Spin dynamics of the superfluid *B* phase and spatially inhomogeneous coherently precessing structures

Yu. G. Makhlin and V. P. Mineev

L. D. Landau Institute of Theoretical Physics, Russian Academy of Sciences, 117334 Moscow, Russia
(Submitted 17 May 1995)
Zh. Éksp. Teor. Fiz. **109,** 441–470 (February 1996)

The equations of the inhomogeneous spin dynamics of the superfluid B phase of ^{3}He are derived. In the hydrodynamic region, a generalization of the Leggett–Takagi two-fluid hydrodynamics to the spatially inhomogeneous case is obtained. Near the critical temperature, approximate equations for the macroscopic description of the collisionless spin dynamics of ^{3}He-B are found. A solution of these equations is obtained that represents a coherently precessing inhomogeneous distribution of the magnetization; as $T\to T_c$, this solution goes over into the recently discovered two-component structure in normal ^{3}He. It is shown that, in contrast to normal helium, the two-component coherently precessing distribution in ^{3}He-B exists both in a weakly inhomogeneous and completely homogeneous external field. An investigation is made of the relaxation of the structure and the temperature region in which it exists. [S1063-7761(96)01002-3]

1. INTRODUCTION

In the B phase of superfluid ^{3}He, the coherently precessing two-domain spin structure that exists in an inhomogeneous magnetic field at temperatures somewhat below the temperature of the superfluid transition, $T<0.8T_c$, has been thoroughly studied. In this structure, the deviation of the magnetization from the equilibrium direction ($\mathbf{S}\|\mathbf{H}$) increases in the direction in which the field decreases and reaches 104° (the angle is fixed by the spin–orbit interaction). The lifetime of such a structure significantly exceeds the time of dephasing of the precession due to the field inhomogeneity. Fomin[2] showed that the coherent precession in the inhomogeneous field is due to the flow of a superfluid spin current, which leads to the formation of a two-domain structure and cancellation of the spatial dependence of the Larmor frequency.

Recently, an analogous structure with coherent spin precession was also observed in a normal Fermi liquid [in a solution of ^{3}He in ^{4}He (Refs. 3 and 4) and in ^{3}He (Ref. 5)] in the collisionless region $\omega\tau>1$. Such a structure can be investigated by means of the equations of the spin dynamics of a normal Fermi liquid:

$$\frac{\partial \mathbf{S}}{\partial t}+\frac{\partial \mathbf{J}_i}{\partial x_i}=\mathbf{S}\times\omega_L, \tag{1}$$

$$\frac{\partial \mathbf{J}_i}{\partial t}+\frac{w^2}{3}\frac{\partial}{\partial x_i}\left(\mathbf{S}-\frac{\chi_n\omega_L}{\gamma^2}\right)=\mathbf{J}_i\times\omega_L+\kappa\frac{\gamma^2}{\chi_n}\mathbf{J}_i\times\mathbf{S}-\frac{\mathbf{J}_i}{\tau_1}. \tag{2}$$

Here $\mathbf{S}$ is the spin density, $\mathbf{J}_i$ is the density of the spin current, χ_n is the magnetic susceptibility, γ is the gyromagnetic ratio for ^{3}He nuclei, and the Larmor frequency $\omega_L=\gamma H$ can depend on the coordinates. Here we have written $w^2=v_F^2(1+F_0^a)(1+F_1^a/3)$, $\kappa=(F_1^a/3-F_0^a)/(1+F_0^a)$, $\tau_1=\tau/(1+F_1^a/3)$, and F_0^a and F_1^a are coefficients of the expansion in spherical harmonics of the spin–spin part of the Fermi-liquid interaction of the quasiparticles. These equations were derived by Leggett[6] from a kinetic equation for the quasiparticle distribution function $\delta\nu(\mathbf{r},\mathbf{k},t)$.

The collisionless spin dynamics of a normal Fermi liquid reduces in a remarkable manner to equations for the spin and current representing the zeroth and first harmonics of the function $\delta\nu$ with respect to $\hat{\mathbf{k}}$. The remaining harmonics are small if the characteristic scale λ of the spatial inhomogeneity of the function $\delta\nu$ is large compared with v_F/ω_m, where $\omega_m=\gamma H_m\sim\omega_L$ is the characteristic frequency corresponding to the molecular field H_m in the Fermi liquid.

The solution of Eq. (2) shows that in the collisionless region $\omega\tau>1$ there is a dissipationless diffusion current due to the Fermi-liquid interaction,

$$\mathbf{J}_i\approx\frac{\chi}{\gamma^2}\frac{w^2}{3\kappa S^2}\left(\mathbf{S}\times\frac{\partial\mathbf{S}}{\partial x_i}\right), \tag{3}$$

and this gives rise to a coherently precessing two-component structure. In contrast to superfluid ^{3}He-B, in the normal liquid a domain with equilibrium orientation of the magnetization is situated in a region of lower fields—the dissipationless diffusion spin current in an inhomogeneous magnetic field flows in the direction opposite to the superfluid spin current.

When the temperature is lowered below T_c, the long-lived induction signal from the precessing spin structure persists in a certain interval of temperatures near T_c and in the B phase of superfluid ^{3}He. Although both the magnitude of the signal and the nature of the relaxation processes are changed, since near T_c the superfluid spin current is small compared with the dissipationless diffusion current, it is to be expected that in this temperature range we shall be dealing with a coherently precessing two-domain structure similar to the one observed in the normal liquid.

The coherently precessing two-domain structures that exist in ^{3}He-B at $T<0.8T_c$ have been studied theoretically in the hydrodynamic region $\omega\tau<1$. To describe the structures that arise by virtue of the dissipationless diffusion spin currents, we need equations for the spin dynamics of a superfluid Fermi liquid in the collisionless regime. The corresponding system of equations consists of evolution equations for the total spin, equations of motion of the order parameter, and a kinetic equation for the distribution function of the excitations. As in a normal Fermi liquid, in the B phase of ^{3}He in the hydrodynamic limit there can be a transition from the kinetic equation to equations for the macroscopic variables $\mathbf{S}_q$ (the spin density) and $\mathbf{J}_q$ (the current of the excitations). As a result of this transition, which is made in the present paper, one obtains a system of equations of two-fluid spin hydrodynamics that generalize the well-known system of Leggett and Takagi[7] to the spatially inhomogeneous case.

In the collisionless regime, it is no longer possible to replace the kinetic equation by equations for the first two harmonics $(\mathbf{S}_q,\mathbf{J}_q)$ of the distribution function $\delta\nu$ with respect to $\mathbf{k}$. However, near T_c one can derive approximate equations that describe the dissipationless collisionless spin dynamics.

In this paper, we derive the equations of the two-fluid spin hydrodynamics (Sec. 3) and also, near the critical temperature, approximate equations of the collisionless spin dynamics of ^{3}He-B (Sec. 4). We find a solution of these equations that describes a two-component coherently precessing structure (Sec. 5). We show that, unlike normal ^{3}He and like ^{3}He-A in the hydrodynamic regime,[8] a domain wall in ^{3}He-B exists near T_c in both a weakly inhomogeneous and in a homogeneous external field.

In Sec. 6, we investigate the stability of the precessing structure. Section 7 is devoted to a study of the relaxation of this structure.

Structure resulting from the dissipationless diffusion spin current can form only near the critical temperature, where it predominates over the superfluid spin current. The strengths of these currents are compared in Sec. 8.

2. DERIVATION OF THE BASIC EQUATIONS

Leggett's system of equations of the spin dynamics of superfluid ^{3}He in local equilibrium consists of the evolution equation for the total spin $\mathbf{S}$ and the equation of motion of the order parameter:

$$\frac{\partial \mathbf{S}}{\partial t}=\mathbf{S}\times\omega_L-\nabla_i\mathbf{J}_i+\mathbf{R}_D, \tag{4}$$

$$\frac{\partial \mathbf{d}}{\partial t}=\mathbf{V}\times\mathbf{d}. \tag{5}$$

The order parameter—the expectation value of the operator for annihilation of a Cooper pair—can be found in terms of the vector $\mathbf{d}(\mathbf{k})$ (triplet pairing):

$$\langle\psi_\alpha\psi_\beta\rangle\sim\{i(\hat{\sigma}\mathbf{d}(\mathbf{k}))\hat{\sigma}_y\}_{\alpha\beta}. \tag{6}$$

Here $\hat{\sigma}=(\hat{\sigma}_x,\hat{\sigma}_y,\hat{\sigma}_z)$ is the vector whose components are the Pauli matrices. For the B phase, the vector $\mathbf{d}(\mathbf{k})$ has the form

$$d_\alpha(\mathbf{k})=\Delta R_{\alpha i}\hat{k}_i e^{i\phi}, \tag{7}$$

where $R_{\alpha i}(\mathbf{n},\theta)$ is a matrix of three-dimensional rotations, $\mathbf{n}$ is the direction of the rotation axis, θ is the rotation angle, and $e^{i\phi}$ is the complex phase factor that is responsible for the ordinary (bulk) superfluidity, in which we are not interested in this paper, so that we shall ignore it in all the expressions given below. In Eqs. (4) and (5), $\mathbf{V}$ is the angular velocity of rotation of the order parameter (see below), $\mathbf{J}_i$ is the spin current, which in local equilibrium is determined by the gradient of the order parameter (see below),

$$\mathbf{R}_D=-\mathbf{n}\,\frac{\partial U_D}{\partial\theta} \tag{8}$$

is the moment of the dipole forces, and

$$U_D=\tfrac{8}{15}\Omega_B^2(\cos\theta+\tfrac{1}{4})^2 \tag{9}$$

is the dipole energy. We shall consider motions with frequencies that are low compared with the value of the order parameter, $\omega\ll\Delta$, and with characteristic spatial scale of variation of the quantities that is large compared with the coherence length. Under such restrictions, the order parameter merely rotates in spin space in accordance with Eq. (5) but is not distorted, remaining the order parameter of the B phase.

To study the dissipative spin hydrodynamics, and also in the nonhydrodynamic (collisionless) case, it is necessary to add to the system (4)–(5) the kinetic equation for the spin part $\delta\nu_k(\mathbf{r},t)$ of the quasiparticle distribution function $\delta\nu_k(\mathbf{r},t)$:

$$\delta\nu_{\alpha\beta}=\tfrac{1}{2}(\delta_{\alpha\beta}+\boldsymbol{\sigma}_{\alpha\beta}\delta\boldsymbol{\nu}). \tag{10}$$

It is convenient to study the quasiparticle kinetics in a coordinate system attached to the order parameter, i.e., rotated relative to the original one by means of the matrix $R_{\alpha i}(\mathbf{r},t)$ at each point $(\mathbf{r},t)$. The obvious advantage of such a device, which is analogous to the Galileo transformation[9] usually employed for the variables associated with superfluid mass transport, is that in the derivation of the equations in the rotated coordinate system it is not necessary to differentiate the vector $\mathbf{d}(\mathbf{k})$. The kinetic equation (in the frame of reference determined by the order parameter) has the form[10]

$$\frac{\partial}{\partial t}\,\delta\boldsymbol{\nu}+\frac{\xi}{E}(\boldsymbol{\nu}_F\cdot\nabla)\delta\boldsymbol{\mu}+\frac{1}{\hbar}\,\delta\boldsymbol{\nu}\times\delta\mathbf{E}=\mathbf{I}(\delta\boldsymbol{\nu}). \tag{11}$$

Here $\mathbf{I}(\delta\nu)$ is the collision integral, and $\delta\mathbf{E}$ is the change (of the spin part) of the local energy of the quasiparticles due to the external and molecular magnetic fields and the motion of the condensate:

$$\frac{1}{\hbar}\,\delta\mathbf{E}=-\left[k_i\mathbf{A}_i+\frac{\xi}{E}\,\mathbf{X}-\left(1-\frac{\xi}{E}\right)\hat{\mathbf{d}}(\hat{\mathbf{d}}\cdot(k_i\mathbf{A}_i-\mathbf{X}))\right], \tag{12}$$

where $\xi\cong v_F(k-k_F)$, $E=\sqrt{\Delta^2+\xi^2}$ is the energy of the Bogolyubov excitations,

$$\mathbf{X}=\mathbf{V}+\boldsymbol{\omega}_L-\frac{\gamma^2}{\chi_{n0}}F_0^\alpha\mathbf{S}, \tag{13}$$

and χ_{n0} is the magnetic susceptibility of the normal liquid without the Fermi-liquid corrections. As in Ref. 10, we shall take into account only the zeroth harmonic of the function of the Fermi-liquid interaction of the quasiparticles. The vectors $\mathbf{V}$ and $\mathbf{A}_i$ are the angular velocity of the rotation of the order parameter and the spin superfluid velocity, respectively. In the laboratory coordinate system, they have the form

$$V_\alpha=\frac{1}{2}e_{\alpha\beta\gamma}R_{\beta j}\frac{\partial}{\partial t}R_{\gamma j}, \tag{14}$$

$$A_{\alpha i}=\frac{1}{2m^*}e_{\alpha\beta\gamma}R_{\beta j}\nabla_i R_{\gamma j} \tag{15}$$

and are related by the "Josephson equation"

$$\frac{\partial}{\partial t}\mathbf{A}_i=\frac{1}{m^*}\nabla_i\mathbf{V}-\mathbf{A}_i\times\mathbf{V}_i, \tag{16}$$

where the last term appears because the rotation group is non-Abelian.

