Quantum tunneling of a domain wall in a weak ferromagnet

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The macroscopic quantum tunneling of a domain wall in weak ferromagnets is investigated theoretically using a simplified description of domain-wall dynamics. An expression for the tunneling rate in the WKB approximation is obtained for the case of a Hamiltonian which is not quadratic with respect to the momentum. The equations of motion along the instanton trajectory are solved analytically for any form of the external potential. Experimental results are discussed and interpreted. [S1063-7761(96)02604-2]

1. INTRODUCTION

The macroscopic quantum tunneling of magnetization is presently a subject of active investigation. In theoretical studies of this phenomenon in small particles[1,2] it was predicted that the quantum behavior of the magnetization should be observed more easily in antiferromagnetic particles than in ferromagnetic particles. Barbara and Chudnovsky[2] showed that the Gamow constant B in the formula for the tunneling rate $\Gamma = A\ \exp(-B)$ in the case of small antiferromagnetic particles should be approximately two orders of magnitude smaller than the corresponding constant B for similar ferromagnetic particles. Thus, the temperature of the transition from a thermally activated process to a quantum regime is also two orders of magnitude higher for antiferromagnets.

The macroscopic tunneling of domain walls has attracted great attention. The tunneling of domain walls has hitherto been investigated theoretically only for the case of ferromagnets.[3] Consideration of weakly ferromagnetic materials has another advantage: there are already well developed approaches[4–7] which enable us to describe domain-wall dynamics at rates of motion on the order of 10^5-10^6 cm/s and thus make it possible to investigate the process of the tunneling of a domain wall through a fairly high, but very narrow barrier.

One experimental manifestation of the macroscopic quantum tunneling of magnetization is that the rate of the magnetization relaxation processes does not decrease to zero when the temperature is lowered, but maintains a finite value, which does not depend on the temperature. Similar variation of the relaxation rate has been observed for many magnetic materials (see Refs. 8 and 9 and the review in Ref. 10), and, in particular, for samples of terbium orthoferrite,[6,11] which is a weak ferromagnet at the temperature of the experiments. In these experiments behavior of the magnetization relaxation rate which is typical of macroscopic quantum tunneling was detected, but the results were analyzed using the theory of magnetization tunneling in small antiferromagnetic particles, which led to definite difficulties (we shall discuss the problem of interpreting the experimental results below). Here it should be noted that the techniques in Ref. 3 cannot be applied to such an analysis due to the serious differences between domain-wall motion in ferromagnetic and weakly ferromagnetic materials.

The description of the tunneling of a domain wall is a complicated theoretical problem: a domain wall is a two-dimensional system with an infinite number of degrees of freedom even on large (in comparison with its thickness Δ) scales. In addition, a description of tunneling through a defect must clearly contain information on the defect itself. If we take into account the enormous variety of defects in real samples, devising an exact theory for each type of defect does not seem wise at the present time. It would be natural to use a phenomenological approach to overcome such problems. Thus, a model has been constructed for tunneling through defects of a specific type, which is characterized by a set of semiphenomenological parameters: the width along the x axis, the characteristic transverse dimensions, the density of the defects in the sample, etc.

As we have already mentioned, one macroscopic manifestation of the influence of defects on the motion of domain walls is the magnetic aftereffect, or, stated differently, the finite magnetic viscosity. The aftereffect has been analyzed in samples with a comparatively small defect density (which is often achieved in high-quality samples) using a familiar concept in physics, in which the movement of a domain wall takes place in the form of fluctuational (thermally activated at high temperatures and quantum at low temperatures) movements of individual small elements of the wall[12] (a similar model has also been employed to analyze the viscous flow of the magnetic flux in superconductors.[13]). Thus, our treatment is naturally associated with the small element of the domain wall which participates directly in the tunneling process. Of course, the motion of this element lags behind the motion of the other elements not immobilized by the defect, and the wall bends in the region of the defect. However, when the radius of curvature is much greater than the thickness of the domain wall, the corresponding energy only has the character of a correction, and to describe the dynamics it is sufficient to treat the domain wall as a flat structure (this approximation is discussed in greater detail in the review in Ref. 14). Although this model, as noted above, offers only a semiphenomenological approach to interpreting the interaction of a domain wall with a defect, it is powerful and allows phenomena to be described faithfully over a broad

