Spontaneous occurrence of a hierarchy of generation masses and quark mixing

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It is shown that under certain conditions dynamical spontaneous breaking of chiral symmetry transforms the generations of interacting massless fermions into a state in which a spectrum with a hierarchy of masses and mixing of the generations corresponds to a minimum of the system energy. Thus, these characteristic features in the spectrum of the standard quark generations can arise directly from the internal properties of the spontaneously arising ground state of the system. [S1063-7761(96)00202-X]

1. INTRODUCTION

The standard model of the electroweak interactions gives a phenomenologically successful description of the observed properties of the generations of fermions and gauge bosons. At the same time, many features of this successful phenomenology give a hint that it could arise from a more general, probably dynamical scheme. For example, soon after the mass of the t quark had been determined,[1] it was finally established that there are large and relatively closely spaced energy scales in different phenomenologically independent sectors of the standard model: the vector bosons, the highest quark generation, and the possible Higgs scalars. This provided serious support for a nonstandard connection between the phenomena and their origin in a single dynamical mechanism.

Another convincing indication in support of the existence of new dynamics beyond the standard model is represented by the clear mass hierarchies in the fermion generations and the elements of the matrix of weak mixing of the quarks. In the standard model, these obvious regularities are regarded as the result of a fortuitous choice of the constants of the Yukawa couplings, among which some are exceptionally small; moreover, there is no dynamics in this choice.

The search for a dynamical explanation of the properties of the standard model began already in the seventies.[2] At that time, the idea of a universal mechanism encompassing all features of the standard model simultaneously had not appeared. However, dynamical mechanisms for each of the phenomena were intensively discussed.

A common energy scale results from the hypothesis of strong coupling between t quarks that acts over distances much less than those characteristic for the electroweak breaking: $\sim M^{-1} \ll M_W^{-1}$ (Refs. 3 and 4). It is then found that the masses of the t quark, the vector bosons, and the composite Higgs scalar have the same order of magnitude.[5] They are all determined by the density of the t condensate and by coupling constants that are ~ 1. However, the distinguished role of the t quark, which is separated from the complete system of fermion generations, makes it difficult to understand the origin of the small masses. It is not clear over what distances, $\sim M^{-1}$ or $\sim M_W^{-1}$, the mechanisms that bring into consideration the masses of the other fermions act.[6]

For many years now there has also been study of the possible dynamics of the formation of a mass hierarchy and mixing. The occurrence of the hierarchies is attributed to different generations of perturbative couplings.[7] The low masses are determined in terms of the high mass as radiative corrections with respect to such interactions.[8] Mixing of the quarks arises through inclusion in the Lagrangian of direct perturbative transitions between the generations.[8]

In this paper, it is assumed (as in Refs. 3–6) that the basic properties of the standard model must be generated by a new dynamics at short distances $\sim M^{-1} \ll M_W^{-1}$, directly in the strong-coupling region. We investigate the possibility and the conditions under which such a dynamics is capable of leading simultaneously to breaking of the symmetries of the electroweak theory, a single scale for the phenomena, a hierarchy of fermion masses, and generation mixing. The heaviest generation is not distinguished in advance. All the effects arise from the action of a single dynamical mechanism—spontaneous breaking of chiral symmetry. However, we shall establish at the same time that different forces participate explicitly in its formation.

We investigate the necessary properties for a simplified model in which we retain at short distances only the part of the interaction that approximates the conjectured strong coupling. In this respect, we follow the idea of Ref. 3. It will be shown that in multiflavor systems dynamical breaking of chiral symmetry is capable of transforming the system into a state corresponding to a mass hierarchy and mixing of the flavors (generations); moreover, the mixing occurs spontaneously, without explicit introduction of transitions into the Lagrangian. These properties are intrinsic properties of the new ground state of the system.

As in Refs. 3–6, in the model we choose (Sec. 2) the strong coupling at distances $\sim M^{-1}$ will be approximated by a four-fermion interaction. The calculations of Ref. 5 show that such an approximation can be regarded as sufficient to establish the qualitative conditions needed for the formation of the properties of the standard model. At the same time, the attainment of quantitative agreement may require allowance for corrections.[9]

In the case of a four-fermion local coupling, we are dealing with a form of the Nambu–Jona-Lasinio (NJL) model,[10] which is often used to represent relativistic dynamical breaking of chiral symmetry.[11] This unrenormalizable model is usually treated in the leading approximation in the number of fermion colors: $N_c \gg 1$. For a system of several flavors, hierarchy properties are absent in this approximation. Only extension of the NJL models in the next order in N_c makes it possible to establish the existence of new solutions for which a spectrum with flavor mass hierarchy and the necessary mixing corresponds to the most stable state of the system. The possibility of such an extension is discussed in Sec. 3.

The states thus obtained could serve as a basis for an understanding of the real systems of quark generations. For this, it is necessary to include in the treatment flavor-changing neutral currents, possible pseudo-Goldstone bosons, and a mass difference of the upper and lower quarks. We defer the transition to more realistic schemes to the future.

In addition, there remains the problem of the transition from the strong-coupling region to the observed $(\sim M_W, m_t)$ region. The considered model corresponds to the dynamics at short distances $\sim M^{-1}$. This separation of the regions is not dictated on physical grounds, but it does facilitate, indeed make possible, the analysis. However, as is well known,[5] this requires fine tuning of the parameters of the model, which must be chosen extremely close $(\sim m_t^2/M^2 \ll 1)$ to a critical point. With regard to this situation, various points of view have been advanced. Our treatment makes the problem somewhat less severe by replacing the proximity factor m_t^2/M^2 by the model-dependent small but constant factor N_c^{-1}, which does not depend on the quantity m_t to be determined.

It was shown in the studies of Ref. 12 that if the perturbative interactions that distinguish the quark generations are generated by a neutral vector (pseudovector) particle, then the mass hierarchy and the main properties of the weak-mixing matrix arise simultaneously and without additional assumptions. The dynamical reconstruction of the situation in Ref. 12 presupposed the existence in the massless system of standard quarks of a ground state of precisely the type found here. Therefore, our treatment supports dynamically the phenomenological picture of Ref. 12.

In Sec. 2, we justify the form of the NJL model that we choose. In Secs. 3 and 4, we derive the equation for the mass spectrum of the fermions in the model. The solution for the highest mass is discussed in Sec. 5. The solutions for the light generations are considered in Sec. 6.