It is readily seen that Eq. (5) is simply a different way of writing the definition (14). Our expression acquires the meaning of an equation if $\mathbf{V}$ in it is expressed in accordance with the relation (13),

$$\delta\boldsymbol{\mu}=\delta\boldsymbol{\nu}-\varphi'\delta\mathbf{E} \tag{17}$$

is the deviation of the quasiparticle distribution function from the local-equilibrium value, $\varphi(E)=-(1/2)\tanh(E/2T)$, $\varphi'=\partial\varphi/\partial E$.

Preparing to go over from the microscopic description of the spin dynamics of the excitations by means of the kinetic equation to a description in terms of the macroscopic variables of the spin and the current of the excitations, we introduce the spin density and spin current in the above coordinate system determined by the instantaneous value of the order parameter:[10]

$$\mathbf{S}=\frac{\hbar}{2}\sum_k\left[\frac{\xi}{E}\delta\boldsymbol{\nu}+\left(1-\frac{\xi}{E}\right)\hat{\mathbf{d}}(\hat{\mathbf{d}}\cdot\delta\boldsymbol{\nu})\right]+\frac{\hbar^2}{2}\sum_k\frac{\varphi\Delta^2}{E^3}\hat{\mathbf{d}}\times(\mathbf{d}\times\hat{\mathbf{X}}), \tag{18}$$

$$\mathbf{J}_i=\frac{\hbar}{2}\sum_k v_{Fi}\left[\delta\boldsymbol{\nu}-\left(1-\frac{\xi}{E}\right)\hat{\mathbf{d}}(\hat{\mathbf{d}}\cdot\delta\boldsymbol{\nu})\right]-\frac{\hbar^2}{2}\sum_2\frac{\varphi\Delta^2}{E^3}v_{Fi}k_j\hat{\mathbf{d}}(\hat{\mathbf{d}}\cdot\mathbf{A}_j)-\frac{\hbar}{2}S_0\mathbf{A}_i, \tag{19}$$

where $S_0=\hbar n/2$ is the maximum spin density, and n is the density of the liquid. The first terms on the right-hand sides of these expressions contain the contribution to the spin density and the spin current from the change in the number of the Bogolyubov quasiparticles, while the second terms contain the contribution from the change in the states of the Cooper pairs. Accordingly, we define the quasiparticle spin $\mathbf{S}^q$ as the first term in (18):

$$\mathbf{S}^q=\frac{\hbar}{2}\sum_k\left[\frac{\xi}{E}\delta\boldsymbol{\nu}+\left(1-\frac{\xi}{E}\right)\hat{\mathbf{d}}(\hat{\mathbf{d}}\cdot\delta\boldsymbol{\nu})\right], \tag{20}$$

and the condensate spin $\mathbf{S}^p$ as the second term (cf. Refs. 7, 10):

$$\mathbf{S}^p=\frac{\hbar^2}{2}\sum_k\frac{\varphi\Delta^2}{E^3}\hat{\mathbf{d}}\times(\mathbf{d}\times\hat{\mathbf{X}})=\frac{\chi_{p0}}{\gamma^2}\mathbf{X}. \tag{21}$$

In local equilibrium ($\delta\mu=0$),

$$\mathbf{S}^q=\frac{\chi_{q0}}{\gamma^2}\mathbf{X},\quad \mathbf{S}=\frac{\chi_0}{\gamma^2}\mathbf{X}, \tag{22}$$

where the susceptibilities of the quasiparticles, the condensate, and the liquid as a whole, without allowance for the Fermi-liquid corrections, are

$$\chi_{q0}=\frac{2Y_2+Y_0}{3}\chi_{n0}, \tag{23}$$

$$\chi_{p0}=\frac{2}{3}(1-Y_2)\chi_{n0}, \tag{24}$$

$$\chi_0=\frac{2+Y_0}{3}\chi_{n0}. \tag{25}$$

The ordinary Yosida function Y_0 and the modified function Y_2 are given by

$$Y_n=\int\left(\frac{\xi}{E}\right)^n(-\varphi')d\xi. \tag{26}$$

Both these functions are equal to 1 in the normal phase and vary from 1 to 0 as the temperature is decreased from T_c to zero.

With regard to the expression for the spin current, it is convenient to identify in it the term that represents the current in local equilibrium,

$$J_{\alpha i}^{\rm loc}=\rho_{\alpha i,\beta j}^s A_{\beta j}, \tag{27}$$

and the term that corresponds to the deviation of the current from the equilibrium value:

$$\delta\mathbf{J}_i=\frac{\hbar}{2}\sum_k v_{Fi}\left[\delta\boldsymbol{\mu}-\left(1-\frac{\xi}{E}\right)\hat{\mathbf{d}}(\hat{\mathbf{d}}\cdot\delta\boldsymbol{\mu})\right]. \tag{28}$$

Thus,

$$\mathbf{J}_i=\mathbf{J}_i^{\rm loc}+\delta\mathbf{J}_i. \tag{29}$$

The expression for the superfluid spin density:

$$\rho_{\alpha i,\beta j}^s=-\frac{\hbar S_0}{2}(1-Y_0)\left(\delta_{\alpha\beta}\delta_{ij}-\frac{1}{5}P_{\alpha i,3j}\right) \tag{30}$$

is obtained after the substitution of $\delta\boldsymbol{\nu}=\phi'\delta\mathbf{E}$ in (19) and calculation of the sums over the momenta. In the normal phase, $\rho_{\alpha i,\beta j}^s$ vanishes, and therefore in the limit $T\to T_c$ the current $\delta\mathbf{J}_i$ goes over into the total spin current of the normal liquid. We have introduced the notation

$$P_{\alpha i,\beta j}=\delta_{\alpha\beta}\delta_{ij}+R_{\alpha i}R_{\beta j}+R_{\alpha j}R_{\beta i}=15\langle n_i n_j \hat{\mathbf{d}}_\alpha \hat{\mathbf{d}}_\beta\rangle_{\mathbf{n}}. \tag{31}$$

Differentiating $\mathbf{S}^q$ and $\delta\mathbf{J}_i$ with respect to the time in accordance with the kinetic equation (11) and separating in the derivative the terms corresponding to the same quantities, we obtain two equations of the spin dynamics. The transition to the laboratory coordinate system in the final equations reduces to replacement of the time and spatial derivatives in accordance with the rule

$$\frac{\partial}{\partial t}\mathbf{W}\rightarrow\frac{\partial}{\partial t}\mathbf{W}+\mathbf{W}\times\mathbf{V}, \tag{32}$$

$$\nabla_i\mathbf{W}\rightarrow\nabla_i\mathbf{W}+m^*\mathbf{W}\times\mathbf{A}_i, \tag{33}$$

where $\mathbf{W}$ is the vector in the spin space, and on the right-hand side we have its components already in the laboratory system. If a derivative acts on a matrix with spin indices, it is necessary to add the contraction corresponding to the vector product with respect to each index:

$$\frac{\partial}{\partial t}D_{\alpha\beta}\rightarrow\frac{\partial}{\partial t}D_{\alpha\beta}-e_{\alpha\mu\nu}V_\mu D_{\nu\beta}-e_{\beta\mu\nu}V_\mu D_{\alpha\nu}, \tag{34}$$

$$\nabla_i D_{\alpha\beta}\rightarrow\nabla_i D_{\alpha\beta}-m^*e_{\alpha\mu\nu}A_{\mu i}D_{\nu\beta}-m^*e_{\beta\mu\nu}A_{\mu i}D_{\alpha\nu}. \tag{35}$$

As a result, we obtain the following equations (see Appendices A and B):

$$\frac{\partial}{\partial t}\mathbf{S}^q=\mathbf{S}^q\times\left(\boldsymbol{\omega}_L-\frac{\gamma^2}{\chi_{n0}}F_0^a\mathbf{S}\right)-\nabla_i\delta\mathbf{J}_i+\frac{Y_0-Y_2}{1-Y_0}\nabla_i\mathbf{J}_i^{\mathrm{loc}}+\mathbf{X}\times\mathbf{B}+e_{\alpha\beta\gamma}\nabla_i D^i_{\beta\gamma}+m^*D^i_{\beta\alpha}A_{\beta i}-\frac{\mathbf{S}^q-(1-\lambda_{LT})\mathbf{S}}{\tau}, \tag{36}$$

$$\frac{\partial}{\partial t}\delta\mathbf{J}_i=\delta\mathbf{J}_i\times\left(\boldsymbol{\omega}_L-\frac{\gamma^2}{\chi_{n0}}F_0^a\mathbf{S}\right)-\frac{v_F^2}{3}\nabla_i\mathbf{S}^q+\frac{\hbar S_0}{2m^*}Y_2\times\nabla_i\left(\boldsymbol{\omega}_L-\frac{\gamma^2}{\chi_{n0}}F_0^a\mathbf{S}\right)-\frac{Y_0-Y_2}{1-Y_0}\mathbf{J}_i^{\mathrm{loc}}\times\mathbf{X}-\frac{1}{2m^*}\frac{Y_0-Y_2}{1-Y_0}\rho^s_{\alpha i,\beta j}\nabla_j V_\beta-D^i_{\beta\alpha}X_\beta+\boldsymbol{\Gamma}_i+\frac{v_F^2}{3}(\nabla_i-m^*\mathbf{A}_i\times)\mathbf{B}-\frac{\delta\mathbf{J}_i}{\tau_1}. \tag{37}$$

For convenience, some of the terms in Eqs. (36) and (37) have been written in vector form and some component by component. It is assumed that it is necessary to take the α component of the vectors in spin space. We have introduced the following notation for the quantities that are expressed in terms of the quasiparticle distribution function:

$$\mathbf{B}=\frac{\hbar}{2}\sum_k\left(1-\frac{\xi^2}{E^2}\right)\hat{\mathbf{d}}(\hat{\mathbf{d}}\cdot\delta\boldsymbol{\nu}), \tag{38}$$

$$D^i_{\alpha\beta}=\frac{\hbar}{2}\sum_k v_{Fi}\left(1-\frac{\xi^2}{E^2}\right)(\hat{\mathbf{d}}\times\delta\nu)_\alpha\cdot\hat{\mathbf{d}}_\beta, \tag{39}$$

$$\boldsymbol{\Gamma}_i=-\frac{\hbar v_F^2}{2}\nabla_j\sum_k\left(n_i n_j-\frac{1}{3}\delta_{ij}\right)\left[\frac{\xi}{E}\delta\boldsymbol{\nu}+\left(1-\frac{\xi}{E}\right)\hat{\mathbf{d}}(\hat{\mathbf{d}}\cdot\delta\boldsymbol{\nu})\right]+\frac{\hbar v_F^2}{2}(\nabla_j+m^*\mathbf{A}_j\times)\sum_k\left(n_i n_j-\frac{1}{3}\delta_{ij}\right)\times\hat{\mathbf{d}}(\hat{\mathbf{d}}\cdot\delta\boldsymbol{\nu})\left(1-\frac{\xi^2}{E^2}\right). \tag{40}$$

The final term in Eq. (36) reflects the Leggett–Takagi relaxation of the spin density of the normal and superfluid components to mutual equilibrium.[7] The density of the spin current relaxes to the equilibrium value given by the instantaneous value of $\mathbf{A}_i$ [the final term in (37)]. This form of the relaxation terms is discussed in Appendix B.