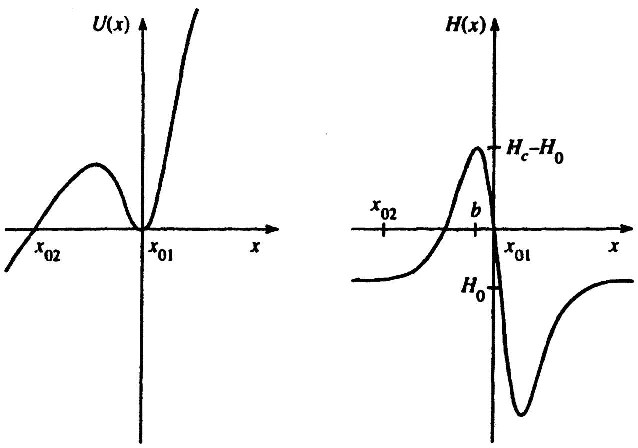

FIG. 1. Left-hand figure – plot of the potential energy $U(x)$ of a domain wall; right-hand figure – corresponding plot of the magnetic field $H(x)$. A defect creates a potential well for the domain wall, as a result of which the domain wall is immobilized by the defect. As long as the external field H_0 does not reach the value H_c, the potential $U(x)$ will have a local metastable minimum at x_{01}.

range of temperatures (in both the thermal-activation region and in quantum region[10,12]), to compare different materials, etc.

Thus, by treating a domain wall as a flat structure not only for simplicity, but also with some justification, we can concentrate on the investigation of the indisputably important question of the relationship between the characteristics of macroscopic quantum tunneling and the nonlinear dynamic properties of the domain wall itself (which have been investigated quite thoroughly[7]).

In this paper we shall use a path-integral method for these purposes, which permits a simple and elegant transition from a classical to a quantum description. It should be noted that this is the simplest method in the present case; the only possible alternative would involve a Hamiltonian formulation, the construction of pairs of canonically conjugate operators, etc., which are complicated tasks in themselves.

Thus, we consider the following model (Fig. 1): a 180-degree domain wall, which, for simplicity, we shall regard as a flat membrane immobilized by the potential of a defect. We apply an external driving field to the sample. Even if the field is not strong enough to completely overcome the potential barrier created by the potential of the defect, a position of the domain wall to the left of the defect (more precisely, to the left of the point x_{02}) is energetically more favorable than a position in the local minimum at x_{01}, where it rests initially, being immobilized by the defect. At zero temperature there are no thermal fluctuations, and, therefore, the motion of the domain wall will obey only the laws of quantum mechanics, which predict a nonzero probability for the wall to tunnel through the barrier.

2. DESCRIPTION OF THE MODEL. LAGRANGIAN FOR A DOMAIN WALL

Let us consider a weak ferromagnet of orthorhombic symmetry (like terbium orthoferrite, $TbFeO_3$). We describe it in the two-sublattice approximation using the ferromagnetism vector $\mathbf{m}$ and the antiferromagnetism vector $\mathbf{l}$. The thermodynamic potential $\Phi(\mathbf{l},\mathbf{m})$ has the form[15]

$$\Phi(\mathbf{l},\mathbf{m})=Jm^2+A(\nabla\mathbf{l})^2-\mathbf{m}\cdot\mathbf{H}+d_1m_xl_z-d_3m_zl_x$$
$$+K_{ac}l_z^2+K_{ab}l_x^2.$$

