A theory of particle diffusion in a magnetic field with strong small-scale random scattering

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The method of functionals is used to average the collisionless Boltzmann equation describing the transport of charged particles in small-scale random and regular magnetic fields. Exact equations for the averaged Green's functions and the vertex operator are derived and solved for the case of strong small-scale random scattering taking place at fairly low particle energies, at which the particles scatter in a single scattering event by an average angle of order unity. The diagrammatic technique is used to determine the main processes of moderate and strong small-scale random scattering. The renormalized Green's functions and the vertex operator are found with strong scattering taken into account, and so are the renormalized kinetic coefficients. The energy dependence of the transport path and the anisotropy of the spatial diffusion tensor are calculated with allowance for moderate and strong small-scale scattering at low particle energies. The diagrammatic technique and the method of functionals differ somewhat from those employed by A. Z. Dolginov and I. N. Toptygin [Sov. Phys. JETP **24**, 1195 (1967); Icarus **8**, 54 (1968); and *Cosmic Rays in Interplanetary Magnetic Fields*, Nauka, Moscow (1983) (in Russian)] and L. I. Dorman and M. E. Katz [*Cosmic Rays: Variations and Space Explorations*, North-Holland, Amsterdam (1974); and Space Sci. Rev. **20**, 529 (1977)] but are convenient for studying strong random scattering and result from the development of methods for obtaining averaged kinetic equations in consistent diffusion theory. [S1063-7761(96)00805-0]

1. INTRODUCTION

The common approach to describing the transport of rapidly moving low-density charged particles in random and regular fields in outer space is to use kinetic equations that are derived in consistent diffusion theory and allow for multiple particle scattering by the inhomogeneities of the random field.[1–6] This approach presupposes separating the spectrum of the magnetic-field inhomogeneities into three components: regular, large-scale random, and small-scale random.

Usually the regular component is assumed to be the magnetic field strength averaged over the entire inhomogeneity spectrum and incorporating chiefly maximum-scale inhomogeneities. The large-scale random magnetic field is the part of the random field with inhomogeneity scales $L_c^* > R_1^*$, where R_1^* is the Larmor radius of the particles in the random component of the field. In interplanetary and interstellar space the regular magnetic field often has the shape of the helically twisted field of an oblate dipole. The large-scale random magnetic field incorporates large-scale MHD perturbations: MHD waves (including shock waves and discontinuities in the magnetic field), magnetic clouds, sectoral structure in the case of the interplanetary magnetic field, and other large-scale structures.[3–9] In this paper the large-scale magnetic field is incorporated into the regular field and is denoted by $\mathbf{H}_0(\mathbf{r},t)$.

Usually the small-scale part of the random magnetic field incorporates magnetic inhomogeneities of scale $L_c < R_1^*$. Such a definition of a small-scale magnetic field is not sufficient, however. The main feature of small-scale scattering is the absence of resonant interaction of particles and MHD waves or the absence of particle trapping in the interaction of particles with shock waves or other steady-state magnetic structures. Stochastization of particle motion, i.e., enhancement of small-scale scattering, may occur due to the motion of the particles in magnetic traps, due to resonance overlap in the phase space, due to violation of the quasi-adiabatic nature of the motion of the particles under rapid variations of the magnetic field, in a strongly turbulent plasma, and in a weakly turbulent plasma in the presence of inhomogeneous conditions.[10–14] Hence, in a highly perturbed medium the scattering closely resembling small-scale scattering may produce large-scale magnetic structures.

Highly irregular perturbations of the magnetic field depending on the level of solar activity can be observed in interplanetary space, and irregular perturbations of the magnetic field can also be observed in interstellar space.[3,6,9,15,16] Consequently, the approximation of cosmic-ray diffusion caused by nonresonant scattering is often used in analyzing experimental data on solar and galactic cosmic rays propagating in interplanetary space.[1–5,9,17–19] This case of cosmic ray propagation is considered below when the analytic results are compared with the experimental data.

It must also be noted that strong small-scale scattering affects the isotropy of the distribution function of the particles and the broadening of resonances in the interaction of the particles and MHD waves in large-scale scattering.[3,6–8] Nonresonant random scattering affects anomalous diffusion in the high-temperature plasma of tokamaks.[20–22]

Initially the kinetic equations were obtained for multiple scattering under the assumption that small-scale scattering was weak,[1-5] which was the result of employing the iteration method to obtain averaged kinetic equations. Dupree,[23] Weinstock,[24] and Rudakov and Tsytovich[25] were the first to broaden the scope of the ordinary iteration procedure, which begins with the unperturbed Green's function of the particle, by allowing in a more consistent manner for higher-order approximations in the random electric field in the turbulent plasma and by using nonlinear kinetic equations, while Klimas and Sandri[26,27] and Völk[28] also allowed for higher-order approximations in the random magnetic field (see also the literature cited in Refs. 6 and 29). Bykov and Toptygin[6] and Vaĭnshteĭn and Kichatinov[30] allowed for higher-order approximations in the small-scale random magnetic field in the relaxation-time approximation and found the averaged transport coefficients. Finally, Dorman *et al.*[31] used the method of functionals to determine the spectrum of the correlation functions of the fluctuation of the distribution of particles scattered by a strong magnetic field.

In this paper the method of functionals is used to derive and solve the nonlinear equations for the average one-particle Green's function and the vertex operator. The equations are used to examine the kinetics of particles undergoing moderate random scattering by an average angle $\lesssim 1$ on a single inhomogeneity of the field and strong random scattering by an average angle $\gtrsim 1$ on a single inhomogeneity. The diagrammatic technique is employed to determine the main processes of moderate and strong small-scale scattering and find the corresponding collision integrals, which are then used to obtain the kinetic and diffusion equations for the averaged distribution function of the charged particles and to determine the averaged transport coefficients. The energy dependence of the transport path and the spatial diffusion coefficient is also studied.

2. THE DIAGRAMMATIC TECHNIQUE AND THE AVERAGED SCATTERING MICROPROCESSES

Following the common approach to studies of the transport of suprathermal particles, we assume the plasma rarefied and the Coulomb collision rate much lower than all the characteristic frequencies in the plasma. We therefore start with the collisionless Boltzmann equation for a particle in a magnetic field. Initially, for completeness of exposition and for establishing the link that exists between the diagrammatic technique and averaged scattering microprocesses in relation to studying the higher-order terms in the random magnetic field, we use the ordinary iteration solution of the Boltzmann equation, which we average over the random-field ensemble by employing the method of functionals.[32] In contrast to Dolginov and Toptygin's work,[1,2] to set up the diagrammatic technique we use the Green's functions that are exact in the regular magnetic field. Since the method differs somewhat from that described in Refs. 4 and 5, including the formulas with functionals, we briefly discuss the iteration solution.

The collisionless Boltzmann equation for the nonaveraged Green's function of a particle, $g(x,x_0)$ (here $x\equiv(\mathbf{r},\mathbf{p},t)$ and $x_0\equiv(\mathbf{r}_0,\mathbf{p}_0,t_0)$), interacting with a magnetic field $\mathbf{H}(\mathbf{r},t)$ has the form

$$\left(\frac{\partial}{\partial t}+\hat{L}_0-\mathbf{H}_1\mathbf{D}\right)g(x,x_0)=\delta(x-x_0), \tag{1}$$

where

$$\hat{L}_0=\mathbf{v}\frac{\partial}{\partial \mathbf{r}}-\mathbf{H}_0\mathbf{D},\quad \mathbf{D}=\frac{e}{c}\left[(\mathbf{v}-\mathbf{u})\frac{\partial}{\partial \mathbf{p}}\right],$$

$\mathbf{v}$ is the particle velocity, $\mathbf{u}$ is the magnetic-field velocity, $\mathbf{H}_1(\mathbf{r},t)$ is the small-scale random field, and

$$\delta(x-x_0)=\delta(\mathbf{r}-\mathbf{r}_0)\delta(\mathbf{p}-\mathbf{p}_0)\delta(t-t_0).$$

For the small-scale random and regular (large-scale random) magnetic fields we have the following relationships:

$$\langle\mathbf{H}\rangle=\langle\mathbf{H}_0+\mathbf{H}_1\rangle=\mathbf{H}_0,\quad \langle\mathbf{H}_1=0,\rangle,$$

where the angle brackets $\langle\cdots\rangle$ stand for averaging over the statistical ensemble of the small-scale random field.

The separation of the total magnetic field into the small-scale random, the large-scale random, and the regular is largely arbitrary and depends on the spectrum and type of the magnetic inhomogeneities and the geometry of the regular magnetic field.[1-7,14,16] In this paper it is assumed that the small-scale random magnetic field incorporates magnetic structures that scatter cosmic-ray particles by a large angle in a single scattering event, e.g., field discontinuities, shock waves, magnetic clouds, and various types of MHD waves leading to resonant scattering in the presence of a strong broadening of wave–particle resonances. The large-scale magnetic field may form as a result of large-scale motion of the medium.[3-7]

The iteration solution of the Boltzmann equation (1) averaged over the small-scale random magnetic field has the form

$$\begin{aligned} G[\boldsymbol{\eta},x,x_0]=&G_0(x,x_0)+\sum_{n=1}^{\infty} i^{-n}\int_0^{t-t_0}d\tau_n\exp\{-\tau_n\hat{L}_0\}\\ &\times\exp\left\{-\tau_n\frac{\partial}{\partial t}\right\}\int d\mathbf{r}_n dt_n\delta(\mathbf{r}-\mathbf{r}_n)\delta(t-t_n)D_\alpha\\ &\times\int_0^{t-t_0}d\tau_{n-1}\exp\{-\tau_{n-1}\hat{L}_0\}\exp\left\{-\tau_{n-1}\frac{\partial}{\partial t}\right\}\\ &\times\int d\mathbf{r}_{n-1}dt_{n-1}\delta(\mathbf{r}-\mathbf{r}_{n-1})\delta(t-t_{n-1})D_\beta\cdots\\ &\times\int_0^{t-t_0}d\tau_1\exp\{-\tau_1\hat{L}_0\}\exp\left\{-\tau_1\frac{\partial}{\partial t}\right\}\\ &\times\int d\mathbf{r}_1 dt_1\delta(\mathbf{r}-\mathbf{r}_1)\delta(t-t_1)D_\gamma\\ &\times\frac{\delta^n\Phi[\boldsymbol{\eta}]G_0(x,x_0)}{\Phi[\boldsymbol{\eta}]\delta\eta_\alpha(\mathbf{r}_n,t_n)\delta\eta_\beta(\mathbf{r}_{n-1},t_{n-1})\cdots\delta\eta_\gamma(\mathbf{r}_1,t_1)}, \end{aligned} \tag{2}$$

where

$$\Phi[\boldsymbol{\eta}]=\left\langle\exp\left\{i\int d\mathbf{r}dt\,\boldsymbol{\eta}(\mathbf{r},t)\mathbf{H}_1(\mathbf{r},t)\right\}\right\rangle$$

is the characteristic functional that fully determines the statistical properties of the small-scale random magnetic field,

$$G_0(x,x_0)=\int_0^{t-t_0} d\tau \exp\{-\tau\hat{L}_0\}\exp\left\{-\tau\frac{\partial}{\partial t}\right\}\delta(x-x_0)$$

$$=\left(\frac{\partial}{\partial t}+\hat{L}_0\right)^{-1}\delta(x-x_0) \quad (3)$$

is the Green's function of a particle in a regular magnetic field, and we sum over repeated vector indices. To eliminate the factor $\frac{1}{2}$ that appears as a result of integrating the appropriate delta function over τ we assume that integration with respect to τ is done from zero to $t-t_0+\Delta$, where Δ is an infinitesimal time interval.

