A theory of electromagnetic emission of two-dimensional magneto-plasma and cyclotron oscillations in a semiconductor heterostructure with a periodic screen

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We develop a rigorous electrodynamic theory of the natural oscillations of a two-dimensional (2D) electron plasma in a semiconductor heterostructure with a lateral metallic strip array in the presence of an external magnetic field directed at right angles to the electron 2D-layer. Oscillations with a wave number corresponding to the center of the Brillouin zone of the periodic structure are accompanied by emission of a homogeneous electromagnetic wave perpendicular to the 2D-layer. We find the frequencies and the dissipative and radiative damping of the cyclotron and magneto-plasma oscillations. The shift of the cyclotron oscillation frequency increases with magnetic field strength. This effect results from electric-field coupling between the cyclotron and plasma motion in the array. We show that the polarizing activity of an array whose period is much shorter than the electromagnetic wavelength halves the radiative damping of the cyclotron oscillations in comparison to the case of homogeneous cyclotron resonance, which explains in full the experimentally observed narrowing of the cyclotron resonance line in a structure with an array. Comparison of our theoretical results with the experimental data suggests that the time of electron relaxation of magneto-plasma oscillations in a GaAs/AlGaAs heterojunction in strong magnetic fields is roughly half the time of electron relaxation of cyclotron oscillations. [S1063-7761(96)01303-X]

1. INTRODUCTION

Experiments in one-photon absorption[1] and emission[2] of electromagnetic waves by two-dimensional (2D) plasmons in a semiconductor GaAs/AlGaAs heterostructure require a metallized array with a period $L \ll \lambda_0$ on the structure's surface, where λ_0 is the electromagnetic wavelength, to ensure that the electromagnetic and plasma oscillations are coupled (Fig. 1). The array provides the coupling between the field of the incident or emitted transverse electromagnetic field, which is homogeneous in the plane of the array, and the longitudinal inhomogeneous electric fields of the plasma oscillations with wave numbers $k_n = 2\pi n/L$ $(n=1,2,3,\dots)$. The position and width of the absorption (or emission) line provides information about the frequency and damping of the plasma oscillations.

The periodic screening of the electric field of the plasma oscillations by the conducting strips of the array usually leads to a shift in the oscillation frequency in comparison to the frequency of plasma 2D-oscillation in a surface-homogeneous structure. Without an external magnetic field, the latter frequency is given in the electrostatic limit and the local approximation for the electron 2D-plasma by the well-known expression[3,4]

$$\omega_p^2 = \frac{N_s e^2 k}{2m^* \varepsilon_0 \bar{\varepsilon}}, \tag{1}$$

where ω_p and k are the plasmon frequency and wave number, N_s is the two-dimensional electron number density in the 2D-plasma, e and m^* are the electron charge and effective mass, ε_0 is the permittivity of free space, and $\bar{\varepsilon}$ is the effective dielectric constant, which depends on the geometry of the structure. If, for instance, the plasma 2D-layer separates two half-spaces with relative dielectric constants ε_1 and ε_2, then[4]

$$\bar{\varepsilon} = \tfrac{1}{2}(\varepsilon_1 + \varepsilon_2).$$

For a structure with a perfectly conducting solid screen positioned at a distance d from the 2D-plasma layer,[5]

$$\bar{\varepsilon} = \tfrac{1}{2}[\varepsilon_1 + \varepsilon_2 \coth(kd)]. \tag{2}$$

Without a screen,[4]

$$\bar{\varepsilon} = \frac{1}{2}\left[\varepsilon_1 + \varepsilon_2 \frac{\varepsilon_3 + \varepsilon_2 \tanh(kd)}{\varepsilon_2 + \varepsilon_3 \tanh(kd)}\right]. \tag{3}$$

In an experimental situation,[1,2] the quantities ε_1, ε_2, and ε_3 in Eqs. (2) and (3) are the relative dielectric constants of GaAs, AlGaAs, and the vacuum, respectively. Since in this case $\varepsilon \simeq \varepsilon_2$ and $\varepsilon_{1,2} \gg \varepsilon_3$, instead of (3) we can, to high accuracy,[2] write

$$\bar{\varepsilon} \simeq \tfrac{1}{2}[\varepsilon_1 + \varepsilon_2 \tanh(kd)], \tag{4}$$

where d is the thickness of the AlGaAs layer.

For a structure with a metallized strip array (Fig. 1), a rigorous thermodynamic theory of plasma 2D-oscillations has been developed in Refs. 6–8 for perfectly conducting strips. The results imply that in the presence of an array, each plasma oscillation with wave number $k_n = 2\pi n/L$ splits into two modes with distinct frequencies $\omega_{pn}^{\pm}$, the modes being characterized by opposite parities of the longitudinal electric field distribution about the center of the gap between the conducting strips. The values of $\omega_{pn}^{\pm}$ monotonically decrease as the array filling factor w/L grows, where w is the conducting strip width. Naturally, when $w/L\rightarrow 1$ and $w/L\rightarrow 0$, the frequencies ω_{pn}^{+} and ω_{pn}^{-} merge, and coincide with the frequencies determined from Eq. (1) at $k=k_n=2\pi n/L$, provided that we take $\bar{\varepsilon}$ in the form (2) or (3), respectively.

The conclusions drawn from the theory of Refs. 6 and 8 are supported by experimental data.[1,2] Batke *et al.*[1] found that the frequency of an observed plasmon with wave number $k=2\pi/L$ coincides, to within the experimental errors, with the frequency specified by Eq. (1) if $\bar{\varepsilon}$ is chosen in the form (2), which agrees with the value of the array filling factor $w/L\rightarrow 1$ in their experiments. On the other hand, the plasmon dispersion law extracted from the experimental data of Okisu *et al.*[2] agrees better with the dispersion law (1) if $\bar{\varepsilon}$ is taken in the form (4) for an open structure, which suggests that the gap width in their array was wider.

