Magnetic moment of a parabolic quantum well in a perpendicular magnetic field

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We study the magnetic moment and susceptibility of an electron gas in a quantum well formed by a parabolic confinement potential. A new method of calculating the magnetic moment based on the representation of this moment as a Fourier integral is suggested. The cases of degenerate and nondegenerate gases are investigated. We show that for a degenerate gas the quantum well contains aperiodic oscillations varying with field B and periodic oscillations with a period equal to the magnetic flux quantum. Finally, we demonstrate that at low temperatures the quantum well splits into Kondo domains.
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1. INTRODUCTION

There has been an upsurge of interest in the properties of electrons in quasi-zero-dimensional systems (quantum wells and quantum dots), especially after it became possible to experimentally investigate wells with widths comparable to the cyclotron radius in heterostructures (superlattices and surface-charge layers). Hybridization of electric, size, and magnetic quantization promises discoveries of new effects in such systems.

A convenient way to model well confinement is to employ a parabolic potential, since the Hamiltonian of electrons with such a potential in a magnetic field is quadratic. With such a Hamiltonian the energy spectrum of the system is simply the sum of the spectra of two independent harmonic oscillators with frequencies that are nonlinear functions of the characteristic frequencies of the confinement potential and the cyclotron frequency. This makes it possible in such systems to use exact analytic functions for the thermodynamic characteristics of the electron gas, which depend on the Hamiltonian parameters. The parabolic potential[1–4] and some other potentials[5,6] have been successfully used for studying the thermodynamic characteristics of low-dimensional systems. It must be noted, however, that nonparabolic models of a well confinement potential[5,6] have been unable to produce analytic formulas, and the symmetric parabolic potential was used by Meyr *et al.*[1] only to obtain the thermodynamic potential in the low-temperature limit ($T \to 0$).

According to Symon's result (see Ref. 7, p. 267, Theorem 11.1), a good approximation for any confinement potential at high energy levels is the parabolic potential. This strongly supports the idea of using a parabolic potential to approximate a quantum-well potential.

According to the generalized Kohn theorem,[8] the electron–electron interaction in a parabolic lateral confinement well has no effect on the well's spectrum. In view of the fact that the quantum-well characteristics studied below depend only on the spectrum, we can ignore the electron–electron interaction inside the well.

General considerations based on the behavior of the density of states suggest that as the magnetic field B varies the thermodynamic potential Ω undergoes oscillations of the de Haas–van Alphen type. These oscillations are caused by the fact that one level belonging to the system of discrete-spectrum levels crosses the chemical potential of the electron gas in the quantum well. Moreover, since the xy plane contains a dimension characteristic of the confinement potential and determined by the effective radius of the well, oscillations of the Aharonov–Bohm type with a period equal to the flux quantum are also possible.

In this paper we study the magnetic moment and magnetic susceptibility of an electron gas (degenerate and nondegenerate) in a magnetic field that is perpendicular to the well's confinement plane. The confinement potential is chosen in the form $U(x_1,x_2)=m^*(\omega_1^2x^2+\omega_2^2y^2)/2$, where m^* is the effective electron mass, and ω_1 and ω_2 are the characteristic frequencies of the potential.

The common approach to calculating the thermodynamic potential and the magnetic moment is to expand these quantities in Fourier series.[9–11] This makes it possible to obtain approximate expressions for the oscillating and monotonic parts of Ω by the methods described in Refs. 9 and 10, which yield, as is known, the same result for a gas of 3D-electrons in the absence of a lateral confinement potential. The success of these methods is guaranteed by the fact that $\Omega(B)$ in such a system of electrons is a periodic function of $1/B$. In the case considered below, adding potential U disrupts periodicity. Direct use of the method based on Poisson's summation formula[9,11] and the method of contour integration of the classical partition function[10] yield divergent Fourier series in our case. The reason for this is the contribution from the poles of the classical partition function

$$Z=\frac{1}{4}\left[\sinh\left(\frac{\hbar\Omega_1}{2T}\right)\sinh\left(\frac{\hbar\Omega_2}{2T}\right)\right]^{-1}$$

at points $2\pi n_i/\hbar\Omega_1$ or $2\pi m_i/\hbar\Omega_2$, where Ω_1 and Ω_2 are the hybrid frequencies of the oscillators. These poles produce the factors $[\sin(\pi m\Omega_2/\Omega_1)]^{-1}$ or $[\sin(\pi m\Omega_1/\Omega_2)]^{-1}$ in the oscillating part of Ω, due to which the Fourier series in m and n always contain high-order terms with extremely small denominators for irrational frequency relationships. We sug-