Here J is the homogeneous exchange constant, A is the inhomogeneous exchange constant, K_{ac} and K_{ab} are the anisotropy constants, $\mathbf{H}$ is the external field, and d_1 and d_3 are the Dzyaloshinskii antisymmetric exchange constants. Minimizing the thermodynamic potential of the system $\Phi(\mathbf{l},\mathbf{m})$ with respect to $\mathbf{m}$ with consideration of the relations

$$\mathbf{l}\cdot\mathbf{m}=0,\quad l^2=1-m^2\approx 1,$$

we obtain Φ in the form (see, for example, Refs. 4, 5, and 15)

$$\Phi=A(\nabla\mathbf{l})^2-\frac{\chi_\perp}{2}(H^2-(\mathbf{H}\cdot\mathbf{l})^2)-M_z^0H_zl_x-M_x^0H_xl_z$$
$$+K_{ac}l_z^2-K_{ab}l_x^2,$$

where $\chi_\perp=M_0/2H_E$ is the transverse susceptibility, and M_z^0 and M_x^0 are quantities equal to the components of the ferromagnetism vector in the $\Gamma_4(G_x A_y F_z)$ and $\Gamma_2(F_x C_y G_z)$ phases. One of two possible types of domain walls appears, depending on the relationship between the anisotropy constants K_{ac} and K_{ab}. They are characterized by the rotation of $\mathbf{l}$ either in the ac plane (an ac-type domain wall, when $K_{ac}<K_{ab}$ holds) or in the ab plane (an ab-type domain wall, when $K_{ab}<K_{ac}$ holds). As the temperature decreases, terbium orthoferrite undergoes several spin-reorientation phase transitions, but at the temperatures at which the measurements in Ref. 11 were carried out, the vector $\mathbf{l}$ lies in the ac plane. Thus, we shall restrict ourselves to the case of an ac-type domain wall, as the type most closely corresponding to the experiment.

Let us consider a flat ac-type domain wall perpendicular to the a axis (the x axis). When the wall is stationary, the vectors $\mathbf{l}$ and $\mathbf{m}$ turn in the ac plane. From a rigorous standpoint, when the wall moves, the vectors $\mathbf{l}$ and $\mathbf{m}$ depart from that plane. However, (see Ref. 4) for $H\leqslant 200$ Oe the angle of departure from the ac plane is less than $0.1°$ and can be neglected.

We introduce a spherical coordinate system such that

$$l_x=\sin\theta\cos\phi,\quad l_y=\sin\theta\sin\phi,\quad l_z=\cos\theta.$$

The dynamics will be described using the Lagrangian formulation. The bulk Lagrangian density for the system under consideration has the form (see, for example, Ref. 7 and the literature cited therein)

$$\mathcal{L}=\frac{\chi_\perp}{2\gamma^2}(\dot{\mathbf{l}})^2-\frac{\chi_\perp}{\gamma}\mathbf{H}\cdot(\mathbf{l}\times\dot{\mathbf{l}})-\Phi.$$

The motion of a domain wall in a weak ferromagnet can be described using soliton perturbation theory, which was described, for example, in Ref. 16 and has been successfully applied to the detailed description of domain-wall dynamics.[4-7] In this case the solution for the free steady motion of a domain wall (boundary) moving with a velocity v without dissipation in the absence of an external field is taken as the zeroth approximation. As was stated above, in the spherical coordinate system $\phi=0$ holds for a flat ac-type domain wall. The solution for the polar angle θ can easily be obtained. We introduce the similarity variable

$$\xi=\frac{x-vt}{\Delta}, \quad \text{where} \quad \Delta=\Delta_0\sqrt{1-\frac{v^2}{c^2}}.$$

Here $\Delta_0=\sqrt{A/(K_{ac}+\chi_\perp H^2)}$ is the width of the stationary domain wall and $c=\gamma\sqrt{A/\chi_\perp}$ is the limiting velocity of the domain wall, which coincides with the velocity of the spin waves. Then the Euler Lagrange equation takes the form

$$\theta''_{\xi\xi}=-\sin\theta\cos\theta.$$

A set of solutions of this equation in the form of an isolated domain wall is well known, and we take the solution in the form

$$\theta_0=-\pi/2+2\arctan e^{\xi}.$$

In our description the external forces acting on the wall are treated as small perturbations. In first order they result in modulation of the velocity and the position of the center of the wall, the modulation rate being of the same order as the perturbation. We are interested only in the position of the wall (in front of or behind the barrier); therefore, we can move over to a description of the domain-wall dynamics in terms of the position of the center of the wall and its velocity.[4,16] Since we obtain corrections to these quantities in first-order perturbation theory, for the distribution of θ we can restrict ourselves to the zeroth approximation θ_0. Thus, we go over to a concise description of the domain wall using the substitution