In addition to the frequency being shifted, the plasma oscillations in an open structure with an array experience additional damping due to radiative decay. Here radiative decay, i.e., interaction of the radiation with transverse electromagnetic fields, is possible only in the plasma mode, which has an antinode of the longitudinal electromagnetic wave (even symmetry) at the center of the array gap. Clearly, only this mode can be observed in experiments[1,2] on the absorption or emission of electromagnetic waves by 2D-plasmons.

In Refs. 6 and 8 it was demonstrated that radiative damping can contribute significantly to the observed emission (absorption) linewidth. The size of this contribution is comparable to that of dissipative damping determined by electron scattering in the 2D-layer. An explanation for the total linewidth of the plasmon resonance observed by Batke *et al.*[1] was given in Ref. 9 with allowance for electron scattering in the 2D-layer, radiative damping of plasmons, and dielectric losses in the layers of the heterostructure.

In the presence of a constant magnetic field $\mathbf{B}_0$ directed at right angles (along the y axis) to the plane of the heterostructure layers, the surface conductivity tensor of the electron 2D-layer at the frequency ω of the external electric field $E\exp[i(\omega t-kx)]$ in the local approximation (the Drude model) has the form

$$\hat{\sigma}=\begin{pmatrix}\sigma_{xx} & \sigma_{xz}\\ \sigma_{zx} & \sigma_{zz}\end{pmatrix} \tag{5}$$

with elements

$$\begin{aligned}\sigma_{xx}&=\sigma_{zz}=\frac{1}{2}(\sigma_{+}+\sigma_{-}),\\ \sigma_{xz}&=-\sigma_{zx}=-\frac{1}{2}(\sigma_{+}-\sigma_{-}),\end{aligned} \tag{6}$$

where

$$\sigma_{\pm}=\frac{e^2N_s\tau}{m^*[1+i(\omega\pm\omega_c)\tau]},$$

$\omega_c=|e|B_0/m^*$ is the cyclotron frequency, and τ is the phenomenological relaxation time of the electron momentum in the 2D-layer.

If the conductivity tensor is taken in the form (5) and (6), a surface-homogeneous structure can exhibit two types of resonance: homogeneous cyclotron resonance with $k=0$ at the frequency $\omega=\omega_c$, and magneto-plasma resonance with $k\neq 0$, which corresponds to excitation of inhomogeneous magneto-plasma oscillations at a frequency specified by[5]

$$\omega^2=\omega_p^2+\omega_c^2. \tag{7}$$

Nonlocal effects excite cyclotron-resonance harmonics in a 2D-system.[10] The coupling of magneto-plasma oscillations and cyclotron-resonance harmonics splits the magneto-plasma resonance line near the frequencies $\omega=n\omega_c$, with $n=2,3,\ldots$ (see Ref. 1).

Homogeneous electron cyclotron motion with a frequency ω_c in the plane of the 2D-layer is accompanied by electromagnetic emission of cyclotron currents into the surrounding medium. As a result, due to radiative damping, the cyclotron resonance line broadens. For instance, for an electron 2D-layer with conductivity (5), (6) in an infinite medium with a relative dielectric constant ε, the equations of electrodynamics make it possible to obtain the following rigorous expression for the complex-valued natural frequency of a homogeneous cyclotron oscillation:

$$\tilde{\omega}=\omega_c+i\left(\gamma_e+\frac{e^2N_s}{m^*}\frac{Z_0}{2\sqrt{\varepsilon}}\right), \tag{8}$$

where $\gamma_e=1/\tau$, and $Z_0\simeq 377$ ohm is the wave impedance of the vacuum. The first term in parentheses in Eq. (8) corresponds to dissipative electron damping, and the second represents the radiative damping of the cyclotron mode.

Magneto-plasma oscillations in a homogeneous structure are nonradiative excitations, with their electron damping varying from $\gamma_e=1/2\tau$ at $B_0=0$ to $\gamma_e=1/\tau$ at $B_0\rightarrow\infty$.

The presence on the surface of the GaAs/AlGaAs heterostructure of a coupling array in the experiment of Batke *et al.*[1] narrows in the cyclotron resonance line by approximately 30%. Zheng *et al.*[11] explained this fact by the action of the array as a linear polarizer for the radiation fields, but they were unable to achieve quantitative agreement with the experimental data of Batke *et al.*[1]

The observed linewidth of the magneto-plasma resonance in a structure with an array[1] changes nonmonotonically with the magnetic field strength in the region where there is interaction with the cyclotron resonance harmonics. However, in strong magnetic fields far from the interaction region (where the local approximation holds), the linewidth of magneto-plasma resonance is almost three times the value related to the electron contribution, $2\gamma_e\simeq 2/\tau$, with the value of τ determined by fitting the experimental and theoretical curves of the cyclotron resonance line in a structure with no array for the same values of the magnetic field strength. The broadening of the magneto-plasma resonance line in a magnetic field has yet to find a physical explanation.

To theoretically study the experimentally observed features mentioned above of the excitation of cyclotron and magneto-plasma resonances in the GaAs/AlGaAs heterostructure with a coupling array, we expand the theory developed in Ref. 8 to incorporate a constant magnetic field di-

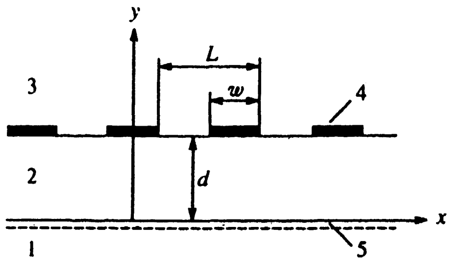

FIG. 1. The structure under investigation and the coordinate system: *1*—GaAs, *2*—AlGaAs, *3*—vacuum, *4*—metallized array, and *5*—two-dimensional electron-plasma layer.

rected at right angles to the plane of the electron 2D-layer. In Sec. 2 we describe the theoretical approach and give the main relationships. In Sec. 3 we discuss the results of calculations and compare them with the experimental data. Section 4 is devoted to concluding remarks originating from our results.