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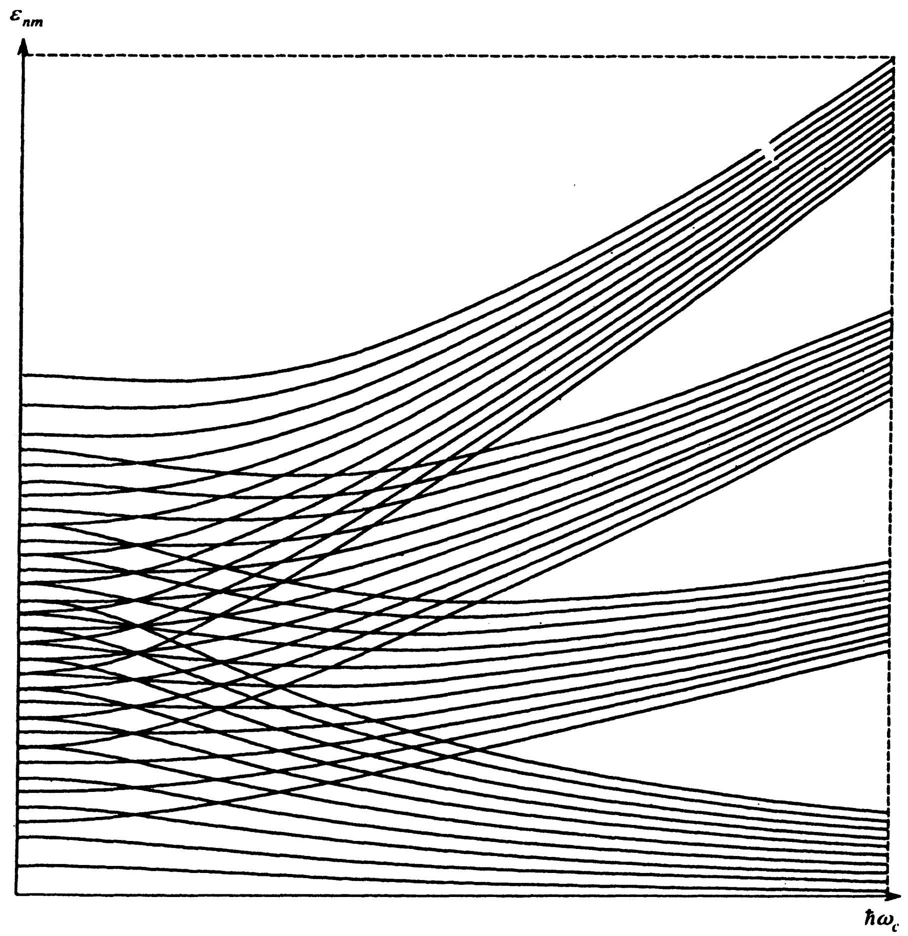

FIG. 1. Dependence of the dispersion laws on cyclotron frequency at $\omega_1=2$ and $\omega_2=5$ (arbitrary units).

gest another method for calculating the thermodynamic characteristics of quasi-zero-dimensional systems based on the Fourier transformation. As shown below, this method makes it possible to obtain exact analytic expressions for the calculated values in the parabolic model of a quantum-well confinement potential.

2. SPECTRUM OF A PARABOLIC WELL IN A PERPENDICULAR FIELD B

The spinless one-particle Hamiltonian of a 2D-electron in a quantum well has the form

$$H=\frac{1}{2m}\left(\mathbf{p}-\frac{e_0}{c}\mathbf{A}\right)^2+\frac{m\omega_1^2x^2}{2}+\frac{m\omega_2^2y^2}{2}, \tag{1}$$

where the vector potential of the field B is $\mathbf{A}=\frac{1}{2}(-By,Bx,0)$. Thus $H(p,q)$, with $q=(x,y)$, is specified by a quadratic form of the phase coordinates p and q; clearly, the matrix M of this form is

$$\mathrm{M}=\begin{Vmatrix} 1/m & 0 & 0 & -\omega_c/2 \\ 0 & 1/m & \omega_c/2 & 0 \\ 0 & \omega_c/2 & m(\omega_1^2+\omega_c^24) & 0 \\ -\omega_c/2 & 0 & 0 & m(\omega_2^2+\omega_c^24) \end{Vmatrix}. \tag{2}$$

By a canonical transformation of the phase space (see Ref. 12, Chap. V, p. 143) the Hamiltonian $H(p,q)$ can be reduced to new phase coordinates P and Q in terms of which H has the canonical form

$$H(P,Q)=P_x^2+P_y^2+\Omega_1^2Q_1^2+\Omega_2^2Q_2^2. \tag{3}$$

As is known (see Ref. 13, §17, p. 240), to find the hybrid oscillation frequencies Ω_1 and Ω_2, we must take the imaginary parts of the eigenvalues λ_i of the matrix IM, where I is the symplectic unity:

$$I=\begin{Vmatrix} 0 & -E \\ E & 0 \end{Vmatrix}, \qquad E=\begin{Vmatrix} 1 & 0 \\ 0 & 1 \end{Vmatrix}. \tag{4}$$

For IM we find

$$\begin{Vmatrix} 0 & -\omega_c/2 & -m(\omega_1^2+\omega_c^2/4) & 0 \\ \omega_c/2 & 0 & 0 & -m(\omega_2^2+\omega_c^2/4) \\ 1/m & 0 & 0 & -\omega_c/2 \\ 0 & 1/m & \omega_c/2 & 0 \end{Vmatrix}. \tag{5}$$

