Spontaneous onset of poloidal rotation of tokamak plasma in banana regime

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The equation describing spontaneous onset of poloidal rotation of tokamak plasma due to intrinsic non-ambipolarity of neoclassical diffusion in banana regime is derived for arbitrary initial conditions. The spin-up rate of poloidal rotation $\nu_p \simeq 0.24\nu/\epsilon S$, the effective inertial mass, and the steady-state rotation rate are found from this equation. Here ν is the ion–ion collision frequency, ϵ the inverse aspect ratio of the tokamak, and S is the orbit squeezing factor.
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1. INTRODUCTION

Tokamak experiments have observed spontaneous onset of poloidal rotation in the edge plasma with concurrent suppression of edge turbulence and improved confinement (*H* mode). In large tokamaks such as Doublet III and Asdex this happens when the edge temperature is sufficiently high to enter the banana regime where ions become trapped.[1,2] In this regime the particles of each species could be divided into two groups with topologically different orbits. In a large aspect ratio tokamak with a strong magnetic field, most of the particles of given species are freely moving along the magnetic field lines thus closely following the magnetic surface (untrapped particles). However, a small group of particles moving along the magnetic field lines with low velocities are trapped in the outer part of the plasma torus since they cannot penetrate the inner part, being mirrored from stronger magnetic field there. These trapped particles deviate from the magnetic surface to a distance much larger than that of the untrapped particles due to their magnetic field curvature drift, forming the banana-like orbit. Figure 1 illustrates how the collision of like particles with different topologies of orbits results in the finite displacement of particles across the magnetic surface and thus in the diffusion of plasma. Here the untrapped particle 2 continues to move after the collision with the trapped particle *1* in the same direction and the trapped particle is turned in the opposite direction.

Under the action of the magnetic field curvature drift (downward in Fig. 1) the trapped particle following the banana-like orbit will be displaced outward from its initial averaged position by the distance much larger than the averaged displacement of untrapped particle across the magnetic surface. Let us note here that the displacement of the like particles just after their collision is zero and it builds up to a finite averaged value only after these particles complete at least one orbital period. Therefore the orbit averaging is necessary to describe properly the plasma diffusion in the rare collision limit (see Eq. (18)).

The nonambipolar diffusion described above generates a radial electric current, leading to the building up of a radial electric field. The radial electric field in turn generates poloidal rotation of plasma. Recently several authors calculated the spin-up rates of poloidal rotation and came out with very different results: $\nu_{\text{pol}} = 1.44\nu$, (Ref. 3), $\nu_{\text{pol}} = \nu/\sqrt{\epsilon}$, (Ref. 4), $\nu_{\text{pol}} \simeq 0.7\nu/\epsilon$ (Ref. 5). In this paper we derive analytically the equation for the time evolution of the poloidal rotation in the asymptotic limit of very large aspect ratio of tokamak, with allowance for the orbit squeezing effect, and obtain values for the spin-up rate of poloidal rotation, effective inertial mass, and steady-state rotation rate.

2. PARTICLE ORBITS AND DRIFT KINETIC EQUATION

For the sake of simplicity we limit ourselves here to an axially symmetric magnetic field with circular magnetic surfaces

$$\mathbf{B}=B_0(\mathbf{e}_\zeta+\Theta\mathbf{e}_\theta)/(1+\epsilon\cos\theta), \tag{1}$$

where $\epsilon=r/R \ll 1$ is the inverse aspect ratio, $\Theta(r) = B_\theta/B_\zeta$ the ratio of the poloidal and toroidal magnetic fields, $\mathbf{e}_r, \mathbf{e}_\theta$ and $\mathbf{e}_\zeta$ are local unit vectors in the radial, poloidal and toroidal directions. We assume here the following ordering of plasma spatial scales:

$$\rho/\Theta L \ll 1, \tag{2}$$

where $\rho = v_T/\omega_B$ is the Larmor radius of ions with the thermal velocity $v_T = \sqrt{2T/m}$ and cyclotron frequency $\omega_B = eB_0/mc$, L the spatial scale of plasma density and temperature profiles.