To facilitate the comparison of our equations with Leggett's equations for the normal liquid[6] and the Leggett–Takagi equations for superfluids,[7] we go over from the equations of motion for $\mathbf{S}^q$ and $\delta\mathbf{J}_i$ to the equations for the total spin current $\mathbf{J}_i$ and

$$\boldsymbol{\eta}=\mathbf{S}^p-\lambda_{LT}\mathbf{S}=(1-\lambda_{LT})\mathbf{S}-\mathbf{S}^q, \tag{41}$$

which characterizes the departure from mutual equilibrium between the condensate spin and the excitation spin. Here, the parameter

$$\lambda_{LT}=\frac{\chi_{p0}}{\chi_0}=\frac{2(1-Y_2)}{2+Y_0} \tag{42}$$

introduced by Leggett and Takagi is the fraction of the total spin corresponding to the superfluid component in equilibrium. The equation for η is obtained by combining Eqs. (4) and (36). To derive an equation for the total current, it is also necessary to calculate the derivative of the equilibrium current $J^{\mathrm{loc}}_{\alpha i}=\rho^s_{\alpha i,\beta j}A_{\beta j}$, differentiating $\mathbf{A}_i$ by means of (16) and then going over to the laboratory system:

$$\frac{\partial}{\partial t}\mathbf{J}_i^{\mathrm{loc}}=\mathbf{V}\times\mathbf{J}_i^{\mathrm{loc}}-\frac{1}{m^*}\rho^s_{\alpha i,\beta j}\nabla_j V_\beta. \tag{43}$$

As a result of the calculations, we obtain the equations

$$\frac{\partial}{\partial t}\boldsymbol{\eta}=\boldsymbol{\eta}\times\left(\boldsymbol{\omega}_L-\frac{\gamma^2}{\chi_{n0}}F_0^a\mathbf{S}\right)+(1-\lambda_{LT})\mathbf{R}_D+\lambda_{LT}\nabla_i\mathbf{J}_i-\frac{1-Y_2}{1-Y_0}\nabla_i\mathbf{J}_i^{\mathrm{loc}}+\mathbf{B}\times\mathbf{X}-e_{\alpha\beta\gamma}\nabla_i D^i_{\beta\gamma}-m^*D^i_{\beta\alpha}A_{\beta i}-\frac{\boldsymbol{\eta}}{\tau}, \tag{44}$$

$$\frac{\partial}{\partial t}\mathbf{J}_i=\mathbf{J}_i\times\left(\boldsymbol{\omega}_L-\frac{\gamma^2}{\chi_{n0}}F_0^a\mathbf{S}\right)-\frac{v_F^2}{3}\left(1+\frac{2+Y_2}{3}F_0^a\right)\nabla_i\mathbf{S}+\frac{\hbar S_0}{2m^*}\frac{2+Y_2}{3}\nabla_i\omega_L-\frac{v_F^2}{2}\frac{\chi_{p0}}{\gamma^2}\times\left\{\frac{1}{3}\delta_{\alpha\beta}\delta_{ij}-\frac{1}{5}P_{\alpha i,\beta j}\right\}\nabla_j V_\beta-\frac{m^*v_F^2}{2}\frac{\chi_{p0}}{\gamma^2}\times\left\{\delta_{\beta\gamma}\delta_{ij}-\frac{1}{5}P_{\beta i,\gamma j}\right\}e_{\alpha\beta\nu}A_{\gamma j}X_\nu+\frac{v_F^2}{3}(\nabla_i\mathbf{B}-m^*\mathbf{A}_i$$

$$\times\mathbf{B})-D^i_{\beta\alpha}X_\beta+\mathbf{\Gamma}_i-\frac{\mathbf{J}_i-\rho^s_{\alpha i,\beta j}A_{\beta j}}{\tau_1}, \tag{45}$$

in which, as in the equations (4) and (5), $\mathbf{A}_{\alpha i}$ is given by the relation (15), and $\mathbf{V}$ and $\mathbf{X}$ by the relations

$$\mathbf{V}=-\boldsymbol{\omega}_L+\frac{\gamma^2}{\chi}\mathbf{S}+\frac{\gamma^2}{\chi_{p0}}\boldsymbol{\eta}, \tag{46}$$

$$\mathbf{X}=\frac{\gamma^2}{\chi_0}\mathbf{S}+\frac{\gamma^2}{\chi_{p0}}\boldsymbol{\eta}, \tag{47}$$

which are obtained by means of (13), (22), and (41). Here

$$\frac{1}{\chi}=\frac{1}{\chi_0}+\frac{F^a_0}{\chi_{n0}}, \tag{48}$$

and χ is the susceptibility of the B phase of ^{3}He.

The system (4), (5), (44), and (45) is not closed, since it contains $\mathbf{B}$, $D^i_{\alpha\beta}$, and $\mathbf{\Gamma}_i$, which, generally speaking, require their own evolution equations; however, these equations contain new quantities that can be expressed in terms of the distribution function of the Bogolyubov quasiparticles. It is necessary to add new equations of motion for new quantities, etc. As a result, an infinite system of equations is obtained.

This difficulty can be overcome in the hydrodynamic limit and also in the collisionless regime near the critical temperature.

3. EQUATIONS OF THE SPIN HYDRODYNAMICS

In the hydrodynamic limit $\omega\tau\ll 1$, $v_F\tau/\lambda\ll 1$ (λ is the characteristic spatial scale) local equilibrium is established over times on the order of the time of collisions between quasiparticles in the gas of the excitations:

$$\frac{1}{\hbar}\delta\nu=-\varphi'\left[\frac{\xi}{E}\mathbf{X}^q+k_i\mathbf{A}_i-\left(1-\frac{\xi}{E}\right)\hat{\mathbf{d}}\cdot(\hat{\mathbf{d}}(k_i\mathbf{A}_i-\mathbf{X}^q))\right], \tag{49}$$

where $\mathbf{X}^q$ is such that the quasiparticle spin calculated in accordance with the expression (20) is

$$\mathbf{S}^q=\frac{\chi_{q0}}{\gamma^2}\mathbf{X}^q. \tag{50}$$

In local equilibrium, the second term in the collision integral (B1) vanishes in the gas of the excitations (49). In the limit of complete local equilibrium in the liquid, $\delta\nu=\hbar\varphi'\delta\mathbf{E}$, the first term is (B1) vanishes.

In accordance with the Leggett–Takagi hypothesis,[7] the relaxation to the local-equilibrium distribution of the spin in the gas of the excitations takes place over shorter times than the spin relaxation between the excitations and the condensate (this is definitely true only near T_c, see Appendix B).

Equation (49), which is the generalization of the corresponding equation of Leggett and Takagi[7] to the inhomogeneous case, means that during the same (short) times the value of the spin current relaxes to its equilibrium value $\rho^s_{\alpha i,\beta j}A_{\beta j}$.

Substitution of (49) in the expressions for $\mathbf{B}$, $D^i_{\alpha\beta}$, and $\mathbf{\Gamma}_i$ gives

$$\mathbf{B}=\frac{\hbar^2N_F}{12}(Y_0-Y_2)\mathbf{X}^q, \tag{51}$$

$$D^i_{\alpha\beta}=\frac{\hbar S_0}{10}(Y_0-Y_2)e_{\alpha\gamma\mu}A^q_{\mu j}P_{\gamma i,\beta j}, \tag{52}$$

$$\Gamma_{\alpha i}=\frac{\hbar S_0}{2}(Y_0-Y_2)e_{\alpha\beta\gamma}A_{\beta j}\left[\frac{1}{3}\delta_{\gamma\mu}\delta_{ij}-\frac{1}{5}P_{\gamma i,\mu j}\right]X^q_\mu. \tag{53}$$

Thus, the system of equations (4), (5), (44), and (45) is closed in the hydrodynamic regime.

In fact, the resulting system is more complicated than it need be. Use of the conditions $\omega\tau\ll 1$ and $v_F\tau/\lambda\ll 1$ enables us to conclude by means of Eqs. (37) and (44) that

$$\frac{|\delta\mathbf{J}_i|}{v_F}\sim\frac{v_F\tau}{\lambda}S, \tag{54}$$

$$|\boldsymbol{\eta}|\sim\max\left\{\tau R_D,\frac{\tau}{\lambda}\frac{|\mathbf{J}_i|}{\sqrt{1-T/T_c}}\right\}. \tag{55}$$

Therefore, in the hydrodynamic limit we can replace Eqs. (37) and (44) by their solutions

$$\begin{aligned}J_{\alpha i}=J^{\rm loc}_{\alpha i}+\tau_l\Bigg\{&-\frac{v_F^2}{3}\nabla_iS^q_\alpha+\frac{\hbar S_0}{2m^*}Y_2\nabla_i\left(\omega_{L,\alpha}-\frac{\gamma^2}{\chi_{n0}}F^\alpha_0S_\alpha\right)\\&-\frac{Y_0-Y_2}{1-Y_0}e_{\alpha\beta\gamma}J^{\rm loc}_{\beta i}X_\gamma-\frac{1}{2m^*}\frac{Y_0-Y_2}{1-Y_0}\rho^0_{\alpha i,\beta j}\nabla_jV_\beta\\&-D^i_{\beta\alpha}X_\beta+\Gamma^\alpha_i+\frac{v_F^2}{3}(\nabla_iB_\alpha-m^*e_{\alpha\beta\gamma}A_{\beta i}B_\gamma)\Bigg\},\end{aligned} \tag{56}$$

$$\begin{aligned}\eta_\alpha=\tau\Bigg\{&(1-\lambda_{LT})R_{D,\alpha}+\left(\lambda_{LT}-\frac{1-Y_2}{1-Y_0}\right)\nabla_iJ^{\rm loc}_{\alpha i}\\&-\frac{1}{2}e_{\alpha\beta\gamma}\nabla_iD^i_{\beta\gamma}+m^*D^i_{\beta\alpha}A_{\beta i}\Bigg\}.\end{aligned} \tag{57}$$

The final system of hydrodynamic equations has the form

$$\frac{\partial\mathbf{S}}{\partial t}=\mathbf{S}\times\boldsymbol{\omega}_L-\nabla_i\mathbf{J}_i+\mathbf{R}_D, \tag{58}$$

$$\frac{\partial\mathbf{d}}{\partial t}=\mathbf{d}\times\left(\boldsymbol{\omega}_L-\frac{\gamma^2}{\chi}\mathbf{S}-\frac{\gamma^2}{\chi_{p0}}\boldsymbol{\eta}\right). \tag{59}$$

Here $\mathbf{J}_i$ and $\boldsymbol{\eta}$ are determined by the expressions (56) and (57), $J^{\rm loc}_{\alpha i}$ by the expression (27), $\mathbf{S}^q$ by the expression (41), $\mathbf{V}$ by the expression (46), $\mathbf{X}$ by (47), and $\mathbf{B}$, $D^i_{\alpha\beta}$, and Γ^i_α by (51), (52), and (53). The system (58)–(59) generalizes the well-known Leggett–Takagi equations[7] to the spatially inhomogeneous case.

4. COLLISIONLESS SPIN DYNAMICS NEAR T_c

In the collisionless regime $\omega\tau\gg 1$ in the general case, the transition from the microscopic description of the spin dynamics by means of a kinetic equation to a macroscopic description is impossible. We note however [see (51), (52), and (53)] that as $T\to T_c$ the factor $(1-(\xi/E)^2)$ makes $\mathbf{B}$ and $D^i_{\alpha\beta}$ vanish, while $\mathbf{\Gamma}_i$ (see Appendix C) tends to

$$\Gamma_i \approx -\frac{\hbar v_F^2}{2} \nabla_j \sum_k \left(n_i n_j - \frac{1}{3} \delta_{ij} \right) \delta \boldsymbol{\nu}. \tag{60}$$

From the equation for Γ_i when $T > T_c$ holds (Ref. 6), we obtain

$$v_F |\Gamma_i| \sim \frac{v_F}{\lambda \omega_m} |\mathbf{J}_i|. \tag{61}$$

Here $\omega_m = F_0^a \omega_L$ is the frequency of the molecular field, and λ is the characteristic spatial scale. Thus, in the normal liquid when $v_F / \lambda \omega_m \ll 1$ holds it is possible to ignore the second harmonic of $\delta\nu$ with respect to $\mathbf{k}$ (the quantity Γ_i) compared with the first (the current $\mathbf{J}_i$). The same is also true for all the higher harmonics of $\delta\nu$. The system of equations for $\mathbf{S}$ and $\mathbf{J}_i$, (4) and (45), is transformed into the closed system (1)–(2).

In the liquid superfluid the terms containing the higher harmonics of the distribution function in Eqs. (44) and (45) will be small either in proportion to $(1 - T/T_c)$ or, as in the normal liquid, in proportion to the parameter $v_F / \lambda \omega_m$. The characteristic inhomogeneity scale is determined by the smaller of these two lengths (see the following section): the inhomogeneity scale in the normal phase (Ref. 4), $\lambda_n = ((\chi/\gamma^2) w^2 / 3\kappa\sigma \nabla \omega_L)^{1/3}$, and the length $\lambda_D = (5/16\kappa)^{1/2} w / \Omega_B$ determined by the dipole interaction. Here we have set $w^2 = v_F^2 (1 + F_0^a)$, $\kappa = -F_0^a / (1 + F_0^a)$, and Ω_B is the frequency of longitudinal resonance in the B phase; near T_c, we have $\Omega_B = \Omega_B^0 \sqrt{1 - T/T_c}$, where $\Omega_B^0 \approx 2\pi \cdot 2 \cdot 10^5$ s^{-1}. With increasing distance from the transition temperature, λ_D decreases and becomes less than λ_n for

$$1 - \frac{T}{T_c} \gtrsim \left(\frac{w}{\lambda_n \Omega_B^0} \right)^2 \frac{5}{16\kappa} \sim 10^{-4}. \tag{62}$$

The estimate was made for magnetic field $H = 413$ Oe and gradient $\nabla H = 0.19$ Oe/cm, corresponding to the conditions of one of the experiments of Ref. 5. At zero pressure, $F_0^a = -0.695$. Recalling also that the nuclear gyromagnetic ratio for helium is $\gamma = 2.04 \times 10^4$ $(\mathrm{Oe \cdot s})^{-1}$ and $\omega_m \approx F_0^a \omega_L$, we obtain the ratio

$$\frac{v_F}{\lambda_D \omega_m} \sim \frac{\Omega_B}{\omega_L} \sim 10^{-2}. \tag{63}$$

The asymptotic behaviors of the Yosida functions near T_c are such that (cf. Ref. 11)

$$1 - Y_0 \approx \frac{7\zeta(3)}{4\pi^2} \left(\frac{\Delta}{T_c} \right)^2, \tag{64}$$

$$1 - Y_2 \approx \frac{\pi}{2} \frac{\Delta}{T_c}. \tag{65}$$

For an estimate, we can set $V \sim X \sim (\gamma^2/\chi) S \sim \omega_L$, $A \sim 1/(\lambda m^*)$, and estimate $\mathbf{B}$, D, Γ using the expressions (51)–(53). Then the contribution of (60) from the higher harmonics of $\delta\nu$ that do not vanish as $T \to T_c$ will be small of order the parameter (63).

