $p(T)=1$, the excitation spectrum undergoes a transformation. Excitations in the form of unscreened vortices coupled into dipoles, corresponding to the trivial solution of Eq. (27), become unstable. Instead, a branch of collective excitations appears, whose concentration depends on temperature according to (29). The coupling of these vortices is exponentially weak and can easily be disrupted by an arbitrarily weak current.

Now let us examine a system of Josephson-coupled planes ($\alpha>0$). The straight lines *1*, *2*, and *3* in Fig. 1 represent the left-hand side of Eq. (28) for different temperatures $T_1<T_2<T_3$. Curve *4* represents the right-hand side, and for negative values of x it moves down and away from the dashed line at an exponentially growing rate. Hence now the condition $p\leqslant 1$ does not guarantee the appearance of nontrivial solutions, as was the case for $\alpha=0$. The first nonzero

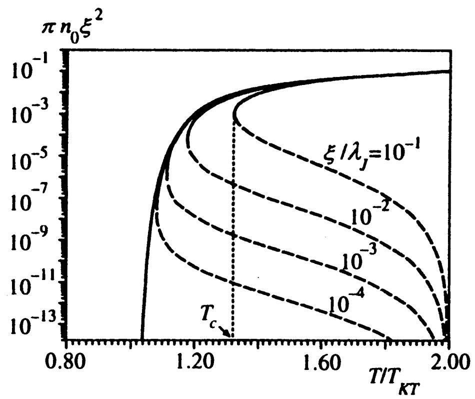

FIG. 2. Numerical solution of Eq. (27) for various values of the constant $\xi(0)/\lambda_J(0)$ of layer coupling. Metastable states exist in the temperature interval from T_c to $T_{c0}=2T_{KT}$. The envelope corresponds to the solution with zero coupling constant. Unstable solutions are depicted by dashed curves.

solution appears at a certain critical temperature, $p(T)=p_c$ (the straight line 2), which corresponds to a finite critical concentration $n_0=n_c$ of collective concentrations. With an increase in temperature (the straight line 3) two nonzero solutions emerge, x_1 and x_2. This fundamentally distinguishes the case of coupled planes from that of uncoupled planes. Since $p_c<1$, the transition always takes place at a temperature $T>T_{KT}$. Of the two new branches of collective excitations, one (x_1) is stable and the other (x_2) unstable. The third branch of trivial solutions $n_0=0$ also remains stable. Equation (28) has three solutions in the temperature interval from T_c to T_{c0}, where the sample goes into the normal state. The discontinuity in the temperature dependence of n_0 and the existence of two stable solutions within a finite temperature range means that the transformation of the excitation spectrum is a first-order transition.

The results of numerical solution of Eq. (27) are depicted in Fig. 2. A dashed curve depicts the unstable solution x_2.

Let us estimate the temperature T_c at which a nontrivial solution appears. At the transition point not only both sides of Eq. (27) but their derivatives are the same. Assuming that $\ln\sqrt{q}/\ln \alpha \ll 1$, we get

$$p(T_c)\approx 1-\frac{\ln\sqrt{q}}{\ln \alpha}<1. \tag{30}$$

The dimensionless collective excitation concentration z at the transition point is

$$\pi\xi_{ab}^2 n_c \sim \left(\frac{\xi_{ab}}{\lambda_J}\frac{\ln \alpha}{\ln\sqrt{q}}\right)^2,$$

and the screening length $\delta \propto (\lambda_J/2)(\ln\sqrt{q}/\ln \alpha)<\lambda_J$.

In a system of Josephson-coupled planes, the collective excitation concentration is certain to jump from zero to a relatively large value n_c corresponding to $\delta<\lambda_J$. The fact that δ cannot be greater than λ_J in a system of coupled layers has a simple explanation. Only dipoles whose length is greater than δ can ensure that the magnetic field of a vortex is screened over distances of order δ. The concentration of long dipoles with $l>\lambda_J$ is exponentially low, $n(l) \propto \exp\{-l/\lambda_J\}$. Equation (21) implies that $n_0 \propto \exp\{-\delta/\lambda_J\}$, while from the definition of δ we find that $\delta/\lambda_J \propto n_0^{-1/2}$. These two relationships can be reconciled only if δ/λ_J is fairly small. Physically this means strong suppression of the probability of creation of long dipoles because of the high energy of Josephson vortices.