This yields the following equation for the eigenvalues of the matrix IM:

$$\lambda^4+(\omega_1^2+\omega_2^2+\omega_c^2)\lambda^2+\omega_1^2\omega_2^2=0. \tag{6}$$

Solving (6) and separating the imaginary parts, we obtain

$$\Omega_{1,2}=\frac{1}{\sqrt{2}}(\omega_1^2+\omega_2^2+\omega_c^2)\pm\sqrt{(\omega_1^2+\omega_2^2+\omega_c^2)^2-4\omega_1^2\omega_2^2}. \quad (7)$$

In accordance with (7) and (3), the spectrum of the quantum well described by the Hamiltonian (1) has the form

$$\varepsilon_{mn}=\hbar\Omega_1(m+1/2)+\hbar\Omega_2(n+1/2), \qquad m,n=0,1,\ldots. \quad (8)$$

The ε_{mn} vs B dependence is depicted in Fig. 1. Clearly, there is strong level crossing in the region $\omega_c\leqslant\omega_1,\omega_2$, while in the field range where $\omega_c\gg\omega_1,\omega_2$ (strong magnetic quantization or a wide well) there is no level crossing. Energy segments containing the same number of levels become narrower as B grows and are separated by energy segments that contain no levels ε_{mn} and broaden as B grows. Asymptotically these levels tend to the Landau levels $\varepsilon_n=\hbar\omega_c(n+1/2)$ as B grows.

3.MAGNETIC MOMENT OF A DEGENERATE ELECTRON GAS

The magnetic moment M of a quantum well can be found by the formula $M=-(\partial\Omega/\partial B)_{\mu,T}$, where the thermodynamic potential Ω has the form

$$\Omega=-T\sum_{m,n=0}^{\infty}\ln\left[1+\exp\frac{\mu-\varepsilon_{mn}}{T}\right]. \quad (9)$$

Using (9) we get

$$-M=\sum_{m,n=0}^{\infty}\frac{d\varepsilon_{mn}}{dB}\left[1+\exp\frac{\varepsilon_{mn}-\mu}{T}\right]^{-1}, \quad (10)$$

where

$$\frac{d\varepsilon_{mn}}{dB}=\frac{m_0}{m^*}\frac{2\mu_B\omega_c[(m+1/2)\hbar\Omega_1-(n+1/2)\hbar\Omega_2]}{\Omega_1^2-\Omega_2^2},$$

with μ_B the Bohr magneton.

Then

$$-\frac{M}{\mu_B}=\frac{2m_0\omega_c}{m^*(\Omega_1^2-\Omega_2^2)}\sum_{n,m=0}^{\infty}\frac{\Omega_1(n+1/2)-\Omega_2(m+1/2)}{1+\exp\{[\hbar\Omega_1(n+1/2)+\hbar\Omega_2(m+1/2)-\mu]/T\}}. \quad (11)$$

We use the reduced frequencies $\alpha=\hbar\omega_c/T$ and $\beta_i=\hbar\Omega_i/T$, $i=1,2$, the parameters $\alpha_i=\exp(-\beta_i/2)$ and $\gamma_i=2\alpha m_0\beta_i/m^*(\beta_1^2-\beta_2^2)$, and the reduced chemical potential $p=(2\mu-\hbar\Omega_1-\hbar\Omega_2)/2T$. Then (7) assumes the form

$$-\frac{M}{2\mu_B}=\gamma_1F_{12}-\gamma_2F_{21}, \quad (12)$$

where

$$F_{12}=\sum_{n=0}^{\infty}\left(n+\frac{1}{2}\right)f_{12}(n),$$

$$F_{21}=\sum_{m=0}^{\infty}\left(m+\frac{1}{2}\right)f_{21}(m).$$

We write the new quantities f_{12} and f_{21} as

$$f_{12}(n)=\frac{1}{2}\exp\left(\frac{p-\beta_1n}{2}\right)\sum_{m=0}^{\infty}\exp\left(-\frac{\beta_2m}{2}\right)\times\left[\cosh\left(\frac{\beta_1n+\beta_2m-p}{2}\right)\right]^{-1},$$

$$f_{21}(m)=\frac{1}{2}\exp\left(\frac{p-\beta_2m}{2}\right)\sum_{n=0}^{\infty}\exp\left(-\frac{\beta_1n}{2}\right)\times\left[\cosh\left(\frac{\beta_1n+\beta_2m-p}{2}\right)\right]^{-1}. \quad (13)$$

We sum the two series in (13) via the following formula:[14]

$$\sum_{k=0}^{\infty}\frac{r^k}{\cosh(kx+a)}=\int_0^{\infty}\frac{\{\cos(at)-r\cos[(a-x)t]\}dt}{(1-2r\cos xt+r^2)\cosh(\pi t/2)}. \quad (14)$$