In Eq. (45), all the terms except the first three and the last are less than the term containing the gradient of the spin density in order of magnitude by a factor $\sqrt{1 - T/T_c}$, like the term $\rho^s A$ in the relaxation term. Ignoring these terms in Eq. (45), we obtain an equation for the current that is identical to Eq. (2) in the normal phase:

$$\frac{\partial \mathbf{J}_i}{\partial t} = \mathbf{J}_i \times \left(\boldsymbol{\omega}_L - \frac{\gamma^2}{\chi_{n0}} F_0^a \mathbf{S} \right) - \frac{v_F^2}{3} (1 + F_0^a) \nabla_i \mathbf{S} + \frac{v_F^2}{3} \frac{\chi_{n0}}{\gamma^2} \nabla_i \boldsymbol{\omega}_L - \frac{\mathbf{J}_i}{\tau_1}. \tag{66}$$

To this equation, it is necessary to append Eq. (4) for the spin:

$$\frac{\partial \mathbf{S}}{\partial t} = \mathbf{S} \times \boldsymbol{\omega}_L - \nabla_i \mathbf{J}_i + \mathbf{R}_D. \tag{67}$$

Despite the smallness of $R_D \sim (1 - T/T_c)$, in this equation it is necessary to retain the dipole moment. In contrast to the terms omitted in Eq. (66) that are small compared with the gradient of the current, the moment has, generally speaking, the same order of magnitude as $\nabla \mathbf{J}_i$, and it is what determines the characteristic scale and structure of the domain wall in the superfluid B phase in the case of weak inhomogeneity of the external field ($\lambda_n > \lambda_D$).

Equation (5) for the order parameter

$$\frac{\partial \mathbf{d}}{\partial t} = \mathbf{d} \times \left(\boldsymbol{\omega}_L - \frac{\gamma^2}{\chi} \mathbf{S} - \frac{\gamma^2}{\chi_{p0}} \boldsymbol{\eta} \right) \tag{68}$$

contains the variable $\boldsymbol{\eta}$. To derive an equation for this variable, we note that in Eq. (44) the terms that contain gradients and $\mathbf{A}_i$ can be omitted, since they are small compared with the dipole moment at temperatures not too close to the transition temperature, i.e., for

$$1 - \frac{T}{T_c} \gtrsim (v_F / \lambda_n \Omega_B^0)^4 \sim 10^{-4}. \tag{69}$$

A problem arises when one considers the term $\mathbf{B} \times \mathbf{X}$. An estimate shows (see below) that this term is of the same order as the term $\boldsymbol{\eta} \times \mathbf{S}$ contained in Eq. (44). Therefore, in contrast to Eq. (45), in which all the terms proportional to $\mathbf{B}$ and D can be omitted, here, strictly speaking, it is necessary to derive an equation for $\mathbf{B}$ that contains new unknown variables, i.e., the system of equations remains unclosed. The proposed method for estimating the term $\mathbf{B} \times \mathbf{X}$ in the equation for η makes it possible to demonstrate that the retention of η in the equations of the collisionless spin dynamics near T_c would be an unnecessary refinement. To estimate the term $\mathbf{B} \times \mathbf{X}$, we use the expression (51) for $\mathbf{B}$:

$$\mathbf{B} = \frac{Y_0 - Y_2}{Y_0 + 2Y_2} \mathbf{S}^q \sim \frac{1}{3} (1 - Y_2) \mathbf{S}^q \tag{70}$$

and the relation

$$\mathbf{X} = \frac{\gamma^2}{\chi_{p0}} \mathbf{S}^p. \tag{71}$$

Finally, we obtain

$$\frac{\partial \boldsymbol{\eta}}{\partial t} = \boldsymbol{\eta} \times \left(\boldsymbol{\omega}_L - \left(\frac{1}{2} + F_0^a \right) \frac{\gamma^2}{\chi_{n0}} \mathbf{S} \right) + \mathbf{R}_D - \frac{\boldsymbol{\eta}}{\tau}. \tag{72}$$

We emphasize once more that writing of the term in the brackets as $1/2+F_0^a$ can be regarded as a pure estimate–we know it only in order of magnitude. The solution of this equation, precessing with frequency ω_p, is

$$\eta=-\frac{\tau}{1+\tilde{\mathbf{u}}\tau^2}\{-\mathbf{R}_D+\tau\tilde{\mathbf{u}}\times\mathbf{R}_D-\tau^2\tilde{\mathbf{u}}(\tilde{\mathbf{u}}\cdot\mathbf{R}_D)\}, \tag{73}$$

where $\tilde{\mathbf{u}}=\boldsymbol{\omega}_L-\boldsymbol{\omega}_P-(1/2+F_0^a)(\gamma^2/\lambda_{n0})\mathbf{S}$. In the case in which we are interested, $(\tilde{\mathbf{u}}\cdot\mathbf{R}_D)=0$ as $\omega\tau\to\infty$ (see the following section) and for finite $\omega\tau$ we have $|\tilde{\mathbf{u}}|\sim O(1/\omega\tau)$. For η this gives an estimate in the collisionless regime:

$$\eta\sim\frac{R_D}{\bar{u}}. \tag{74}$$

Using (74), we compare the last two terms in Eq. (68):

$$\frac{\eta\chi}{\chi_{p0}S}\sim\left(\frac{\Omega_B^0}{\omega_L}\right)^2\sqrt{1-\frac{T}{T_c}}, \tag{75}$$

and this makes it possible to omit η in this equation. Retention of the term with η, which is small in accordance with (75), would be excessive precision, especially since in the derivation of Eq. (66) we have omitted several terms that are small only as $\sqrt{1-T/T_c}$. Note, incidentally, that retention of η is necessary in the study of dissipation in the homogeneous case, i.e., Leggett–Takagi dissipation. Because of the term $\mathbf{B}\times\mathbf{X}$ in Eq. (44), the Leggett–Takagi dissipation in the collisionless regime can be taken into account only qualitatively even near T_c.

Thus, the approximate equations of the collisionless spin dynamics of the B phase near T_c have the form

$$\frac{\partial\mathbf{S}}{\partial t}=\mathbf{S}\times\boldsymbol{\omega}_L-\nabla_i\mathbf{J}_i+\mathbf{R}_D, \tag{76}$$

$$\frac{\partial\mathbf{J}_i}{\partial t}=\mathbf{J}_i\times\left(\boldsymbol{\omega}_L-\frac{\gamma^2}{\chi_{n0}}F_0^a\mathbf{S}\right)-\frac{v_F^2}{3}(1+F_0^a)\nabla_i\mathbf{S}+\frac{v_F^2}{3}\frac{\chi_{n0}}{\gamma^2}\nabla_i\boldsymbol{\omega}_L-\frac{\mathbf{J}_i}{\tau_1}, \tag{77}$$

$$\frac{\partial\mathbf{d}}{\partial t}=\mathbf{d}\times\left(\boldsymbol{\omega}_L-\frac{\gamma^2}{\chi}\mathbf{S}\right). \tag{78}$$

5. TWO-DOMAIN STRUCTURE NEAR THE TRANSITION

We shall seek solutions of the equations of the spin dynamics near the transition temperature that correspond to precession of the spin and the spin current with a frequency ω_P constant throughout the volume of the liquid, i.e., such that at each point we have $\partial\mathbf{S}/\partial t=\mathbf{S}\times\boldsymbol{\omega}_P$, $\partial\mathbf{J}_i/\partial t=\mathbf{J}_i\times\boldsymbol{\omega}_P$. In connection with the experiment, we shall be interested in a situation in which the magnetic field and its (constant) gradient are directed along the $\hat{z}$ axis, and we shall assume that all quantities depend only on the one coordinate z. We assume that the precession frequency ω_P is also directed along the $\hat{z}$ axis and is close to the Larmor frequency (which we assume is slowly varying within the container).

Then the spatial dependence of the spin and spin current is determined by the system of equations

$$\mathbf{S}\times(\boldsymbol{\omega}_L-\boldsymbol{\omega}_P)=\nabla\mathbf{J}+\mathbf{R}_D=0, \tag{79}$$

$$\mathbf{J}\times(\boldsymbol{\omega}_L-\boldsymbol{\omega}_P+\kappa\mathbf{S})-\frac{w^2}{3}\frac{\partial(\mathbf{S}-\boldsymbol{\omega}_L)}{\partial z}-\frac{\mathbf{J}}{\tau_1}=0, \tag{80}$$

where $\mathbf{R}_D$ is the dipole moment, and $\mathbf{J}=\mathbf{J}_3$ is the component of the spin current along the $\hat{z}$ axis. In this and the following sections, we shall use a system of units in which $\chi=\gamma^2$.

As in the spatially homogeneous case (see Ref. 12), to investigate periodic solutions of Eqs. (76)–(78) it is convenient to parametrize the matrix $\hat{R}$ of the order parameter by Euler angles:

$$\hat{R}=\hat{R}_z(\alpha)\hat{R}_y(\beta)\hat{R}_z(\gamma). \tag{81}$$

We define unit vectors $\hat{\zeta}=\hat{R}\hat{z}$ and $\hat{\beta}=(\hat{z}\times\hat{\zeta})/|\hat{z}\times\hat{\zeta}|$, which rotate together with the order parameter $\mathbf{d}(\mathbf{k})$, and introduce notation for the projections of the spin density: $S_\beta=(\mathbf{S}\cdot\hat{\boldsymbol{\beta}})$, $S_\zeta=(\mathbf{S}\cdot\hat{\zeta})$, and $P=S_z-S_\zeta$. The matrix of the order parameter can also be specified by the direction of the rotation axis n and the rotation angle θ. Then

$$n_i=-\frac{1}{2\sin\theta}e_{ikl}R_{kl}(\alpha,\beta,\gamma), \tag{82}$$

$$\cos\theta=\frac{\cos\beta+\cos\Phi+\cos\beta\cos\Phi-1}{2}, \tag{83}$$

$\Phi=\alpha+\gamma$, and the dipole energy is given by

$$U_D=\tfrac{8}{15}\Omega_B^2(\cos\theta+\tfrac{1}{4})^2,$$

where Ω_B is the frequency of longitudinal resonance in the B phase. It attains the minimum value at $\theta=\theta_L=\cos^{-1}(-1/4)$.

In these variables the equations for the spin (76) and order parameter (78) (without dissipative terms) have the form (cf. Ref. 12)

$$\frac{\partial}{\partial t}S_z=-\frac{\partial U_D}{\partial\Phi}-\nabla J_z, \tag{84}$$

$$\frac{\partial}{\partial t}P=-\nabla\mathbf{J}\cdot(\hat{z}-\hat{\zeta}), \tag{85}$$

$$\frac{\partial}{\partial t}S_\beta=\frac{\sin\beta}{(1+\cos\beta)^2}\left(\frac{\cos\beta}{1-\cos\beta}P+S_z\right)\times\left(\frac{P}{1-\cos\beta}-S_z\right)-\nabla\mathbf{J}\cdot\hat{\beta}-\frac{\partial U_D}{\partial\beta}, \tag{86}$$

$$\frac{\partial}{\partial t}\alpha=-\omega_L+\frac{1}{\sin^2\beta}(P\cos\beta+S_z(1-\cos\beta)), \tag{87}$$

$$\frac{\partial}{\partial t}\beta=S_\beta, \tag{88}$$

$$\frac{\partial}{\partial t}\Phi=-\omega_L+\frac{1}{1+\cos\beta}(2S_z-P). \tag{89}$$

The search for solutions corresponding to precession of $\mathbf{S}$ and $\mathbf{n}$ with a frequency ω_P constant in the complete volume and θ=const means fulfillment of the conditions

$$\dot{\alpha}=-\omega_P=\text{const},\quad \dot{\beta}=\dot{\Phi}=\dot{S}_z=\dot{P}=\dot{S}_\beta=0. \tag{90}$$

In the homogeneous case, such solutions were investigated in Ref. 12.

Using (88), (89), (87), and (90), we can readily find that

$$S_z=\omega_L \cos \beta+(\omega_L-\omega_P)(1-\cos \beta), \tag{91}$$

$$P=(\omega_L-2\omega_P)(1-\cos \beta), \tag{92}$$

$$S_\beta=0. \tag{93}$$

The vanishing of S_β means that the vector **S** lies in the plane of the vectors $\hat{z}$ and $\hat{\zeta}$. Below, using the equations that describe the precessing structure and the boundary condition of absence of a spin current at the boundary of the container (for a container bounded on at least one side), we shall show that the position of this plane does not depend on z, while the current vector is directed perpendicular to this plane.

Indeed, the conditions $\dot{\beta}=\dot{\Phi}=0$ and Eq. (83) have the consequence that $\dot{\theta}=0$, and, since $\dot{\theta}=\mathbf{n}\cdot(\mathbf{S}-\omega_L)$ (see Ref. 13), and the dipole moment (8), which is equal to

$$\mathbf{R}_D=-\mathbf{n}\frac{\partial U_D}{\partial \theta},$$

is proportional to the vector **n**, it follows from Eq. (79) that $\nabla \mathbf{J}\perp(\mathbf{S}-\boldsymbol{\omega}_L)$.

On the other hand, the condition $\dot{P}=0$ entails $\nabla\mathbf{J}\perp(\hat{z}-\hat{\zeta})$. These two conditions of orthogonality for $\nabla\mathbf{J}$ mean that either $\nabla\mathbf{J}$ is perpendicular to the plane $(\hat{z},\hat{\zeta})$ or $\mathbf{S}=\omega_L\hat{\zeta}$ and $\nabla\mathbf{J}$ lie in the plane perpendicular to $(\hat{z}-\hat{\zeta})$.