We can now transform the solution (2) by introducing a new functional argument:

$$\xi[\eta,\mathbf{r},t]=-i\frac{\delta \ln\Phi[\eta]}{\delta\eta(\mathbf{r},t)}. \quad (4)$$

If the distribution of the small-scale random magnetic field is Gaussian, the relationship between the functional arguments ξ and η is linear; in the more general case the functional argument $\xi[\eta,\mathbf{r},t]$ is a nonlinear functional of $\eta(\mathbf{r},t)$.

For the remainder of this section we assume that the distribution of the small-scale random magnetic field is Gaussian. In this case the relationship linking the functional derivatives with respect to the functional arguments ξ and η has the form

$$\frac{\delta}{\delta\eta_\gamma(\mathbf{r},t)}=i\int d\mathbf{r}_1dt_1B_{\alpha\gamma}(\mathbf{r}_1,t_1;\mathbf{r},t)\frac{\delta}{\delta\xi_\alpha(\mathbf{r}_1,t_1)}, \quad (5)$$

where

$$B_{\alpha\beta}(\mathbf{r},t;\mathbf{r}_1,t_1)=-\frac{\delta^2 \ln\Phi[\eta]}{\delta\eta_\alpha(\mathbf{r},t)\delta\eta_\beta(\mathbf{r}_1,t_1)}$$

is the correlation tensor of the small-scale random magnetic field.

Transforming the solution (2) via (4) and (5), we get

$$G[\xi,x,x_0]=G(x,x_0)+\sum_{n=1}^{\infty}\int_0^{t-t_0}d\tau_n$$

$$\times\exp\{-\tau_n\hat{L}_0\}\exp\left\{-\tau_n\frac{\partial}{\partial t}\right\}$$

$$\times\int d\mathbf{r}_ndt_n\delta(\mathbf{r}-\mathbf{r}_n)\delta(t-t_n)D_\alpha\int_0^{t-t_0}d\tau_{n-1}$$

$$\times\exp\{-\tau_{n-1}\hat{L}_0\}\exp\left\{-\tau_{n-1}\frac{\partial}{\partial t}\right\}$$

$$\times\int d\mathbf{r}_{n-1}dt_{n-1}\delta(\mathbf{r}-\mathbf{r}_{n-1})\delta(t-t_{n-1})D_\beta\cdots$$

$$\times\int_0^{t-t_0}d\tau_1\exp\{-\tau_1\hat{L}_0\}\exp\left\{-\tau_1\frac{\partial}{\partial t}\right\}$$

$$\times\int d\mathbf{r}_1dt_1\delta(\mathbf{r}-\mathbf{r}_1)\delta(t-t_1)D_\gamma\int d\mathbf{r}_n'dt_n'$$

$$\times\int d\mathbf{r}_{n-1}'dt_{n-1}'\cdots\int d\mathbf{r}_1'dt_1'B_{\delta\alpha}(\mathbf{r}_n',t_n';\mathbf{r}_n,t_n)$$

$$\times B_{\mu\beta}(\mathbf{r}_{n-1}',t_{n-1}';r_{n-1},t_{n-1})\cdots B_{\nu\gamma}(\mathbf{r}_1't_1';\mathbf{r}_1,t_1)$$

$$\times\frac{\delta^n\Phi[\xi]G_0(x,x_0)}{\Phi[\xi]\delta\xi_\delta(\mathbf{r}_n',t_n')\delta\xi_\mu(\mathbf{r}_{n-1}',t_{n-1}')\cdots\delta\xi_\nu(\mathbf{r}_1',t_1')}. \quad (6)$$

In this expansion we transform the functional derivatives of $\Phi[\xi]$ with respect to ξ by expressing them in terms of the functional derivatives of $\ln\Phi[\xi]$ and allowing for the fact that in a Gaussian random magnetic field

$$\frac{\delta \ln\Phi[\xi]}{\delta\xi_\alpha(\mathbf{r},t)}=\int d\mathbf{r}_1dt_1S_{\alpha\gamma}(\mathbf{r},t;\mathbf{r}_1,t_1)\xi_\gamma(\mathbf{r}_1,t_1), \quad (7)$$

where the function $S_{\alpha\gamma}$ (see Ref. 32), which does not enter into the final result, obeys the following relationship:

$$\int d\mathbf{r}_1dt_1S_{\gamma\beta}(\mathbf{r},t;\mathbf{r}_1,t_1)B_{\beta\alpha}(\mathbf{r}_1,t_1;\mathbf{r}',t')$$

$$=\int d\mathbf{r}_1dt_1B_{\alpha\beta}(\mathbf{r}',t';\mathbf{r}_1,t_1)S_{\beta\gamma}(\mathbf{r}_1,t_1;\mathbf{r},t)$$

$$=\delta\alpha\gamma\delta(\mathbf{r}-\mathbf{r}')\delta(t-t').$$

In the solution (6) the nth term in the series is a polylinear functional of the nth degree containing terms proportional to the argument ξ raised to different powers. In this expansion of the Green's function $G[\xi]$ we are interested only in terms that contain the zeroth and first power of the functional argument ξ. Also, for the sake of convenience in setting up the diagrammatic technique, we replace the inverse Boltzmann operator (3) with the Green's function $G_0(x,x_0)$ via the following relationship:

$$\int_0^{t-t_0}d\tau \exp\{-\tau\hat{L}_0\}\exp\left\{-\tau\frac{\partial}{\partial t}\right\}G(x,x_0)$$

$$=\int dx_1G_0(x,x_1)G(x_1,x_0), \quad (8)$$

where

$$G_0(x,x_0)=\theta(t-t_0)\delta(\mathbf{r}-\Delta\mathbf{r}(t-t_0)-\mathbf{r}_0)\delta(\mathbf{p}-\Delta\mathbf{p}(t-t_0)$$

$$-\mathbf{p}_0), \quad (9)$$

and $\Delta\mathbf{r}(t-t_0)$ and $\Delta\mathbf{p}(t-t_0)$ are the variations of position $\mathbf{r}$ and momentum $\mathbf{p}$ of a particle in the regular magnetic field $\mathbf{H}_0$ during $t-t_0$.

After transformation, expansion (6) becomes

$$G[\xi,x,x_0]=G_0(x,x_0)$$

$$+\int dx_1G_0(x,x_1)D_{1\alpha}\xi_\alpha(\mathbf{r}_1,t_1)G_0(x_1,x_0)$$

$$+\int dx_2G_0(x,x_2)D_{2\alpha}\int dx_1G_0(x_2,x_1)$$

$$\times D_{1\beta}B_{\beta\alpha}(\mathbf{r}_1,t_1;\mathbf{r}_2,t_2)G_0(x_1,x_0)$$

$$+\int dx_3G_0(x,x_3)D_{3\alpha}\int dx_2G_0(x_3,x_2)D_{2\beta}$$

$$\times\int dx_1G_0(x_2,x_1)D_{1\gamma}[B_{\beta\alpha}(\mathbf{r}_2,t_2;\mathbf{r}_3,t_3)$$

$$\times\xi_\gamma(\mathbf{r}_1,t_1)+B_{\gamma\alpha}(\mathbf{r}_1,t_1;\mathbf{r}_3,t_3)\xi_\beta(\mathbf{r}_2,t_2)$$

$$
\begin{aligned}
&+B_{\gamma\beta}(\mathbf{r}_1,t_1;\mathbf{r}_2,t_2)\xi_\alpha(\mathbf{r}_3,t_3)]G_0(x_1,x_0)\\
&+\int dx_4 G_0(x,x_4)D_{4\alpha}\int dx_3 G_0(x_4,x_3)D_{3\beta}\\
&\times\int dx_2 G_0(x_3,x_2)D_{2\gamma}\\
&\times\int dx_1 G_0(x_2,x_1)D_{1\delta}[B_{\beta\alpha}(\mathbf{r}_3,t_3;\mathbf{r}_4,t_4)\\
&\times B_{\delta\gamma}(\mathbf{r}_1,t_1;\mathbf{r}_2,t_2)\\
&+B_{\gamma\alpha}(\mathbf{r}_2,t_2;\mathbf{r}_4,t_4)B_{\delta\beta}(\mathbf{r}_1,t_1;\mathbf{r}_3,t_3)\\
&+B_{\gamma\beta}(\mathbf{r}_2,t_2;\mathbf{r}_3,t_3)B_{\delta\alpha}(\mathbf{r}_1,t_1;\mathbf{r}_4,t_4)]\\
&\times G_0(x_1,x_0)+\cdots, \qquad (10)
\end{aligned}
$$

where

$$
\mathbf{D}_1=\frac{e}{c}\left[(\mathbf{v}_1-\mathbf{u})\frac{\partial}{\partial \mathbf{p}_1}\right],
$$

etc.