2. STATEMENT OF THE PROBLEM; BASIC RELATIONSHIPS

To rigorously solve for the electromagnetic spectrum of magneto-plasma oscillations in a structure with an array (Fig. 1) we must find a self-consistent solution of the Maxwell equations with allowance for the response of the electron 2D-plasma in an external magnetic field directed along the y axis. The array is considered to be an infinite periodic sequence of perfectly conducting strips of zero width located in the plane $y=d$ and having infinite length in the z direction. In accordance with the conditions of the experiment of Batke *et al.*,[1] we assume that the desired electric and magnetic fields are constant along the array strips (in the z direction).

In the absence of a magnetic field, transverse electric (*TE*) and transverse magnetic (*TM*) waves can exist independently in the structure. Since *TE*-waves have a vanishing E_x-component, they cannot interact with longitudinal plasma oscillations in the x-direction, which were observed at $B_0=0$ in the experiments described in Refs. 1 and 2.

In the presence of an external magnetic field, the *TM*- and *TE*-waves become coupled due to Hall currents in the electron 2D-layer. Here we must consider a hybrid electromagnetic field with six nonzero components of the electric (E_i) and magnetic (H_i) fields ($i=x,y,z$).

Using Floquet 's theorem for fields in a periodic structure,

$$\begin{Bmatrix} E_z \\ H_z \end{Bmatrix}(x+L,y)=\begin{Bmatrix} E_z \\ H_z \end{Bmatrix}(x,y)\exp(-ikL),$$

we write the following expression for the z-components of the electric and magnetic fields:

$$\begin{Bmatrix} E_z \\ H_z \end{Bmatrix}(x,y)=\sum_{m=-\infty}^{\infty}\begin{Bmatrix} E_{zm} \\ H_{zm} \end{Bmatrix}(y)\exp(-i\beta_m x), \tag{9}$$

where

$$\begin{Bmatrix} E_{zm} \\ H_{zm} \end{Bmatrix}(y)=\frac{1}{L}\int_{-L/2}^{L/2}\begin{Bmatrix} E_z \\ H_z \end{Bmatrix}(x,y)\exp(i\beta_m x)dx$$

is the amplitude of the spatial Fourier harmonics, and

$$\beta_m=k+\frac{2\pi m}{L},$$

with k the wave number reduced to the center of the first Brillouin zone. We represent the amplitudes of the Fourier harmonics in the media 1, 2, and 3 (see Fig. 1) in the form

$$\begin{Bmatrix} E_{zm}^{(1)} \\ H_{zm}^{(1)} \end{Bmatrix}(y)=\begin{Bmatrix} A_m \\ A'_m \end{Bmatrix}\exp(\alpha_m^{(1)}y), \qquad y\leqslant 0, \tag{10}$$

$$\begin{Bmatrix} E_{zm}^{(2)} \\ H_{zm}^{(2)} \end{Bmatrix}(y)=\begin{Bmatrix} B_m \\ B'_m \end{Bmatrix}\sinh(\alpha_m^{(2)}y) +\begin{Bmatrix} C_m \\ C'_m \end{Bmatrix}\cosh(\alpha_m^{(2)}y), \quad 0\leqslant y\leqslant d, \tag{11}$$

$$\begin{Bmatrix} E_{zm}^{(3)} \\ H_{zm}^{(3)} \end{Bmatrix}(y)=\begin{Bmatrix} D_m \\ D'_m \end{Bmatrix}\exp(-\alpha_m^{(3)}y), \qquad y\geqslant 0, \tag{12}$$

where $(\alpha_m^j)^2=\beta^2-k_0^2\varepsilon_j$, $k_0=\bar{\omega}\sqrt{\varepsilon_0\mu_0}$, μ_0 is the magnetic constant, $\varepsilon_j=\varepsilon'_j-i\varepsilon''_j$ is the complex–valued relative dielectric constant of the jth medium ($j=1,2,3$), and A_m, A'_m, B_m, B'_m, C_m, C'_m, D_m, and D'_m are constants. In accordance with Maxwell's equations with the time dependence of the fields given by $\exp(i\bar{\omega}t)$ the x-components of the electric and magnetic fields can be written as

$$E_{xm}^{(j)}=-\frac{i\bar{\omega}\mu_0}{k_0^2\varepsilon_j}\frac{\partial H_{zm}^{(j)}}{\partial y}, \qquad H_{xm}^{(j)}=\frac{i\bar{\omega}\varepsilon_0}{k_0^2}\frac{\partial E_{zm}^{(j)}}{\partial y}. \tag{13}$$

Now we write the boundary conditions at the surfaces:

$$\begin{aligned} &E_{xm}^{(1)}=E_{xm}^{(2)}, \qquad H_{xm}^{(2)}-H_{xm}^{(1)}=-j_{zm}(0), \\ &E_{zm}^{(1)}=E_{zm}^{(2)}, \qquad H_{zm}^{(2)}-H_{zm}^{(1)}=j_{xm}(0), \end{aligned} \tag{14}$$

at $y=0$, and

$$\begin{aligned} &E_{xm}^{(2)}=E_{xm}^{(3)}, \qquad H_{xm}^{(3)}-H_{xm}^{(2)}=-j_{zm}(d), \\ &E_{zm}^{(2)}=E_{zm}^{(3)}, \qquad H_{zm}^{(3)}-H_{zm}^{(2)}=j_{xm}(d), \end{aligned} \tag{15}$$

at $y=d$, where $j_{xm}(0)$, $j_{zm}(0)$ and $j_{xm}(d)$, $j_{zm}(d)$ are the spatial Fourier harmonics of the components of the surface current density in the planes $y=0$ and $y=d$, respectively. For the current density in the plasma 2D-layer we have

$$\begin{bmatrix} j_{xm}(0) \\ j_{zm}(0) \end{bmatrix}=\hat{\sigma}\begin{bmatrix} E_{xm}(0) \\ E_{zm}(0) \end{bmatrix}, \tag{16}$$

where $E_{xm}(0)$ and $E_{zm}(0)$ are the spatial Fourier harmonics of the components of the electric field in the plane $y=0$, and the elements of the surface conductivity tensor $\hat{\sigma}$ are expressed by Eqs. (6) with $\bar{\omega}$ replacing ω.