If $\mathbf{S}=\omega_L\hat{\zeta}$, then it follows from (91) and (92) that either the spin vector is directed along the magnetic field ($\beta=0$, $\hat{\zeta}=\hat{z}$) or the Larmor frequency of the external field is homogeneous and equal to the precession frequency. We are not interested in either of these cases. Thus, $\nabla\mathbf{J}\|\hat{\beta}$.

In particular, this means that $\nabla\mathbf{J}_z=0$, i.e., by virtue of the boundary conditions

$$J_z=0. \tag{94}$$

With regard to Eq. (80) for the current, the final term in it leads to energy dissipation (see also Sec. 8). Therefore, a precessing solution with nonvanishing spin current can exist only in the collisionless limit $\tau_1\to\infty$. For finite τ_1, the solution can be assumed to be a rapidly precessing solution with a frequency that changes slowly because of the dissipation.

Thus, to determine the distribution of the magnetization in the precessing structure, we shall use Eq. (95) in the form

$$\mathbf{J}\times\mathbf{u}=\frac{w^2}{3}\nabla(\mathbf{S}-\boldsymbol{\omega}_L), \tag{95}$$

where $\mathbf{u}=\boldsymbol{\omega}_L-\boldsymbol{\omega}_P+\kappa\mathbf{S}$.

From this equation and Eq. (79), it is readily found that $\nabla(\mathbf{J}\cdot(\mathbf{S}-\boldsymbol{\omega}_L))=0$, and, hence, $\mathbf{J}\perp(\mathbf{S}-\boldsymbol{\omega}_L)$. Recalling that $J_z=0$, we conclude that **J** is perpendicular to the plane $(\hat{z},\hat{\zeta})$. Therefore, $\nabla\mathbf{S}$ lies in this plane [see (95)], and, hence, the plane itself (like the plane containing ω_L and **S**) does not rotate in the motion in space along the $\hat{z}$ axis.

The vector **u** lies in the plane $(\hat{z},\hat{\zeta})$, and therefore $\mathbf{J}\perp\mathbf{u}$. Hence and from (95) we immediately obtain an expression for the spin current:

$$\mathbf{J}=\frac{w^2}{3u^2}\mathbf{u}\times\nabla(\mathbf{S}-\boldsymbol{\omega}_L). \tag{96}$$

Since we are interested in solutions with precession frequency close to the Larmor value ($|\omega_L-\omega_P|\ll\omega_L$; hitherto we have not used this), we can assume $\mathbf{u}\sim\kappa\mathbf{S}$. In addition, in the region in which gradients of the spin density are important they greatly exceed the gradient of the Larmor frequency (this will be seen from what follows; cf. also the solution for the normal liquid[4]). Therefore, for the current we obtain the approximate expression

$$J_\beta=\frac{w^2}{3\kappa}\nabla\beta. \tag{97}$$

Note that in general the angle β is not equal to the angle between **S** and the magnetic field. However, it follows from (91) and (92) that in the limit $|\omega_L-\omega_P|\ll\omega_P$ these angles are equal.

From (84) and (86), using (91), (92), (94), and (97), we obtain two equations for determining the spatial dependence of β and Φ:

$$\frac{\partial U_D}{\partial \Phi}=0, \tag{98}$$

$$\frac{w^2}{3\kappa}\beta''+\omega_P(\omega_L-\omega_P)\sin\beta+\frac{\partial U_D}{\partial\beta}=0. \tag{99}$$

Since

$$\frac{\partial U_D}{\partial\Phi}=-\frac{8}{15}\Omega_B^2\left(\cos\theta+\frac{1}{4}\right)\sin\Phi(1+\cos\beta),$$

the condition (98) is satisfied in the following cases:[12]

$$1)\ \cos\theta=-\tfrac{1}{4},\quad 2)\ \Phi=0,$$

$$3)\ \Phi=\pi,\quad 4)\ \beta=\pi. \tag{100}$$

The solution corresponding to case 4) is of no interest to us. The solutions 2) for $\beta<\theta_L$ and 3) are unstable in the homogeneous case,[12] and we shall not consider them.

In case 1),

$$\cos\beta+\cos\Phi+\cos\beta\cos\Phi=\tfrac{1}{2}. \tag{101}$$

This means $\cos\beta>-1/4$, i.e., $\beta<\theta_L$. In this case, the dipole energy is stationary with respect to β as well, i.e., $\partial U_D/\partial\beta=0$.

In case 2), $\beta=\theta$ and

$$\frac{\partial U_D}{\partial\beta}=-\frac{16}{15}\Omega_B^2\sin\beta\left(\cos\beta+\frac{1}{4}\right).$$

Thus, to determine the spatial dependence of β we obtain the equation

$$\frac{w^2}{3\kappa}\beta''=-\omega_P(\omega_L-\omega_P)\sin\beta+\frac{16}{15}\Omega_B^2\sin\beta \times\left(\cos\beta+\frac{1}{4}\right)\Theta(\beta-\theta_L), \tag{102}$$

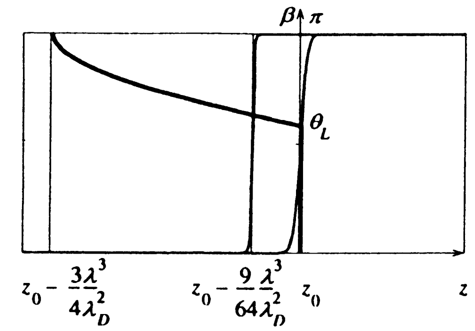

FIG. 1. Dependence of the angle of inclination of the magnetization on the coordinate for the two-domain structure in the normal (right-hand curve) and superfluid (left-hand curve, plotted for $\lambda_D \approx 0.11\lambda$) liquid. The domain wall in the superfluid liquid is narrower and displaced relative to the wall in the normal state. The heavy curve is the set of points of inflection (see the text).

where the step function Θ is nonvanishing (and equal to unity) only for positive values of its argument. If we use the characteristic thickness of the domain wall in the normal liquid

$$\lambda_n=\left(\frac{w^2}{2\kappa S\nabla\omega}\right)^{1/3} \tag{103}$$

and the dipole length

$$\lambda_D=\left(\frac{5}{16\kappa}\right)^{1/2}\frac{w}{\Omega_B}, \tag{104}$$

then this equation takes the form

$$\beta''=-\frac{z-z_0}{\lambda_n^3}\sin\beta+\frac{1}{\lambda_D^2}\sin\beta\left(\frac{1}{4}+\cos\beta\right)\Theta(\beta-\theta_L), \tag{105}$$

where $\omega_L=\omega_p+\nabla\omega_L(z-z_0)$, i.e., z_0 is the point where the local Larmor frequency of the external field is equal to precession frequency.

We are interested in a solution of Eq. (105) with boundary condition $\beta'=0$, indicating the absence of a current. For $\lambda_n \ll \lambda_D$, this solution is practically indistinguishable from the solution for the normal liquid[4] and is a two-domain structure with domain wall thickness of order λ_n (Fig. 1).

In the opposite limiting case $\lambda_n \gg \lambda_D$, the solution is modified (see Fig. 1). As before, this solution describes a two-domain structure. In one of the domains (situated in the region of weak fields) the spin has the equilibrium value, while in the other it differs in sign from the equilibrium value. The thickness of the domain wall in order of magnitude is λ_D, and the wall itself is displaced from the point z_0 into the region of weak fields (i.e., by virtue of the decrease in the size of the equilibrium domain). Besides this solution, which describes rotation of the magnetization through the angle π, there also exist other solutions, which describe rotation through 3π, 5π, etc., and also solutions that are not stable (see the following section).

The solution $\beta(z)$ in Fig. 1 has one point of inflection. Indeed, it can be seen from Eq. (105) that $\beta''=0$ either at the point $z=z_0$ [if $\beta(z_0)\leq\theta_L$] or at a point z where $z-z_0=(\lambda_n^3/\lambda_D^2)(\cos\beta+1/4)<0$. This means that the point of inflection is the point of intersection of our solution with the heavy curve in Fig. 1. It is readily seen that the solution has precisely one point of inflection.

To calculate the position of the domain wall itself relative to the point z_0, where the local Larmor frequency of the external field is equal to ω_p, we note that Eq. (105) is the equation of motion of a particle of unit mass in the potential

$$U(\beta)=-\frac{z-z_0}{\lambda_n^3}\cos\beta+\frac{1}{2\lambda_D^2}\left(\cos\beta+\frac{1}{4}\right)^2\Theta(\beta-\theta_L).$$

Here, the role of the time is played by z, and the potential itself depends on the time. The solution in which we are interested corresponds to a situation in which at large negative z the particle is for a certain time at the maximum of the potential at $\beta=0$ and then hops to the point $\beta=\pi$. If $\lambda_n \gg \lambda_D$, then in the region of the "hop" the potential depends weakly on the time, and therefore we can use the energy conservation law $U(0)=U(\pi)$, from which it follows that the wall is situated at the point

$$z_w=z_0-\frac{9}{64}\frac{\lambda_n^3}{\lambda_D^2}. \tag{106}$$

This corresponds to displacement of the frequency to the point of the wall

$$\omega_P=\omega_L+\frac{3}{20}\frac{\Omega_B^2}{\omega_L}. \tag{107}$$

The thickness of the wall can be estimated as the hopping time.

The thickness λ_D of the domain wall remains finite in the limit of a homogeneous field: $\nabla\omega_L\to 0$. This means that a two-domain structure can arise even in a homogeneous external field.

A two-domain structure can be observed in experiments using continuous and pulsed NMR. In the case of continuous NMR, the precession frequency ω_p is equal to the frequency of the rf transverse field, and the (absolute) position of the domain wall can be readily found from (107).

In pulsed NMR experiments, the pulse of the transverse field deflects the magnetization in the complete container through a certain angle from the direction of the magnetic field (the $\hat{z}$ axis), and after the transverse field has been switched off a two-domain structure is formed as a result of the flowing of the spin currents. The position (absolute, not relative to z_0 as above) of the domain wall in the two-domain structures in a normal Fermi liquid[4] and in ^{3}He-B in the hydrodynamic region[2] can be uniquely determined from the initial data by means of the conservation law for the longitudinal component of the magnetization in the normal liquid or the integral of P over the volume of the sample in ^{3}He–B. In the case in which we are interested, neither of these quantities is conserved, and the determination of the position of the wall requires additional calculations.

Uniformly precessing domain in the hydrodynamic regime. We note that an equation analogous to (105) arises in the investigation of the structure of a uniformly precessing domain (HPD) in the region of angles $\beta\geq\theta_L$. In this region $\alpha'=\Phi=0$ (Ref. 2), and the solution can be found from the

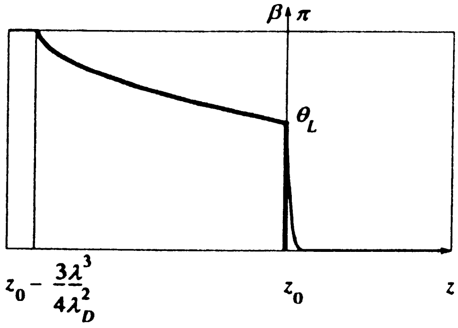

FIG. 2. Dependence of the angle of inclination of the magnetization on the coordinate for a uniformly homogeneously precessing domain in the region of angles $\beta \gg \theta_L$. For $\theta_L < \beta < \pi$, this curve almost exactly coincides with the heavy line—the set of points of inflection.

condition of minimization of the free energy, which is the sum of the gradient energy, the spectroscopic energy in the inhomogeneous external field, and the dipole energy:

$$F=\frac{\chi}{\gamma^2}\int dV\left\{\frac{c_\parallel^2}{2}\beta'^2-\omega_P\nabla\omega_L\cos\beta(z-z_0)+\frac{8}{15}\Omega_B^2\left(\cos\beta+\frac{1}{4}\right)^2\right\}, \tag{108}$$

where $c_\parallel$ is a parameter that characterizes the spin-wave spectrum (see Ref. 2). The corresponding Euler–Lagrange equation has the form

$$\beta''=\frac{z-z_0}{\lambda^3}\sin\beta-\frac{1}{\lambda_D^2}\sin\beta\left(\cos\beta+\frac{1}{4}\right), \tag{109}$$

where $\lambda=(c_\parallel^2/\omega_P\nabla\omega_L)^{1/3}$, and $\lambda_D = (\sqrt{15}/4)(c_\parallel/\Omega_B)$. This equation differs from (105) in having the opposite sign of β''. It corresponds to motion in the potential

$$U(\beta)=\frac{z-z_0}{\lambda^3}\cos\beta-\frac{1}{2\lambda_D^2}\left(\cos\beta+\frac{1}{4}\right)^2\Theta(\beta-\theta_L).$$

If it is assumed that this equation describes a homogeneously precessing domain for angles $\beta<\theta_L$ as well, then we can obtain the solution[1)] shown in Fig. 2, which differs qualitatively from the solution shown in Fig. 1. This solution has three points of inflection, which can be found as in the previous case and which correspond to intersection of the solution with the heavy line in Fig. 2, and it also has a broad plateau. The behavior of this solution can be understood as follows: At large positive z, the particle is at the maximum of the potential at $\beta=0$. At $z\sim z_0$, the values of the potential at the points $\beta=0$ and $\beta=\theta_L$ are comparable[2)] [while $U(\pi)$ remains much smaller], and the particle hops to the position $\beta=\theta_L$. After this, with decreasing z, the position of the maximum of the potential, and with it the particle as well, is gradually shifted toward $\beta=\pi$. Thus, the particle moves almost along the heavy line. In the region, the spatial variation of the Larmor frequency is balanced by the moment of the dipole forces (cf. Ref. 2). Finally, for

$$z=z_0-\frac{3}{4}\frac{\lambda^3}{\lambda_D^2} \tag{110}$$

[i.e., for $\omega_L=\omega_P-(4/5)\Omega_B^2/\omega_L$] the particle reaches the point π and after this remains at it. This solution reflects in a qualitatively correct manner the structure of the uniformly precessing domain for $\beta\geq\theta_L$. The solution has the form of a three-domain structure: In the region of strong fields, there is an equilibrium domain, adjoining which there is a domain of thickness $3\lambda^3/4\lambda_D^2$, in which the angle between the magnetization and the field varies slowly from $\theta_L\approx 104°$ to 180°, while in the region of the weakest fields there is a domain in which the magnetization has the direction opposite to the equilibrium direction.