2. CHOICE OF THE MODEL

As potential that models the strong coupling in the region $\sim M \gg M_W$, we consider the four-fermion interaction of n flavors (generations) of L and R chiral quarks with N_c colors α, β:

$$V = \sum_{\alpha,\beta=1}^{N_c} \sum_{i,i'=1}^{n} \lambda_{ii'} (\bar{\psi}_{Li}^{\alpha} \psi_{Ri'}^{\alpha})(\bar{\psi}_{Ri'}^{\beta} \psi_{Li}^{\beta}),$$

$$\lambda_{ii'} \sim M^{-2}. \tag{1}$$

When the constant $\lambda_{ii'} = \lambda_0$ does not depend on the flavor indices i, i', we have a symmetric $U_L(n) \times U_R(n) \times SU(N_c)$ form. It has frequently been used[13,14] to investigate problems raised by the hypothesis of a t condensate.

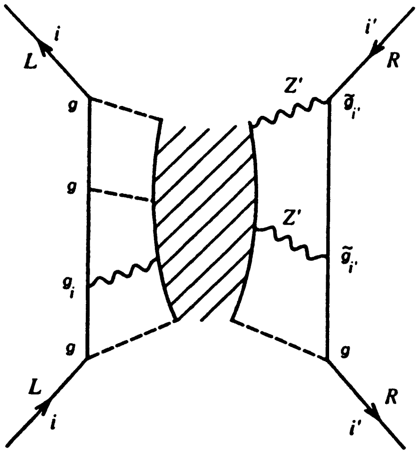

FIG. 1. Quark–antiquark amplitude in the theory with vector coupling (the hatched region symbolizes the strong coupling).

The exact symmetry of the problem (1) is $[U_L(1)]^n \times [U_R(1)]^n \times SU(N_c)$. There are no transitions between i or i', and therefore mixing can arise only spontaneously. This is one of the physical reasons that dictates the choice of (1).

A second reason is as follows. A Fierz transformation (see Ref. 11) carries (1) into a combination of vector–vector $L \times R$ couplings (including color). Therefore, (1) can be understood as a local effective $L \times R$ potential for a theory in which all interactions are generated exclusively by vector fields.[1)] Indeed, any number of vector exchanges between the massless L and R fermions (Fig. 1) generates precisely a vector–vector coupling in the local limit. Flavor conservation then means that all vector interactions are diagonal with respect to them.

Since the possible dynamical theory explaining the properties of the standard model can be expected to be a gauge theory, the potential (1) is a very acceptable model of the local low-energy limit of this theory. It was shown in the studies of Ref. 12 that to explain the main properties of the standard model it is sufficient if, in addition to gluons and weak vector bosons, there are just neutral vector fields Z'_μ. This then means that one can consider conserved flavors, and the constants $\lambda_{ii'}$ are the elements of a diagonal matrix in the space of the R and L flavor indices i, i' and the chiralities α, $\alpha' = R, L$:[2)]

$$\lambda_{\alpha i, \alpha' i'}^{\alpha_1 i_1, \alpha'_1 i'_1} = \lambda_{ii'} \delta_{ii_1} \delta_{i' i'_1} \delta_{\alpha\alpha_1} \delta_{\alpha'\alpha'_1} (1 - \delta_{\alpha\alpha'}). \tag{2}$$

In addition, the neutral scalar form (1) is also distinguished by the following property: Among all flavor-conserving local forms of coupling of two quarks, it alone makes a contribution to lowest order in N_c to the equation for the fermion masses [see Eq. (6)].

To conclude this section, we discuss the possible nature of the dependence of $\lambda_{ii'}$ on the indices i, i'. For this, we again return to the studies of Ref. 12.

In them, it was shown phenomenologically that the hierarchy and structure of the weak-mixing matrix can be simul-

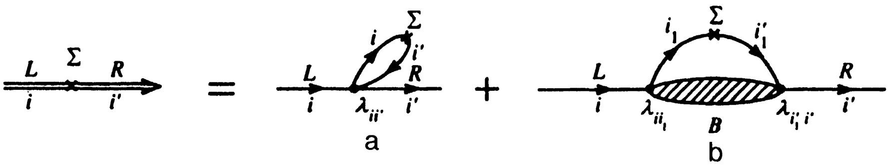

FIG. 2. Equation for the mass operator.

taneously and in detail reproduced if the flavor-distinguishing interaction has a perturbative nature and is realized by neutral vector fields:

$$V_i^{(f)}=g_i(\bar{\psi}_{Li}^{(f)}\gamma_\mu\psi_{Li}^{(f)})Z'_\mu,$$
$$\tilde{V}_i^{(f)}=\tilde{g}_i^{(f)}(\bar{\psi}_{Ri}^{(f)}\gamma_\mu\psi_{Ri}^{(f)})Z'_\mu,\quad f=u,\,d. \tag{3}$$

Attempts on this basis to construct a dynamical picture of the occurrence of the quark masses and mixing lead to the assumption of critical phenomena of the type considered in this paper. At the same time, the constant $\lambda_{ii'}$ arises in the form of a series in g_i and $\tilde{g}_i$ (Ref. 12) (here, different families of U and D are not considered):

$$\lambda_{ii'}=\lambda_0+\delta\lambda_{ii'}=\lambda_0+ag_i+b\tilde{g}_{i'}+cg_i\tilde{g}_{i'}+\cdots,$$
$$|\delta\lambda_{ii'}|\ll\lambda_0. \tag{4}$$

The diagrams of Fig. 1 illustrate and even prove the expansion (4) if all interactions between the quarks are realized solely by the vector fields, while the perturbative participation of the couplings (3) introduces into the system a difference between the flavors.

In what follows, we shall assume that $\lambda_{ii'}$ have properties similar to those in the expression (4), since it qualitatively reproduces the picture of the standard model. The particular form of (3) is not used.

Note that expressions of the type (3) and (4) must fix the difference of the operators ψ_{Li}, $\psi_{Ri'}$ with respect to the flavor indices i, i' and determine the flavor basis in which the constant $\lambda_{ii'}$ in (1) is expressed.

Thus, the situation with which we begin our study is close to a symmetric case: All flavors are equivalent, and a highest generation is not distinguished.

3. EQUATION FOR THE MASS MATRIX (GAP EQUATION)

Spontaneous breaking of chiral symmetry has the consequence that the equations for the self-energies of the massless fermions have solutions with masses that are not equal to zero. The equation for the masses is an analog of the gap equation in superconductivity, and this name is also used in relativistic problems of the spontaneous breaking of chiral symmetry.[5,10,11] If there is to be mixing, the mass matrix of the system of fermions must be nondiagonal. The gap equation becomes a system that determines not only the masses but also the mixing parameters.