Performing lengthy but otherwise simple transformations involving Eqs. (10)–(16), we arrive at the following relationship between the electric field and the surface current density in the lattice plane $y=d$:

$$\begin{bmatrix} E_{xm}(d) \\ E_{zm}(d) \end{bmatrix}=\hat{G}_m\begin{bmatrix} j_{xm}(d) \\ j_{zm}(d) \end{bmatrix}. \tag{17}$$

The surface impedance tensor

$$\hat{G}_m=\begin{pmatrix} G_{mxx} & G_{mxz} \\ G_{mzx} & G_{mzz} \end{pmatrix} \tag{18}$$

has the following elements:

$$G_{mxx}=iZ_0\frac{\alpha_m^{(2)}}{k_0\varepsilon_2}\frac{\Phi_{1m}}{\Phi_{1m}\Phi_{2m}+\varepsilon_2\Phi_{3m}^2},$$

$$G_{mxz}=-G_{mzx}=Z_0\frac{\Phi_{3m}}{\Phi_{1m}\Phi_{2m}+\varepsilon_2\Phi_{3m}^2},$$

$$G_{mzz}=-iZ_0\frac{k_0}{\alpha_m^{(2)}}\frac{\Phi_{2m}}{\Phi_{1m}\Phi_{2m}+\varepsilon_2\Phi_{3m}^2},$$

where

$$\Phi_{1m}=\coth(\alpha_m^{(2)}d)-\Theta_{1m}\Theta_{3m}+\frac{\alpha_m^{(3)}}{\alpha_m^{(2)}},$$

$$\Phi_{2m}=\coth(\alpha_m^{(2)}d)-\Theta_{2m}\Theta_{3m}+\frac{\alpha_m^{(2)}\varepsilon_3}{\alpha_m^{(3)}\varepsilon_2},$$

$$\Phi_{3m}=\sigma_{xz}Z_0\Theta_{3m},$$

with

$$\Theta_{1m}=\coth(\alpha_m^{(2)}d)+\frac{\alpha_m^{(2)}\varepsilon_1}{\alpha_m^{(1)}\varepsilon_2}-i\sigma_{xx}Z_0\frac{\alpha_m^{(2)}}{\varepsilon_2 k_0},$$

$$\Theta_{2m}=\coth(\alpha_m^{(2)}d)+\frac{\alpha_m^{(1)}}{\alpha_m^{(2)}}+i\sigma_{xx}Z_0\frac{k_0}{\alpha_m^{(2)}},$$

$$\Theta_{3m}=\frac{\coth^2(\alpha_m^2 d)-1}{(\sigma_{xz}Z_0)^2+\Theta_{1m}\Theta_{2m}\varepsilon_2}.$$

Allowing for (17), we can write an expansion like (9) for the electric field in the lattice plane as

$$\begin{bmatrix} E_x(x,d) \\ E_z(x,d) \end{bmatrix}=\sum_{m=-\infty}^{\infty}\hat{G}_m\begin{bmatrix} j_{xm}(d) \\ j_{zm}(d) \end{bmatrix}\exp(-i\beta_m x), \tag{19}$$

where

$$\begin{bmatrix} j_{xm}(d) \\ j_{zm}(d) \end{bmatrix}=\frac{1}{L}\int_{-L/2}^{L/2}\begin{bmatrix} j_x(x,d) \\ j_z(x,d) \end{bmatrix}\exp(i\beta_m x)dx, \tag{20}$$

with $j_x(x,d)$ and $j_z(x,d)$ the components of the surface current density in the plane $y=d$. Obviously, $j_x(x,d)=j_z(x,d)=0$ for $w/2<|x|\leqslant L/2$, where w is the width of a conducting strip in the array.

Substituting (20) into the expansion (19) and using the boundary conditions $E_x(x,d)=E_z(x,d)=0$ at a perfectly conducting strip for $|x|\leqslant w/2$, we arrive at the following system of two integral equations for the components of the surface current density of the array strips:

$$\sum_{m=-\infty}^{\infty}\exp(-i\beta_m x)\int_{-w/2}^{w/2}\hat{G}_m\begin{bmatrix} j_x(x',d) \\ j_z(x',d) \end{bmatrix}\times\exp(i\beta_m x')dx'=0. \tag{21}$$

To solve this equation numerically, we approximate the distributions of the components of the surface current density on the interval $-w/2\leqslant x\leqslant w/2$ by the following expansions:

$$j_x(x,d)=\sum_{n=1}^{N}p_n\varphi_n(x),\qquad j_z(x,d)=\sum_{n=0}^{N-1}q_n\psi_n(x), \tag{22}$$

where

$$\varphi_n=\sqrt{1-\left(\frac{2x}{w}\right)^2}\,U_n\left(\frac{2x}{w}\right), \tag{23}$$

$$\psi_n=\frac{T_n(2x/w)}{\sqrt{1-(2x/w)^2}}, \tag{24}$$

p_n and q_n are unknown constants, and $T_n(2x/w)$ and $U_n(2x/w)$ are, respectively, Chebyshev polynomials of the first kind of order n, and of the second kind of order $n-1$. The weighting functions $[1-(2x/w)^2]^{\pm 1/2}$ explicitly allow for the specific features of the distribution of the surface current density at the edges of a perfectly conducting strip (the Meixner conditions):[12]

$$j_x(x,d)\sim\sqrt{1-\left(\frac{2x}{w}\right)^2},\qquad j_z(x,d)\sim\frac{1}{\sqrt{1-(2x/w)^2}}$$