Let us now set up a diagrammatic technique that uses, in contrast to the one developed by Dolginov and Toptygin[1,2] only the Green's functions that are exact in the regular magnetic field and is topologically close to the one developed in Refs. 32 and 33. With the terms in (10) we associate the following diagrams:

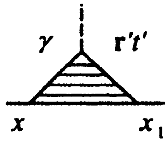
$$
G_0(x,x_0)\sim \underset{x}{}\!\longleftarrow\!\underset{x_0}{},\ \int dx\ldots D_\alpha\ldots\sim \overset{\alpha}{\underset{x}{\cdot}},
$$

$$
B_{\alpha\beta}(\mathbf{r},t;\mathbf{r}_1,t_1)\sim \overset{\alpha}{\underset{\mathbf{r}t}{}}\text{-----}\overset{\beta}{\underset{\mathbf{r}_1t_1}{}},\quad G(x,x_0)\sim \underset{x}{}\!\longleftarrow\!\underset{x_0}{}.
$$

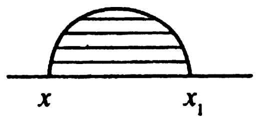
Next, with the functional argument we associate the times sign:

$$
\xi_\alpha(\mathbf{r},t)\sim \times^{\alpha}_{\mathbf{r}t}.
$$

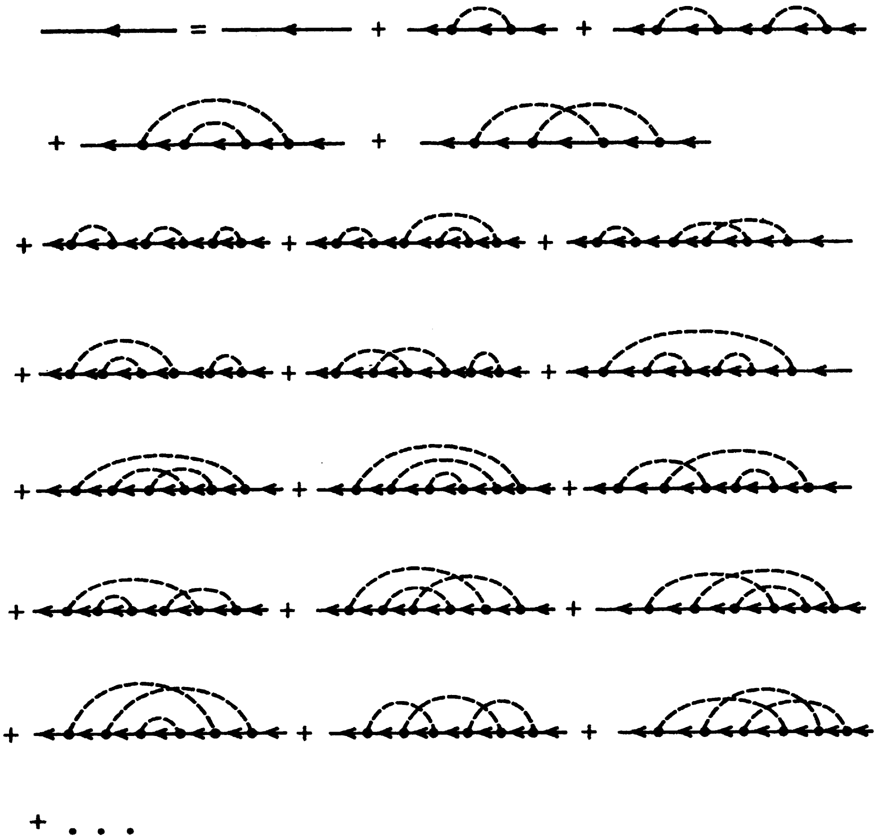
With the vertex operator $\hat{\Gamma}_\gamma(x,x_1;\mathbf{r}',t')$ (see Eq. (18) below) we associate a triangle,

$\hat{\Gamma}_\gamma(x,x_1;\mathbf{r}',t')$ ~ 

while with the mass operator $\hat{q}(x,x_1)$ (see Eq. (20) below) we associate a semicircle,

$\hat{q}(x,x_1)$ ~ 

We assume integration in the diagrams integration over internal variables. Using the diagrammatic technique, we can represent the expansion of the Green's function (10) with the functional argument $\xi=0$ in the following way:

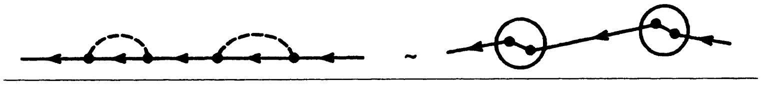


(11)

Separation of the resulting set of diagrams into weakly coupled and tightly coupled is done in the ordinary manner.[1,2,32]

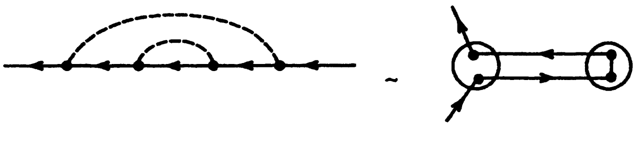
By using expansions (10) and (11) we can qualitatively separate the entire set of terms and the corresponding averaged elementary scattering microprocesses by the value of the average angle of particle deviation in a single scattering microprocess. For terms of order $\langle \mathbf{H}_1^4 \rangle$ and the corresponding averaged elementary scattering microprocesses, a separation similar to the one performed in Ref. 33 is done in the following way:

(a) a small-angle random scattering microprocess,[1,2]



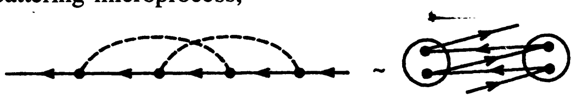
(12)

(b) a simple strong scattering microprocess,



(13)

(c) a complex strong scattering microprocess,



(14)

The diagrams and microprocesses shown in order of increasing average deviation angle.

3. EXACT EQUATIONS FOR THE GREEN'S FUNCTION AND THE VERTEX AND MASS OPERATORS

Averaging the Boltzmann kinetic equation by the method of functionals over the small-scale random magnetic field $\mathbf{H}_1$, we arrive at an equation similar to the Schwinger equation[34] for the functional $G[\boldsymbol{\eta},x,x_0]$, which becomes the averaged Green's function at $\boldsymbol{\eta}=0$ (see Refs. 4, 5, and 31). The averaged equation has the form

$$\left(\frac{\partial}{\partial t}+\hat{L}_0\right)G[\boldsymbol{\eta},x,x_0] = -iD_\alpha\left\{\frac{\delta G[\boldsymbol{\eta},x,x_0]}{\delta\eta_\alpha(\mathbf{r},t)}+G[\boldsymbol{\eta},x,x_0]\frac{\delta\ln\Phi[\boldsymbol{\eta}]}{\delta\eta_\alpha(\mathbf{r},t)}\right\} + \delta(x-x_0).$$

Let us write this equation in integral form:

$$G[\boldsymbol{\eta},x,x_0]=G_0(x,x_0)-i\int_0^{t-t_0}d\tau\,\exp\{-\tau\hat{L}_0\} \times\exp\left\{-\tau\frac{\partial}{\partial t}\right\}D_\alpha\left\{\frac{\delta G[\boldsymbol{\eta},x,x_0]}{\delta\eta_\alpha(\mathbf{r},t)} +G[\boldsymbol{\eta},x,x_0]\frac{\delta\ln\Phi[\boldsymbol{\eta}]}{\delta\eta_\alpha(\mathbf{r},t)}\right\}. \quad (15)$$

We then go over to the functional argument $\boldsymbol{\xi}[\boldsymbol{\eta};\mathbf{r},t]$ (Eq. (4), assuming the distribution of the small-scale random field $\mathbf{H}_1$ non-Gaussian and the relationship between the functional arguments $\boldsymbol{\xi}$ and $\boldsymbol{\eta}$ nonlinear. We also introduce the inverse functional $G^{-1}[\boldsymbol{\xi}]$ by the following formulas:

$$\int dx_1 G[\boldsymbol{\xi},x,x_1]G^{-1}[\boldsymbol{\xi},x_1,x_0]=\delta(x-x_0),$$

$$\int dx_1 G^{-1}[\boldsymbol{\xi},x,x_1]G[\boldsymbol{\xi},x_1,x_0]=\delta(x-x_0). \quad (16)$$

After going over to the new functional argument, we can use Eqs. (4), (5) and (16) to write the equation for the Green's functional $G[\boldsymbol{\xi}]$ in the form

$$G[\boldsymbol{\xi},x,x_0] =G_0(x,x_0)+\int_0^{t-t_0}d\tau\,\exp\{-\tau\hat{L}_0\} \times\exp\left\{-\tau\frac{\partial}{\partial t}\right\}D_\alpha\left\{G[\boldsymbol{\xi},x,x_0]\xi_\alpha(\mathbf{r},t) -\int d\mathbf{r}_1dt_1B_{\alpha\beta}[\boldsymbol{\xi};\mathbf{r},t;\mathbf{r}_1,t_1]\int dx' \times\int dx''G[\boldsymbol{\xi},x,x']\hat{\Gamma}_\beta[\boldsymbol{\xi},x',x'';\mathbf{r}_1,t_1]G[\boldsymbol{\xi},x'',x_0]\right\}, \quad (17)$$

where the vertex operator is defined as

$$\hat{\Gamma}_\beta[\xi,x_1,x_2;\mathbf{r},t]=\frac{\delta G^{-1}[\xi,x_1,x_2]}{\delta\xi_\beta(\mathbf{r},t)}, \tag{18}$$

and at $\xi=0$ the correlation functional $B_{\alpha\beta}[\xi;\mathbf{r},t;\mathbf{r}_1,t_1]$ transforms into the correlation tensor $B_{\alpha\beta}[\mathbf{r},t;\mathbf{r}_1,t_1]$.