It was shown in Refs. 14 and 15 that in reality the thickness of the intermediate domain is somewhat less [and equal to $(3-\sqrt{6})\lambda^3/4\lambda_D^2$], since at angles β close to π there may be a rearrangement of the solution into an energetically more advantageous configuration that is not exactly precessing.

6. STABILITY OF TWO-DOMAIN PRECESSING STRUCTURE

In the preceding section, we derived an equation that determines the spatial dependence of the magnetization in a two-domain structure. This equation has various solutions. Of course, in a real liquid only solutions that are stable with respect to small perturbations can be realized. In this section, we shall show that the two-domain structure in a normal liquid[4] is stable (see also Ref. 16), and we shall then give qualitative arguments supporting the stability of the structure in ^{3}He-*B*.

For the following arguments, we shall find it convenient to use an expression for the diffusion spin current. To derive it, we make the assumption that the characteristic frequencies $\delta\omega$ of the motions in the frame of reference rotating with the precession frequency are small compared with the reciprocal time $1/\tau_1$ between collisions of the quasiparticles:

$$\tau_1\delta\omega\ll 1. \tag{111}$$

In the normal liquid, the role of $\delta\omega$ is played by $\lambda\nabla\omega_L$, the change of the Larmor frequency of the external field over the characteristic length. In superfluid ^{3}He-*B*, as we have seen in the previous section, there is one further characteristic frequency: Ω_B^2/ω_L.

Using (111), we can cancel $\dot{\mathbf{J}}_i$ and $\mathbf{J}_i\times\omega_L$ in (77). Solving the obtained equation for $\mathbf{J}_i$, we obtain for the spin current the expression

$$\mathbf{J}_i\cong-\frac{w^2\tau_1/3}{1+(\kappa S\tau_1)^2}[\nabla_i(\mathbf{S}-\boldsymbol{\omega}_L)+\kappa\tau_1\nabla_i(\mathbf{S}-\boldsymbol{\omega}_L)\times\mathbf{S}+(\kappa\tau_1)^2\mathbf{S}(\mathbf{S}\nabla_i(\mathbf{S}-\boldsymbol{\omega}_L))]. \tag{112}$$

The conditions $\omega_L\tau_1\gg 1$ and $\lambda\nabla\omega_L\tau_1\ll 1$ enable us to simplify this expression. First, the characteristic scale of variation of the Larmor frequency is large compared with the scale over which the magnetization changes: $|\nabla_i\mathbf{S}|\sim\omega_L/\lambda\gg 1/(\tau_1\lambda)\gg\nabla_i\omega_L$. Thus, $\nabla(\mathbf{S}-\omega_L)$ can be replaced by $\nabla\mathbf{S}$. Further, each subsequent term in the brackets in (112) is, generally speaking, greater than the preceding term by a factor $\kappa\omega_L\tau_1$. Therefore, we omit the first term and keep the second one, since in the case in which we are interested the absolute magnitude of the magnetization is almost constant in space, corresponding to the fact that $\mathbf{S}$ and $\nabla\mathbf{S}$ are almost orthogonal, i.e., the third term is anomalously small. A direct estimate shows that retention of the second term in (112) given the fact that we have omitted the term proportional to $\nabla\omega_L$ in the third is not excessive precision: The omitted term is small compared with the second term in accordance with the parameter $\lambda\nabla\omega_L\tau_1$. Thus, as a result of the simplification we obtain for the current the expression

$$\mathbf{J}_i \cong -\frac{w^2}{3\kappa S^2}[\nabla_i\mathbf{S}\times\mathbf{S}+\kappa\tau_1\mathbf{S}(\mathbf{S}\nabla_i\mathbf{S})]. \tag{113}$$

Stability in the normal liquid. We show that the final term in (113) is not important for our study. It leads to rapid diffusion equalization of the absolute magnitude of the magnetization in space, and in the absence of gradients of $S=|\mathbf{S}|$ vanishes. Indeed, it follows from the evolution equation for the spin in the normal liquid and the expression (113) for the current that

$$\frac{\partial}{\partial t}(S^2)=\frac{w^2\tau_1}{6S^2}[(\nabla S^2)^2+2S^2\nabla^2(S^2)]. \tag{114}$$

It can be seen from this equation that the characteristic time for smoothing variations in the distribution of S^2 is

$$T\sim\frac{\lambda^2}{w^2\tau_1}. \tag{115}$$

In the collisionless regime in the normal liquid, this time is small compared with the reciprocal characteristic frequency $(\lambda\nabla\omega_L)^{-1}$, and in superfluid ^{3}He-B (for $\lambda_D\ll\lambda_n$) it is small compared with the reciprocal characteristic frequency ω_L/Ω_B^2.

Thus, considering small oscillations near the two-domain solution, we can assume that $S^2=$const and that the spin current is given by

$$\mathbf{J}_i\cong\frac{w^2}{3\kappa S^2}\mathbf{S}\times\nabla_i\mathbf{S}. \tag{116}$$

The spin evolution equation takes the form

$$\frac{\partial\mathbf{S}}{\partial t}=\mathbf{S}\times\boldsymbol{\omega}_L-\frac{w^2}{3\kappa S^2}\mathbf{S}\times\nabla^2\mathbf{S}. \tag{117}$$

This equation has the energy integral

$$H=\int dz\left[\frac{(\mathbf{S}-\boldsymbol{\omega}_L)^2}{2}-\frac{w^2}{6\kappa S^2}(\nabla_i\mathbf{S})^2\right]. \tag{118}$$

Equation (117) is the Landau–Lifshitz equation for an isotropic magnet in a magnetic field but with negative (!) (for $\kappa>0$) coefficient of the gradient term. This is a Hamiltonian equation with the energy (118) as Hamiltonian and the usual commutation relation for the spin: $[S_\alpha(x),S_\beta(y)]=ie_{\alpha\beta\gamma}S_\gamma(x)\delta(x-y)$ (see, for example, Ref. 17). Quasiperiodic solutions of Eq. (117) for the spin dynamics were investigated in Ref. 18.

The absolute magnitude of the magnetization is an integral of Eq. (117). The total longitudinal magnetization $\int dzS_z$ is also an integral of Eq. (117). For given magnitude of the spin and total longitudinal magnetization, the energy is, to within constant terms,

$$H=\int dz\left[(\boldsymbol{\omega}_P-\boldsymbol{\omega}_L)\mathbf{S}-\frac{w^2}{6\kappa S^2}(\nabla_i\mathbf{S})^2\right], \tag{119}$$

where to the energy we have added the longitudinal magnetization with coefficient ω_p, corresponding to the transition to a frame of reference rotating with this frequency. The expression for the energy can be rewritten in terms of the spherical coordinates α and β of the vector $\mathbf{S}$:

$$H=\int dz\left[(\omega_P-\omega_L)S\cos\beta-\frac{w^2}{6\kappa}((\nabla\beta)^2+(\nabla\alpha)^2\sin^2\beta)\right]. \tag{120}$$

The extrema of the energy (120) give stationary (in the frame of reference rotating with frequency ω_p) solutions of Eq. (117). The solution proposed in Ref. 4 realizes a local maximum of the energy (119). It can be seen from (120) that the maximum is attained when $\nabla\alpha=0$. The distribution of the angle β is given by the equation obtained by varying the energy with respect to β:

$$(\omega_L-\omega_P)S\sin\beta+\frac{w^2}{3\kappa}\beta''=0. \tag{121}$$

The second variation of the energy is

$$\delta^2H=\int dz\left[(\omega_L-\omega_P)S\cos\beta(\delta\beta)^2-\frac{w^2}{3\kappa}(\delta\beta')^2-\frac{w^2}{6\kappa}\delta(\alpha'^2\sin^2\beta)\right]. \tag{122}$$

For $\kappa>0$, a domain with $\beta=0$ is situated in the region of weaker fields $(\omega_L<\omega_p)$, i.e., $(\omega_L-\omega_p)\cos\beta<0$, and the expression (122) is nonpositive for all deviations from the two-domain solution. At the same time, the only perturbation that does not change the energy is a uniform rotation around the $\hat{z}$ axis, under which α changes but $\nabla\alpha$ and β do not.

In view of the conservation of energy, the fact that the solution in Ref. 4 realizes a local maximum of (120) ensures its stability.

Thus, the solution that realizes the local maximum is stable. Walls with rotation of the magnetization through 3π, 5π, etc.,[16] also correspond to local maxima of (120).

In a liquid with negative κ, the coefficient of the gradient term in (119) is positive, and a solution corresponding to a local minimum of the sum of the spectroscopic and gradient energy is realized.

The eigenfrequencies of small oscillations superposed on the two-domain structure in a normal liquid were calculated by Fomin.[16]

Stability in the superfluid B phase. In superfluid ^{3}He-B near T_c, the energy of the dipole–dipole interaction is also added to the energy. The investigation of the stability of the two-domain structure is a complicated problem. However, qualitative arguments offer hope that this structure is stable.

We note, first, that if we freeze the degrees of freedom associated with the change in the absolute magnitude of the spin and Φ [i.e., we assume that Φ is a quantity that is completely determined by the angle β in accordance with the rules (100) and (101) of the previous section], then the problem takes the same form as in the normal liquid except that it is necessary to add the dipole term to the energy. As before, the solution realizes a local maximum of this energy and is stable.

On the other hand, in the homogeneous case the solutions 1) and 2) of the previous section are stable with respect to homogeneous perturbations of all the variables, including S and Φ.[12]

In principle, oscillations near the solution in the homogeneous case are not independent even for frozen S and Φ, and a careful investigation of the stability problem is needed. However, the arguments given above offer hope that the result will be positive.

7. RELAXATION OF THE TWO-DOMAIN STRUCTURE

To study the relaxation, it is necessary to take into account the dissipative terms in the equations. The energy of the liquid is (near the transition temperature, we ignore the gradient energy associated with the spatial variation of the order parameter)

$$E=\int dz\,\frac{\chi}{\gamma^2}\left\{\frac{(\boldsymbol{\sigma}-\boldsymbol{\omega}_L)^2}{2}+\frac{3}{2}\left(\frac{\gamma^2}{\chi}\right)^2\frac{\mathbf{J}^2}{w^2}+\Omega_B^2\langle(\hat{\mathbf{k}}\cdot\hat{\mathbf{d}}(\mathbf{k}))^2\rangle\right\}. \tag{123}$$

Here the first two terms correspond to the energy of a Fermi liquid with nonequilibrium values of the spin density and spin current (cf. Ref. 4), and the third term is the dipole energy. The averaging in this term is over the directions of the unit vector $\hat{\mathbf{k}}$. Calculating the time derivatives of the energy by means of the equations of motion (76), (77), and (68), we obtain for the dissipation

$$\frac{dE}{dt}=-\int dz\left\{\frac{\gamma^2}{\chi}\frac{3\mathbf{J}^2}{w^2\tau_1}+\frac{\gamma^2}{\chi_{p0}}(\eta\cdot\mathbf{R}_D)\right\}. \tag{124}$$

The first term here corresponds to the dissipation in the normal liquid, while the second is associated with the Leggett–Takagi mechanism.[7] To estimate η, it is necessary to use the equation of motion (72).

For the structure in which we are interested, the second term in (124) is small compared with the first, and we shall not take it into account. The main relaxation mechanism is associated with the flowing of the diffusion spin currents.

In the case $\lambda_n \gg \lambda_D$, the calculations lead to the expression

$$\frac{dE}{dt}=-\frac{\chi}{\gamma^2}\frac{w^2}{3\kappa^2\lambda_D}\frac{C}{\tau_1}, \tag{125}$$

where C is a number of order unity that is determined by the exact shape of the wall:

$$C=\lambda_D\int dz\beta'^2. \tag{126}$$

Assuming that the wall thickness λ_D is small compared with the length L of the vessel, for the energy of the system in the leading approximation in λ_D/L we obtain

$$E=\frac{2\chi}{\gamma^2}\omega_L^2 l, \tag{127}$$

where l is the length of a domain with the nonequilibrium value of the magnetization.