as $|x|\to w/2$. After substituting the expansions (22) into Eqs. (21), we can use the standard Galerkin procedure[13] with (23) and (24) taken as the basis functions. This makes it possible to go from the system of two integral equations (21) for the functions $j_x(x,d)$ and $j_z(x,d)$ to the following system of $2N$ homogeneous algebraic equations for the coefficients p_n and q_n:

$$\begin{aligned} &\sum_{n=1}^{N}A_{kn}p_n+\sum_{n=0}^{N-1}B_{kn}q_n=0\quad(k=1,2,\ldots,N),\\ &\sum_{n=1}^{N}C_{kn}p_n+\sum_{n=0}^{N-1}D_{kn}q_n=0\quad(k=N+1,\ldots,2N), \end{aligned} \tag{25}$$

where

$$\begin{aligned} A_{kn}&=ni^n\sum_{m=-\infty}^{\infty}\frac{G_{mxx}}{\beta_m^2}J_k\left(\frac{\beta_m w}{2}\right)J_n\left(\frac{\beta_m w}{2}\right),\\ B_{kn}&=\frac{w}{2}i^{n+1}\sum_{m=-\infty}^{\infty}\frac{G_{mxz}}{\beta_m}J_k\left(\frac{\beta_m w}{2}\right)J_n\left(\frac{\beta_m w}{2}\right),\\ C_{kn}&=ni^n\sum_{m=-\infty}^{\infty}\frac{G_{mzx}}{\beta_m}J_{k-N-1}\left(\frac{\beta_m w}{2}\right)J_n\left(\frac{\beta_m w}{2}\right),\\ D_{kn}&=\frac{w}{2}i^{n+1}\sum_{m=-\infty}^{\infty}G_{mzz}J_{k-N-1}\left(\frac{\beta_m w}{2}\right)J_n\left(\frac{\beta_m w}{2}\right). \end{aligned} \tag{26}$$

The nth Bessel function of the first kind $J_n(\beta_m w/2)$ emerges in Eqs. (25) as a result of using explicit expressions for the Chebyshev polynomials,[13]

$$T_n(\zeta)=\cos(n\arccos\zeta),$$

$$U_n(\zeta)=\frac{\sin(n\arccos\zeta)}{\sqrt{1-\zeta^2}},$$

in the expansions (22) and their subsequent integration when substituted into Eqs. (21).

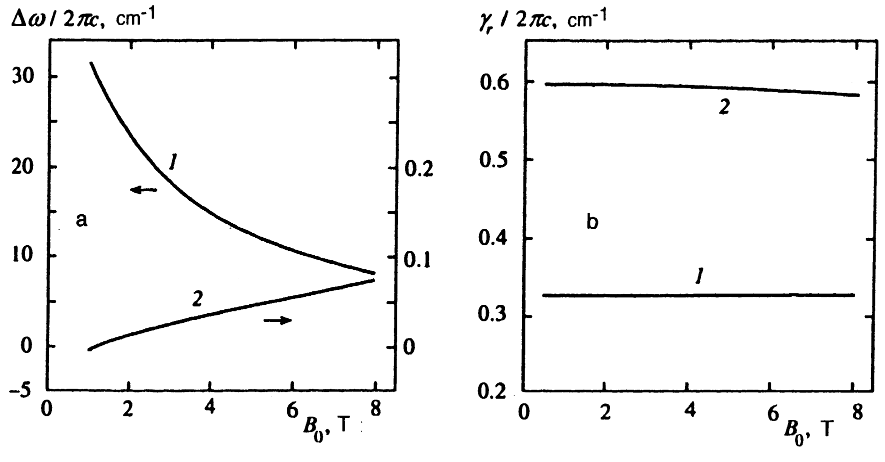

FIG. 2. $\Delta\omega$ (a) and γ_r (b) for magneto-plasma oscillations (curves *1*) and cyclotron oscillations (curves *2*) as functions of the magnetic field strength in the structure depicted in Fig. 1 with the following parameters:[1] $\varepsilon_1 = 12.8$, $\varepsilon_2 = 11.0$, $\varepsilon_3 = 1$, $N_s = 6.7\times 10^{11}\,\mathrm{cm}^{-2}$, $L = 8.72\times 10^{-5}$ cm, $w/L = 0.9$, $d = 8\times 10^{-6}$cm, $m^* = 0.071 m_e$; $c = 1/\sqrt{\varepsilon_0\mu_0}$ is the speed of light.

The dispersion relation, which establishes the relationship between the frequency and the reduced wave number of the natural oscillations of the system, is determined by setting the determinant of the system of homogeneous linear equations (20) to zero. For a fixed real value of the wave number, the roots of the dispersion equation yield the complex-valued frequencies $\tilde{\omega} = \omega + i\gamma$, where the real part ω corresponds to the natural oscillation frequency, and the imaginary part γ is the damping coefficient for these oscillations, the latter generally being related to dissipative processes in the system and to the electromagnetic radiation emitted by the structure. Obviously, if we ignore all dissipative damping mechanisms, then $\gamma = \gamma_r$, where γ_r is the radiative damping coefficient.

The Galerkin procedure and the series in (26) both converge. Below we present the results of numerical calculations in which $N = 5$ and only terms with $|m| \leq 100$ are retained in the series in (26). This ensures that the error in the natural frequencies is no higher than 1%.