Let us introduce the mass operator $\hat{q}[\xi]$ and derive the main relationships for it. To this end we write Eq. (17) in the form

$$G[\xi,x,x_0]=G_0(x,x_0)+\int_0^{t-t_0}d\tau\,\exp\{-\tau\hat{L}_0\}\times\exp\left\{-\tau\frac{\partial}{\partial t}\right\}\int dx_1\hat{q}[\xi,x,x_1]G[\xi,x_1,x_0], \tag{19}$$

where the mass operator $\hat{q}[\xi]$ meets the following condition:

$$\hat{q}[\xi,x,x_1]=D_\alpha\xi_\alpha(\mathbf{r},t)\delta(x-x_1)-D_\alpha\int d\mathbf{r}'dt'B_{\alpha\beta}[\xi;\mathbf{r},t;\mathbf{r}',t']\times\int dx_2G[\xi,x,x_2]\hat{\Gamma}_\beta[\xi,x_2,x_1;\mathbf{r}',t']. \tag{20}$$

Next we obtain the differential relationship that exists between the vertex and mass operators. Muitiplying Eq. (19) by $G^{-1}[\xi,x,x_1]$, integrating the product with respect to x_0, applying the Boltzmann operator $(\partial/\partial t+\hat{L}_0)$ to the result, and differentiating the resulting expression with respect to the functional argument $\xi(\mathbf{r}',t')$, we get

$$\hat{\Gamma}_\beta[\xi;x,x_1;\mathbf{r}',t']=-\frac{\delta\hat{q}[\xi,x,x_1]}{\delta\xi_\beta(\mathbf{r}',t')}. \tag{21}$$

We derive an equation for the vertex operator $\hat{\Gamma}_\beta[\xi]$. To this end we differentiate the expression (20) for the mass operator $\hat{q}[\xi]$ with respect to the functional argument $\xi(\mathbf{r}',t')$ and allow for the formula for the derivative of the Green's function,

$$\frac{\delta G[\xi,x,x_1]}{\delta\xi_\beta(\mathbf{r}',t')}=-\int dx_3\int dx_2G[\xi,x,x_3]\times\hat{\Gamma}_\beta[\xi;x_3,x_2;\mathbf{r}',t']G[\xi,x_3,x_1].$$

As a result we arrive at the following equation for the vertex operator:

$$\hat{\Gamma}_\gamma[\xi;x,x_1;\mathbf{r}',t']=-\delta(\mathbf{r}-\mathbf{r}')\delta(t-t')D_\gamma\delta(x-x^1)-D_\alpha\int d\mathbf{r}_2dt_2B_{\alpha\beta}[\xi;\mathbf{r},t;\mathbf{r}_2,t_2]\int dx_3\times\int dx_4\int dx_5G[\xi,x,x_5]\hat{\Gamma}_\gamma\times[\xi;x_5,x_4;\mathbf{r}'t']G[\xi,x_4,x_5]\hat{\Gamma}_\beta\times[\xi;x_3,x_1;\mathbf{r}_2,t_2]+D_\alpha\int d\mathbf{r}_2dt_2B_{\alpha\beta}[\xi;\mathbf{r},t;\mathbf{r}_2,t_2]\times\int dx_3G[\xi,x,x_1]\frac{\delta\hat{\Gamma}_\beta[\xi;x_3,x_1;\mathbf{r}_2,t_2]}{\delta\xi_\gamma(\mathbf{r}',t')}+D_\alpha\int d\mathbf{r}_2dt_2\int dx_3G[\xi,x,x_3]\hat{\Gamma}_\beta\times[\xi;x_3,x_1;\mathbf{r}_2,t_2]\frac{\delta B_{\alpha\beta}[\xi;\mathbf{r},t;\mathbf{r}_2,t_2]}{\delta\xi_\gamma(\mathbf{r}',t')}. \tag{22}$$

Equations (17) and (20) constitute a closed system of equations for the exact Green's functional $G[\xi,x,x_0]$ and the exact vertex operator $\hat{\Gamma}_\gamma[\xi;x,x_1;\mathbf{r}',t']$.

The first term on the right-hand side of Eq. (22) for the vertex operator $\hat{\Gamma}_\gamma[\xi]$ (the one with the delta functions) provides the main contribution in scattering in a weak small-scale random magnetic field[1–5] and in a small-scale random field of moderate strength[35] with the average angle $\langle\alpha\rangle$ of scattering by a single inhomogeneity $\lesssim 1$, where $\langle\alpha\rangle=R_1/L_c$, $R_1=cp/e<H_1^2>^{1/2}$ is the Larmor radius in the random magnetic field, and L_c is the size of a small-scale magnetic inhomogeneity.

The second term on the right-hand side of Eq. (22) reflects the complex processes of strong small-scale random scattering of type (14) corresponding to diagrams with crossed dashed lines. In such a process the particle oscillates between inhomogeneities of the small-scale random magnetic field and the scattering is of a quasi-resonant nature (see Sec. 5.2).

The third term on the right-hand side of Eq. (22) refines the complex processes of strong random scattering, and allowing for this term means stepping outside the scope of the iteration approximation in the vertex operator $\hat{\Gamma}_\gamma[\xi]$. Allowing for this term corresponds to allowing for processes of strong small-scale random scattering in traps formed by random inhomogeneities of the magnetic field, processes more complicated than those of type (55) (see Sec. 5.2) taken into account in this paper.

The fourth term on the right-hand side of Eq. (22) is related to the non-Gaussian nature of the distribution of the small-scale random magnetic field. Allowing for it means taking into account processes of strong random scattering when the distribution function of the small-scale random field broadens. Since the magnetic field in outer space is formed as a result of collective movements of the plasma, the effect of the non-Gaussian nature of the distribution of the random magnetic field can be appreciable. Thus, when the distribution of the random magnetic field is non-Gaussian, we must allow for strong random scattering, and in the kinetic equation we must allow for the terms like the last term on the right-hand side of Eq. (22).

4. MODERATE SMALL-SCALE RANDOM SCATTERING

4.1. The kinetic equation and averaged scattering microprocesses

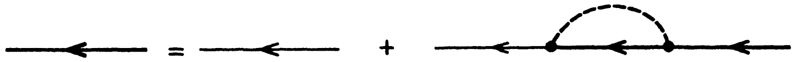
In moderate small-scale random scattering the average angle of scattering by a single magnetic inhomogeneity does not exceed unity. Then all the terms on the right-hand side of Eq. (22) for the vertex operator except the first can be ignored and we arrive at an expression for the vertex operator in the zeroth approximation:

$$\hat{\Gamma}^0_\gamma(x,x_1;\mathbf{r}',t') = -\delta(\mathbf{r}-\mathbf{r}')\delta(t-t')D_\gamma\delta(x-x_1). \quad (23)$$

Substituting this vertex operator into the equation for the Green's functional (17), assuming the functional argument ξ zero, and replacing the inverse Boltzmann operator by the unbroadened Green's function $G_0(x,x_0)$, we arrive at the following nonlinear kinetic equation:[31,36]

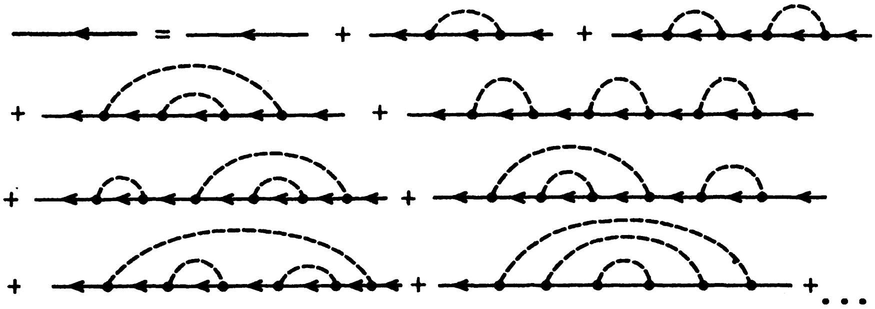
$$G(x,x_0) = G_0(x,x_0) + \int dx_1 G_0(x,x_1)D_{1\alpha} \times \int dx_2 B_{\alpha\beta}(\mathbf{r}_1,t_1;\mathbf{r}_2,t_2) \times G(x_1,x_2)D_{2\beta}G(x_2,x_0). \quad (24)$$

Diagrammatically this equation can be written as

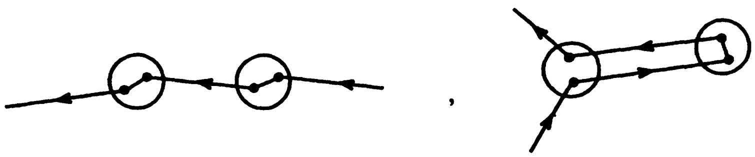


(25)

The iteration solution of this equation is represented by diagrams as



This solution implies that Eq. (25) allows for the averaged scattering microprocesses



(26)

in various combinations. The left diagram in (26) corresponds to weak small-scale scattering, while the right corresponds to the simple process of strong random scattering.

4.2. The Green's function in the small-time-interval approximation

To solve the nonlinear kinetic equation (24) we linearize it by employing the solution of the linear kinetic equation. The Green's function $G(x_1,x_2)$ in the collision term in (24) is convolved in time and position with the correlation tensor $B_{\alpha\beta}(\mathbf{r}_1,t_1;\mathbf{r}_2,t_2)$, so that we take the Green's function in the collision term in the small-time-interval approximation, $t_1-t_2 \ll L_c/v$, which is a refinement of the approximation used in Ref. 35.

To obtain the Green's function in the small-time-interval approximation we use the solution of the linear kinetic equation found in the ordinary iteration approximation in which for the zeroth approximation we take the Green's functions that ignore broadening due to random scattering and take into account only small-scale random scattering.[1-5] In calculating such Green's functions we assume the regular magnetic field fairly weak, as a result of which we can put $\mathbf{H}_0=0$ in the collision term.

With allowance for these approximations, the linear kinetic equation for the Green's function in the small-time-interval approximation has the form

$$\frac{\partial G_1}{\partial t} + \mathbf{v}\frac{\partial G_1}{\partial \mathbf{r}} = \omega_1^2 m^2 d_\alpha \int_0^{t-t_0} d\tau b_{\alpha\beta}(|-\mathbf{u}|\tau)d_\beta G_1(\mathbf{r} - \mathbf{v}\tau,\mathbf{p},t-t_0-\tau,\mathbf{p}_0,\mathbf{r}_0) + \delta(x-x_0), \quad (27)$$

where

$$b_{\alpha\beta}(\mathbf{x}) = \frac{1}{3}\left(\psi(x)\delta_{\alpha\beta} + \psi_1(x)\frac{x_\alpha x_\beta}{x^2}\right) \quad (28)$$

is the (normalized to unity), correlation tensor of the random magnetic field,

$$\mathbf{d}=\left[(\mathbf{v}-\mathbf{u})\frac{\partial}{\partial \mathbf{r}}\right],\quad \mathbf{r}=\frac{\mathbf{r}_1+\mathbf{r}_2}{2},\quad \mathbf{x}=\mathbf{r}_2-\mathbf{r}_1,$$

and $\omega_1(\mathbf{r})=e<\mathbf{H}_1^2(\mathbf{r})>^{1/2}/mc$ is the Larmor frequency of particle rotation in the small-scale random magnetic field. Since the conditions of particle scattering described by Eq. (27) are homogeneous, we assume that the Green's function in this equation depends only on the difference of arguments, $G_1(\mathbf{r}-\mathbf{r}_0-\mathbf{v}\tau,t-t_0-\tau)$. We now introduce the Fourier transform of the Green's function $G_1(\mathbf{k})$:

$$G_1(\mathbf{r}-\mathbf{r}_0)=(2\pi)^{-3}\int d\mathbf{k}G_1(\mathbf{k})\exp\{i\mathbf{k}(\mathbf{r}-\mathbf{r}_0)\}.$$