Note that Ref. 14 has a misprint: the denominator in (14) contains b instead of x. We introduce the following notation:

$$q_i=(1-2\alpha_i\cos\beta_it+\alpha_i^2)^{-1}, \quad i=1,2. \quad (15)$$

Then, using (14), we can easily arrive at the following:

$$f_{12}(n)=\exp\left(\frac{p-\beta_1n}{2}\right)\int_0^{\infty}dtq_2\times\frac{\cos[(p-\beta_1n)t]-\alpha_2\cos[(p-\beta_1n+\beta_2)t]}{\cosh(\pi t)},$$

$$f_{21}(m)=\exp\left(\frac{p-\beta_2m}{2}\right)\int_0^{\infty}dtq_1\times\frac{\cos[(p-\beta_2m)t]-\alpha_1\cos[(p-\beta_2m+\beta_1)t]}{\cosh(\pi t)}. \quad (16)$$

Substituting (16) into the expressions for F_{12} and F_{21}, interchanging the order of summation and integration (which is possible because of uniform convergence of the series), using a formula that follows from Ref. 14,

$$\sum_{k=1}^{\infty}\left(k+\frac{1}{2}\right)a^k\cos(kx-\delta)=\frac{1}{2}\frac{(1-a^2)\cos\delta-a\cos(x-\delta)+a^2\cos(2x+\delta)+a^3\cos(x+\delta)}{(1-2a\cos x+a^2)^2},\tag{17}$$

and carrying out lengthy but otherwise simple transformations, we obtain

$$F_{12}=\frac{1}{2}\exp\frac{p}{2}\int_0^{\infty}\frac{q_1^2q_2dt}{\cosh(\pi t)}\{[(1-2\alpha_1^2)\cos pt-\alpha_1\cos[(p-\beta_1)t]+\alpha_1^2\cos[(p-\beta_1)t]+\alpha_1^3\cos[(p+\beta_1)t]]-\alpha_2[(1-2\alpha_1^2)\cos[(p+\beta_2)t]-\alpha_1\cos[(p-\beta_2-\beta_1)t]+\alpha_1^2\cos[(p+2\beta_1+\beta_2)t]+\alpha_1^3\cos[(p+\beta_1+\beta_2)t]]\}.\tag{18}$$

The expression for F_{21} can be obtained from F_{12} by interchanging the subscripts: $1\leftrightarrow 2$ Substituting (18) into (12), we arrive at the Fourier transform of the magnetic moment. The expression for the magnetic moment found in this manner is an exact analytic formula for the M vs B dependence and is convenient for studying particular cases.

4.MAGNETIC MOMENT OF A WIDE QUANTUM WELL

Let us examine another important case: a wide quantum well ($\omega_c \gg \omega_1,\omega_2$). Below we show that when we are dealing with a wide well, we can go from the exact formula with the Fourier integral to the ordinary approximation in which the magnetic series is expressed by a Fourier series. This is possible, obviously, because the levels $\varepsilon_{nm}(B)$ do not cross in the region where $\omega_c \gg \omega_1,\omega_2$ (see Fig. 1). The following approximate formulas can be derived from (7) in this case:

$$\Omega_1\simeq\omega_c,\qquad \Omega_2\simeq\frac{\omega_1\omega_2}{\omega_c},\qquad \Omega_2\ll\Omega_1.\tag{19}$$

Hence $\alpha_1\simeq 0$, $\alpha_2\simeq 1$, $\gamma_2 \ll \gamma_1$, $F_{21}=o(F_{12})$, $q_1\simeq 1$, and $\gamma_1\simeq m_0/m^*$. Then Eq. (18) leads to the following estimate:

$$F_{12}\simeq\frac{1}{2}\exp\frac{p}{2}\int_0^{\infty}\frac{dt}{\cosh(\pi t)}\frac{\cos(pt)-\alpha_2\cos[(p-\beta_2)t]}{1-2\alpha_2\cos(\beta_2 t)+\alpha_2^2},\tag{20}$$

Combining (14) and (20) we get

$$F_{12}\simeq\frac{1}{2}\sum_{n=0}^{\infty}[1+\exp(\beta_2 n-p)]^{-1}.\tag{21}$$

The series in (21) can be transformed into the Fourier series if Poisson's formula $F(x)=[1+\exp(x-p)]^{-1}$ in employed. After performing elementary transformation we obtain

$$F_{12}\simeq\frac{1}{4}\left[1+\exp\left(\frac{\hbar\Omega_1+\hbar\Omega_2-2\mu}{2T}\right)\right]^{-1}+\frac{T}{2\hbar\Omega_1}\mathscr{F}_0\left(\frac{2\mu-\hbar\Omega_1-\hbar\Omega_2}{2T}\right)+\frac{\pi T}{\hbar\Omega_2}\sum_{n=1}^{\infty}\frac{\sin[\pi n(2\mu-\hbar\Omega_1-\hbar\Omega_2)/\hbar\Omega_2]}{\sinh(2\pi^2 nT/\hbar\Omega_2)},\tag{22}$$

where $\mathscr{F}_0$ is the Fermi integral,

$$\mathscr{F}_0=\int_0^{\infty}\frac{dy}{1+\exp(y-x)}.$$

From (22) and (12) it follows that

$$-\frac{M}{\mu_B}\simeq\frac{m_0}{m^*}\left\{\frac{1}{2}\left[1+\exp\left(\frac{\hbar\Omega_1+\hbar\Omega_2-2\mu}{2T}\right)\right]^{-1}+\frac{T}{\hbar\Omega_1}\mathscr{F}_0\left(\frac{2\mu-\hbar\Omega_1-\hbar\Omega_2}{2T}\right)+\frac{2\pi T}{\hbar\Omega_2}\times\sum_{n=1}^{\infty}\frac{\sin[\pi n(2\mu-\hbar\Omega_1-\hbar\Omega_2)/\hbar\Omega_2]}{\sinh(2\pi^2 nT/\hbar\Omega_2)}\right\}.\tag{23}$$