3. RESULTS OF CALCULATIONS; DISCUSSION

In this section we give the results of numerical calculations of the natural frequencies and damping coefficients of the magneto-plasma and cyclotron modes for a vanishing wave number k, which corresponds to the center of the first Brillouin zone for the periodic structure depicted in Fig. 1. These excitations were manifested as magneto-plasma and cyclotron resonances in the experiments of Batke *et al.*[1]

As in a vanishing magnetic field,[8] the presence of an array splits each magneto-plasma oscillation with wave number $k_n = 2\pi n/L$ ($n = 1,2,3$), which corresponds to $k = 0$ in the reduced band structure, into two oscillations with distinct frequencies $\omega_n^{\pm}$. As noted in the Introduction, at $B_0 = 0$ one oscillation is nonradiative, since one node of the amplitude distribution of the longitudinal electric field E_x is at the center of an array gap. Since an external magnetic field lowers the symmetry of the problem in comparison to the geometric symmetry of the lattice, for $B \neq 0$ the distribution of the field of the natural oscillations cannot be characterized by a symmetry with a definite parity in relation to the center of the array gap. The two oscillations, with frequencies ω_n^+ and ω_n^-, can therefore theoretically experience radiative damping. However, calculations with parameters with characteristic of the experiment of Batke *et al.*[1] have shown that radiative damping of the "nonradiative" mode is several orders of magnitude weaker than that of the "radiative" mode, so that the former has essentially no effect on the position and shape of the magneto-plasma resonance line in Ref. 1. Furthermore, radiative damping of magneto-plasma oscillations drops dramatically as n increases, so that magneto-plasma resonances with $n \geq 2$ were weak in the experiments of Batke *et al.*[1] For this reason we restrict our discussion to the "radiative" magneto-plasma mode $n = 1$, and drop the subscript 1 for the sake of simplicity.

We start by discussing the results obtained without allowing for electron scattering ($\tau \to 0$) and dielectric losses ($\varepsilon_j'' = 0$). Figure 2(a) depicts the frequency variation $\Delta\omega = \omega - \omega_c$ for the magneto-plasma and cyclotron oscillations as a function of the external magnetic field strength. The magneto-plasma oscillation frequency varies with the magnetic field strength in good agreement with (7) if ω_p in that formula is interpreted as the frequency of plasma oscillations in an array structure in zero magnetic field. The frequency shift $\Delta\omega$ of the cyclotron mode in the array structure grows with magnetic field strength, but proves to be a factor of 100 smaller than for the magneto-plasma mode in the given range of magnetic field strengths. As Fig. 2(b) implies, the radiative damping of magneto-plasma oscillations is essentially independent of the magnetic field, while the radiative damping of cyclotron oscillations decreases only slightly in strong magnetic fields.

The nature of the dependence of the frequency and radiative damping of cyclotron oscillations on the magnetic field strength can be explained as follows. In a structure with a periodic array, a homogeneous cyclotron oscillation is accompanied by higher spatial harmonics of the electric field, which excite forced plasma oscillations with wave numbers $k = 2\pi n/L$. As a result, an additional "restoring force" acting on the electrons appears that is related to the induced separation of charge in the plasma 2D-layer, which leads to renormalization of the cyclotron oscillation frequency. As the magnetic field strength grows, the cyclotron frequency approaches the natural frequency of magneto-plasma oscillations. In this connection the intensity of excitation of forced

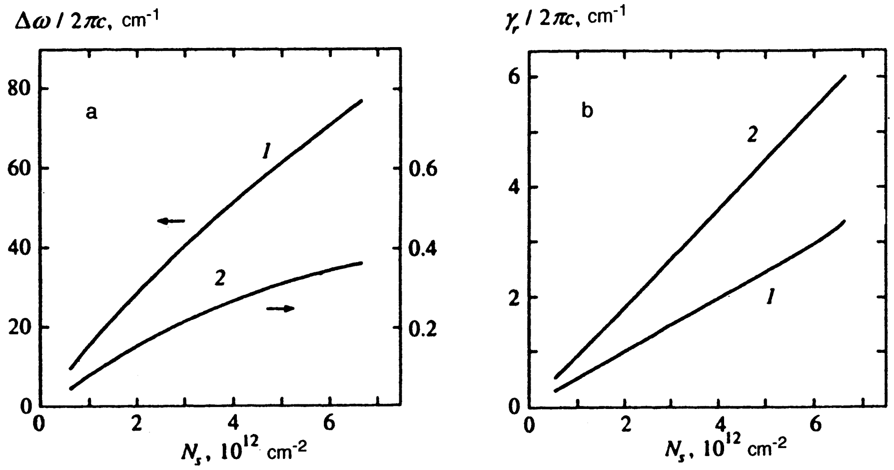

FIG. 3. $\Delta\omega$ (a) and γ_r (b) for magneto-plasma oscillations (curves *1*) and cyclotron oscillations (curves *2*) as functions of the surface electron number density N_s at $B_0=6$ T. The other parameters of the structure are the same as in Fig. 2.

plasma oscillations and, hence, the variation of the cyclotron oscillation frequency increase. Since in strong magnetic fields an appreciable fraction of the cyclotron oscillation energy is contained in nonradiative forced plasma oscillations, the radiative damping of cyclotron oscillations decreases.

The assumption of a role for forced plasma oscillations in the renormalization of the cyclotron oscillation frequency is supported by the curves in Fig. 3(a). As the surface electron number density grows, the 2D-plasma becomes more "rigid," which increases the cyclotron oscillation frequency. A strong (essentially linear) dependence of the radiative damping coefficient on N_s [Fig. 3(b)] indicates that the homogeneous component of the cyclotron current provides the dominant contribution to radiative damping, in accordance with Eq. (8). As in the case with $B_0=0$, the frequency and radiative damping of magneto-plasma oscillations increase with N_s (Fig. 3).