Taking (28) into account, we can write Eq. (27) for the Fourier transform $G_1(\mathbf{k})$ as follows:

$(\partial\partial t+i\mathbf{k}\mathbf{v})G1(\mathbf{k},t-t0)$

$$=\frac{\omega_1^2}{3}m^2 d_\alpha\int_0^{t-t_0}d\tau\psi(|\mathbf{v}-\mathbf{u}|\tau)d_\alpha \times\exp\{-i\mathbf{k}\mathbf{v}\tau\}G_1(\mathbf{k},t-t_0-\tau)+\delta(\mathbf{p}-\mathbf{p}_0)\delta(t-t_0). \tag{29}$$

For small time intervals we can write

$$\exp\{-i\mathbf{k}\mathbf{v}\tau\}G_1(\mathbf{k},t-t_0-\tau)\approx G_1(\mathbf{k},t-t_0).$$

Substituting this into Eq. (29) yields

$$\left(\frac{\partial}{\partial t}+i\mathbf{k}\mathbf{v}\right)G_1(\mathbf{k},t-t_0) =\frac{\omega_1^2}{3}m^2 d_\alpha\int_0^{t-t_0}d\tau\psi(|\mathbf{v}-\mathbf{u}|\tau)d_\alpha G_1(\mathbf{k},t-t_0) +\delta(\mathbf{p}-\mathbf{p}_0)\delta(t-t_0). \tag{30}$$

Note that the solution of the new equation holds both for small time intervals $t-t_0\ll L_c/v$ and for large time intervals $t-t_0\gg L_c/v$.

We select the correlation function $\psi(k_0x)$ for the power-like spectrum of a small-scale random magnetic field in the form[31]

$$\psi(k_0x)=\left[2^{(\nu-1)/2}\Gamma\left(\frac{\nu-1}{2}\right)k_0^2x\right]^{-1} \times\frac{\partial}{\partial x}[(k_0x)^{(\nu+3)/2}K_{(\nu-1)/2}(k_0x)],$$

with $k_0=L_c^{-1}$, where $K_m(k_0x)$ is a modified Bessel function of the second kind, and ν is the spectral index of the correlation function of the random magnetic field. Expanding $\psi(k_0x)$ in a power series for small values of the argument and substituting the expansion into Eq. (30), we finally arrive at the linear kinetic equation for small time intervals:

$$\left(\frac{\partial}{\partial t}+i\mathbf{k}\mathbf{v}\right)G_1(\mathbf{k},t-t_0)=\frac{\omega_1^2}{3}m^2(t-t_0)d^2G_1(\mathbf{k},t-t_0) +\delta(\mathbf{p}-\mathbf{p}_0)\delta(t-t_0). \tag{31}$$

To solve this equation for small time intervals we employ the operator method.[35–37] First we introduce the function

$$g_1(\mathbf{k})=\exp\{i\mathbf{k}\mathbf{v}(t-t_0)\}G_1(\mathbf{k}).$$

The equation for g_1 can then be written as

$$\frac{\partial g_1}{\partial t}=\exp[i\mathbf{k}\mathbf{v}(t-t_0)]\frac{\omega_1^2}{3}m^2(t-t_0)\mathbf{d}^2\exp[-i\mathbf{k}\mathbf{v}(t-t_0)]g_1. \tag{32}$$

We apply the operator $\mathbf{d}^2$ to the exponential in this equation and, considering first the case of small velocities ($v\ll c$ and $u\ll c$), ignore terms in the result proportional to v/c and u^2/c^2. This yields

$$\frac{\partial g_1}{\partial t}=\left\{-\frac{\omega_1^2}{3}(t-t_0)^3[(\mathbf{v}-\mathbf{u})\mathbf{k}]^2+i\frac{2\omega_1^2}{3}(t-t_0)^2((\mathbf{v}-\mathbf{u})\mathbf{k})-i\frac{2\omega_1^2}{3}(t-t_0)^2m[(\mathbf{v}-\mathbf{u})\mathbf{k}]\mathbf{d}+\frac{\omega_1^2}{3}(t-t_0)m^2\mathbf{d}^2\right\}g_1. \tag{33}$$

The second term on the right-hand side of this equation is related to the decrease in the average particle velocity in the scattering process and has a small effect on the factor of p in the final dependence of the transport path Λ on p (see Refs. 35 and 36). The third is related to the correlation between the particle's position in the $\mathbf{r}$-space and the direction of the particle momentum $\mathbf{p}$. The relative contribution of this term for small time intervals $t-t_0\ll L_c/v$ is fairly low.[36] The last term is related to the diffusion over the angles and absolute value of momentum in $\mathbf{p}$-space. Since the main purpose of this study is to examine diffusion processes in angle space over distances of the order of the correlation length L_c, we can ignore this term, too, because (a) it is balanced by higher-order commutators of $\mathbf{d}$ and $\mathbf{v}$ and $\mathbf{p}$, and (b) the accelerating processes over distances of order L_c are weak. The last term provides a large contribution to the higher-order harmonics of the distribution function.

Thus, in Eq. (33) we allow only for the first term on the right-hand side, the term that describes diffusion of particles in the $\mathbf{r}$-space. If we allow only for this term, the solution of Eq. (31) can be written as

$$G_1(\mathbf{k},t-t_0)=\exp\left\{-i\mathbf{k}\mathbf{v}(t-t_0)-\frac{\omega_1^2}{12}(t-t_0)^4[(\mathbf{v}-\mathbf{u})\mathbf{k}]^2\right\}G_0(\mathbf{k},t\rightarrow t_0), \tag{34}$$

where

$$G_0(\mathbf{k},t-t_0)=\theta(t-t_0)\delta(\mathbf{p}-\mathbf{p}_0)\exp\{-i\mathbf{k}\mathbf{v}(t-t_0)\}.$$

When particle velocities are high ($v\sim c$), the above reasoning concerning the relative value of the terms in Eq. (33) remains valid and Eq. (33) assumes the form

$$\frac{\partial g_1}{\partial t}=-\frac{\omega_1^2}{3}(t-t_0)^3\left(1-2\frac{\mathbf{uv}}{v^2}\right)[\mathbf{vk}]^2 g_1.$$

In this case the solution of Eq. (31) for the Green's function differs only slightly from (34) for $u\ll c$, and the transport path obtained in this approximation coincides with (42) (see Sec. 4.3 below).

4.3. The coefficient of diffusion in the phase space and the transport path

The linearized kinetic equation (25) can be written in differential form as follows:

$$\frac{\partial G}{\partial t}+\mathbf{v}\frac{\partial G}{\partial \mathbf{r}}-\mathbf{H}_0\mathbf{D}G=\mathrm{Coll}\ G, \tag{35}$$

where

$$\mathrm{Coll}\ G=D_\alpha\int d\mathbf{r}_1 d\mathbf{p}_1 dt_1 B_{\alpha\beta}(\mathbf{r},t;\mathbf{r}_1,t_1)\times G_1(\mathbf{r},\mathbf{p},t;\mathbf{r}_1,\mathbf{p}_1,t_1)D_\beta G(\mathbf{r},\mathbf{p},t;\mathbf{r}_0,\mathbf{p}_0,t_0)+\delta(x-x_0).$$

This linearized equation allows for processes of weak small-scale scattering and simple processes of small-scale scattering (26) in combinations that provide the largest contribution to the collision term in Eq. (24).

Let us examine the widely encountered case of an isotropic small-scale random magnetic field with the correlation tensor[1-5]

$$B_{\alpha\beta}(\mathbf{r},\mathbf{k})=\frac{A_\nu(\mathbf{r})k^2}{(k_0^2+k^2)^{2+\nu/2}}\left(\delta_{\alpha\beta}-\frac{k_\alpha k_\beta}{k^2}\right), \tag{36}$$

where

$$A_\nu(\mathbf{r})=\frac{\Gamma(2+\nu/2)k_0^{\nu-1}\langle\mathbf{H}_1^2(\mathbf{r})\rangle}{3\pi^{3/2}\Gamma((\nu-1)/2)},$$

and $\mathbf{r}=\frac{1}{2}(\mathbf{r}_1+\mathbf{r}_2)$. We write the collision term in (35) as[1-5]

$$\mathrm{Coll}\ G=D_\alpha T_{\alpha\beta}D_\beta G+\delta(x-x_0), \tag{37}$$

where

$$T_{\alpha\beta}=I_0\delta_{\alpha\beta}+I_1\frac{(\mathbf{v}-\mathbf{u})_\alpha(\mathbf{u}-\mathbf{v})_\beta}{(\mathbf{v}-\mathbf{u})^2}.$$

Allowing for the relation between $B_{\alpha\beta}(\mathbf{r},t;\mathbf{r}_1,t_1)$ and $B_{\alpha\beta}(\mathbf{r},\mathbf{k})$ (see Refs. 1-3) and substituting $B_{\alpha\beta}(\mathbf{r},\mathbf{k})$ and the function $G_1(\mathbf{k},t-t_1)$ (Eq. (34)) into the collision term Coll$G$ (Eq. (37)), we first calculate the sum $\delta_{\alpha\beta}T_{\alpha\beta}$, which can be transformed into

$$\delta_{\alpha\beta}T_{\alpha\beta}=2\pi^{3/2}A_\nu\int_0^\infty d\tau\int_0^\infty dx\left[\frac{x^{(\nu-3)/2}}{\Gamma(1+\nu/2)}-\frac{k_0^2x^{(\nu-1)/2}}{\Gamma(2+\nu/2)}\right]\exp\left\{-\frac{|\mathbf{v}-\mathbf{u}|^2\tau^2}{4x}-k_0^2x\right\}\times\left[1+\frac{\omega_1^2\tau^4|\mathbf{v}-\mathbf{u}|^2}{12x}\right]^{-1}. \tag{38}$$

In the last factor we replace x with the value

$$x_m=\frac{|\mathbf{v}-\mathbf{u}|\tau}{2k_0},$$

which we find from the condition that the expression in braces in the integrand is at its maximum, since at such values x_m the exponential factor provides the largest contribution to the integral (38). Then for the sake of simplicity we replace one τ in the last factor in (38) with $\tau=(vk_0)^{-1}$, which has a small effect on the final result (cf. Ref. 35).