The first two terms in the braces describe the monotonic part of the magnetic moment, and the last term describes the oscillating part. Note that the series in (23) diverges as $T\to 0$. This situation is similar to the behavior of M as $T\to 0$ in 3D-systems.[15,16]

5. MAGNETIC MOMENT OF A NONDEGENERATE ELECTRON GAS AND A DEGENERATE GAS IN THE $T=0$ LIMIT

The free energy F of a nondegenerate electron gas is given by the expression

$$F=-NT\ln\frac{e}{N}\times\sum_{m,n=0}^{\infty}\exp\left[\frac{\hbar\Omega_1(n+1/2)+\hbar\Omega_2(m+1/2)}{T}\right],\tag{24}$$

where N is the number of particles in the well. Summing the series in (24) we get

$$F=NT\ln\left(\frac{4N}{e}\sinh\frac{\hbar\Omega_1}{2T}\sinh\frac{\hbar\Omega_2}{2T}\right).\tag{25}$$

Let us calculate the magnetic moment of the quantum well, $M=-(\partial F/\partial B)_T$:

$$-\frac{M}{N\mu_B}=\frac{m_0}{m^*}\frac{\omega_c}{\Omega_1^2-\Omega_2^2}\left[\Omega_1\coth\frac{\hbar\Omega_1}{2T}-\Omega_2\coth\frac{\hbar\Omega_2}{2T}\right].\tag{26}$$

Using (26), we can find the susceptibility per electron:

$$\chi^{\text{dia}}=-\frac{1}{3}\left(\frac{m_0}{m^*}\right)^2\frac{\mu_B^2}{T}.\tag{27}$$

The paramagnetic susceptibility per electron can be calculated in the standard way:

$$\chi^{\text{para}}=\mu_B^2/T.\tag{28}$$

Now let us take a degenerate gas at $T=0$. Note that for values of B at which there is no level crossing, the number of electrons in the well is equal to the number of levels ε_{mn}

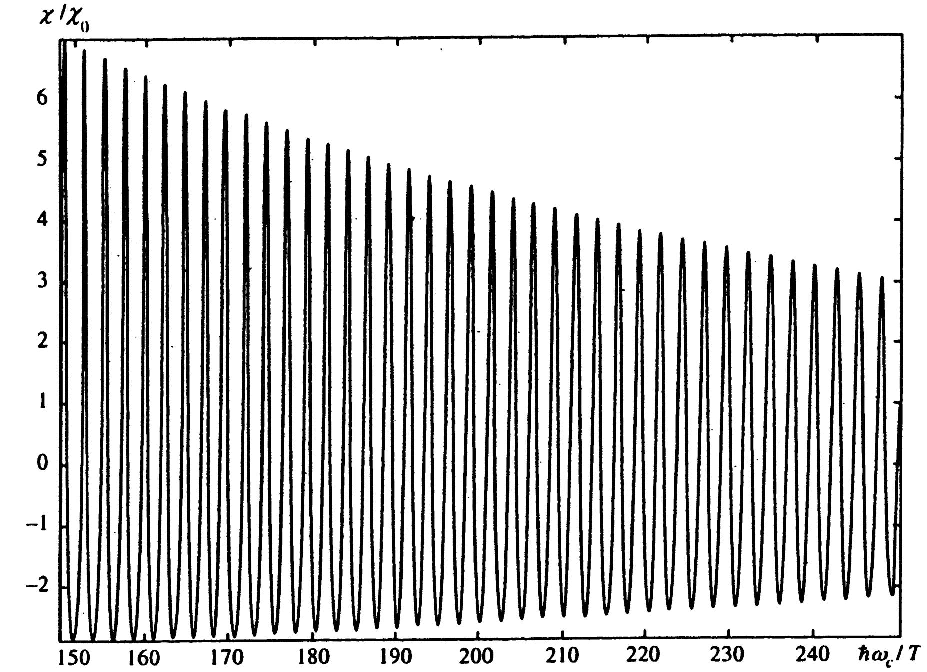

FIG. 2. Oscillations of magnetic susceptibility with variations of the cyclotron frequency; $\hbar\omega_1/T=50$, $\hbar\omega_2/T=40$, and $\mu/T=1000$.

below the Fermi level, since the levels in this case are not degenerate. This sets the case of a quantum well apart from the 3D-case[9,16] and from the quasi-two-dimensional case,[4] where the density of states on the levels is high ($\gg 1$) and, consequently, the number of filled levels is low. In our case, with the number of electrons N in the well being roughly 1000, the number of filled levels is of the same order of magnitude, so that the ratios $\mu/\hbar\Omega_1$ and $\mu/\hbar\Omega_2$ are also equal to N in order of magnitude.