5. THE EFFECT OF TRANSPORT CURRENT ON THE PHASE TRANSITION

In the preceding section we described the phase transition associated with the appearance of collective excitations. For a system of isolated superconducting planes this is a BKT transition. In a system of Josephson-coupled layers, the screened two-dimensional vortices remain united into dipoles by means of Josephson vortices and cannot move apart to an infinitely large distance.

Experimentally such transitions are studied by analyzing the current–voltage characteristics of layered superconductors measured at different temperatures. In the process a transport current flows through the sample, and this current can change the collective excitation spectrum and the nature of the phase transition.

In contrast to the zero-current situation, the dipole energy contains an additional term that takes into account the work of the external current source:

$$\mathcal{F}_I(l)=\frac{\phi_0}{c}Il \sin \theta,$$

where θ is the angle between the dipole's arm and the direction of the current **I** in the layer. If this term is taken into account, the minimal work (24) spent on creating a dipole ceases to monotonically increase with the dipole length. It reaches its maximum at a length l_c, which has the smallest value for dipoles perpendicular to the current ($\theta=\frac{1}{2}\pi$), and can be found from the equation

$$K_1\left(\frac{l_c}{\delta}\right)=\frac{\delta}{\xi_{ab}}\frac{I-I_c}{I_{\rm GL}}, \tag{31}$$

where $I_{\rm GL}(T)=cTp(T)/\phi_0\xi_{ab}(T)$ is the Ginzburg–Landau current, which disrupts dipoles of length ξ_{ab}, and

$$I_c(T)=I_{\rm GL}(T)\frac{2\xi_{ab}(T)}{\lambda_J(T)}. \tag{32}$$

A further increase in the dipole arm length leads to a decrease in the dipole's energy. This means that the system becomes thermally unstable against avalanche creation of dipoles with arm lengths $l>l_c$, whose vortices can freely move under the action of the current. The steady-state concentration n_f of free vortices emerges as a result of dynamic equilibrium between free-vortex creation and annihilation:

$$\frac{dn_f}{dt}=\Gamma_f-\mu n_f^2. \tag{33}$$

According to Refs. 29 and 30, the rate Γ_f at which free vortices are created is determined by the energy of a vortex dipole of critical length l_c as follows:

$$\Gamma_f = B \exp\left[-\frac{\mathcal{F}(l_c)}{T}\right] I_0\left(\frac{l_c\phi_0 I}{cT}\right),$$

where I_0 is the modified Bessel function.

The equations for l_c and n_f contain the screening length δ, which is related to the collective excitation concentration n_0. Hence Eqs. (31) and (33) must be solved simultaneously with Eq. (25) for n_0. In the presence of a transport current, however, the rate Γ at which collective excitations are created changes. Here the magnitude of Γ is determined by various processes that depend on the ratio between l_c and δ.

If the screening length δ is shorter than l_c, then Γ is determined by the least work that must be done to form a dipole with an arm length δ, as was established in Sec. 4. The difference compared with the zero-current case is that here Γ acquires an additional factor related to the dipole–current interaction energy,

$$\oint \frac{d\theta}{2\pi} \exp\left[-\frac{\mathcal{F}_I(\delta)}{T}\right] = I_0\left(\frac{\delta\phi_0 I}{cT}\right).$$

The equation for the concentration $z=\pi n_0 \xi_{ab}^2$ of collective excitations assumes the form

$$z^{1-p} = q \exp\left(-\frac{\xi_{ab}}{\lambda_J}\sqrt{\frac{2p}{z}}\right) I_0^{1/2}\left(\frac{I}{I_{\mathrm{GL}}}\sqrt{\frac{2p}{z}}\right). \qquad (34)$$

In this case not all collective excitations are free: a fraction of these remain coupled to dipoles because of the tension of Josephson vortices.