Note that for the characteristic values of the parameters in the experiment of Batke *et al.*[1] the shift in the cyclotron oscillation frequency due to variations in B_0 or N_s in an array structure proves to be of the order of the measurement errors and so were not observed by Batke *et al.*[1] The physical reason for the effect being so small lies in the fact that in the system considered here, the electron 2D-plasma is separated from the array by a layer of insulator, so that the spatial harmonics of the array field have a small effect on homogeneous cyclotron oscillations. The situation changes little if the thickness d of the insulating layer is made smaller, because here the perfectly conducting array strips have a strong screening effect on the tangential electric fields associated with charge separation in the plasma 2D-layer. It stands to reason, however, that in other structures with strong time-dependent modulation of the surface charge density in the plasma 2D-layer, the above renormalization of the cyclotron oscillation frequency due to perturbations of homogeneous electron motion caused by the time-dependent potential of forced plasma oscillations can be much more pronounced. For instance, Kotthaus *et al.*[14] observed an increase in the positive shift of the cyclotron resonance frequency with magnetic field strength in an electron 2D-system with a periodically modulated equilibrium density. In this case the higher spatial harmonics of the field appear directly in the plasma 2D-layer because of the interaction of an external homogeneous electric field and the steady-state periodic profile of the electron number density. Existing approximate theoretical models[15] do not explain the increase observed by Kotthaus *et al.*[14] in the shift of the cyclotron resonance frequency with magnetic field strength.

Figure 4 depicts the frequency shift and radiative damping of magneto-plasma and cyclotron oscillations as functions of the array filling factor w/L. The frequency of magneto-plasma oscillations behave in the same way as in a

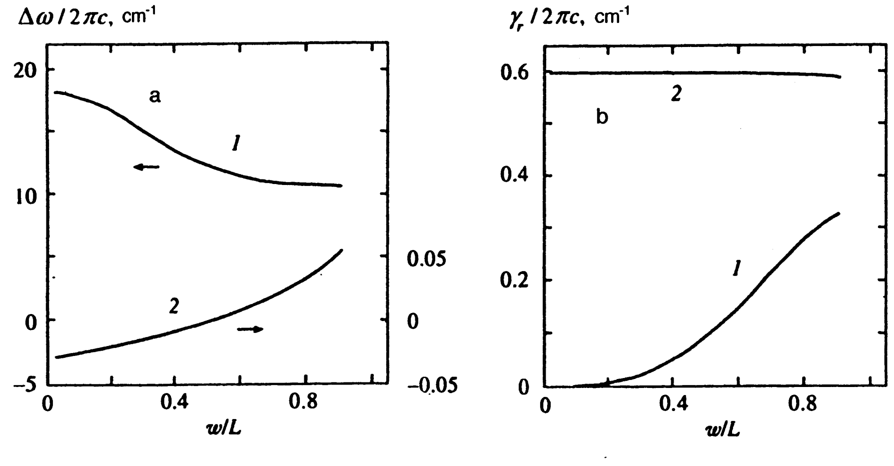

FIG. 4. $\Delta\omega$ (a) and γ_r (b) for magneto-plasma oscillations (curves *1*) and cyclotron oscillations (curves *2*) as functions of the array filling factor w/L at $B_0=6$ T. The other parameters of the structure are the same as in Fig. 2.

zero magnetic field, decreasing from $\underset{\sim}{\omega}$ at $w/L=0$ to $\bar{\omega}$ at $w/L=1$, where $\underset{\sim}{\omega}$ and $\bar{\omega}$ are the frequencies of magneto-plasma oscillations with a wave vector $k=2\pi/L$ in surface-homogeneous structures with, respectively a free interface and a perfectly conducting screen in the plane $y=d$. Radiative damping of magneto-plasma oscillations, as for the case where $B_0=0$, increases dramatically as $w/L\rightarrow 1$ because of the growing coupling of the field of these oscillations and the homogeneous electric field of the emitted electromagnetic wave.

The shift in the frequency of cyclotron oscillations changes sign at $w/L\simeq 0.5$. In other words, it can be said that the "cyclotron oscillator" considered here becomes "softer" (for $w/L<0.5$) or more "rigid" (for $w/L>0.5$) than the "cyclotron oscillator" with purely homogeneous electron motion due to the coupling of the homogeneous motion and the plasma oscillations in the electron 2D-layer. A change in the sign of $\Delta\omega$ indicates a change in the phase relationship between the homogeneous cyclotron motion and the additional "restoring force" caused by the motion of electrons in the 2D-layer in the time-dependent Coulomb potential of the plasma oscillations. Indeed, calculations show that the phase shift between the homogeneous component of the field and the total field of the higher spatial harmonics at each point in the plane of the electron 2D-layer changes by π as the array filling factor varies from $w/L<0.5$ to $w/L>0.5$.

The radiative damping coefficient for cyclotron oscillations is essentially independent of the value of w/L [see Fig. 4(b)], which supports the conclusions of Ref. 11 that the width of the cyclotron resonance curve is independent of the array parameters for $L\ll\lambda_0$ and amounts to half the radiative damping coefficient for a homogeneous cyclotron oscillation in a structure without an array. Such a decrease in radiative damping is caused by the fact that for $L\ll\lambda_0$ and $d<L$ the array acts mainly as a linear polarizer. The conducting strips of the array effectively screen the z-component of the circularly polarized electric field accompanying the homogeneous cyclotron oscillation, and the z-component of the cyclotron current in the electron 2D-layer is balanced by the oppositely-directed longitudinal image current in the perfectly conducting array strips. This results in emission of a linearly polarized homogeneous wave with its electric field directed transversely to the array strips.

Homogeneous cyclotron motion in a structure without an array can be represented by a sum of homogeneous linear currents having the same amplitude, flowing in the z- and x-directions, and shifted in phase relative to one another by $\pi/2$. Here the radiative damping of the cyclotron oscillations in (8) is related to the electromagnetic emission by both current components. The presence of an array hinders emission by currents flowing in the z-direction, and lowers radiative damping by a factor of two.