$$\theta(t)=\theta_0\left(\frac{x-x_0(t)}{\Delta(t)}\right),$$

where

$$\Delta(t)=\Delta_0\sqrt{1-\left(\frac{\dot{x}_0(t)}{c}\right)^2},$$

and $x_0(t)$ is the coordinate of the center of the domain wall. We assume that the external field is directed along the c axis:

$$\mathbf{H}=(0,\ 0,\ H).$$

Integrating the bulk Lagrangian density over x in the range $[-D,D]$ (D is the domain diameter, $D\gg\Delta$), we obtain the Lagrangian for a unit of the wall surface:

$$L=-mc^2\sqrt{1-v^2/c^2}-U(x_0). \tag{1}$$

Here $v=\dot{x}_0$, $mc^2=2\sqrt{A(K_{ac}+\chi_\perp H^2}$, and $U(x_0)=-\int 2M_z^0H(x_0)dx_0$, where $H(x_0)$ is the total external field acting on the wall, which includes the external driving field and the effective field created by the defect.

Since we are considering an absolutely flat wall, on whose surface all points are equivalent, it is simple to obtain the Lagrangian: it is only necessary to multiply the surface Lagrangian density by the area of the wall. We shall henceforth assume that all such quantities (Lagrangians, Hamiltonians, etc.) are for a unit area of the domain wall unless otherwise specified.

3. WKB APPROXIMATION FOR THE TUNNELING RATE: FORMULATION IN TERMS OF PATH INTEGRALS

We consider the evolution of a quantum-mechanical system described by the Lagrangian (1), which is found at the time t_0 in a lowest-energy state (roughly speaking, it is stationary) at the metastable minimum x_{01} of the potential $U(x_0)$. Thus, in our problem it is convenient to use the approach developed in the problem of the decay of a metastable vacuum.[17] Since a vacuum decays, its formally calculated energy has an imaginary part, which is proportional to the tunneling rate. The energy of a vacuum state (including its imaginary part) can easily be found by means of functional integration in Euclidean space–time (i.e., after the replacement $t=i\tau$). A program of action was proposed in this form in Refs. 17 and 18.

Now we set about carrying out this program. From the Lagrangian description of a domain wall we go over to the Hamiltonian formalism. It should be stressed that the Lagrangian (1) has the same form as the Lagrangian for classical relativistic particle motion in an external field $U(x_0)$. The corresponding Hamiltonian is well known:

$$H(p,x_0)=c\sqrt{p^2+m^2c^2}+U, \tag{2}$$

where the canonical momentum is

$$p=\frac{mv}{\sqrt{1-v^2/c^2}}. \tag{3}$$

The amplitude of the transition in imaginary time from the state $|x\rangle$ to the state $|y\rangle$ is usually represented in the form of a functional integral with respect to the Wiener measure[19,20]

$$\rho(x,y)=\int_{(q=x)}^{(q=y)}Dq\,\exp(-S_E/\hbar),$$

where

$$S_E=\int_0^T L\,d\tau$$

is the Euclidean action, i.e., the action in the imaginary time $\tau=-it$. However, this form of the integral is applicable only for Hamiltonians which are quadratic with respect to the momentum (see, for example, Ref. 21), and since we have a pseudorelativistic Hamiltonian, we must start from the very beginning, i.e., we must start out from the Hamiltonian form of the functional integral for the transition amplitude:

$$\rho(x,y)=\int_{(q=x)}^{(q=y)}Dq\int_{(-\infty)}^{(+\infty)}\frac{Dp}{2\pi\hbar}\exp\left[\frac{1}{\hbar}\int_0^T d\tau(ip\dot{q}-H(p,q))\right]. \tag{4}$$

To calculate the integral (4), we use the WKB approximation. Expanding the functional

$$S[p,q]=\int_0^T d\tau\,(ip\dot{q}-H(p,q))$$

with accuracy to the second order with respect to the variations δq and δp (and, accordingly, to second order in $\hbar$), we obtain a Gaussian integral, which is easily calculated by the Laplace method.[19]