In the opposite case, $\delta > l_c$, the rate Γ at which collective excitations are created is directly linked to the rate Γ_f of free-vortex creation. In a current, dipoles whose length l is no less than l_c begin to increase in length at a rate $v \sim I - I_c$. If there were no collisions between dipoles, the length of each dipole would reach δ and Γ would become equal to Γ_f. But by the time l reaches δ, some of the dipoles are able to collide with others, as a result of which their concentration decreases due to the annihilation of vortices and antivortices belonging to different dipoles. And Γ decreases accordingly.

Let us estimate Γ in a simple model. In the steady state, the variation of the dipole concentration at lengths $l > l_c$ is described by the transport equation

$$v\frac{dn(l)}{dl} = -\mu n_f n(l) + \Gamma_f \delta(l - l_c). \qquad (35)$$

The left-hand side of the equation describes the variation in $n(l)$ due to transport in the size space. The first term on the right-hand side allows for the decrease in concentration due to vortex annihilation. Here it is assumed that all long dipoles with $l > l_c$ are oriented in an energetically favorable direction, perpendicular to the current, and a collision of two vortices is followed by the creation of one dipole with a length equal to the sum of the dipole lengths prior to the collision. We drop the arrival term in (35) since we assume that a single collision of two long dipoles forms one dipole with a total length $l > \delta$, and we are interested in processes in the length interval from l_c to δ. Neither do we take into account the interaction between dipoles with small lengths, $l < l_c$, since such dipoles have an essentially equiprobable orientation in the layer and, after averaging over the angle θ, prove to contribute little to the variation of $n(l)$.

Equation (35) makes it possible to estimate the rate Γ of creation of collective excitations:

$$\Gamma = \int_\delta^\infty v \frac{dn(l)}{dl} dl = \Gamma_f \exp\left(-\frac{\delta - l_c}{L}\right), \qquad (36)$$

where $L = v/\mu n_f$ is the mean free path of the vortices.

The two situations just described correspond to different phase transition types. We start by analyzing the case $\delta < l_c$, corresponding to a relatively weak current flowing in the sample. In this case the phase transition to a collective state is fully described by Eq. (34).

At high enough temperatures and concentrations z, the argument of the Bessel function is small, the value of the function is close to unity, and the solution $z(T)$ of Eq. (34) is independent of current I.

At low concentrations of collective excitations the Bessel function can be replaced with its asymptotic value at large values of the argument. Equation (34) in this case assumes the form

$$\left(\frac{7}{8} - p\right) \ln(z) = \ln(q') - \alpha'(T)\sqrt{\frac{2p}{\pi z}}, \qquad (37)$$

where

$$\alpha'(T) = \frac{\xi_{ab}}{\lambda_J} - \frac{I}{2I_{\mathrm{GL}}}.$$

This equation can be analyzed graphically, as we did with Eq. (28) in Fig. 1. Here we must allow for the temperature dependence of the function $\alpha'(T)$, where $I_{\mathrm{GL}} \sim \xi^{-3}(T)$. As the temperature is increased, the second term in $\alpha'(T)$ grows much faster than the first. For a given temperature there exists a critical current $I_c(T)$ [Eq. (32)] above which α' is positive. For such values of I the curve *4* in Fig. 1 bends upward and nontrivial solutions emerge.