Let us calculate $\Omega_0=\Omega(T=0)$ by the formula

$$-\Omega_0=\sum_{m=0}^{M}\sum_{n=0}^{N_0}\{\varepsilon_F-\hbar\Omega_1(n+\tfrac{1}{2})-\hbar\Omega_2(m+\tfrac{1}{2})\}, \quad (29)$$

where $M=[(\varepsilon_F-\hbar\Omega_1(n+\tfrac{1}{2})-\hbar\Omega_2/2)\hbar\Omega_2]$, and $N_0=[(\varepsilon_F-\hbar\Omega_1-\hbar\Omega_2/2)\hbar\Omega_1]$ (here ε_F is the Fermi energy, and the square brackets stands for the integral part of a number). From (29) we obtain

$$-\Omega_0=\sum_{n=0}^{N_0}\left(\varepsilon_F-\frac{\hbar\Omega_1}{2}-\frac{\hbar\Omega_2}{2}\right)\times\left[\frac{\varepsilon_F-\hbar\Omega_1(n+1/2)-\hbar\Omega_2/2}{\hbar\Omega_2}\right]-\hbar\Omega_2\sum_{n=0}^{N_0}n\left[\frac{\varepsilon_F-\hbar\Omega_2(n+1/2)-\hbar\Omega_1/2}{\hbar\Omega_1}\right]. \quad (30)$$

We see that for field strengths B satisfying the equalities

$$\varepsilon_F=\hbar\Omega_1(n+1/2)+\hbar\Omega_2/2,$$
$$\varepsilon_F=\hbar\Omega_2(n+1/2)+\hbar\Omega_1/2. \quad (31)$$

the quantity (30) has discontinuities in its integral part, which means that the derivative $\partial\Omega_0/\partial B$ at the points specified by (31) has delta-like peaks. Of course, when $T\neq 0$ these oscillations have a finite height and width, and their peaks shift. In estimating the monotonic part of Ω_0 we can ignore the fractional part of the sums in (30). Then, to within terms $\sim(\hbar\Omega_1/\varepsilon_F)^2$, we have

$$\Omega_0^m\simeq-\frac{\varepsilon_F}{24\hbar^2\Omega_1\Omega_2}[4\varepsilon_F^2-\hbar^2(\Omega_1+\Omega_2)^2]. \quad (32)$$

Allowing for the fact that $\omega_1\omega_2=\Omega_1\Omega_2$, from (32) we obtain the following estimates:

$$-\frac{M^m}{\mu_B}\simeq\frac{m_0}{m^*}\frac{\varepsilon_F\omega_c}{\hbar\omega_1\omega_2},$$
$$\chi_{\text{dia}}^m\simeq-\frac{\mu_B^2\varepsilon_F}{3\hbar^2\omega_1\omega_2}\left(\frac{m_0}{m^*}\right)^2\frac{1}{N}. \quad (33)$$

The paramagnetic susceptibility is calculated in the standard manner:[9]

$$\chi^{\text{para}}=-\frac{1}{N}\frac{\partial^2\Omega_0}{\partial\varepsilon_F^2}(B\to 0)=\frac{\mu_B^2\varepsilon_F}{N\hbar^2\omega_1\omega_2}. \quad (34)$$

In both (33) and (34) the susceptibility is per electron.

6. OSCILLATIONS OF THE MAGNETIC MOMENT FOR $T\neq 0$

In this section we discuss the oscillations of the susceptibility and of the magnetic moment for a wide quantum well. From (23) it follows that in the low-temperature limit, where

$$\frac{\hbar\Omega_2}{2\pi^2T}\sinh\frac{2\pi^2T}{\hbar\Omega_2}\to 1,$$

the maximum of the oscillating part is $(-M/\mu_B)^{\text{osc}}\sim m_0/m^*$, and from (33) it follows that

$$\frac{M^{\mathrm{osc}}}{M^{m}} \sim \frac{\hbar\omega_1\omega_2}{\varepsilon_F\omega_c} \ll 1.$$

To find the susceptibility per electron, we differentiate with respect to B in (23) only the rapidly oscillating factors in the series. Then

$$\chi^{\mathrm{osc}} \simeq -\left(\frac{2\pi\mu_B}{\hbar\Omega_2}\frac{m_0}{m^*}\right)^2 \frac{\mu T}{N\hbar\omega_c} \times \sum_{n=1}^{\infty} n\frac{\cos[\pi n(2\mu-\hbar\Omega_1-\hbar\Omega_2)/\hbar\Omega_2]}{\sinh(2\pi^2 nT/\hbar\Omega_2)}. \tag{35}$$

Estimating in (35) the oscillation amplitude in the low-temperature limit, we obtain

$$\chi^{\mathrm{osc}} \simeq -2\left(\frac{m_0\mu_B}{m^*}\right)^2 \frac{\mu}{N\hbar^2\omega_1\omega_2}. \tag{36}$$

From (33) and (36) it follows that $\chi^{\mathrm{osc}}/\chi^{m} \simeq 6$.