The stationary points of the functional $S[p,q]$ correspond to the classical trajectories x_0 and p_0 (in imaginary time), where the first variations of $S[p,q]$ with respect to δq and δp are equal to zero:

$$p_0=\frac{im\dot{x}_0}{\sqrt{1+\dot{x}_0^2/c^2}}, \tag{5a}$$

$$i\dot{p}_0+\partial U(x_0)/\partial x_0=0. \tag{5b}$$

The integral (4) is now rewritten in the form

$$\rho(x,y)=\exp\left(-\frac{S_{\rm cl}(x,y)}{\hbar}\right)\int\int\frac{D\xi\,D\eta}{2\pi\hbar}\exp\left(-\frac{1}{2\hbar}\delta^2 S\right), \tag{6}$$

$$\xi=q-x_0,\quad \eta=p-p_0.$$

Here $S_{\rm cl}(x,y)$ is the action on the classical trajectory which begins at the point $x_0=x$ at $\tau=0$ and ends at the point $x_0=y$ at $\tau=T$:

$$S_{\rm cl}=\int_0^T d\tau\left(mc^2\sqrt{1+\frac{\dot{x}_0^2}{c^2}}+U(x_0)\right), \tag{7}$$

and

$$\frac{1}{2}\delta^2 S=\int_0^T\left(\frac{1}{2m}\eta^2\bar{u}^2+\frac{1}{2}\xi^2 v^2-i\eta\dot{\xi}\right)d\tau, \tag{8}$$

where

$$v^2=\left.\frac{\partial^2 U}{\partial q^2}\right|_{q=x_0(\tau)},$$

$$\bar{u}^2=\frac{m^3c^3}{(p_0^2+m^2c^2)^{3/2}}.$$

Although the integral (6) has a Gaussian form, the integration over η does not bring it into the standard form; the norm in the finite-dimensional approximation has the form

$$\prod_{k=1}^{N}\sqrt{\frac{m}{2\pi\hbar\varepsilon}}\frac{1}{(1+\dot{x}_k^2/c^2)^{3/4}},\quad \varepsilon=T/N.$$

This difficulty is easily avoided by performing the substitution

$$\theta=\int_0^\tau[\bar{u}(\tau')]^2 d\tau'.$$

Then (6) is rewritten in the form

$$\rho(x,y)=e^{-S_{\rm cl}/\hbar}\int\int\frac{D\xi D\eta}{2\pi\hbar}\exp\left[-\frac{1}{\hbar}\int_0^\Theta\left(\frac{\eta^2}{2m}+\frac{\xi^2 v^2}{2\bar{u}^2}-i\eta\xi'_\theta\right)d\theta\right],$$

where

$$\Theta=\int_0^T[\bar{u}(\tau)]^2 d\tau.$$

and the integration over η brings the integral into the standard form

$$\rho(x,y)=\exp\left\{-\frac{S_{\rm cl}}{\hbar}\right\}\int_{\xi=0}^{\xi=0}D\xi\exp\left[-\frac{1}{\hbar}\int_0^\theta\left(\frac{m}{2}(\xi'_\theta)^2+\frac{v^2}{2\bar{u}^2}\xi^2\right)d\theta\right], \tag{9}$$

with ordinary Wiener normalization.

Now we can use the approach proposed in Refs. 17 and 18. Motion in the imaginary time τ in the potential U is equivalent to motion in real time in the reversed potential $U\to -U$. To calculate the ground-state energy, we must find the amplitude $\rho(x,y)$ for $x=y=x_{01}$ and $T\to\infty$. These conditions are satisfied by two classical trajectories: the trajectory $q\equiv x_{01}$ and the trajectory $q=x_{\rm inst}$, which begins at the point x_{01} for $\tau\to-\infty$, passes the point x_{02} at $\tau=0$, and ends at the point x_{01} for $\tau\to+\infty$ (it is called the instanton trajectory). The tunneling rate is equal to the ratio between the values of the integral (9) on the trajectories $q\equiv x_{01}$ and $q=x_{\rm inst}$ (this is because the trajectory $q\equiv x_{01}$ determines the normalization of the functional integral; for further details see Ref. 17). Selecting the arbitrary additive constant in the potential U such that $U(x_{01})=-mc^2$ (the action on the trajectory $x_0\equiv x_{01}$ is then equal to zero), for the tunneling rate in the WKB approximation we obtain the expression