The dependence of the concentration of free excitations obtained as a result of solving the system of equations for n_0 and n_f numerically is depicted in Fig. 3 for the following values of the coupling constants between the layers: $\xi_{ab}(0)/\lambda_J = 0.01$ (Fig. 3a), and $\xi/\lambda_J = 0.0$ (Fig. 3b). Let us examine the temperature behavior of n_0 for a system of coupled layers. The solutions of interest correspond to low currents and belong to the same type as the zero-current solutions, the only difference being that the right-hand stability limit of a metastable state is current-dependent and can be found from the equation

$$I = I_c(T). \qquad (38)$$

Curve *1* in Fig. 3a is a typical solution. Solutions of this group lie between curves *0* and *2*, with the first corresponding to a zero current and the second to the first critical current I_{c1} at which the region of metastable states disappears. The left and right limits of metastability merge at the temperature

$$T_g = \frac{8T_{c0}}{7T_{c0} + T_{\mathrm{KT}}} > T_{\mathrm{KT}}, \qquad (39)$$

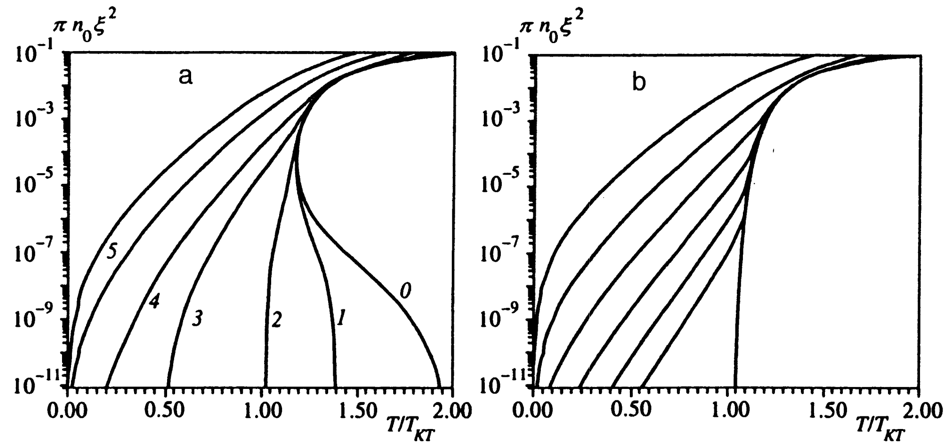

FIG. 3. The temperature dependence of collective excitation concentrations for different currents: (a) $\xi/\lambda_J=0.01$, with the corresponding values of the current described in the text, and (b) $\xi/\lambda_J=0$. The higher the curve, the greater the current.

and the magnitude of the critical current I_{c1} is determined by the relationship

$$I_{c1}=I_c(T_g).$$

For low-current solutions the right-hand stability limit for a metastable state corresponds to the resistance transition temperature, since the Josephson coupling of excitations is disrupted by the transport current.

When a current I higher than I_{c1} flows through the sample, the case $\delta>l_c$ is realized. The solutions corresponding to such currents describe a second-order phase transition in which the collective excitation concentration grows from zero continuously. The temperature at which nontrivial solutions emerge is given by Eq. (38) and monotonically decreases as the current grows. There is a critical value I_{c2} of current at which the transition temperature is zero:

$$I_{c2}=I_c(0)$$

(curve *3* in Fig. 3a). When $I>I_{c2}$, we have a second-order phase transition at zero temperature but with a different temperature dependence of z. At low temperatures, $T<T_g$, the collective excitation concentration is always current-dependent and monotonically increases with I. At temperatures $T>T_g$ the solutions are essentially identical with the zero-current one if the current is lower than a certain value

$$I_{c3}\sim\sqrt{q}I_{\mathrm{GL}}(T_g), \qquad (40)$$

and monotonically grow with current when $I>I_{c3}$ (curves *4* and *5*, respectively, in Fig. 3a).

Numerical calculation of $z(T,I)$ for a zero coupling constant is illustrated by Fig. 3b. In this case there is critical current and

$$I_{c1}=I_{c2}=0.$$

The temperature dependence z is similar to that depicted by curves *4* and *5* in Fig. 3a with a zero temperature of the phase transition. As Fig. 3b shows, at low temperatures the functions $z(T,I)$ corresponding to different currents are represented, on a log–log scale, by straight lines and can be written as

$$z(T,I)\simeq z(T)\left(\frac{I}{I_{\mathrm{GL}}}\right)^{2p(T)}.$$

We note once more that in a system of free superconducting planes there is a nontrivial solution $z(T)$ for any arbitrarily low current and at any temperature.