Thus, the oscillating part of the magnetic moment is small compared to the monotonic part, while for the susceptibility the opposite is true. A diagrams built from Eq. (35) is depicted in Fig. 2.

The above formulas describe the oscillations of $M(B)$ and $\chi(B)$ as functions of the field strength B. Oscillations of an entirely different type are also possible. We introduce the effective radius of a quantum well by the formula

$$\mu-\frac{\hbar\Omega_1}{2}-\frac{\hbar\Omega_2}{2}=\frac{m^*\omega_1\omega_2R^2}{2}. \tag{37}$$

The magnetic flux through a circle with such a radius is $\Phi=\pi BR^2$. The argument of the sine in (23) and the cosine in (35) can then be transformed into

$$\pi n\frac{2\mu-\hbar\Omega_1-\hbar\Omega_2}{\hbar\Omega_2}=\frac{m^*\omega_1\omega_2\Phi}{\hbar B\Omega_2}n. \tag{38}$$

Substituting $\Omega_2 \simeq \omega_1\omega_2/\omega_c$, we get

$$\pi n\frac{2\mu-\hbar\Omega_1-\hbar\Omega_2}{\hbar\Omega_2} \simeq 2\pi n\frac{\Phi}{\Phi_0}, \tag{39}$$

where $\Phi_0=2\pi\hbar c/e_0$ is the magnetic flux quantum. Combining (23) and (35), we obtain

$$\left(-\frac{M}{\mu_B}\right)^{\mathrm{osc}} \sim \sum_{n=1}^{\infty}\frac{\sin(2\pi n\Phi/\Phi_0)}{\sinh(2\pi^2 nT/\hbar\Omega_2)}, \quad \chi^{\mathrm{osc}} \sim \sum_{n=1}^{\infty}\frac{\cos(2\pi n\Phi/\Phi_0)}{\sinh(2\pi^2 nT/\hbar\Omega_2)}. \tag{40}$$

Thus, the magnetic moment and the susceptibility oscillate with a period equal to Φ_0. The amplitude of these periodic oscillations, estimated against the monotonic quantity M or χ, is the same as of the quantities considered above.

7. DISCUSSION

Studies of the magnetic properties of an electron gas in a quantum well for the case of a degenerate gas show that the susceptibility and magnetic moment are oscillating functions of the magnetic field strength B. Two types of oscillations can occur. First, oscillations of the de Haas–van Alphen type, which are not periodic in $1/B$ (in contrast to Landau's diamagnetism) because of the nonlinear B-dependence of the hybrid frequencies Ω_1 and Ω_2. In addition, as analysis of the oscillations for the case of a wide quantum well has shown (Sec. 6), there can also be oscillations of the Aharonov–Bohm type with a period equal to the magnetic flux quantum. Estimates of the amplitude of susceptibility oscillations show that for both types $\chi^{\mathrm{osc}}/\chi^{m} \gg 1$.

For both a nondegenerate gas (Eqs. (27) and (28)) and a degenerate gas in the low-temperature limit (Eqs. (33) and (34)) the ratio of the diamagnetic part of the susceptibility to the paramagnetic part has the form

$$\frac{\chi^{\mathrm{dia}}}{\chi^{\mathrm{para}}}=-\frac{1}{3}\left(\frac{m_0}{m^*}\right)^2.$$

Thus, for a quasi-zero-dimensional system the value of this ratio is the same as for the 3D case (Landau's diamagnetism). Note that the expressions (27) and (28) for the susceptibility of a nondegenerate gas in a well are the same as in the 3D case.[9] But in the low-temperature limit, Eqs. (33) and (34) imply that the diamagnetic and paramagnetic susceptibilities strongly depend on the characteristic frequencies ω_1 and ω_2 of the lateral confinement potential of the well. We also note that here the well susceptibility per electron, χ^{w}, is high compared to Landau's diamagnetism χ^{L}:

$$\frac{\chi^{w}}{\chi^{L}} \simeq \frac{\varepsilon_F^2}{\hbar^2\omega_1\omega_2} \gg 1.$$

Note that except for some values of B at which the hybrid quantization levels cross (Fig. 1), the levels ε_{nm} are nondegenerate. This sets a quantum well apart from Landau's diamagnetism and from the case of a quasi-two-dimensional layer in a magnetic field,[4] where each level in the spectrum carries many states because of their strong degeneracy.