$$\Gamma=A\exp(-B), \tag{10}$$

$$\frac{B}{A_\omega}=\frac{S_{\rm inst}}{\hbar}, \tag{11}$$

where $S_{\rm inst}$ is the value of the function $S_{\rm cl}$ on the instanton trajectory and A_ω is the surface area of the tunneling element of the domain wall. To calculate the pre-exponential factor A we must find the values of the Gaussian functional integral

$$\mathcal{Y}=\int_{(\xi=0)}^{(\xi=0)}D\xi\exp\left[-\frac{1}{\hbar}\int_0^\theta\left(\frac{m}{2}(\xi'_\theta)^2+\frac{v^2}{2\bar{u}^2}\xi^2\right)d\theta\right] \tag{12}$$

on trajectories close to $q\equiv x_{01}$ and $q=x_{\rm inst}$ to within a normalizing factor common to both trajectories. For greater clarity in the ensuing calculations we go over to the variable τ in the integral (12):

$$\mathcal{Y}=\int_{(\xi=0)}^{(\xi=0)}D\xi\exp\left[-\frac{1}{\hbar}\int_0^T\left(\frac{m}{2\bar{u}^2}\dot{\xi}^2+\frac{v^2}{2}\xi^2\right)d\tau\right]. \tag{13}$$

The exponent in (13) is the scalar product $\xi\cdot\hat{W}\xi$, where the Sturm–Liouville operator has the form

$$\hat{W}=-\frac{\partial}{\partial\tau}\left(\frac{1}{\bar{u}^2}\frac{\partial}{\partial\tau}\right)+\frac{v^2}{m}. \tag{14}$$

Expanding ξ in the complete set of the orthonormalized eigenfunctions β_k of $\hat{W}$, we can bring the scalar product in the exponent into the form

$$\sum m\lambda_k C_k^2,$$

where the λ_k are the eigenvalues corresponding to the functions β_k, and the C_k are expansion coefficients (which are the integration variables in each integral). In this case the functional integral (13) breaks down into a product of Gaussian integrals, and integrating over C_k with the measure

$$\prod_k dC_k\sqrt{\frac{m}{2\pi\hbar}},$$

we can take the integral (12). Here we obtain the product of the eigenvalues λ_k, i.e, the determinant of the operator $\hat{W}$, to within the normalizing factor N. However, caution must be exercised here: on the instanton trajectory the operator $\hat{W}$ has a zeroth eigenvalue, which corresponds to the eigenfunction

$$\xi_1=\frac{\dot{x}_{\text{inst}}}{\sqrt{\nu}},\quad \nu=\int_{-\infty}^{+\infty}[\dot{x}_{\text{inst}}(\tau)]^2 d\tau.$$

Using $dC_1=\sqrt{\nu}d\tau$ and integrating with respect to C_1, we obtain (to within the normalizing factor N)

$$\mathcal{Y}_{\text{inst}}=N\sqrt{\frac{m\nu}{2\pi\hbar}}\det{}'\left[-\partial_\tau\left\{\frac{1}{\bar{u}^2}\partial_\tau\right\}+\frac{v^2}{m}\right],$$

where $\det'$ indicates that the zeroth eigenvalue should be discarded. Calculating $\mathcal{Y}$ on the trajectory $q\equiv x_{01}$, for the factor $A=\mathcal{Y}[q\equiv x_{01}]/\mathcal{Y}_{\text{inst}}$ we obtain (compare Ref. 17)

$$A=\sqrt{\frac{m\nu}{2\pi\hbar}}\left|\frac{\det'[\partial_\tau\{(1/\bar{u}^2)\partial_\tau\}+(v_{\text{inst}})^2/m]}{\det[-\partial_\tau^2+\omega^2]}\right|^{-1/2}, \tag{15}$$

where

$$\omega^2=\frac{1}{m}\frac{\partial^2 U(x)}{\partial x^2}\bigg|_{x=x_{01}},$$

$$(v_{\text{inst}})^2=\frac{\partial^2 U(x)}{\partial x^2}\bigg|_{x=x_{\text{inst}}}.$$