In NJL problems, the gap equation is written in the form of an expansion with respect to the large number N_c of colors. At the same time, the parameter $\lambda N_c M^2$ remains finite in the limit $N_c\to\infty$. Usually, the solution is restricted to just the leading term in N_c (Refs. 10 and 11). As we shall see below, in such an approximation the system of n generations of quarks interacting with the potential (1) and form of $\lambda_{ii'}$ of the type (3) does not have a solution with a mass hierarchy $m_1\gg m_2\gg\ldots\gg m_n$. However, the necessary properties do arise when the next two orders in N_c are taken into account.

In the given approximation, the gap equation is represented graphically by Fig. 2. If the inverse propagator with nondiagonal mass matrix Σ is written in the form[3)]

$$(G^{-1})_{ii'}=\Sigma_{ii'}-\hat{p}\delta_{ii'}, \tag{5}$$

then the equation for the matrix Σ is

$$\Sigma_{ii'}=\frac{\beta_{ii'}}{M^2}\int\frac{d^4p}{4i\pi^2}\,\mathrm{Tr}\,G_{ii'}(p,\Sigma)f\left(\frac{p^2}{M^2}\right)+\frac{1}{2N_c}\sum_{i_1,i'_1}\int\frac{d^4q}{\pi^2 i}G_{i_1i'_1}\times(-q,\Sigma)B_{ii_1}^{i'i'_1}(q^2,\Sigma)\frac{\beta_{i'i'_1}}{M^2}. \tag{6}$$

The first term is the single-loop term (Fig. 2a), and the second is the contribution $\sim N_c^{-1}$ (Fig. 2b). In Eq. (6), $N_c^{-1}B\beta/M^2$ is the quark–antiquark interaction amplitude, which in the present approximation is a function of only the energy variable, and

$$\beta_{ii'}=\frac{\lambda_{ii'}M^2N_c}{8\pi^2}\to\mathrm{const},\quad N_c\to\infty,\quad M^2\to\infty \tag{7}$$

is a dimensionless parameter that determines the interaction strength. The integral of the single-loop contribution in (6) contains the cutoff function $f(p^2/M^2)$ explicitly; the second term also depends on the cutoff through the dimensionless amplitude $B(q^2)$.

The fact is that the quadratic divergences of the unrenormalizable model necessitate the use of a cutoff in the divergent integrals. It is known[10,11] that the arbitrariness in the cutoff does not change the qualitative results in the case of integration in simple loops but can give rise to a change by 100% for multiloop contributions with overlapping of divergences. This is the reason why studies of NJL models have been restricted to the single-loop contribution in (6). In the N_c^{-1} term in (6), there is also an arbitrariness in how the divergent contributions are summed. However, the direct diagram summation of the contributions of the same N_c^{-1} order leads to the form (6) irrespective of the choice of the cutoff f. Here the summation does not change the order of magnitude; the result is the same as the magnitude of each term.

The actual calculation of B will be done in Sec. 4. Here we discuss the key point in this problem, which makes it possible to extend unambiguously and consistently the gap equation of the unrenormalizable model to the next order in N_c^{-1} and to do this despite the arbitrariness in the cutoff and summation of the multiloop divergent contributions.

FIG. 3. Diagrams of the lowest approximation in N_c of the quark–antiquark scattering amplitude.

At the first sight, the amplitude B calculated in the zeroth order in N_c^{-1} should be used in the expression (6). This order includes only the braid of the simple loops shown in Fig. 3. Any other contribution contains extra powers of N_c^{-1}. Therefore, the B contributions that depend on more than the one variable q^2 (i.e., on s,t,u) can be ignored: In (6), they give terms $\sim N_c^{-2}, N_c^{-3}, \ldots$. When the $B(q^2)$ diagrams in Fig. 3 are summed, a very delicate situation does, however, arise. The denominators of the expression for $B(q^2)$ vanish with respect to q^2, demonstrating the existence of scalar and pseudoscalar bound states. When the gap equation (6) has a solution, one of the pseudoscalar particles must necessarily be a Goldstone state breaking complete (independent of the indices i,i') chiral symmetry (1). A complete set of Goldstone particles must be present in the symmetric limit $\lambda_{ii'}=\lambda_0$. At the same time, if allowance is made for the N_c^{-1} term in Eq. (6) and there are no corresponding contributions in the denominators (the zeroth order for B is being used), the resulting masses do not vanish and are quantities $\sim N_c^{-1}M^2$. The necessary contributions can appear in the B denominators from the contributions of loops with internal insertions (Fig. 4). Their order of magnitude is precisely $\sim \pm N_c^{-1}M^2$, and they do not depend on q^2 for the momenta $|q^2| \ll M^2$ of interest to us in (6). In the B denominators such contributions affect only the expressions for the masses of resonances and could make the corresponding Goldstone terms vanish [when (6 is satisfied].

At the same time, it is clear that in the unrenormalizable theory it is not possible to calculate in a convincing manner the contributions of multiloop diagrams of complicated form, especially since in the present situation we are dealing with the summation of only the parts that diverge quadratically simultaneously for all contours of integration, and the summation of the contributions of a given order can be done in different ways with complete dependence of the result on the form of the cutoff. However, there is no need for this calculation.

The point is that the Goldstone "masses" must be equal to zero when the spontaneous symmetry breaking has occurred, i.e., the gap equation has a solution. Therefore, in the symmetric limit $\lambda_{ii'}=\lambda_0$ a solution of Eq. (6) can be found by setting equal to zero the expressions in the B denominators that correspond to the masses of the Goldstone particles. For $\lambda_{ii'}=\lambda_0$, this operation fixes, as it were, the necessary supplement to the contributions of Figs. 4 and leads to a unique and consistent answer. When $\delta\lambda_{ii'}$ is included perturbatively, as should be done in accordance with the representation (4), a unique procedure can also be identified for the total interaction (1). We shall demonstrate this in the following section.

Thus, the gap equation of the NJL model can be consistently defined in the two leading orders in $N_c \gg 1$.