We then perform transformations in the sum $\delta_{\alpha\beta}T_{\alpha\beta}$ that are the inverse of the previous transformations. As a result we get

$$\delta_{\alpha\beta}T_{\alpha\beta}=\frac{2\pi^2A_\nu}{|\mathbf{v}-\mathbf{u}|k_0^\nu(1+\nu/2)}\left[\left(1+\frac{2}{\nu}\right)\int_0^\infty\frac{dz\exp(-z)}{(1+\beta^2z^2)^{\nu/2}}-\int_0^\infty\frac{dz\exp(-z)}{(1+\beta^2z^2)^{1+\nu/2}}\right], \tag{39}$$

where $\beta=\omega_1/\sqrt{6}k_0|\mathbf{v}-\mathbf{u}|$. Note that the method used for calculating (39) is a refinement of the stationary phase method.

Similar calculations can be done for the sum

$$\frac{(\mathbf{v}-\mathbf{u})_\alpha(\mathbf{v}-\mathbf{u})_\beta T_{\alpha\beta}}{(\mathbf{v}-\mathbf{u})^2},$$

with the final expression substituted into the formula for I_0.

As a result of transforming the collision term we arrive at a kinetic equation for the distribution function with an averaged collision term:

$$\frac{\partial F}{\partial t}+\mathbf{v}\frac{\partial F}{\partial \mathbf{r}}-\mathbf{H}_0\mathbf{D}F=\mathrm{Coll}\ F, \tag{40}$$

where

$$\mathrm{Coll}\ F=\frac{c^2}{e^2}\left(\mathbf{D}\frac{p^2\chi(\omega_1,\mathbf{u})}{2\Lambda_0|\mathbf{v}-\mathbf{u}|}\mathbf{D}\right)F(\mathbf{r},\mathbf{p},t),$$

with the additional function $\chi(\omega_1,\mathbf{u})$ having the form

$$\chi(\omega_1,\mathbf{u})=\int_0^\infty\frac{dz\exp(-z)}{(1+\omega_1^2z^2/6k_0^2|\mathbf{v}-\mathbf{u}|^2)^{\nu/2}}+\frac{\nu\omega_1^2}{4k_0|\mathbf{v}-\mathbf{u}|^2}\int_0^\infty\frac{dz\exp(-z)z^2}{(1+\omega_1^2z^2/6k_0^2|\mathbf{v}-\mathbf{u}|^2)^{1+\nu/2}}, \tag{41}$$

and Λ_0 being the transport path with allowance for processes of moderate random scattering,[1-5]

$$\Lambda_0=3\Gamma\left(\frac{\nu-1}{2}\right)\frac{c^2p^2}{\sqrt{\pi}\Gamma(\nu/2)e^2L_c<\mathbf{H}_1^2>}.$$

The transport path with moderate random scattering can be found by passing to the diffusion approximation in the kinetic equation (40). Substituting the expansions of the distribution function

$$F(\mathbf{r},\mathbf{p},t)=\frac{1}{4\pi}N(\mathbf{r},p,t)+\frac{3}{4\pi v^2}\mathbf{J}(\mathbf{r},p,t)\mathbf{v}$$

and the additional function

$$\chi(\omega_1,\mathbf{u})=\chi(\omega_1)+\frac{\mathbf{uv}}{v^2}\chi_1(\omega_1)$$

into the kinetic equation (40) and allowing only for terms proportional to N, uN, u^2N, $\mathbf{J}$, and $u\mathbf{J}$, we arrive at a system of equations for the particle number density N and the current $\mathbf{J}$. This system coincides with the one for the same quantities but derived with allowance only for weak small-scale random scattering.[1-5] What changes is only the formula for the transport path Λ, which with allowance for the above approximations in the velocity $\mathbf{u}$ is $\mathbf{u}$-independent and has the form

$$\Lambda_1=\Lambda_0\chi^{-1}(\omega_1). \tag{42}$$

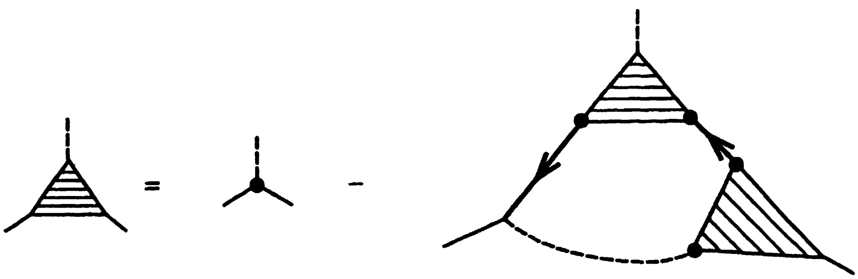
We see that at high particle energies ($p\to\infty$) the transport path Λ is proportional to p^2, while for low particle energies ($p\to 0$) it is proportional to p. Such a transition to a smoother Λ vs p dependence ($\Lambda \propto p$) occurs when the Larmor radius of particles in a random magnetic field, R_1, is of the order of the inhomogeneity size L_c.

5. STRONG PARTICLE SCATTERING IN A SMALL-SCALE RANDOM MAGNETIC FIELD

5.1. Solving the equation for the vertex operator

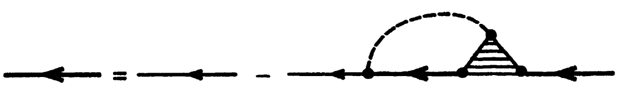
The description of strong random scattering of charged particles occurring at low energies and at average angles of scattering $\langle\alpha\rangle$ by a single magnetic inhomogeneity $\gtrsim 1$ requires using the vertex operator $\hat{\Gamma}_\gamma(\xi)$, which takes into account the higher-order approximations in the random magnetic field. To this end one must solve the exact equation for the vertex operator (22), allowing for the terms on the right-hand side proportional to the correlation tensor of the small-scale random magnetic field.

To simplify the solution process, we assume that the distribution of the small-scale random magnetic field is Gaussian. In solving Eq. (22) for the vertex operator we use the iteration method. The final result shows that such an approach yields a solution that is close to the exact solution. If we allow for these approximations, the last two terms on the right-hand side of Eq. (22) contribute nothing. Then diagrammatically Eq. (22) for the vertex operator $\hat{\Gamma}_\gamma(x,x_1;\mathbf{r}',t')$ without the last two terms on the right-hand side can be represented as



(43)

The vertex operator obtained as a result of solving this equation must be substituted into Eq. (17) for the Green's function. At $\xi=0$ Eq. (17) can be expressed diagrammatically as



(44)

This equation allows for complex processes of strong random scattering. Passing in (44) from the integral form to the differential, we obtain

$$\left(\frac{\partial}{\partial t}+\hat{L}_0\right)G(x,x_0)=\delta(x-x_0)-D_\alpha\int d\mathbf{r}'dt'\int dx_1 \times\int dx_2 B_{\alpha\beta}(\mathbf{r},t;\mathbf{r}',t') \times G(x,x_1)\hat{\Gamma}_\beta(x_1,x_2;\mathbf{r}',t')G(x_2,x_0). \tag{45}$$

We begin the iteration procedure of solving Eq. (43) for the vertex operator by substituting into the right-hand side of this equation the vertex operator in the zeroth approximation in the small-scale random magnetic field $\hat{\Gamma}^0_\gamma(x,x_1;\mathbf{r}',t')$ in the form (23). Instead of the two vertex operators $\hat{\Gamma}_\gamma$ and $\hat{\Gamma}_\beta$ we can substitute into the right-hand side of Eq. (43) the initial operators in the zeroth approximation, $\hat{\Gamma}^0_\gamma$. Continuation of this iteration procedure leads to an approximate iteration equation of the type (51).

It is more convenient, however, to substitute the initial operator $\hat{\Gamma}^0_\gamma$ into the integral convolution instead of substituting the operator $\hat{\Gamma}_\gamma(x_5,x_4;\mathbf{r}',t')$ into Eq. (43). The result is a linear equation for the vertex operator:

$$\hat{\Gamma}_\gamma(x,x_1;\mathbf{r}',t')=\hat{\Gamma}^0_\gamma(x,x_1;\mathbf{r}',t') - D_\alpha\int d\mathbf{r}_2 dt_2\int dx_3\int dx_4 \times\int dx_5 B_{\alpha\beta}(\mathbf{r},t;\mathbf{r}_2,t_2) \times G(x,x_5)\hat{\Gamma}^0_\gamma(x_5,x_4;\mathbf{r}',t') \times G(x_4,x_3)\hat{\Gamma}_\beta(x_3,x_1;\mathbf{r}_2,t_2). \tag{46}$$

At low particle energies a plurality of collisions with inhomogeneities of the small-scale random magnetic field occur with the particles being scattered at large angles. Thus, the fraction of the random magnetic field scattering the particles at small angles diminishes. The distance traveled by a particle between collisions with large-angle scattering decreases, too. This makes it possible to use in the integrand of Eq. (46) for the vertex operator $\hat{\Gamma}_\gamma$ the free-particle Green's function at $\mathbf{H}_0=0$:

$$G_0(x,x_1,\mathbf{H}_0=0)=\theta(t-t_1)\,\delta(\mathbf{p}-\mathbf{p}_1)\times\delta(\mathbf{r}-\mathbf{r}_1-\mathbf{v}(t-t_1)). \tag{47}$$

We also note the asymptotic nature of this behavior of the scattering-particle Green's function in the integrand, i.e., as $\mathbf{p}\rightarrow 0$,

$$G(x,x_1)\rightarrow G_0(x,x_1,\mathbf{H}_0=0).$$

This resembles the asymptotic freedom of quarks in QCD (see Refs. 38–40).