In the normal liquid, there is no dipole energy, and the longitudinal component of the total magnetization is conserved.[4] Therefore, in the process of relaxation of the two-domain structure the absolute magnitude of the magnetization is reduced in both domains, and the position of the domain wall is changed in such a way as to ensure conservation of the longitudinal component. In the superfluid liquid, the magnetization is not conserved. In addition, the deviation of the absolute magnitude of the magnetization from the equilibrium value is due to the strong increase in the dipole energy. Therefore, in the process of relaxation the absolute magnitude of the magnetization does not change, but the position of the domain wall does—the equilibrium domain grows. For an uniformly precessing domain, this causes the frequency to decrease with time. In our case, the frequency increases.

Since l is uniquely determined by z_0 [see (106)], and z_0 determines the precession frequency, for the variation of the frequency in time we obtain

$$\frac{d\omega_P}{dt}=\frac{\lambda_D\nabla\omega_L}{6}\left(\frac{w}{\kappa\omega_L\lambda_D}\right)^2\frac{C}{\tau_1}. \tag{128}$$

The frequency is a linear function of the time.

8. FORMATION OF TWO-DOMAIN STRUCTURE; COMPETITION OF THE SPIN CURRENTS

As the temperature is lowered, the induction signal from the precessing structure disappears, but at lower temperatures it is restored. In this region, a homogeneously precessing domain is observed. A structure forms as a consequence of the competition between the spin currents: after application of a pulse (in pulsed NMR experiments), the magnetization and order parameter, deflected from the equilibrium values, begin to precess in the inhomogeneous external field. This gives rise to gradients and spin currents. In the normal liquid,[4] it is a consequence of the Fermi-liquid effects that the spin current of the z component of the magnetization is then directed toward the weak fields, while in the superfluid liquid in the hydrodynamic regime[2] the spin current $\mathbf{J}_i^{\mathrm{loc}}$ flows in the direction of the stronger fields. As a result, different two-domain structures arise—in the normal liquid, the equilibrium domain is in the region of weak fields, while in the case of a uniformly precessing domain it is in the region of strong fields. We estimate the magnitude of these two currents in order to understand the temperatures at which a solution is realized.

We shall assume that the pulse of the transverse field has deflected the magnetization through an angle of order 90°. To estimate the gradients that arise in the case of precession in the inhomogeneous field, we can use Eqs. (76) and (5). The gradient of the order parameter at time δt after the pulse is $\nabla R_{\alpha i}=\delta t e_{\alpha\beta\gamma}R_{\beta i}(\nabla\omega_L)$, and hence $\mathbf{A}=\delta t\nabla\omega_L/2m^*$. Thus, for the hydrodynamic contribution to the spin current we obtain the estimate

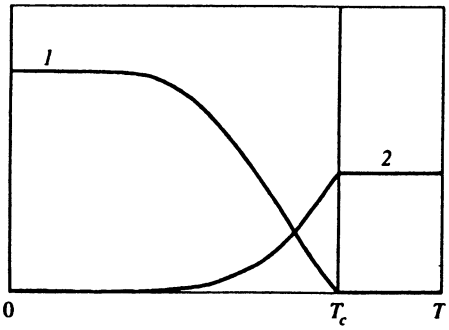

FIG. 3. Coefficients that determine the relative importance of the hydrodynamic spin current J_i^{loc} (curve 1) and the nonequilibrium spin current δJ_i (curve 2) as functions of the temperature.

$$J^{loc}=\frac{2\hbar S_0}{5m^*}(1-Y_0)\nabla\omega_L\delta t. \tag{129}$$

The nonequilibrium current can be estimated from Eq. (37). Ignoring the terms proportional to Y_0-Y_2, which are small in the complete region of temperatures, for $\omega_p\tau_l>1$ we can solve the equation for the current:

$$\delta\mathbf{J}=\frac{\mathbf{u}}{u^2}\times\left[\frac{v_F^2}{3}\nabla\mathbf{S}^q+\frac{\hbar S_0}{2m^*}Y_2\frac{\gamma^2}{\chi_{n0}}F_0^a\nabla\mathbf{S}\right]. \tag{130}$$

Here $\mathbf{u}=\omega_L-\omega_P-(\gamma^2/\chi_{n0})F_0^a\mathbf{S}$. Substituting this expression in the previous one, we arrive at the estimate

$$\delta J=-\frac{\hbar S_0}{2m^*}\left(\frac{\chi}{\chi_{n0}}\right)^2\frac{(1+F_0^a)^2(1-\lambda_{LT}+Y_2F_0^a)}{-F_0^a}\nabla\omega_L\delta t. \tag{131}$$

The coefficients in the expressions for the two contributions to the current are shown in Fig. 3. They are equal at $1-T/T_c\sim0.15$. Thus, near the transition the structure described in the previous sections arises, and at low temperatures a uniformly precessing domain must be formed. In the region of intermediate temperatures, the competition between the spin currents evidently leads to a chaotic picture, and the relaxation occurs more rapidly than any of the uniformly precessing states can be established.

9. CONCLUSIONS

In this paper, we have derived the equations of the spatially inhomogeneous spin dynamics of the superfluid B phase of ^{3}He. In the hydrodynamic regime, we have obtained a generalization of the Leggett–Takagi equations to the inhomogeneous case. As in the homogeneous case, the corresponding system of equations consists of evolution equations for the spin density and the order parameter, while the spin current is determined in the hydrodynamic limit by the values and gradients of the spin and the order parameter. These equations make it possible to take into account the dissipation in the leading order in the small parameters $\omega\tau$ and $v_F\tau/\lambda$.

In the collisionless region, we have derived approximate equations for the spin dynamics near the transition temperature. They are equations for macroscopic variables: the densities of the spin and of the spin current and the order parameter. They can be used to investigate the inhomogeneous spin structures that arise.

The coherently precessing solution of these equations near the transition temperature found in this paper shows that in the region in which the energy of the dipole–dipole interaction is already important a two-domain structure can exist analogous to the one that exists in the normal Fermi liquid. In one of the domains, the magnetization is parallel to the magnetic field, while in the other it is antiparallel. The shift of the frequency due to the dipole moment has the consequence that the precession frequency differs from the local Larmor frequency at the point at which the domain wall is situated. In the process of relaxation, the antiparallel domain decreases, while the precession frequency increases.

Recently, a new long-lived coherently precessing structure was found in ^{3}He-B at temperatures appreciably below the transition temperature.[19] The increase with time of the precession frequency as this structure[3)] relaxes suggests that the equilibrium domain is situated in the region of weaker fields, as in the two-component structures in a normal liquid and in superfluids near T_c. It is natural to assume that in this case too the dissipationless spin current associated with the Fermi-liquid interaction leads to the formation of such a structure. However, the estimates given in the paper show that the role of this current is small at low temperatures. The question of the reasons for the formation of the new coherently precessing state at low temperatures remains open.

We thank I. A. Fomin, Yu. Bun'kov, and especially V. V. Dmitriev for numerous helpful discussions and for allowing us to be acquainted with the results of their studies before publication. This work was done with partial financial support by the International Science Foundation (Grants No. MG1000 and No. MG1300) and also by the Forschungszentrum Jülich GmbH in the framework of the Landau Scholarship Program (Yu. G. M.).

APPENDIX A

In this appendix, we derive the equations (36) and (37) of the evolution of the spin of the Bogolyubov quasiparticles and the nonequilibrium spin current. The spin of the normal component is determined by [cf. (18)]

$$\mathbf{S}^q=\frac{\hbar}{2}\sum_k\left[\frac{\xi}{E}\delta\boldsymbol{\nu}+\left(1-\frac{\xi}{E}\right)\hat{\mathbf{d}}(\hat{\mathbf{d}}\cdot\delta\boldsymbol{\nu})\right]. \tag{A1}$$

The time derivative of the distribution function of the Bogolyubov quasiparticles (after the gauge transformation) is determined by the kinetic equation (11), and the order parameter $\mathbf{d}$ in the ("moving") frame of reference determined by the gauge transformation does not depend on the time. Using this, we calculate the time derivatives of $\mathbf{S}^q$ and $\delta\mathbf{J}_i$ in the "moving" frame of reference. We give the derivation of the equation of motion for the quasiparticle spin; the derivation of the equation for the current is similar. Thus,

$$\frac{\partial}{\partial t}\mathbf{S}^q=\frac{\hbar}{2}\sum_k\left[\frac{\xi}{E}\left\{\frac{\xi}{E}(\mathbf{v}_F\nabla)(\varphi'\delta\mathbf{E}-\delta\boldsymbol{\nu})+\delta\mathbf{E}\times\delta\boldsymbol{\nu}\right\}\right.$$
$$+\left(1-\frac{\xi}{E}\right)\hat{\mathbf{d}}\left(\hat{\mathbf{d}}\cdot\left\{\frac{\xi}{E}(\mathbf{v}_F\nabla)(\varphi'\delta\mathbf{E}-\delta\boldsymbol{\nu})\right.\right.$$
$$\left.\left.\left.+\delta\mathbf{E}\times\delta\boldsymbol{\nu}\right\}\right)\right].\qquad\text{(A2)}$$

In this expression, we distinguish four components: parts not containing gradients; those containing components of $\delta\mathbf{E}$ and $\delta\boldsymbol{\nu}$, one of which is even in the momenta and the other odd); a part containing the gradients of the distribution function; and a part containing the gradients of $\delta\mathbf{E}$. We shall calculate them separately.

For the first component, we obtain, separating the term $\mathbf{S}^q\times\mathbf{X}$,

$$\left(\frac{\partial}{\partial t}\mathbf{S}^q\right)^{(1)}=\frac{\hbar}{2}\sum_k\left[\frac{\xi}{E}\left\{\frac{\xi}{E}\delta\boldsymbol{\nu}\times\mathbf{X}+\left(1-\frac{\xi}{E}\right)\delta\boldsymbol{\nu}\times\mathbf{d}(\hat{\mathbf{d}}\cdot\mathbf{X})\right\}\right.$$
$$\left.+\left(1-\frac{\xi}{E}\right)\hat{\mathbf{d}}\left(\hat{\mathbf{d}},\delta\boldsymbol{\nu},\frac{\xi}{E}\mathbf{X}\right)\right]$$
$$=\frac{\hbar}{2}\sum_k\left[\frac{\xi^2}{E^2}\delta\boldsymbol{\nu}\times\mathbf{X}+\frac{\xi}{E}\left(1-\frac{\xi}{E}\right)\right.$$
$$\left.\times\{\delta\boldsymbol{\nu}-\hat{\mathbf{d}}(\hat{\mathbf{d}}\cdot\delta\boldsymbol{\nu})\}\times\mathbf{X}\right]$$
$$=\frac{\hbar}{2}\sum_k\left[\frac{\xi}{E}\delta\boldsymbol{\nu}+\left(1-\frac{\xi}{E}\right)\hat{\mathbf{d}}(\hat{\mathbf{d}}\cdot\delta\boldsymbol{\nu})\right.$$
$$\left.-\left(1-\frac{\xi^2}{E^2}\right)\hat{\mathbf{d}}(\hat{\mathbf{d}}\cdot\delta\boldsymbol{\nu})\right]\times\mathbf{X}=(\mathbf{S}^q-\mathbf{B})\times\mathbf{X},\qquad\text{(A3)}$$

where $\mathbf{B}$ is given by the expression (38). Here we have used the identity[7]

$$\mathbf{A}\times\mathbf{B}+\mathbf{B}\times\hat{\mathbf{d}}(\hat{\mathbf{d}}\cdot\mathbf{A})+\hat{\mathbf{d}}\times\mathbf{A}(\hat{\mathbf{d}}\cdot\mathbf{B})+\hat{\mathbf{d}}(\hat{\mathbf{d}},\mathbf{B},\mathbf{A})=0,$$

where $(\mathbf{d},\mathbf{B},\mathbf{A})$ denotes the triple product.