4. CALCULATION OF THE QUARK–ANTIQUARK INTERACTION AMPLITUDE

Our task is to calculate the sum of the contributions of the diagrams in Fig. 3. The problem is exceptionally simple in terms of matrices analogous to the representation (2). We introduce matrices with respect to the flavor and chiral indices: $\beta^{\alpha_1 i_1,\alpha'_1 i'_1}_{\alpha i,\alpha' i'}$, $G_{\alpha i,\alpha_1 i_1}(p,\Sigma)$. Then for the single-loop contribution $A(q^2,\Sigma)$ we obtain the matrix expression

$$A(q^2,\Sigma) = -\int \frac{d^4p}{2i\pi^2}\,\mathrm{Tr}\left\{\frac{\beta}{M^2}\,G(p,\Sigma)G(p-q,\Sigma)\right\} f\left(\frac{p^2}{M^2}\right). \tag{8}$$

In the leading approximation in N_c, the amplitude $B(q^2,\Sigma)$ is

$$B=(1-A)^{-1}. \tag{9}$$

The condition $|\delta\lambda_{ii'}| \ll \lambda_0$ (4) enables us to seek the matrix that is the inverse of $1-A$ by using perturbation theory with respect to the symmetric system $\delta\lambda=0$:

$$B=B_0+B_0\delta A B_0\ldots, \quad B_0=(1-A_0)^{-1}. \tag{10}$$

In the $U_L(n)\times U_R(n)$-symmetric situation, the unknown mass matrix $\Sigma^{(0)}_{ii'}$ can be represented in diagonal form. This can be achieved by a simultaneous rotation of the R and L systems of flavor bases and can be done with an arbitrary mass matrix. We begin with the calculation of the symmetric limit $\delta\lambda=0$.

We shall consider solutions that are distinguished by the number n' of spontaneously arising massive fermions $(n' \leq n)$. For simplicity, we assume that the masses of all these states are the same m_1. Such a state corresponds to breaking of the chiral and flavor symmetries in the following way:

$$U_L(n)\times U_R(n) \to U_L(n-n')\times U_R(n-n')\times SU_V(n'). \tag{11}$$

Then $n-n'$ quarks remain massless. Now this set of solutions is sufficient to study the situations in which we are interested. In this case, the matrix elements of A and B are diagonal with respect to the flavors, $i=i_1$, $i'=i'_1$, and do not depend on the indices i, i'. The flavors are conserved. Only

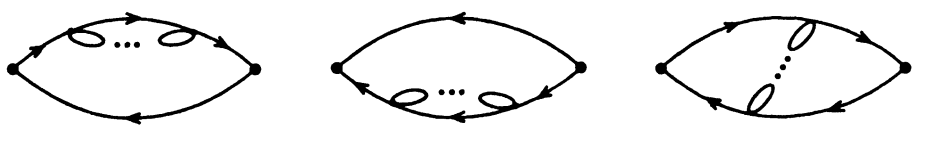

FIG. 4. The N_c^{-1} correction to the simple loop of the amplitude B.

scattering of the massive quarks by one another contributes to the N_1^{-1} term in (6). We seek the amplitude B_0 in the form that, in accordance with (8), holds for A_0:

$$(1-A_0)^{\alpha_1\alpha_1'}_{\alpha\alpha'}=(1-\delta_{\alpha\alpha'})(1-\delta_{\alpha_1\alpha_1'})(A_+\delta_{\alpha\alpha_1}+A_-),$$

$$(B_0)^{\alpha_1\alpha_1'}_{\alpha\alpha'}=(1-\delta_{\alpha\alpha'})(1-\delta_{\alpha_1\alpha_1'})(B_+\delta_{\alpha\alpha_1}+B_-). \quad (12)$$

The equation $(1-A_0)B_0=1$ enables us to find the chirality-changing part B_-. Only it contributes to the N_c^{-1} term of the gap equation (6):

$$B_-=-\frac{A_-}{(A_++2A_-)A_+}=\frac{1}{2}\left(\frac{1}{A_++2A_-}-\frac{1}{A_+}\right). \quad (13)$$

The denominators in the expression (13) are the propagators of a composite scalar particle with mass $m_H \simeq 2m_t$ and a Goldstone state of broken symmetry.[10] These denominators are equal to

$$A_++2A_-=\frac{1}{M^2}\left[\Gamma\left(\frac{m_1^2}{M^2}\right)M^2+\frac{1}{4}\beta_0(-q^2)+\frac{1}{2}\beta_0(4m_1^2-q^2)I(m_1^2,q^2)\right],$$

$$A_+=\frac{1}{M^2}\left[\Gamma\left(\frac{m_1^2}{M^2}\right)M^2+\frac{1}{4}\beta_0(-q^2)+\frac{1}{2}\beta_0(-q^2)\times I(m_1^2,q^2)\right]. \quad (14)$$

Here

$$\beta_0=\frac{\lambda_0 M^2 N_c}{8\pi^2},$$

$$I(m_1^2,q^2)=\int\frac{d^4p}{\pi^2 i}\frac{1}{(m_1^2-p^2)[m_1^2-(p-q)^2]}f\left(\frac{p^2}{M^2}\right), \quad (15)$$

and the function $\Gamma(m_1^2/M^2)$ has the form

$$\Gamma\left(\frac{m_1^2}{M^2}\right)=1-\frac{\beta_0}{M^2}\int\frac{d^4p}{\pi^2 i}\frac{1}{m_1^2-p^2}f\left(\frac{p^2}{M^2}\right)+O\left(\frac{1}{N_c}\right). \quad (16)$$

For the very simple and frequently used cutoff function $f=\vartheta(M^2-|p^2|)$, we obtain

$$\Gamma\left(\frac{m_1^2}{M^2}\right)=1-\beta_0\left(1-\frac{m_1^2}{M^2}\ln\frac{M^2}{m_1^2}\right)+O\left(\frac{1}{N_c}\right). \quad (17)$$

As we have already emphasized, the qualitative results obtained when the contributions of the simple loops are used do not depend on the choice of the cutoff.

The single-loop contribution (8) is responsible for the appearance of only the first two terms of the expressions (16) and (17). The third term represents in the B_0 denominators the possible contribution $\sim N_c^{-1}$ from the multiloop corrections of Fig. 4 discussed in Sec. 3. The single-loop terms (8) are identical to the gap equation (6) in the leading N_c approximation if $\lambda=\lambda_0$. This part of the function $\Gamma(m_1^2/M^2)$ is equal to zero when m_1 is the solution of the single-loop gap equation (Fig. 2a). At the same time, the magnitude of this part is of order N_c^{-1} when allowance is made in (6) for the first term on the right. For this reason, we have augmented (16) and (17) with contributions that could arise only from the following approximations in N_c^{-1}. On the other hand, the expression $\Gamma(m_1^2/M^2)M^2$ determines in (14) the "mass" of the bound state, which after symmetry breaking becomes a pseudoscalar Goldstone particle. If breaking does indeed occur, i.e., the root of Eq. (6) satisfies $m_1^2>0$, then a genuine Goldstone boson with vanishing mass must also appear. In the single-loop approximation, this process can be followed analytically. Therefore, the solution of Eq. (6) should also be sought with a Goldstone mass $\Gamma(m_1^2/M^2)M^2$ equal to zero.