6. CURRENT–VOLTAGE CHARACTERISTICS

The resistance behavior of type II superconductors is governed by the motion of magnetic vortices. In layered superconductors in the absence of a magnetic field, the only vortices of thermally excited dipoles that can move under a current are those whose Josephson and magnetic attraction can be overcome by the Lorentz force.

Dipoles whose length l is greater than l_c meet this condition. In contrast to Refs. 21 and 31, the critical length l_c is given by Eq. (31), which was obtained with allowance for the transformation that the excitation spectrum undergoes as a result of the phase transition. This made it possible to describe the superconductor resistance caused by the motion of vortices of concentration n_f (see Ref. 32),

$$\rho_f=\rho_n 2\pi\xi^2 n_f, \qquad (41)$$

over a broad range of currents and temperatures below T_{c0}. Here ρ_n is the resistance of the material in the normal state.

The results of calculations of $n_f(I)$ are depicted as solid curves in Figs. 4a and b. For the sake of comparison, the dashed lines show the behavior of the collective excitation concentration n_0. At low temperatures, $T<T_g$, the concentrations n_0 and n_f coincide. At high temperatures the curves representing the n_0 vs I dependence start at finite values at $I=I_c(T)$, while the n_f vs I curves start at zero.

The difference between the $n_f(I,T)$ functions for systems of coupled and free superconducting layers is due only to the existence of a critical current $I_c(T)$. A curve representing $n_f(I,T)$ for coupled layered superconductors and built as a function of $I-I_c$ is similar to a curve in Fig. 4b. Hence all the properties of the resistance and the current–voltage char-

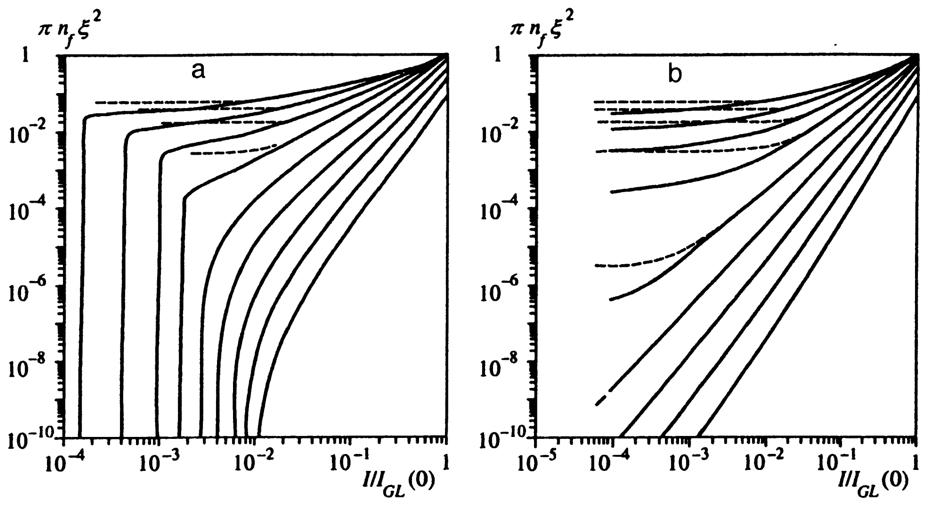

FIG. 4. The current dependence of the free-vortex concentration $\pi n_f \xi^2$ for different temperatures. The calculation was done for parameter values $T_{KT}=1$, $T_{c0}=2$, and $q=0.1$ and temperatures ranging from 0.5 (the lower curve) to 1.7 (the upper curve) in steps of 0.15. For the sake of comparison, the dashed curves represent the current dependence of the collective excitation concentration $\pi n_0 \xi^2$ calculated for the same temperatures: (a) $\xi/\lambda_J=0.01$, and (b) $\xi/\lambda_J=0$.

acteristics of a system of free superconducting planes can be carried over to a system of coupled planes simply by substituting $I-I_c$ for I.