Two important conclusions follow. First, the chemical potential lies high on the energy axis, since the number of occupied levels ε_{mn} below it is, to order of magnitude, equal to the number of particles in the system, and at $\omega_1 \sim \omega_2 \sim \omega_c$ it can be higher than these characteristic frequencies by a factor of several hundred. Second, under perturbations (say, an impurity potential) the nondegenerate hybrid levels can only shift along the energy axis, but no states whose energies lie between the hybrid quantization levels can split away from these levels. Hence the density of states between the levels ε_{nm} is zero. Finally, the last statement indicates that at $T=0$ the Fermi level ε_F coincides with one of the hybrid levels. Then, except for a small number of points of degeneracy of the spectrum mentioned earlier, the Fermi level experiences discontinuities $\Delta\varepsilon_F=\hbar\Omega_2$ when B varies, since Ω_2 is always lower than Ω_1.

As is known,[15] a wide quantum well is stable when $\partial H/\partial B>0$. If this inequality is violated, the system splits into Kondo domains. Hence the condition of thermodynamic instability for the amplitude of the specific susceptibility χ_0 of such a well assumes the form

$$\chi_0 \simeq \frac{2\mu_B^2\mu}{\hbar^2\omega_1\omega_2}\left(\frac{m_0}{m^*}\right)^2 \frac{1}{V_{\rm eff}} > \frac{1}{4\pi},$$

with V_{eff} the effective volume of the well. The effective volume of the well can be estimated at $4\pi R_0^3/3$, where the radius R_0 is defined by a formula for the number of electrons N derived from Chaplik's expression[17] in the Thomas–Fermi approximation:

$$N = \frac{a^* R_0^3 m^{*2}\omega_1\omega_2}{\hbar^2} \sum_{j=1}^{\infty} \frac{8R_0}{\lambda_j^2(2R_0+\lambda_j a^*)},$$

with a^* the effective Bohr radius, and λ_j the zeros of the Bessel function J_0. For typical values of the parameters ($m^*=0.1m_0$, $\omega_1\sim\omega_2\sim 10^{12}$ s^{-1}, $\mu\sim 500\hbar\omega_1$, and $V_{eff}\sim 10^{-15}$ cm^{-3}), the previous formula implies that $\chi \gg 1/4\pi$. If we assume, as usual,[18] that the range of magnetic fields B in which the domains can exist is specified by the inequalities $B_1 < H_0 < B_2$, where $H_0 = B$ on the flat outer boundary of the well (perpendicular to the field) and the size of the region $\Delta B = B_2 - B_1$ is, to order of magnitude, the distance between the peak values of the susceptibility $\chi(B)$, then an estimate of the size can be made. The distance between the peaks on the χ vs B curve is

$$\Delta B \sim \frac{m^* c}{e_0} \frac{\hbar\omega_1\omega_2}{\mu - \hbar\omega_c}.$$

For a well with the above parameters, $\Delta B \sim 10$ G. From the formula for ΔB it follows that ΔB is a slowly varying function of B.

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[1] Y. Meyr, O. Entin-Wohlman, and Y. Gefen, Phys. Rev. B **42**, 8351 (1990).
[2] A. Feldman and A. H. Kahn, Phys. Rev. B **1**, 4584 (1970).
[3] R. Merlin, Solid State Commun. **64**, 99 (1987).
[4] V. A. Geĭler, V. A. Margulis, A. G. Nesmelov, and I. I. Chuchuev, Fiz. Tverd. Tela (Leningrad) **36**, 1994 (1994) [Phys. Solid State **36**, 1090 (1994)].
[5] B. Huckestein and R. Kümmel, Phys. Rev. B **38**, 8215 (1988).
[6] B. Huckestein and R. Kümmel, Z. Phys. B **66**, 475 (1987).
[7] H. L. Cycon, R. G. Frese, W. Kirsch, and B. Symon, *Schrödinger Operators. With Application to Quantum Mechanics and Global Geometry*, Springer, Berlin (1987).
[8] L. Brey, N. E. Johnson, and B. I. Halperin, Phys. Rev. B **40**, 10647 (1989).
[9] L. D. Landau and E. M. Lifshitz, *Statistical Physics*, Part 1, 3rd ed., Pergamon Press, Oxford (1980).
[10] A. M. Perelomov, *Generalized Coherent States and Their Application* [in Russian], Nauka, Moscow (1987).
[11] E. M. Lifshitz, M. Ya. Azbel', and M. I. Kaganov, *Electron Theory of Metals* [in Russian], Nauka, Moscow (1971).
[12] I. A. Malkin and V. I. Man'ko, *Dynamic Symmetries and Coherent States of Quantum Systems* [in Russian], Nauka, Moscow (1979).
[13] Ya. V. Tatarinov, *Lectures on Classical Dynamics* [in Russian], Moscow Univ. Press., Moscow (1984).
[14] A. P. Prudnikov, Yu. A. Brychkov, and O. I. Marichev, *Integrals and Series* [in Russian], Nauka, Moscow (1981).
[15] Ya. M. Blanter, M. I. Kaganov, and D. V. Posvyanskiĭ, Usp. Fiz. Nauk **165**, No. 2, 213 (1995) [Phys. Usp. **38** (1995)].
[16] D. Shoenberg, *Magnetic Oscillations in Metals*, Cambridge Univ. Press, Cambridge (1984).
[17] A. A. Abrikosov, *Fundamentals of the Theory of Metals*, Elsevier, New York (1988).

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