Before we turn to the solution, we consider how the nonsymmetric expression (10) for $B(q^2)$ is calculated. For an arbitrary assumed spectrum of the symmetric problem, i.e., for a general form of the symmetry breaking, this is a rather complicated procedure. We consider it for the case $n'=1$ in the expression (11). If we require precisely the occurrence of a hierarchical solution, then the possibility $n'=1$ is the basis.

The problem requires expansion of the expression (8) with respect to the parameters of the deviation from the values λ_0 and Σ_0 of the symmetric system. The matrix $\lambda=\lambda_0+\delta\lambda$ is now represented in some quite definite basis. For example, this may be a basis in which (1) explicitly conserves the flavors, or a basis in which certain more fundamental couplings are diagonal like (3). The complete matrix B should be sought in such a basis. For the case $n'=1$, we use the general representation of a matrix that possesses one large eigenvalue m_1:

$$\Sigma_{ii'}=\Sigma_0+\delta\Sigma_{ii'}, \quad |\delta\Sigma_{ii'}|\ll\Sigma_0=\frac{m_1}{n}. \quad (18)$$

In such a basis, the symmetric part of the matrix $1-A$ can be found from (8) in the form

$$(1-A_0)^{\alpha_1 i_1,\alpha_1' i_1'}_{\alpha i,\alpha' i'}=(1-\delta_{\alpha\alpha'})(1-\delta_{\alpha_1\alpha_1'})\{[A_1\delta_{ii_1}\delta_{i'i_1'}+A_2(\delta_{ii_1}+\delta_{i'i_1'})+A_3]\times\delta_{\alpha\alpha_1}+(1-\delta_{\alpha\alpha_1})A_4\}. \quad (19)$$

The functions $A_s(q^2,m_1^2)$ can be readily found by substituting in (8) the propagator (5) for $\Sigma_{ii'}=\Sigma_0$. We seek the inverse matrix $B_0(1-A_0)$ in a similar form:

$$(B_0)^{\alpha_1 i_1,\alpha_1' i_1'}_{\alpha i,\alpha' i'}=(1-\delta_{\alpha\alpha'})(1-\delta_{\alpha_1\alpha_1'})\{[B_1\delta_{ii_1}\delta_{i'i_1'}+B_2(\delta_{ii_1}+\delta_{i'i_1'})+B_3]\delta_{\alpha\alpha_1}+B_4\}. \quad (20)$$

We have

$$B_1=A_1^{-1},$$

$$B_3=-\frac{(A_3-A_4)B_1+2[A_2+n(A_3-A_4)]B_2}{A_1+2nA_2+n^2(A_3-A_4)},$$

$$B_2=-\frac{A_2B_1}{A_1+nA_2},$$

$$B_4=-A_4\frac{B_1+2nB_2+n^2B_3}{A_1+2nA_2+n^2(A_3+A_4)}. \quad (21)$$

For the chirality-changing component B_4, we obtain the previous expression (13)–(15): $B_4 \equiv B_-$. All the components B_i are needed in the calculation of the correction in the expression (10) (see Sec. 6). However, as regards the structure of the denominators, the amplitude B is now extremely complicated. It includes the scattering of not only massive quarks by one another but also the scattering of massless quarks. Generally speaking, the denominators for arbitrary bound states cannot be analyzed in the model (1). Therefore, the treatment in Sec. 6 will have a general formal nature: What properties are found if everything is consistent (the masses of all bound states are greater than or equal to zero). In the chirality-changing part B_4, which is important for the gap equation, only massive quarks participate in the symmetric case. This makes the analysis unambiguous.

5. SOLUTION OF THE GAP EQUATION FOR THE SYMMETRIC CASE $\lambda_{ii'}=\lambda_0$

Substitution of (13) in the second term on the right-hand side of Eq. (6) leads to the expression

$$\frac{n'}{N_c}F(m_1^2)=-\frac{2n'}{N_c}\int\frac{d^4q}{\pi^2 i}\frac{m_1^2}{m_1^2-q^2}\times\frac{I(m_1^2,q^2)}{[(4m_1^2-q^2)I(m_1^2,q^2)-q^2/2][(-q^2)I(m_1^2,q^2)-q^2/2]}. \quad (22)$$

The integral can be calculated when $\ln(M^2/m_1^2)\gg 1$; as a result, we obtain

$$F(m_1^2)\simeq-\frac{2\sqrt{2}}{\ln(M^2/m_1^2)}. \quad (23)$$

This accuracy is completely sufficient for our main aim, which is to prove the existence of solutions of a definite type.

The representation of B_- as the difference of the propagators of two bound states [see (13)] explains the sign $F<0$. In this case, the gap equation preserves the critical properties in the N_c^{-1} approximation too: It has a solution $m_1^2\geq 0$ only for $\beta_0\geq 1$. In the symmetric problem, Eq. (6) is diagonal with respect to all the mass flavors and can be reduced to the simple form

$$1-\beta_0^{-1}=\frac{m_1^2}{M^2}\ln\frac{M^2}{m_1^2}+\frac{n'}{\beta_0 N_c}|F(m_1^2)|. \quad (24)$$

The condition for a phase transition (i.e., the existence of solutions $m_1^2\geq 0$),

$$\beta_0\geq 1, \quad (25)$$

remains the same as for the leading order of NJL models.[10]

Thus, we are dealing here with a second-order phase transition: m_1^2 increases with β_0 from the value zero and not abruptly. The situation would be quite different if the sign of the second term were negative. A solution $m_1^2>0$ could exist for all β. The system would have a first-order transition (see the discussion in Ref. 14).

In the limit $N_c\to\infty$, $\beta_0=\text{const}$, there arises for each flavor the n'-independent single-loop equation that is well known in the NJL models. Then the value of the mass does not depend on the number of massive particles. One can compare the energies of the different solutions by calculating for them the shifts of the Dirac negative-energy states:

$$E_n^{n'}[m_1(n')]-E_n(0)=-2n'm_1^2(n')O(\Lambda^2)N_cV. \quad (26)$$

Here $O(\Lambda^2)$ is a quadratically divergent integral over the momenta, and V is the spatial volume. It can be seen from (26) that if the mass m_1 does not depend on n' then the system with maximum number of massive fermions, $n'=n$, has the lowest energy. This system will be the ground state in the leading order in N_c. A mass hierarchy is absent, and the inclusion of $\delta\lambda_{ii'}\ll\lambda_0$ does not change the nature of the spectrum.