As Fig. 4b shows, the curves clearly contain sections where the function $n_f(I)$ and hence the resistance $\rho_f(I)$ can be approximated to high accuracy by the power-like function

$$n_f(I,T) \sim \rho_f(I,T) \sim [I-I_c(T)]^{a(T)-1}, \tag{42}$$

with an exponent $a(T)$ that is current-independent. At low temperatures, $T<T_g$, and in the high-current range at high temperatures, $a(T)-1 \approx 2p(T)$. This result is consistent with those of other researchers,[19,21] who found the free vortex concentration n_f without allowing for the transformation of the excitation spectrum. This coincidence is not accidental, since in such ranges the screening length δ is greater than l_c and the transformation of the spectrum has essentially no effect on the unpairing of the dipoles by the current.

The situation differs in the high-temperature branches of $n_f(I,T)$. When the currents diminish, the curves representing the function $n_f(I,T)$ deviate from the high-current asymptotic behavior (42) and become flatter. The reason is that here the length l_c proves to be greater than the screening length δ, and the magnetic attraction between vortices becomes exponentially small. In this range of currents and temperatures the function $n_f(I,T)$ can also be represented by Eq. (42); but the exponent a is a slowly varying function of the current. The current dependence of a shows up as a smearing of the Nelson–Kosterlitz jump in the temperature dependence of a as the current grows (Fig. 5b).

The above dependence of the resistance ρ_f on the current and temperature make it possible to estimate the current–voltage characteristics of layered superconductors:

$$E \sim \rho_f(I,T)[I-I_c(T)] \sim [I-I_c(T)]^{a(T)}. \tag{43}$$

The properties of these current–voltage characteristics are completely determined by the behavior of $n_f(I,T)$ and require no additional explanation. Let us discuss them from the experimental viewpoint.

The threshold nature of the current–voltage characteristics resulting from the presence of a critical current $I_c(T)$ manifests itself in the tendency of the electric field strength E to vanish as I approaches $I_c(T)$. This is reflected in the E vs I dependence: the branch of the current–voltage characteristics with a fixed temperature T makes a sharp dip near

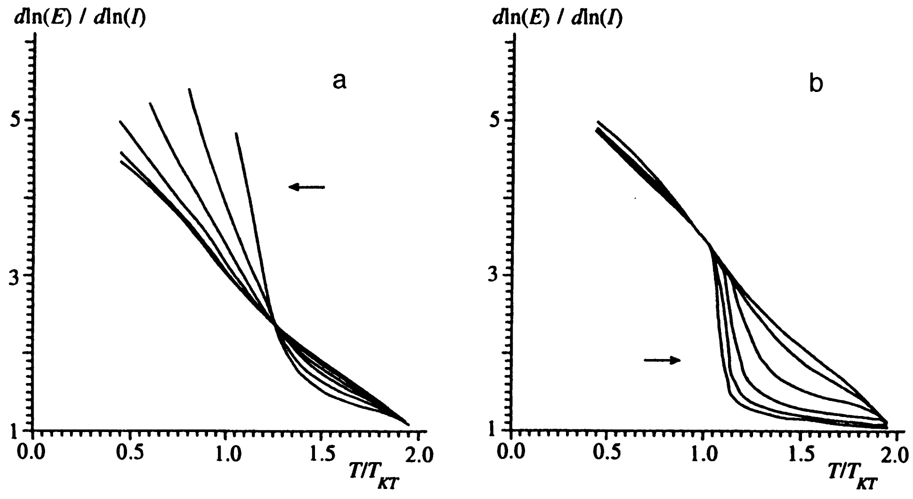

FIG. 5. The temperature dependence of $\partial \ln E/\partial \ln I$ for different currents and with $T_{c0}=2T_{KT}$: (a) $\xi/\lambda_J=0.01$, and (b) $\xi/\lambda_J=0$. The arrow shows the direction in which the curves shift as the current increases.