The equations of domain-wall motion on the instanton trajectory are assigned by (5a) and (5b), which are rewritten in the form

$$\frac{m\ddot{x}_0}{(1+\dot{x}_0^2/c^2)^{3/2}}=U(x_0). \tag{16}$$

The first integral, i.e., the energy, has the form

$$\frac{mc^2}{\sqrt{1+\dot{x}_0^2/c^2}}+U(x_0)=E. \tag{17}$$

At the point x_{01} the velocity vanishes; therefore, the potential U must be replaced by $\tilde{U}=U-mc^2$ [we assume $U(x_{01})=0$], and then from (17) we obtain the quadrature for $x_{\text{inst}}(\tau)$:

$$\int_{x_{01}}^{x_{\text{inst}}}\frac{dx}{\pm c\sqrt{(mc^2/\tilde{U})^2-1}}=\tau, \tag{18}$$

where the sign is chosen in accordance with the direction of motion.

The expression for S_{inst} can be obtained in the same manner:

$$S_{\text{inst}}=2\int_{x_{02}}^{x_{01}}\sqrt{2m\tilde{U}(x)-[\tilde{U}(x)]^2/c^2}\,dx. \tag{19}$$

Here it must be taken into account that the results (18) and (19) have meaning only under the condition

$$\max_{x\in[x_{02},x_{01}]} U(x)<mc^2;$$

otherwise, because of (17), the instanton solution will not exist, and the tunneling rate in the WKB approximation will be equal to zero. Actually, of course, a domain wall can still tunnel in this case, but the rate of the process will be of the next order with respect to $\hbar$, i.e., the tunneling will be suppressed to a considerable degree.

4. COMPARISON WITH EXPERIMENT. INFLUENCE OF DISSIPATION

For a comparison with the experimental results presented in Ref. 11, the potential of the defect must be specified. Let us use the simplest potential as a model: we assume that the height of the potential barrier is low and that the total field acting on the domain wall between the points x_{01} and x_{02} can be represented in the form

$$H(x)=H_c\left(1-\frac{(x-b)^2}{2a^2}\right)+H_0, \tag{20}$$

where H_c is the coercive force of the defect, a and b are, respectively, its width and the position of its center, H_0 is the external field (in our case $H_0<0$, i.e., the domain wall moves from right to left, Fig. 1). Since the height of the barrier is low, we restrict ourselves to the case of $U\ll mc^2$. We then have

$$\frac{B}{A_\omega}=\frac{2}{\hbar}\int_{x_{02}}^{x_{01}}\sqrt{2mU(x)}\,dx.$$

We select the normalization of the potential and the position of the origin of coordinates along the x axis so that $x_{01}=0$ and $U(x_{01})=0$. Then, from (20) we obtain

$$U(x)=\frac{M_z^0 H_c}{3a^2}x^3+\frac{M_z^0 H_c}{a}x^2\sqrt{2\left(1+\frac{H_0}{H_c}\right)} \tag{21}$$

and

$$b=-a\sqrt{2(1+H_0/H_c)},\quad x_{02}=-3a\sqrt{2(1+H_0/H_c)}.$$

The calculation of B gives

$$B/A_\omega\simeq(16/\hbar)a\sqrt{mH_cM_z^0a}(1+H_0/H_c)^{5/4},$$

where A_ω is the area of the tunneling element of the domain wall. Table I in Ref. 11 presents the values of B for four different values of the external field H. After constructing the dependence of $B^{4/5}$ on H, we can evaluate H_c from the slope of the straight line obtained. A least-squares approximation gives $H_c\simeq 600$ Oe. How does this value relate to the other data?