When N_c is reduced, n'-dependent solutions $m_1(n')$ appear. Such solutions can readily be found even analytically in the limit $\ln(M^2/m_1^2)\gg 1$, when the first term on the right-hand side of Eq. (24) can be ignored. Then the solution is represented by the expression

$$\frac{m_1^2(n')}{M^2}\simeq\exp\left[-2\frac{\sqrt{2}n'}{(\beta_0-1)N_c}\right]. \quad (27)$$

The region of parameters in which it is valid can be found from the condition

$$(\beta_0-1)N_c\ll\frac{2\sqrt{2}n'}{\ln\left[\frac{\beta_0N_c}{2\sqrt{2}n'}\ln^2\frac{\beta_0N_c}{2\sqrt{2}n'}\right]}\sim\frac{2\sqrt{2}n'}{\ln N_c}. \quad (28)$$

This condition, in turn, overlaps the region in which the solution $n'=1$ (27) is stable:

$$(\beta_0-1)N_c<\frac{2\sqrt{2}}{\ln 2}. \quad (29)$$

It can be seen that the existence of a stable solution $n'=1$, when in the symmetric problem the spectrum consists of one massive fermion and $n-1$ massless fermions, is entirely possible.

It is this solution that is the most convenient for the occurrence of a mass hierarchy. The inclusion of a weak breaking of the $U_L(n-1)\times U_R(n-1)$ symmetry leads to small masses of the other flavors too: $m_s\ll m_1$. A choice of the breaking $\delta\lambda_{ii'}$ of the type (4) makes it possible to reproduce the successive steps of the hierarchy. The flavors s that are diagonal in the masses are superpositions of states with indices i and i'. This circumstance leads to 100% mixing.[12]

The problem of comparing the vacuum energies for the different quark sectors coupled by the interaction (1) can also be investigated in a different way.[5,14] For this, we represent the four-fermion coupling (1) in terms of Yukawa interactions by means of functional integration with respect to auxiliary scalar fields $\varphi_{ii'}$: $(\lambda/4)(\bar{\psi}\psi)^2 \to \bar{\psi}\psi\varphi - \varphi^2/\lambda$ ($2n^2$ real components). This set of fields depends on the nature of the spontaneous transition, i.e., on n'. For $n'=n$, one can take as the auxiliary fields a $SU_V(n)$ scalar ϕ, a pseudoscalar $\bar{\phi}$, and $SU_V(n)$ vectors (pseudo) $\phi^{(\tau)}$ and $\bar{\phi}^{(\tau)}$; for the case $n'=1$, one should introduce $SU_L(n)\times SU_R(n)$ spinors $\varphi_{ii'}$. For $\beta>1$, the vacuum expectation values become, respectively, ϕ=const ($n'=1$) with potential on the scale M

$$V(\phi)=(\bar{\psi}\psi)\phi-\frac{n}{\lambda_0}\phi^2, \quad \phi=\frac{2}{n}\sum_i \varphi_{ii},$$

$$\bar{\psi}\psi=\sum_i \bar{\psi}_i\psi_i, \quad \psi_i=\psi_{Li}+\psi_{Ri}, \tag{30}$$

and the component of the double spinor φ_{11}=const ($n'=n$) with potential

$$V(\varphi_{11})=\bar{\psi}_{L1}\psi_{R1}\varphi_{11}+\bar{\psi}_{R1}\psi_{L1}\varphi_{11}^{+}-\frac{1}{\lambda_0}|\varphi_{11}|^2. \tag{31}$$

Integrating over ψ and $\bar{\psi}$, we determine the effective vacuum potential of the fields ϕ and φ_{11} on the scale $\mu \ll M$. In the limit $\ln(M^2/\mu^2)\gg 1$, we can restrict ourselves to the terms up to ϕ^4 and $|\varphi|^4$ inclusively. In the single-loop approximation, this problem is presented in detail in Refs. 5 and 14.

It can be asserted that the N_c^{-1} corrections mainly influence the part of the effective potential that is quadratic in ϕ^2 or $|\varphi_{11}|^2$. These corrections change the vacuum expectation values $\langle\phi\rangle$ or $\langle\varphi_{11}\rangle$, i.e., the calculated masses of the fermions $m_1=h\langle\phi\rangle$ and $h\langle\varphi_{11}\rangle$. [The induced Yukawa constants are $h\sim[\ln M^2/\mu^2]^{-1}$ (Ref. 5).] As we already know, the transition $\beta=1$ is a second-order phase transition, and at this point the coefficient of ϕ^2, $|\varphi_{11}^2|$ simply changes sign. The numerical changes from the N_c^{-1} corrections should not affect the qualitative picture: The nature of the transition is not changed in the N_c^{-1} approximation. The use of vacuum expectation values calculated in terms of the fermionic masses $\langle\varphi\rangle=m_1/h$ known from (24) makes it possible to confirm the results of the analysis of the Dirac shifts.

The state $n'=1$ convenient for the occurrence of a hierarchy can very well be the ground state of a many-flavor quark system.

6. PROPERTIES OF THE EQUATIONS FOR THE LIGHT GENERATIONS

Here the program of actions appears obvious. On the basis of the solution $n'=1$ of the symmetric problem, it is necessary to use perturbation theory, expanding in the parameter

$$\delta\beta_{ii'}=\frac{\delta\lambda_{ii'}M^2N_c}{8\pi^2}, \tag{32}$$

and in the nonsymmetric part of the mass matrix $\delta\Sigma_{ii'}$ (18). This must lead to a linear system, the solution of which determines the spectrum of the light particles.

We shall solve, as it were, the opposite problem: What properties of the constants $\delta\beta_{ii'}$ characterizing the interaction at the scale M ensure the occurrence of the known features of the quark spectrum—the mass hierarchy $m_1\gg m_2\gg$... and weak mixing? The investigation is severely complicated by the abundance of small parameters: $\delta\beta/\beta_0$, m_1^2/M^2, N_c^{-1}, m_2^2/m_1^2, ... Depending on the ratios between them, different possibilities arise. The number of parameters and possibilities increases with the transition to the lighter states. We restrict ourselves to investigating the first approximation, i.e., the second (heaviest) generation. We first of all consider the derivation of the equation.