Allowing for electron scattering in the 2D-plasma, dielectric losses in the layers of the GaAs/AlGaAs heterostructure, and radiative damping, we calculated the frequency and the width of the cyclotron resonance curve for characteristic values of the parameters of the experiment conducted by Batke *et al.*:[1] $m^*=0.071m_e$, $N_s=6.7\times 10^{11}\text{cm}^{-2}$, $\varepsilon=12.8$ (GaAs), $\varepsilon_2=11.0$ (AlGaAs), $\varepsilon=1$, $L=8.72\times 10^{-5}$cm, $d=8\times 10^{-6}$cm, and $w/L\sim 1$ in a magnetic field $B_0=5$ T. Dielectric losses were taken into account by introducing imaginary parts $\varepsilon''_{1,2}$ into the dielectric constants $\varepsilon_{1,2}$. The quantities $\varepsilon''_{1,2}\simeq 0.27$ were obtained by fitting the experimental and theoretical widths of the plasma resonance curve in zero magnetic field.[8] For the phenomenological relaxation time in Eq. (6) we took the value $\tau=\tau_c=4.5\times 10^{-12}$ s, obtained by Batke *et al.*[1] by fitting the experimental and theoretical cyclotron resonance curves in a surface-homogeneous structure without an array.

When calculated with the above values of the parameters, the frequency and total linewidth of the cyclotron resonance in the GaAs/AlGaAs heterostructure with a metallized array were found to coincide with the experimental values to within the experimental errors. Note that Zheng *et al.*[11] achieved agreement between the experimental and theoretical data by using the value $\tau=5.5\times 10^{-2}$ s in their calculations. Our results appear more realistic from the physical standpoint, since we believe that it is improbable that the presence of an array can change the electron relaxation time of cyclotron oscillations.

The experimentally observed[1] linewidth of a magneto-plasma resonance in strong magnetic fields (6–8 T), when the interaction between magneto-plasma oscillations and cyclotron-resonance harmonics can be ignored, exceeds the plasma-resonance linewidth at $B_0=0$ by a factor greater than two. The above results [see Fig. 2(b)] show that radiative damping of magneto-plasma oscillations is essentially independent of the magnetic field strength. It is natural then to assume that $\varepsilon''_{1,2}$ are also essemtially independent of the magnetic field. Note that here the dissipative damping γ_d of magneto-plasma oscillations, related to dielectric losses, decreases as B_0 (or the oscillation frequency) grows. Here the broadening of the magneto-plasma resonance line can be related only to the increase in electron damping of the magneto-plasma oscillation. As noted earlier, the electron damping γ_e of such oscillations, related to electron scattering in the 2D-plasma, increases with the magnetic field strength, in an approximation in which the relaxation time is B_0-independent, from $\gamma_e=1/2\tau$ at $B_0=0$ to $\gamma_e=1/\tau$ as $B_0\rightarrow\infty$. Consequently, the experimentally observed broadening of the magneto-plasma resonance line by a factor greater than two cannot be explained entirely by this effect, since electron damping determined only one-third of the magneto-plasma resonance linewidth at $B_0=0$ (see Ref. 8). A rigorous calculation of the damping $\gamma=\gamma_e+\gamma_d+\gamma_r$ for a structure with the above values of the parameters has shown that the best agreement between our theoretical value of the damping coefficient for magneto-plasma oscillations and the experimental value of the halfwidth of the resonance line observed by Batke *et al.*[1] in strong magnetic fields (6–8 T) is achieved when the value $\tau=\tau_{mp}\simeq 2.2\times 10^{-12}$ s is used in calculations. Here the calculated frequency of magneto-plasma oscillations was found to coincide, to within the experimental errors, with the experimental value of the frequency of magneto-plasma resonance. The above value of τ_{mp}, which can be named the magneto-plasma relaxation time, is approximately half the cyclotron relaxation time τ_c

less than one-third the dc electron-momentum relaxation time $\tau_{dc} \simeq 7 \times 10^{-12}$ s. In our theory the quantity τ is purely phenomenological, so that it is impossible to establish the physical reason why τ_{mp} differs so markedly from τ_c and τ_{dc}.

4. CONCLUSION

We have developed a rigorous electrodynamic theory of electromagnetic emission of magneto-plasma and cyclotron oscillations in a semiconductor heterostructure with a coupling array.

We found that the frequency of radiative magneto-plasma oscillations in a structure with an array increases with magnetic field strength in good agreement with Eq. (7), which was obtained for a surface-homogeneous structure, if ω_p is interpreted as the frequency of plasma oscillations in a structure with an array but without an external magnetic field.

The frequency shift $\Delta\omega = \omega - \omega_c$ for the cyclotron mode increases with magnetic field strength, but the shift is small. For characteristic values of parameters of previous cyclotron-resonance experiments involving a GaAs/AlGaAs heterostructure with an array, the resonance frequency shift was found to be of the same order as the measurement errors. We suggest a physical picture of this phenomenon that explains the cyclotron-resonance frequency shift in a structure with an array as caused by electrodynamic coupling of homogeneous cyclotron motion and inhomogeneous forced magneto-plasma oscillations. We propose a hypothesis according to which the effect consists of an increase, experimentally observed by Kotthaus *et al.*,[14] in the shift of the cyclotron resonance frequency as the magnetic field becomes stronger in a heterostructure with modulation of the equilibrium electron number density. We also note that the existing approximate theoretical models provide no explanation of the magnetic-field dependence of the shift in the cyclotron resonance frequency.

The radiative damping of magneto-plasma oscillations grows considerably as the array spacing becomes narrower because of the increasing coupling of the oscillation field and the homogeneous electric field of the emitted electromagnetic wave. When the array period is much shorter than the electromagnetic wavelength, the radiative damping of cyclotron oscillations is essentially independent of the array filling factor w/L and amounts to half the radiative damping of homogeneous cyclotron motion in a structure without an array. This is because such an array acts as an effective linear polarizer for the radiation fields.

Using our theory, we calculated the total cyclotron resonance linewidth for the characteristic values of the parameters of well-known experiments.[1] The derived value of the linewidth was found to coincide with the experimental value to within the experimental errors.

The calculated and experimental values of the total linewidth for magneto-plasma resonance in strong magnetic fields ($\omega_c \simeq 2\omega_p$) coincide only if we assume that the effective electron relaxation time for magneto-plasma oscillations is approximately half the cyclotron relaxation time, and less than one-third the dc electron-scattering time.

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