Substituting the simplest vertex operator $\hat{\Gamma}^0_\gamma$ (Eq. (23)) and the Green's function $G_0(\mathbf{H}_0)=0$ (Eq. (47)) into the right-hand side of Eq. (36) and integrating, we obtain

$$\hat{\Gamma}_\gamma(x,x_1;\mathbf{r}',t')=\hat{\Gamma}^0_\gamma(x,x_1;\mathbf{r}',t') + D_\alpha\int d\mathbf{r}_2 dt_2\int dt_3 B_{\alpha\beta}(\mathbf{r},t;\mathbf{r}_2,t_2)\,\theta(t - t')\,\theta(t'-t_3)\,\delta(\mathbf{r}-\mathbf{r}'-\mathbf{v}(t-t'))D_\gamma\hat{\Gamma}_\beta(\mathbf{r}' - \mathbf{v}(t'-t_3),\mathbf{p},t_3;x_1;\mathbf{r}',t'). \tag{48}$$

Next, substituting the zeroth-approximation vertex operator $\hat{\Gamma}^0_\beta$ for $\hat{\Gamma}_\beta$ on the right-hand side of this equation for the operator $\hat{\Gamma}_\gamma$, we arrive at an expression for the first-order vertex operator $\hat{\Gamma}^1_\gamma$. To refine this expression for $\hat{\Gamma}^1_\gamma$ we substitute it into the right-hand side of Eq. (45) for the Green's function. When integrating over internal variables and transforming the resulting collision term, we ignore in the integrand the action of the operators $\mathbf{D}$ on the velocities of the particles moving between two collisions in which a particle is scattered by a large angle. Here we assume that the path that a particle traverses between two collisions is fairly short, $v(t-t_1)\ll L_c$, so that as the particle energy decreases an ever increasing number of small-scale and low-strength magnetic inhomogeneities of the random magnetic field scatter the particles by a large angle.

We write the intermediate integrals of the correlation functions in the form

$$\int_0^\infty d\tau_1\int_{\tau_1}^\infty d\tau_2\int_0^\infty d\tau_3\,\psi(v\tau_2)\,\psi(v(\tau_1+\tau_3)) = C_\nu\int_0^\infty d\tau\psi(v\tau).$$

Then the function C_ν can be expressed as

$$C_\nu=\int_0^\infty d\tau_1\int_{\tau_1}^\infty d\tau_2\int_{\tau_1}^\infty d\tau_3\psi(v\tau_1)\,\psi(v\tau_3) \times\left[\int_0^\infty d\tau\psi(v\tau)\right]^{-1}. \tag{49}$$

Setting the magnetic field velocity $\mathbf{u}$ to zero in the above expression for the operator $\hat{\Gamma}^1_\gamma$ and using the commutation relations for the $\mathbf{d}$ operators,

$$[d_\alpha,d_\beta]=m^{-1}e_{\alpha\beta\gamma}d_\gamma \quad \text{at } \mathbf{u}=0,$$

and the duality relations

$$e_{\alpha\beta\gamma}d_\alpha d_\beta=m^{-1}d_\gamma \quad \text{at } \mathbf{u}=0,$$

where $e_{\alpha\beta\gamma}$ is the Levi-Civita symbol, we arrive at an expression for the vertex operator in the first iterative approximation:

$$\hat{\Gamma}^1_\gamma(x,x_1;\mathbf{r}',t';\mathbf{u}=0)=\hat{\Gamma}^0_\gamma(x,x_1;\mathbf{r}',t';\mathbf{u}=0) + C_\nu\frac{L_c^2}{3R_1^2}[m^2\mathbf{d}^2(\mathbf{u}=0)+1] \times\hat{\Gamma}^0_\gamma(x,x_1;\mathbf{r}',t';\mathbf{u}=0). \tag{50}$$

For the next iteration we substitute the vertex operator $\hat{\Gamma}^1_\gamma$ obtained in (50) into the right-hand side of Eq. (48) and carry out transformations similar to those just discussed. The result is an expression for the vertex operator in the second iterative approximation:

$$\hat{\Gamma}^2_\gamma(x,x_1;\mathbf{r}',t';\mathbf{u}=0)=\hat{\Gamma}^0_\gamma(x,x_1;\mathbf{r}',t';\mathbf{u}=0) + C_\nu\frac{L_c^2}{3R_1^2}[m^2\mathbf{d}^2(\mathbf{u}=0)+1]\left\{1 + C_\nu\frac{L_c^2}{3R_1^2}[m^2\mathbf{d}^2(\mathbf{u}=0)+1]\right\} \times\hat{\Gamma}^0_\gamma(x,x_1;\mathbf{r}',t';\mathbf{u}=0).$$

Allowing for (50), we can write the above expression as

$$\hat{\Gamma}_\gamma^2(x,x_1;\mathbf{r}',t';\mathbf{u}=0)=\hat{\Gamma}_\gamma^0(x,x_1;\mathbf{r}',t';\mathbf{u}=0)$$
$$+C_\nu\frac{L_c^2}{3R_1^2}[m^2\mathbf{d}^2(\mathbf{u}=0)+1]$$
$$\times\hat{\Gamma}_\gamma^1(x,x_1;\mathbf{r}',t';\mathbf{u}=0). \qquad (51)$$

Substituting this expression for $\hat{\Gamma}_\gamma^2$ into the right-hand side of Eq. (48) and performing the necessary transformations, we arrive at the operator $\hat{\Gamma}_\gamma^3$ expressed in terms of $\hat{\Gamma}_\gamma^2$. We can assume that by continuing this iteration procedure indefinitely we arrive at a situation in which the left- and right-hand sides of equations like (51) contain the same vertex operators $\hat{\Gamma}_\gamma^n$, where $n \gg 1$. Thus, for the vertex operator we have an equation similar to the Dyson equation.[34] The solution of this equation has the form

$$\hat{\Gamma}_\gamma(x,x_1;\mathbf{r}',t';\mathbf{u}=0)=\left[1+C_\nu\frac{L_c^2}{3R_1^2}(\mathbf{L}^2-1)\right]^{-1}$$
$$\times\hat{\Gamma}_\gamma^0(x,x_1;\mathbf{r}',t';\mathbf{u}=0), \qquad (52)$$

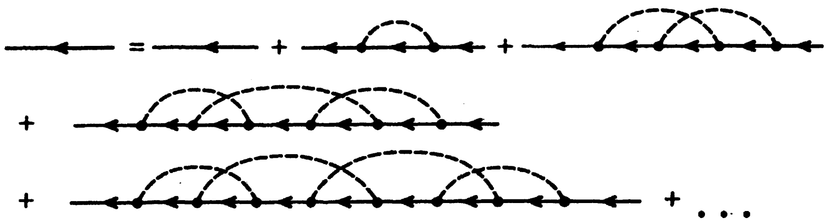
where $\mathbf{L}=-i[\mathbf{p}(\partial/\partial\mathbf{p})]$. When this operator equation used, it is convenient to employ the integral transformations and relationships of Ref. 37.

5.2. The diagrammatic technique and microprocesses of strong random scattering

Let us write the equations for the Green's function and the vertex operator in diagrammatic form. The iterative solution of Eq. (46) for the vertex operator in which we allow for the first four terms in the expansion in the random magnetic field can be represented diagrammatically as

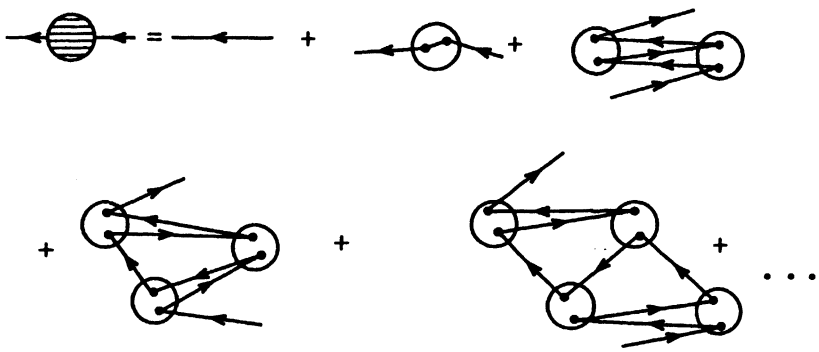
(53)

Substitution of this vertex operator into Eq. (44) transforms it into a closed equation for the Green's function only, and in diagrammatic form the equation becomes



(54)

Allowing for the correspondence that exists between the elements of the diagrammatic technique and the averaged elementary scattering microprocesses (12)–(14), we obtain the following types of processes taken into account in the expansion (53) of the vertex operator and in Eq. (54):



(55)

The first two processes in expansion (55) are allowed for by the kinetic equation in the small-angle approximation and ensure the transition of the small-scale collision term at high energies to the small-angle expression. The other processes are related to complicated strong small-scale random scattering.

As the diagrammatic representation (55) shows, the motion of particles in strong random scattering is of a "trapping" nature, since in the latter two processes a particle moves between inhomogeneities of the random magnetic field along almost closed paths; for the two-dimensional case this feature of random scattering was discussed in Ref. 41.

From (55) we also see that strong scattering processes corresponding to diagrams with an even number of correlation tensors (the third and fifth processes) result in particle scattering into the forward hemisphere, while processes with an odd number of correlation tensors (the fourth process) result in scattering into the backward hemisphere.

5.3. The transport path with strong scattering

Equation (45) for the Green's function requires substituting into its collision term the vertex operator for a nonzero magnetic field velocity, $\mathbf{u} \neq 0$. To find this operator we must solve Eq. (46) iteratively, using the vertex operator $\hat{\Gamma}^0_\gamma(x,x_1;\mathbf{r}',t')$, the correlation tensor $B_{\alpha\beta}(\mathbf{r},t;\mathbf{r}_2,t_2)$, and the operator $\mathbf{D}$ at $\mathbf{u}\neq 0$. But since the vertex operator obtained from (46) is substituted into the collision term in Eq. (45) in which conversion to the diffusion approximation has been achieved, in one of the vertex operators, precisely $\hat{\Gamma}^0_\gamma(x_5,x_4;\mathbf{r}',t')$, we can ignore terms proportional to $\mathbf{u}$ in the second term on the right-hand side of Eq. (46), as well as in the operator $\mathbf{D}$ and in the correlation tensor $B_{\alpha\beta}(\mathbf{r},t;\mathbf{r}_2,t_2)$. This is possible because with the adopted system of approximations in the velocity $\mathbf{u}$, the particle number density N, and the current $\mathbf{J}$, the magnetic field velocity $\mathbf{u}$ in the above-mentioned operators $\hat{\Gamma}_\gamma$ and $\mathbf{D}$ and the tensor $B_{\alpha\beta}$ contributes nothing to the final collision term in the diffusion approximation.