To simplify the expressions, we consider symmetric constants $\lambda_{ii'}=\lambda_{i'i}$ (parity conservation) and symmetric mass matrices $\Sigma_{ii'}$ corresponding to them. The general case differs only technically. Besides $\beta_{ii'}$, a dependence on the flavor indices enters the equations for the second generation through the tensors

$$\delta_{ii'},\Sigma_{ii'},\Delta_{ii'}(\Sigma^2)=\delta_{ii'}\ \mathrm{Tr}\ \Sigma^2-(\Sigma^2)_{ii'},(\Sigma\Delta(\Sigma^2))_{ii'}. \tag{33}$$

This is readily understood if we calculate the propagator from the expression (5):

$$G_{ii'}(p,\Sigma)=\hat{p}\{(p^2)^{n-1}\delta_{ii'}-(p^2)^{n-2}\Delta_{ii'}(\Sigma^2)+\cdots\}\frac{1}{D(p^2)}+\{(p^2)^{n-1}\Sigma_{ii'}-(p^2)^{n-2}(\Sigma\Delta(\Sigma^2))_{ii'}+\cdots\}\frac{1}{D(p^2)},$$

$$D(p^2)=(m_1^2-p^2)(m_2^2-p^2)\cdots(m_n^2-p^2). \tag{34}$$

The omitted terms contain the small mass m_3. The ratio of their order of magnitude to the terms retained in (34) is of order m_2m_3/m_1^2 when $p^2\sim m_1^2$. The number of tensors increases in the transition to the subsequent generations.

Substituting (34) in the expression (8) and making the expansion $\beta\to\beta_0+\delta\beta$, $\Sigma\to\Sigma_0+\delta\Sigma$, we calculate the correction to the simple loop δA. It can be written in the form

$$(\delta A(q^2))_{\alpha i,\alpha',i'}^{\alpha_1 i_1,\alpha_1' i_1'}=(1-\delta_{\alpha\alpha'})(1-\delta_{\alpha_1\alpha_1'})\times\Bigg\{\Bigg[\frac{M^2}{m_1^2}\delta\beta_{ii'}\delta_{ii_1}\delta_{i'i_1'}+a^{(1)}(q^2,\delta\Sigma)\delta_{ii_1}\delta_{i'i_1'}+a^{(2)}(q^2)\frac{\delta_{\delta\Sigma}(\Delta(\Sigma^2))_{ii'}}{m_1^2}\Bigg]\delta_{\alpha\alpha_1}+(1-\delta_{\alpha\alpha_1})\Bigg[a^{(3)}(q^2)\frac{\delta\Sigma_{ii'}}{m_1}+a^{(4)}(q^2)\frac{\delta_{\delta\Sigma}(\Sigma\Delta\Sigma^2)_{ii_1}}{m_1^3}\Bigg]\Bigg\}\frac{m_1^2}{M^2}. \tag{35}$$

The term $a^{(1)}$ leads to a contribution to the gap equation (6) that does not depend on the indices, and therefore it can be ignored. All such contributions must be included in the part of (6) that does not depend on the indices—the equation of the symmetric problem (24); they determine additional corrections to m_1. The leading parts of the remaining terms $a^{(i)}$ are

$$a^{(2,3,4)}(q^2)=\beta_0\int\frac{d^4p}{\pi^2 i}\frac{1}{(m_1^2-p^2)[m_1^2-(p-q)^2]}\left[-\frac{(p-q)p}{p^2},1,-\frac{m_1^2}{p^2}\right], \quad (36)$$

and $a^{(i)}$ depends on $\ln(M^2/m_1^2)$. The small factor m_1^2/M^2 in (35) is compensated by similar factors in the denominators of the amplitudes B_0 in (10). This can be clearly seen from the expressions (14) and (17), which give the denominators of the amplitudes B_3 and B_4 from the expression (21). This assertion is also true for the other B_0 components. Therefore, the term $\delta\beta_{ii'}$ in the expression (35) is significantly enhanced when $m_1^2/M^2 \ll 1$. Its appearance is a result of the quadratic divergence of the model and the presence in it of corrections $\sim M^2$, which for the first time are not compensated by the gap equation in our analysis. Therefore, the enforced smallness of $\delta\beta_{ii'}$ [see (40) and (41)] is probably a model effect.

We substitute (35) and (20) in the expression (10) and select the chirality-changing components δB. We integrate the resulting correction to the amplitude with respect to the momentum in the N_c^{-1} term (6). The single-loop term in (6) is also expanded with respect to $\delta\Sigma$ and $\delta\beta$. We distinguish in the equation for m_1 the part that does not depend on the flavor indices explicitly (it can contain complete sums over i and i'); this equation will now have an accuracy better than (24). The procedure we have described leads us to the required system of equations. In this cumbersome expression, we exhibit only the qualitatively important terms. We represent the remainder symbolically, retaining their main features. We have

$$\frac{\delta\Sigma_{ii'}}{m_1}\left[1-\beta_0^{-1}-\frac{m_1^2}{M^2}\ln\frac{M^2}{m_1^2}+\frac{1}{N_c\beta_0}f^{(1)}\right]+\frac{\delta_{\delta\Sigma}(\Sigma\Delta(\Sigma^2))_{ii'}}{m_1^3}\left[\frac{m_1^2}{M^2}\ln\frac{M^2}{m_1^2}+\frac{1}{N_c\beta_0}f^{(2)}\right]+\frac{1}{N_c\beta_0}f^{(3)}\left[\sum_i\frac{\delta\Sigma_{ii'}}{m_1}+\sum_{i'}\frac{\delta\Sigma_{ii'}}{m_1}\right]=-\frac{\delta\beta_{ii'}}{\beta_0}-\frac{1}{N_c}\frac{M^2}{m_1^2}f^{(4)}\left[\sum_{i'}\frac{\delta\beta_{ii'}}{\beta_0}+\sum_i\frac{\delta\beta_{ii'}}{\beta_0}\right]. \quad (37)$$

Here the functions $f^{(i)}$ are finite but may depend on $\ln(M^2/m_1^2)$ through the influence of the factors (36). For example, we write down two of them, which determine the terms that are most discussed in what follows:

$$f^{(1)}=\frac{\beta_0}{2}\left(\frac{m_1^2}{M^2}\right)^2\int\frac{d^4q}{\pi^2 i}\frac{[B_1(q^2)+nB_2(q^2)]^2}{(m_1^2-q^2)m_1^2}a^{(3)}(q^2),$$

$$f^{(4)}=\frac{\beta_0}{2}\left(\frac{m_1^2}{M^2}\right)\int\frac{d^4q}{\pi^2 i}\frac{B_1(q^2)+nB_2(q^2)}{(m_1^2-q^2)m_1^2}n^2B_4(q^2). \quad (38)$$

It is readily verified that all the $f^{(i)}$, represented as functions of m_1, do not depend on the number of generations n. Their actual form is unimportant for the further discussion.