The relaxation time τ_R for the trapped ion distribution due to collisions is much greater than the period τ_B associated with their motion

$$\tau_R \simeq \epsilon/\nu \gg \tau_B \simeq r/\Theta v_T\sqrt{\epsilon}, \tag{3}$$

where ν is the ion–ion collision frequency. Neglecting collisions in a first approximation we obtain from the equations

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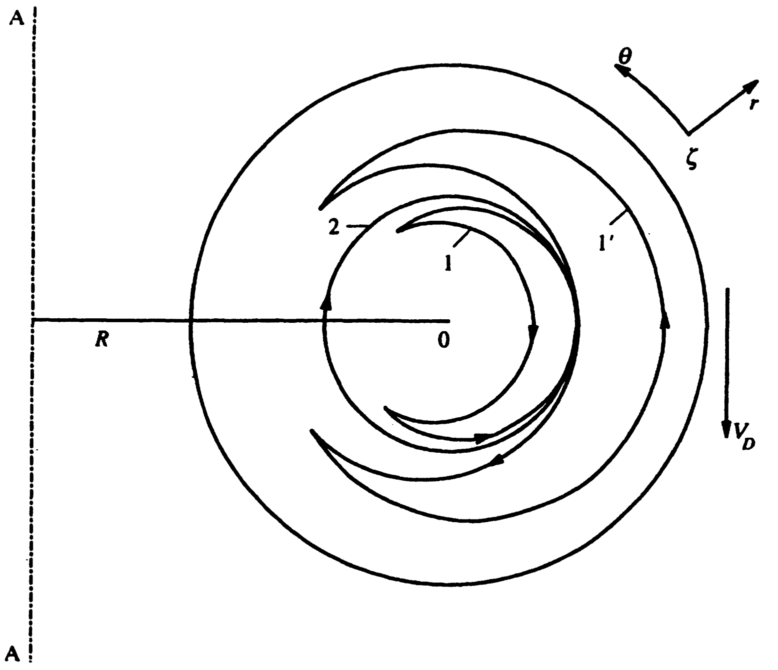

FIG. 1. Collision of counterstreaming trapped (*1*) and untrapped (*2*) ions resulting in displacement of new orbit (*1′*) of trapped ion much larger than that for electrons in similar case.

of motion of guiding centers an additional constant of the motion besides the energy $\mathscr{E}$ and the adiabatic invariant μ—the generalized toroidal momentum of a particle[6]

$$J=m\omega_B\int_0^r\Theta(r)dr-mv_\parallel(1+\epsilon\cos\theta). \tag{4}$$

Here the longitudinal velocity of a particle is expressed in terms of $\mathscr{E}$, μ, and the electrostatic potential $\Phi(r,t)$ of plasma:

$$v_\parallel=\sigma[2(\mathscr{E}-e\Phi(r,t)-\mu B(r,\theta))/m]^{1/2},\quad \sigma=\pm1. \tag{5}$$

We assume here that the radial electric current due to nonambipolar neoclassical diffusion results in a building up of the radial electric field, thus driving the poloidal rotation of plasma. Therefore we allowed for temporal evolution of this radial electric field on a time scale that, as we show later, is much smaller than that of the neoclassical diffusion $\tau_D = 1/\nu\sqrt{\epsilon}(\rho/\Theta L)^1$, but larger than the trapped ion distribution relaxation time τ_R. Under this condition we consider below the plasma density and temperature profiles to be stationary. Expanding the expression for J to second order in the radial displacement $(r-r_0)$ of the particle from the starting point $(r_0,0)$, we obtain the particle orbit in the form[6]

$$r-r_0=2\sqrt{\frac{\mu B_0}{m}\epsilon S}\,\frac{\kappa+\sigma(\kappa^2-\sin^2\theta/2)^{1/2}}{\Theta\omega_B S}, \tag{6}$$

where

$$\kappa^2=m(\Delta v_\parallel)^2/4\mu B_0\epsilon S,\quad \Delta v_\parallel=v_\parallel(r_0,0)+V_E(r_0,t)/\Theta,$$

$$V_E(r,t)=(c/B_0)\partial\Phi(r,t)/\partial r\,;$$

and $S=1+(\partial V_E/\partial r)/\omega_B\Theta^2$ is the orbit squeezing factor.[7,8] According to the condition (3) the zero-order distribution functions of trapped and untrapped ions can be written as functions of the integrals of the motion[1]

$$f_t=f_{0t}(\mu,\mathscr{E},J),\quad f_u=f_{0u}(\mu,\mathscr{E},J;\sigma). \tag{7}$$

The shape of these distribution functions is governed by infrequent particle collisions and can be found from the equation for the first-order distribution function f_1:

$$(\Theta v_\parallel+V_E)\frac{\partial f_1}{r\partial\theta}-\frac{\mu}{m}(\mathbf{b}\nabla)B\frac{\partial f_1}{\partial v_\parallel}=\mathrm{St}(f_0), \tag{8}$$

where $\mathbf{b}=\mathbf{B}/B$. Using the expression (6) for the particle orbit, we substitute

$$v_\parallel=-\frac{V_E}{\Theta}+2\sigma\sqrt{\frac{\mu B_0}{m}\epsilon S}(\kappa^2-\sin^2\theta/2)^{1/2} \tag{9}$$

into Eq. (8), with the result[1]