Since Zhang *et al.*.[11] presented only the value of the anisotropy constant

$$K_{ac}=1\times10^5 \ \text{erg/cm}^3,$$

for the theoretical evaluations we use the typical parameters of a terbium orthoferrite sample:

$$A=1\times10^{-7} \ \text{erg/cm}, \quad M_z^0=10 \ \text{G},$$

$$c=2\times10^6 \ \text{cm/s},$$

whence we have

$$\Delta_0=10^{-6} \ \text{cm}, \quad \sigma_0=mc^2=4\sqrt{AK_{ac}}=0.4 \ \text{erg/cm}^2.$$

Let us consider a nonmagnetic inclusion (defect) with an area A_ω and a length d. If the demagnetization poles are neglected, the decrease in the energy of a domain wall containing this nonmagnetic inclusion will be equal to the sum of the changes in the exchange energy and the anisotropy energy. The energy minimum of the domain wall is achieved when the defect is located at the middle of the thickness of the wall. Then the change in the surface energy density of the domain wall gives us the maximum height $U_{\max}^{(0)}$ of the potential barrier for a vanishing external field. Calculating it under the condition $d\ll\Delta_0$ (then a must have a value of the order of the wall thickness Δ_0), we obtain

$$U_{\max}^{(0)}=mc^2\frac{d}{2\Delta_0}.$$

On the other hand, from (21) we can also find that this quantity equals

$$U_{\max}^{(0)}\simeq 4M_z^0H_ca.$$

Hence we can obtain $d\simeq10$ Å.

The value of A_ω can be calculated from the condition $B=30$. In this case we obtain

$$A_\omega\simeq10^3 \ \text{Å}^2. \tag{22}$$

On the other hand, Zhang *et al.*,[11] presented values of the energy U of the potential barrier for different fields. An external field $H_0=-75$ Oe scarcely lowers the barrier (since H_0/H_c is small in this case). Since $U_{\max}^{(0)}$ is equal to U/A_ω, we can obtain A_ω by another method. In this case

$$A_\omega\simeq4\cdot10^3 \ \text{Å}^2, \tag{23}$$

which agrees quite well with the value (22), if we take into account the relative crudeness of our model potential. Next, we evaluate the volume of the tunneling element of the domain wall V:

$$V=\frac{U}{2M_z^0H_c}\simeq7\cdot10^5 \ \text{Å}^3.$$

Since the width of the barrier $a\simeq\Delta_0=100$ Å, we have

$$A_\omega=V/a\simeq7\cdot10^3 \ \text{Å}^2. \tag{24}$$

As we see, the three independent evaluations of A_ω associated with different tunneling characteristics give fairly close values.

Several remarks should be made here. Zhang *et al.*,[11] interpreted their results using the theory of magnetization tunneling in small antiferromagnetic particles. After obtaining a value $V\simeq8\times10^4$ Å^3 for the volume of the tunneling element of the domain wall, they naturally considered it unjustifiably small and found it difficult to identify the type of defect immobilizing the wall. However, in the case of the tunneling of a domain wall, the quantity which they evaluated is not the volume of the tunneling element, but the volume of the defect (in the case of a strictly single-domain particle, in which a domain wall does not form even during magnetization reversal, both quantities coincide and are equal to the volume of the particle). The fundamental difference between these cases is that when $d\ll\Delta_0$ holds the width of the barrier equals Δ_0, rather than d.

5. CONCLUSIONS

The macroscopic quantum tunneling of a domain wall in a weak ferromagnet has been investigated theoretically. Although the Hamiltonian describing the motion of the domain wall is not quadratic with respect to the momentum, a formalism similar to Ref. 17, which is based on functional integration, can be developed in the WKB approximation. The equations of motion for the instanton trajectory have been solved in quadratures for any form of the potential barrier, whence it is easy to obtain the quadrature for the action on the instanton trajectory. Thus, the most important exponential term in the expression for the tunneling rate can be written out in the form of a one-dimensional integral. An expression for the pre-exponential factor has been obtained in the form of the ratio between the determinants of two second-order elliptic operators. An analysis of the experimental data shows that the theory gives reasonable values for different characteristics of the process of the macroscopic tunneling of a domain wall.

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