In the second component, it is convenient to separate the term $m^*\mathbf{J}_i^q\times\mathbf{A}_i$, which corresponds to lengthening the divergence of the spin current of the quasiparticles:

$$\mathbf{J}_i^q=\frac{\hbar}{2}\sum_k v_{Fi}\left[\delta\boldsymbol{\nu}-\left(1-\frac{\xi}{E}\right)\hat{\mathbf{d}}(\hat{\mathbf{d}}\cdot\delta\boldsymbol{\nu})\right]$$
$$=\rho^q_{\alpha i,\beta j}A_{\beta j}+\delta\mathbf{J}_i,\qquad\text{(A4)}$$

where

$$\rho^q_{\alpha i,\beta j}=\frac{\hbar S_0}{2}\left(Y_0\delta_{\alpha\beta}\delta_{ij}-\frac{Y_0-Y_2}{5}P_{\alpha i,\beta j}\right).\qquad\text{(A5)}$$

After analogous transformations, we obtain

$$\left(\frac{\partial}{\partial t}\mathbf{S}^q\right)^{(2)}=m^*\mathbf{J}_i^q\times\mathbf{A}_i m^*D^i_{\alpha\beta}A_{\beta i},$$

where D is determined by the expression (39). For the third component, we obtain the expression

$$\left(\frac{\partial}{\partial t}\mathbf{S}^q\right)^{(3)}=-\nabla_i\mathbf{J}_i^q-\nabla_i\frac{\hbar}{2}\sum_k v_{Fi}\left(1-\frac{\xi^2}{E^2}\right)$$
$$\times[\hat{\mathbf{d}}(\hat{\mathbf{d}}\cdot\delta\boldsymbol{\nu})-\delta\boldsymbol{\nu}]$$
$$=-\nabla_i\mathbf{J}_i^q+e_{\alpha\beta\gamma}\nabla_iD^i_{\beta\gamma},$$

and the fourth does not contain $\delta\boldsymbol{\nu}$, and therefore all the sums over the momenta can be calculated:

$$\left(\frac{\partial}{\partial t}\mathbf{S}^q\right)^{(4)}=\frac{\hbar}{2}S_0Y_2\nabla_i\mathbf{A}_i.$$

Adding these contributions, we obtain the expression for the derivative of the spin of the quasiparticles in the moving coordinate system:

$$\left(\frac{\partial}{\partial t}\mathbf{S}^q\right)^m=(\mathbf{S}^q-\mathbf{B})\times\mathbf{X}-(\nabla_i+m^*\mathbf{A}_i\times)\mathbf{J}_i^q$$
$$+e_{\alpha\beta\gamma}\nabla_iD^i_{\beta\gamma}+m^*+m^*D^i_{\alpha\beta}A_{\beta i}$$
$$+\frac{\hbar}{2}S_0Y_2\nabla_i\mathbf{A}_i.\qquad\text{(A6)}$$

Going over to the laboratory frame of reference [see (32) and (33)] and expressing $\mathbf{J}_i^q$ in terms of $\delta\mathbf{J}_i$ by means of (A4), we arrive at Eq. (36). The relaxation terms are discussed in the following appendix.

APPENDIX B

To derive the relaxation terms, we use the form of the collision integral given in Ref. 7:

$$\mathbf{I}(\delta\boldsymbol{\nu})=-\frac{1}{\tau_{\mathrm{CE}}}(\delta\boldsymbol{\nu}-\varphi'\delta\mathbf{E})-\frac{1}{\tau'}(\delta\boldsymbol{\nu}-\varphi'\delta\mathbf{F}).\qquad\text{(B1)}$$

The form of the first term on the right-hand side was proposed by Combescot and Ebisawa[20] and corresponds to relaxation of the normal component to relative equilibrium with the condensate for a fixed state of this last (i.e., for fixed $\delta\mathbf{E}$). The form of this term is determined by requiring that it vanish in local equilibrium. The second term corresponds to collisions of the quasiparticles with each other, i.e., to redistribution of the spin between the quasiparticles subject to conservation of the total spin of the normal component. The form of this term is due to the fact that in such collisions the spin is conserved but not the spin current—the current relaxes to its equilibrium value. Accordingly,

$$\frac{1}{\hbar}\delta\mathbf{F}=-\left[k_i\mathbf{A}_i+\frac{\xi}{E}\mathbf{X}^q-\left(1-\frac{\xi}{E}\right)\hat{\mathbf{d}}(\hat{\mathbf{d}}\cdot(k_i\mathbf{A}_i-\mathbf{X}^q))\right],\qquad\text{(B2)}$$

where

$$\mathbf{S}^q=\frac{\chi_{q0}}{\gamma^2}\mathbf{X}^q\qquad\text{(B3)}$$

[cf. (50)]. It is readily verified that this term in the collision integral does not contribute to the relaxation of the quasiparticle spin. In the transition to the normal liquid we have $\tau_{\mathrm{CE}}\to\infty$, $\tau'\to\tau_n^0$, and the relaxation is described by the second term. During the relaxation process, the total spin is conserved (we note in this connection that we work under the assumption of an infinite longitudinal spin relaxation time T_1, cf. Ref. 6).

For the collisional term in the equation for the quasiparticle spin, we obtain

$$\left(\frac{\partial}{\partial t}\mathbf{S}^q\right)_{\text{coll}}=-\frac{1}{\tau_{\text{CE}}(T)}(\mathbf{S}^q-\mathbf{S}^{q,\text{loc}}), \quad \text{(B4)}$$

where $\mathbf{S}^{q,\text{loc}}$ is the value of the quasiparticle spin for $\delta\boldsymbol{\nu}=\varphi'\delta\mathbf{E}$, i.e., in local equilibrium. Since $\mathbf{S}^{q,\text{loc}}=(\chi_{q0}/\chi_{p0})\mathbf{S}^p$, we can, going over in (B4) to the variables $\mathbf{S}^q$ and $\mathbf{S}$, reduce the relaxation term to the form

$$\left(\frac{\partial}{\partial t}\mathbf{S}^q\right)_{\text{coll}}=-\frac{1}{\lambda_{\text{LT}}\tau_{\text{CE}}(T)}(\mathbf{S}^q-(1-\lambda_{\text{LT}})\mathbf{S}). \quad \text{(B5)}$$

This corresponds to Eq. (36) for $\tau=\lambda_{\text{LT}}\tau_{\text{CE}}$. Note that in the limit $T\to T_c$ this relaxation time tends to the lifetime of the quasiparticles of the normal Fermi liquid: $\tau\to\tau_n^0$ (Refs. 21 and 22).

For the relaxation of the spin current, we obtain

$$\left(\frac{\partial}{\partial t}\delta\mathbf{J}_i\right)_{\text{coll}}=-\frac{1}{\tau_1}\delta\mathbf{J}_i, \quad \text{(B6)}$$

where $1/\tau_1=1/\tau_{\text{CE}}+1/\tau'$. Near the transition temperature, $\tau_1\approx\tau'\approx\tau_n^0$.

APPENDIX C

On the transition to the case of the normal liquid, for example, in Eqs. (18) and (19), the following difficulty arises. In this limit, the expression ξ/E goes over into sign ξ and in the expressions terms that depend on the order parameter remain even in the normal liquid. The reason is that the matrix of the Bogolyubov transformation used in the derivation of the expressions tends to the identity matrix as $T\to T_c$ only when $\xi>0$.

The Bogolyubov transformation used in the derivation of the expressions in the paper is a transformation that diagonalizes the energy operator of the quasiparticles in the absence of perturbations:

$$\varepsilon_k=\begin{pmatrix}\xi_k\delta_{\alpha\beta} & \Delta^+_{k,\alpha\beta}\\ \Delta_{k,\alpha\beta} & -\xi_{-k}\delta_{\alpha\beta}\end{pmatrix}. \quad \text{(C1)}$$

Under this transformation, the distribution function (and, similarly, the energy operator of the quasiparticles) is replaced by

$$\nu_k=U_k n_k U_k^+, \quad \text{(C2)}$$

where the unitary matrix U_k is chosen in the form

$$U_k=\begin{pmatrix}u_k & \nu_k\\ -\upsilon_k^+ & u_k\end{pmatrix}, \quad \text{(C3)}$$

and the matrices u_k and υ_k are such that

$$u_{k,\alpha\beta}=\sqrt{(E+\xi)/2E}\,\delta_{\alpha\beta}, \quad \text{(C4)}$$

$$\upsilon_{k,\alpha\beta}=\Delta^+_{k,\alpha\beta}/\sqrt{2E(E+\xi)}. \quad \text{(C5)}$$

Here $E=\sqrt{\Delta^2+\xi^2}$ is the energy of the Bogolyubov excitations, and

$$n_{kk'}=\begin{pmatrix}\langle a^+_{k\alpha}a_{k'\beta}\rangle & \langle a^+_{k\alpha}a^+_{-k'\beta}\rangle\\ \langle a_{-k\alpha}a_{k'\beta}\rangle & \langle a_{-k\alpha}a^+_{-k'\beta}\rangle\end{pmatrix}. \quad \text{(C6)}$$

To avoid the difficulty noted above, one can use a Bogolyubov transformation that goes over to the identity as $T\to T_c$. This transformation is identical to (C3) for $\xi>0$ and differs from it by the replacement of ξ by $|\xi|$ and reversal of the sign of the components nondiagonal with respect to the particle—hole index (i.e., a change in the sign of υ) when $\xi<0$. Such a substitution is possible because of the nonuniqueness of the Bogolyubov transformation: For each pair $(\mathbf{k},-\mathbf{k})$, there are two eigenstates of the Bogolyubov Hamiltonian with energy E and two with energy $-E$. Therefore, transformations that interchange these two two-dimensional subspaces and also rotate them separately do not change the diagonal form of the Hamiltonian, and this guarantees the necessary freedom. The Bogolyubov transformation that we have just described is essentially a transformation, not to quasiparticles with two branches of the spectrum $\varepsilon(\mathbf{k})=\pm E$, but to quasiparticles with spectrum $\varepsilon(\mathbf{k})=\pm\text{sign}(\xi)E$ (see Ref. 21), going over as $T\to T_c$ into $\pm\xi$. These two branches of the spectrum in the normal liquid correspond to quasiparticles and quasiholes.

It can be shown that the change of the Bogolyubov transformation has the consequence that in all the expressions of the paper it is necessary to replace ξ/E by $|\xi|/E$. In the approach to the transition temperature, this ratio tends to unity irrespective of the sign of ξ.

[1)] In reality, in the region of angles $\beta<\theta_L$ the angle α depends on the coordinates, and Φ is not equal to zero but is determined by the equation $\cos\theta=-1/4$. However, the dependence of β on the coordinate for the solution corresponding to the domain wall of Ref. 2 is qualitatively the same as in Fig. 2. The angle α changes by a finite amount within the domain wall near z_0.

[2)] For $\beta<\theta_L$, the right-hand side of Eq. (109) does not contain a dipole term, the equation has the same form as in the "normal liquid," and the transition from $\beta=0$ to $\beta=\theta_L$ occurs over the corresponding length λ.

[3)] Yu. M. Bun'kov and S. R. Zakazov, private communication.

[1] A. S. Borovik-Romanov, Yu. M. Bun'kov, V. V. Dmitriev *et al.*, Zh. Éksp. Teor. Fiz. **88**, 2025 (1985) [Sov. Phys. JETP **61**, 1199 (1985)]; A. S. Borovik-Romanov, Yu. M. Bun'kov, V. V. Dmitriev *et al.*, Zh. Éksp. Teor. Fiz. **96**, 1109 (1989) [Sov. Phys. JETP **69**, 630 (1989)].

[2] I. A. Fomin, Zh. Éksp. Teor. Fiz. **88**, 2039 (1985) [Sov. Phys. JETP **61**, 1207 (1985)].

[3] G. Nunes (Jr.), C. Jin, D. L. Hawthorne *et al.*, Phys. Rev. B **46**, 9082 (1992).

[4] V. V. Dmitriev and I. A. Fomin, Pis'ma Zh. Éksp. Teor. Fiz. **59**, 352 (1994) [JETP Lett. **59**, 378 (1994)].

[5] V. V. Dmitriev, S. R. Zakazov, and V. V. Moroz, Pis'ma Zh. Éksp. Teor. Fiz. **61**, 309 (1995) [JETP Lett. **61**, 324 (1995)].

[6] A. J. Leggett, J. Phys. C **3**, 448 (1970).

[7] A. J. Leggett and S. Takagi, Ann. Phys. (N.Y.) **106**, 79 (1977).

[8] I. A. Fomin, in *Helium 3*, edited by W. P. Halperin and L. P. Pitaevskii (Elsevier Science Publishers, B. V., 1990), p. 609.
[9] O. Betbeder-Matibet and P. Nozières, Ann. Phys. (N.Y.) **51**, 932 (1969).
[10] R. Combescot, Phys. Rev. B **13**, 126 (1976); Phys. Rev. A **10**, 1700 (1974).
[11] D. Vollhardt and P. Wölfle, *The Superfluid Phases of Helium 3* (Taylor and Francis, London, 1990).
[12] I. A. Fomin, Zh. Éksp. Teor. Fiz. **84**, 2109 (1983) [Sov. Phys. JETP **57**, 1227 (1983)].
[13] W. P. Brinkman and H. Smith, Phys. Lett. A **53**, 43 (1975).
[14] J. S. Korhonen and G. E. Volovik, Pis'ma Zh. Éksp. Teor. Fiz. **55**, 358 (1992) [JETP Lett. **55**, 362 (1992)].
[15] Yu. M. Bun'kov and G. E. Volovik, Zh. Éksp. Teor. Fiz. **103**, 1619 (1993) [JETP **76**, 794 (1993)].
[16] I. A. Fomin, Zh. Éksp. Teor. Fiz. **108**, 642 (1995) [JETP **81**, 347 (1995)].
[17] L. D. Faddeev and L. A. Takhtadzhyan, *Hamiltonian Methods in the Theory of Solitons* (Springer-Verlag, Berlin, 1987) [Russ. original, Nauka, Moscow, 1986}.
[18] L. P. Lévy, Phys. Rev. Lett. **31**, 7077 (1985).
[19] Yu. M. Bun'kov, S. N. Fisher, A. M. Guénault, and G. R. Pickett, Phys. Rev. Lett. **69**, 3092 (1992).
[20] R. Combescot and H. Ebisawa, Phys. Rev. Lett. **33**, 810 (1974).
[21] P. Bhattacharyya, C. J. Pethick, and H. Smith, Phys. Rev. B **15**, 3367 (1977).
[22] P. Bhattacharyya, C. J. Pethick, and H. Smith, Phys. Rev. Lett. **35**, 473 (1975).

Translated by Julian B. Barbour