The coefficient of $\delta\Sigma_{ii'}$ differs only in the term $\sim N_c^{-1}$ from the gap equation (24): The function $f^{(1)}$ is very different from the expression (22). However, in the limit $N_c\to\infty$, the first term of (37) drops out, and it turns out that the remaining equation does not have a solution. The fact is that the differential of the matrix $\Sigma\Delta(\Sigma^2)$ at the point (18) satisfies identically the conditions

$$\sum_i\delta_{\delta\Sigma}(\Sigma\Delta(\Sigma^2))_{ii'}=\sum_{i'}\delta_{\delta\Sigma}(\Sigma\Delta(\Sigma^2))_{ii'}\equiv 0, \quad (39)$$

and this is impossible for arbitrary $\delta\beta_{ii'}$ and does not have a sensible physical interpretation. This is one more piece of evidence for the absence of a hierarchical solution in the single-loop approximation (see the discussion of Sec. 5).

In the opposite limit, $m_1^2/M^2\ll N_c^{-1}$, the left-hand side of Eq. (37) contains a small quantity $N_c^{-1}(\delta\Sigma/m_1)$. It is obvious that $\delta\beta$ on the right-hand side of (37) must have the same order of smallness:

$$|\delta\beta|\leqslant N_c^{-1}\left|\frac{\delta\Sigma}{m_1}\right|. \quad (40)$$

The second term on the right-hand side strengthens the bound even more:

$$|\delta\beta|\sim\frac{m_1^2}{M^2}\left|\frac{\delta\Sigma}{m_1}\right|. \quad (41)$$

Then the dependence of the solution (37) on the flavor indices in the lowest order of this procedure has the specific nature

$$\delta\Sigma_{ii'}=\delta\Sigma_i+\delta\Sigma'_{i'},\quad \delta\Sigma_i\sim\frac{M^2}{m_1^2}\sum_{i'}\delta\beta_{ii'},\quad \delta\Sigma'_{i'}\sim\frac{M^2}{m_1^2}\sum_i\delta\beta_{ii'}. \quad (42)$$

In the proof of (42), we have again omitted in the equation the contributions that do not contain the indices explicitly, relating them to the equation for m_1. The higher orders of the expansion in $\delta\Sigma$ and $\delta\beta$ lead to the appearance of a dependence on i, i' of general form.

A solution beginning with terms of the type $\delta\Sigma_i+\delta\Sigma'_{i'}$ is attractive from the physical point of view. For then the elements of the matrix $\delta\Sigma_{ii'}$ are related in the following manner to the spectrum of states:

$$\frac{|\delta\Sigma|}{m_1}\sim\frac{|\delta\Sigma'|}{m_1}\sim\sqrt{\frac{m_2}{m_1}}. \quad (43)$$

The presence in the mass matrix of parameters like the square root of a mass ratio implies that the ratio V_{us}/V_{ud} of the elements of the matrix of the weak mixing is also of the same order of magnitude. The numerical correspondence was noted long ago at the phenomenological level.[15] The correspondence of the steps of the hierarchies in the mass matrix and mixing matrix was investigated in Ref. 12.

On the other hand, if the masses of the light generations arise from radiative corrections (the radiative hypothesis, see Ref. 8 and the references in that study), then the appearance of square roots means that the series for the mass matrix contains the roots of radiative corrections, i.e., odd powers of the coupling constants of the conjectured perturbative interaction. Such a series differs from a normal perturbation theory but is entirely possible in a situation in which fields that do and do not distinguish flavor act simultaneously. An expression of the form (42) could be specific evidence of the physical complexity of the spectrum problem, of the participation of different forces in the phenomenon.

In such a situation, smallness of the flavor-dependent part of the coupling constant $\delta\beta_{ii'}$ is also natural. The expansion

$$\beta_{ii'}=\beta_0+O\left(\frac{m_1^2}{M^2},N_c^{-1}\right)\left(a\sqrt{\frac{m_2}{m_1}}+b\,\frac{m_2}{m_1}+\dots\right) \qquad (44)$$

is also unlike an ordinary perturbative expansion. But it can be understood if both perturbative and nonperturbative forces act simultaneously and the latter do not distinguish the flavors. The distinguishing part of the effective interaction (44) can then contain an additional smallness in the form of the ratio of the weak to the strong components of the field.

In Refs. 12, we proposed a qualitative scenario for realizing this situation in a system close to the standard model. The existence of a ground state with one massive fermion in the symmetric problem of n flavors was a necessary hypothesis for the subsequent semiphenomenological recovery of the properties of the spectrum. The answer reduces to a situation in which practically all the qualitative features of the quark spectrum, including the properties of the weak-mixing matrix, are reproduced if the weak perturbative component of the forces is represented by a chiral vector coupling.

7. CONCLUSIONS

We consider once more the problem of the fine tuning of the parameters of the system, the need for which is widely discussed in connection with the problem of the dynamical occurrence of fermion masses (see Refs. 5, 13, and 14). Equation (24) makes this disagreeable procedure somewhat less problematic, since the constant β_0 can now be displaced from the critical point $\beta_0=1$ by an amount $[N_c\ln(M^2/m_1^2)]^{-1}$, whereas in the single-loop approximation the deviation was $(m_1^2/M^2)\ln(M^2/m_1^2)$. The difference is particularly impressive if one substitutes the values used in Ref. 5: $M\geqslant 10^{15}$ GeV, the "grand unification" energy, and $N_c=3$.

Of course, the problem that we have solved is far from the properties of the real quarks. Many features of the standard model are not reflected in it. Nevertheless, an attempt has been made to take into account in part the possible conditions at high energies. The appearance under these circumstances of solutions that make it possible to reproduce the properties of the real quarks seems promising.

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[1)]These fields would have to develop a strong coupling precisely in the region $\sim M$ or have masses $\sim M$. The vector fields could include gluons that carry color but do not distinguish flavors and vector fields that distinguish i or i'.

[2)]The terms independent of the indices in the expressions (2) for the matrix elements signify their presence for all values of the indices. The same holds for the expressions (4), (12), (18)–(20), and (35).

[3)]In the order N_c-1, the form (5) is approximate. In accordance with (6), for $G^{-1}=\Sigma-Z\hat{p}$ and $p^2\to\infty$ we have $\Sigma\approx\text{const}+O(M_1^2/N_cp^2)$, $Z\approx 1+O(N_c^{-1})$. The influence of the deviations from the case $\Sigma=\text{const}$, $Z=1$ reduces to an unimportant redetermination of M^2 or to corrections of order M_1^2/N_cM^2.

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