Assuming that all the above approximations are valid, we can write the following expression for the vertex operator substituted into (45):

$$\hat{\Gamma}_\gamma(x,x_1;\mathbf{r}',t')=\left[1+C_\nu\frac{L_c^2}{3R_1^2}(\mathbf{L}^2-1)\right]^{-1}\times\hat{\Gamma}^0_\gamma(x,x_1;\mathbf{r}',t'). \tag{56}$$

Substituting this vertex operator into the collision term in Eq. (45), replacing the Green's function G with $G_0(x,x_1,\mathbf{H}_0=0)$, and integrating, we arrive at a kinetic equation for the distribution function F of type (40) with the collision term

$$\mathrm{Coll}F=D_\alpha\int_0^{t-t_0}d\tau B_{\alpha\beta}((\mathbf{v}-\mathbf{u})\tau)\times\left[1+C_\nu\frac{L_c^2}{3R_1^2}(\mathbf{L}^2-1)\right]^{-1}D_\beta F(\mathbf{r}-\mathbf{v}\tau,\mathbf{p},t-t_0-\tau). \tag{57}$$

Thus, in momentum space the resulting collision term does not usually coincide with the diffusion term, especially at small angles of deviation of a particle from the initial direction, in which case large eigenvalues of the operator $\mathbf{L}^2$ play an essential role.

For large time intervals $t-t_0\gg L_c/v$ and small gradients of $\mathbf{H}_0$, $\mathbf{u}$, and Λ, we can pass to the diffusion approximation in the kinetic equation for the distribution function $F(\mathbf{r},\mathbf{p},t)$ of type (40) with a collision term (57). As a result we arrive at the ordinary diffusion equation in the small-angle approximation.[1-5] Only the formula for the transport path will change: with allowance for strong random scattering we have

$$\Lambda_2=\Lambda_0\varphi^{-1}(p), \tag{58}$$

where

$$\varphi(p)=\left[1+C_\nu\frac{L_c^2}{3R_1^2}\right]^{-1}, \tag{59}$$

while without strong random scattering the transport path is

$$\Lambda_0=3c^2p^2/2e^2\langle\mathbf{H}_1^2\rangle L_c\int_0^\infty dy\,\psi(y).$$

The formulas for Λ_2 show that at high particle energies ($p\to\infty$) we have $\Lambda_2\to\Lambda_0$, and $\Lambda_0\propto p^2$. When the particle energy is low ($p\to 0$), the transport path Λ_2 tends to

$$C_\nu L_c\Big/2\int_0^\infty dy\,\psi(y),$$

which is a quantity of the order of the size L_c of an inhomogeneity in the small-scale random magnetic field. Such behavior of the transport path Λ_2 for low particle energies agrees with the phenomenological results obtained by Dolginov and Toptygin,[1-3] the main model results,[4] and the experimental data[9] for nonresonant scattering of charged particles in a random magnetic field.

6. THE TRANSPORT PATH WITH MODERATE AND STRONG RANDOM SCATTERING

The foregoing discussion implies that when moderate and strong small-scale random scattering are taken into account in the diffusion approximation, the equation for the particle number density retains it form,

$$\frac{\partial N}{\partial t}=\frac{\partial}{\partial r_\alpha}\kappa_{\alpha\beta}\frac{\partial N}{\partial r_\beta}-\mathbf{u}\frac{\partial N}{\partial \mathbf{r}}+\frac{p}{3}\operatorname{div}\mathbf{u}\frac{\partial N}{\partial p}, \tag{60}$$

and the spatial diffusion coefficient does not change form either:

$$\kappa_{\alpha\beta}=\frac{\Lambda v}{3}\left(1+\frac{\Lambda^2}{R_0^2}\right)^{-1}\left(\delta_{\alpha\beta}+\frac{\Lambda^2}{R_0^2}h_\alpha h_\beta-\frac{\Lambda}{R_0}e_{\alpha\gamma\beta}h_\gamma\right),\quad R_0=\frac{cp}{cH_0}. \tag{61}$$

The one thing that does change is the expression for the transport path Λ: at low particle energies $\Lambda\propto p$ for moderate random scattering and $\Lambda\sim L_c$ for strong small-scale random scattering.

As the particle energy decreases, moderate random scattering is transformed into strong scattering, with the result that when the particle energy varies over a wide range we need to allow for all types of small-scale scattering processes. This can be done approximately if one takes into account the fact that in the phenomenological theory with magnetic inhomogeneities of different scales, the transport scattering cross sections on the inhomogeneities of each scale can be added,[1-3] with each transport cross section being inversely proportional to the respective transport path. Here, taking into account (42) and (58), we add the transport paths.

Thus, an interpolation formula for the transport path Λ that allows for moderate and strong random scattering can be approximately written as

$$\Lambda=\Lambda_0\left[\chi^{-1}(\omega_1)+C_\nu\frac{L_c^2}{3R_1^2}\right]. \tag{62}$$

The concentrations of inhomogeneities in the random magnetic field, which give rise to moderate and strong random scattering, are assumed equal.

The function $\chi(\omega_1)$ (Eq. (41)) contains integrals that can be expressed in terms of the Struve function $H_{-\mu}(y)$ and the Neumann function $Y_{-\mu}(y)$ (see Ref. 42), which are cumbersome for analysis. Hence we express $\chi(\omega_1)$ using Laguerre's interpolation quadrature formula,[43] which is transformed into an exact expression at the extreme values of parameter q:

$$\int_0^\infty \frac{dz\exp(-z)}{(1+q^2z^2)^\mu}=\left[1+\frac{2\Gamma(\mu)q}{\sqrt{\pi}\Gamma(\mu-1/2)}\right]^{-1}.$$

The maximum error introduced by the formula amounts to several percent.

After applying this formula, we arrive at a simplified expression for the transport path with moderate and strong small-scale scattering:

$$\Lambda=\Lambda_0\left[1+\frac{4\Gamma(\nu/2)}{5\sqrt{6\pi}\Gamma((\nu-1)/2)}\frac{L_c}{R_1}+C_\nu\frac{L_c^2}{3R_1^2}\right]. \tag{63}$$

Using this formula for Λ, we examine the anisotropy of the spatial diffusion tensor $\kappa_{\alpha\beta}$, which is important in explaining the high degree of isotropy in the distribution of cosmic rays in quiet periods in interplanetary and interstellar space.[3,7-9]

We start with the case of a strong regular magnetic field ($R_0\ll\Lambda$). To this end we use Eq. (61) for the spatial diffusion tensor with a transport path (63). In deriving Eq. (3) we assumed the effect of the regular magnetic field on the motion of particles between collisions with strong scattering to be weak (see Sec. 5.1). However, the formula for the transport path at low particle momenta ($p\to 0$) should not change too drastically in the case of a strong regular magnetic field, since in the presence of such a field basically only the system of random magnetic "traps" yielding strong scattering changes.

Thus, using (61), we can find the ratio between the part of the diffusion tensor transverse to the regular magnetic field $\kappa_\perp$ and the longitudinal part $\kappa_\parallel$:

$$\frac{k_\perp}{k_\parallel}\approx\frac{R_0}{\Lambda},\quad \Lambda\gg R_0.$$

Substituting the path Λ from (63), we find that $\kappa_\perp/\kappa_\parallel$ attains its maximum at

$$\omega_1=\nu_c\sqrt{3/C_\nu},$$

where $\nu_c=v/L_c$ is the random collision rate. This means that in the event of strong small-scale random scattering with the Larmor frequency in a regular magnetic field being close to the random scattering rate there is quasiresonant scattering, which leads to an increase in the transverse diffusion of particles and additional isotropy of scattering.

In the presence of a weak regular magnetic field ($R_0\gg\Lambda$), we define the ratio of the transverse part of the diffusion tensor to the total diffusion tensor as

$$\frac{\kappa_\perp}{\kappa_\parallel+\kappa_\parallel}\approx\frac{\Lambda}{R_0}.$$

In this case, at $\omega_1=\mathrm{v}_c\sqrt{3/C_\nu}$ the ratio $\kappa_\perp/(\kappa_\parallel+\kappa_\parallel)$ is at its minimum, which explains the increase in the magnetization of particles in a regular magnetic field as the particle energy decreases to a point where the frequencies ω_1 and $\nu_c\sqrt{3/C_\nu}$ coincide.

7. DISCUSSION AND CONCLUSION

The method of functionals and the diagrammatic technique show that as the energy of particles scattered in a small-scale random magnetic field decreases in the presence of a regular magnetic field, there is a transition from weak scattering to moderate small-scale random scattering, in which a particle scatters from a single magnetic inhomogeneity at an average angle $\lesssim 1$. A further decrease in the particle energy radically changes the scattering: it is transformed into strong small-scale scattering, with scattering angle $\gtrsim 1$. However, the diffusion equations for the particle number density N and the current $\mathbf{J}$ and the expression for the spatial diffusion tensor $\kappa_{\alpha\beta}$, which now allow for moderate and strong scattering, do not change. Only the expression for the transport path Λ changes.

When moderate small-scale scattering is present, there is random "broadening" of the integrand Green's function in the diffusion coefficient in momentum space, with the transport path becoming proportional to p at low particle momenta.

When strong small-scale scattering is present, the interaction of a particle with the random magnetic field acquires a "trapping" nature, with the particles moving between the inhomogeneities, which form the traps, along almost closed paths. In this case as the particle energy decreases, the transport path asymptotically tends to a quantity of order L_c, and the expansion of the iteration solution for the vertex operator in the random magnetic field is similar, as the diagrammatic technique implies, to the expansion of the total interaction in the interactions with an increasing number of magnetic traps.

The solution of the equation for the vertex operator also implies that at low energies the strong small-scale scattering is "parton-like" i.e., a particle moves freely between collisions with strong scattering. This is probably a feature of the interaction of a particle and a random magnetic field that makes it possible to use the iteration procedure in solving the equation for the vertex operator. The case at hand bears a strong resemblance to the asymptotic freedom of quarks in QCD, where when the momentum transferred to a quark tends to infinity, the interaction with one-gluon exchange becomes infinitesimal and the iteration procedure can be employed to obtain the renormalization factor.[38–40]

Also, for strong small-scale scattering the collision term (57) is not of the diffusion type in the momentum space because of the presence of inverse momentum operators, with the result that the kinetic equation in the phase space cannot be reduced to the Fokker–Planck equation. When a strong regular magnetic field is present and the Larmor frequency in a regular magnetic field coincides with the strong random scattering rate, transverse diffusion of the particles increases and additional isotropy of scattering emerges.

In view of the large number of processes leading to the stochastization of particle scattering, the small-scale random magnetic field can be defined as an effective random magnetic field with characteristics determined by comparing the averaged scattering characteristics derived analytically with the experimental data.

Since in obtaining the iteration solution for the vertex operator we ignored the magnetic field velocity **u** in the integrand of (46), we therefore ignored the accelerating processes on distance of order of the inhomogeneity size L_c, processes related to strong small-scale random scattering.

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