$I_c(T)$. In the experiments reported in Refs. 10, 11, 33, and 34 such behavior is observed for the low-temperature branches of current–voltage characteristics. In the high-temperature branches the exponent $a(T,I)$ decreases as the temperature drops.

We see two reasons for such a discrepancy between theory and experiment. First, at current values I that exceed the critical value $I_c(T)$ considerably, the main process determining the resistance behavior of the superconductor is free-vortex motion. When the current approaches the critical value, the field strength E associated with vortex motion decreases, and other mechanisms, such as Aslamazov–Larkin fluctuations and creep, come to the fore and lead to an almost linear slope of the respective sections of the current–voltage characteristics. In a number of real current–voltage characteristics[10,11] such a change in the mechanism of the resistance behavior reveals itself in the form of breaks in the branches of the current–voltage characteristics for low field strengths E. As T_{c0} is approached, the contribution of the alternative mechanisms to the resistance behavior becomes greater and may mask the feature in the current–voltage characteristics related to the disruption of vortex dipoles.

Another reason for this discrepancy may be that in real samples at temperatures higher than T_g the superconductive coupling between the layers can break down. In this case E vanishes only at $I=0$. Accordingly, the high-temperature branches of the current–voltage characteristics must have a slope close to a linear one.

As noted in the Introduction, the temperature dependence of the exponent $a(T)$ is one of the most characteristic manifestations of phase transitions of the BKT type. Indeed, experiments involving various high-T_c materials[1,35,36] clearly demonstrate the existence of the universal Nelson–Kosterlitz jump. In other studies[23,34,36] the jump has proved to be greatly distorted. Such a large spread in the results can probably be linked to the lack of a rigorous method for extracting $a(T)$ from current–voltage characteristics. This statement is illustrated by Figure 5a, which shows the temperature dependence of $\partial \ln(E)/\partial \ln(I)$ at different values of current for the calculated current–voltage characteristics [Eq. (42)]. The curves resemble the experimental dependences $a(T)$ (see Refs. 23, 34, and 36). Clearly, this quantity is not the exponent in the current–voltage characteristics. On the other hand, the temperature dependence $a(T)=\partial \ln(E)/\partial \ln[I-I_c(T)]$ for a constant value of $I-I_c(T)$ demonstrates a universal behavior, as in Fig. 5b.

Thus, in interpreting measurements of the current–voltage characteristics of layered superconductors one must allow for the presence of an internal critical current $I_c(T)$ and the dependence of the exponent a on the current.

7. CONCLUSION

We have described a phase transition in layered superconductors with Josephson coupling between the layers. The transition is related to a transformation of the excitation spectrum, a process that manifests itself in the screening of the magnetic interaction of vortices, as in a BKT transition, but proceeds as a first-order transition. We have shown that a transport current changes the nature of the phase transition, narrowing the stability region for a metastable state, and transforms it into a second-order phase transition when $I>I_c$. Transformation of the excitation spectrum affects the concentration n_f of free vortices, which determine the resistance behavior of the superconductor. The result is the appearance of the universal jump in the temperature dependence of the exponent $a(T)$ in the current–voltage characteristics. The jump broadens as the current grows. We have suggested a method for building the dependence of $a(T)$ on the current–voltage characteristics that reflects the features of the E vs I dependence [Eq. (43)] for layered superconductors with Josephson coupling.

Remaining within the framework of this approximation, we were unable to examine the renormalization of the interaction between layers induced fluctuations in the vicinity of the phase transition. We assumed that the coupling between the layers disappears at T_{c0}. However, the results of our analysis can easily be extended to the case where the coupling disappears at a temperature $T_{2D} < T_{c0}$. The important and challenging problem of the transformation of Josephson coupling in layered superconductors should become the topic of further investigations, both theoretical and experimental.

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