$$2\sigma\Theta\sqrt{\frac{\mu B_0}{m}\epsilon S}(\kappa^2-\sin^2\theta/2)^{1/2}\frac{\partial f_1}{r\partial\theta}=\mathrm{St}(f_0), \tag{10}$$

where the collision term (with allowance for orbit squeezing) takes the form

$$\mathrm{St}(f_0)=\frac{\nu}{S\epsilon}A(x)\sigma\left(\kappa^2-\sin^2\frac{\theta}{2}\right)^{1/2}\frac{\partial}{\partial\kappa^2}\times\left\{\sigma\left(\kappa^2-\sin^2\frac{\theta}{2}\right)^{1/2}\left(\frac{\partial f_0}{\partial\kappa^2}+2x\epsilon Sf_0\right)-\sqrt{2x\epsilon S}\left(\frac{V_E}{\Theta v_T}+\frac{U_\parallel}{v_T}\right)f_0\right\}, \tag{11}$$

where

$$A(x)=(3\sqrt{\pi}/4)(\eta-\eta'-\eta/2x)x^{-3/2},\quad \eta'=d\eta/dx,$$

$$\eta(x)=(2/\sqrt{\pi})\int_0^x\exp(-x)\sqrt{x}\,dx,\quad x=2\mu B_0/v_T^2,$$

$U_{\|}$ is the field-aligned velocity of plasma flow. From the necessary condition to solve the equation (10)[1]

$$\int_0^{2\pi}\left(\kappa^2-\sin^2\frac{\theta}{2}\right)^{-1/2}\mathrm{St}(f_0)d\theta=0, \tag{12}$$

we obtain the zero-order distribution function:[1]

$$\begin{aligned}f_0=&\frac{n(r)}{\pi^{3/2}v_T^3}\exp\left[-x(1+2\kappa^2\epsilon S)+\frac{\pi\sigma}{2v_T}\sqrt{2x\epsilon S}\left(\frac{V_E}{\Theta}\right.\right.\\&\left.\left.+U_{\|}\right)\eta(\kappa^2-1)\int_1^{\kappa^2}\frac{d\xi}{\sqrt{\xi}E(1/\xi)}\right]\left\{1-\frac{\sigma\sqrt{2x\epsilon}}{\Theta\omega_B\sqrt{S}}\right.\\&\times v_T\left[\frac{d\ln n}{dr}+\left(x(1+2\kappa^2\epsilon S)-\frac{3}{2}\right)\frac{d\ln T}{dr}\right]\\&\left.\times\left[\left(\kappa^2-\sin^2\frac{\theta}{2}\right)^{1/2}-\frac{\pi\eta}{4}\int_1^{\kappa^2}\frac{d\xi}{\sqrt{\xi}E(1/\xi)}\right]\right\},\end{aligned} \tag{13}$$

where $\eta(x)=1$ for $x>0$ and $\eta(x)=0$ for $x<0, n(r)$ and $E(1/\kappa^2)$ is the complete elliptic integral of the second kind. Note that due to the temporal evolution of the electric field, this distribution depends on time via parameters κ and $V_E(r,t)$ entering the expression (13). The time dependence of the field-aligned velocity of plasma flow is neglected here due to conservation of the toroidal angular momentum of the plasma.

3. NEOCLASSICAL INERTIA OF POLOIDAL ROTATION IN THE BANANA REGIME

To describe the temporal evolution of the radial electric field, which produces poloidal rotation of the plasma, we write the equation for the second-order distribution function

$$\sigma\Theta\sqrt{2x\epsilon S}v_T\left(\kappa^2-\sin^2\frac{\theta}{2}\right)^{1/2}\frac{\partial f_2}{r\partial\theta}=\mathrm{St}(\delta f)-\frac{\partial f_0}{\partial t}, \tag{14}$$

where the time derivative of f_0 is calculated with the help of expressions (13) for f_0 and (6) for κ^2 dependence on $V_E(r,t)$:

$$\begin{aligned}\frac{\partial f_0}{\partial t}=&-\frac{2\sigma\dot{V}_E}{\Theta v_T}\sqrt{2x\epsilon S}\left[\left(\kappa^2-\sin^2\frac{\theta}{2}\right)^{1/2}\right.\\&\left.-\frac{\pi\eta}{4}\int_1^{\kappa^2}\frac{d\xi}{\sqrt{\xi}E(1/\xi)}\right]f_0.\end{aligned} \tag{15}$$

Here $\dot{V}_E=\partial V_E/\partial t$. Moreover, while writing the expression for the collisional term $\mathrm{St}(f_1)$ on the right-hand side of Eq. (12), we have taken into account that the dominant contribution to the first-order distribution function f_1 comes from the general solution of Eq. (10) $\delta f_u=\delta f_u(x,\kappa^2,\sigma,t)$ and not from its particular solution. The explicit form for the distribution function δf is found from the necessary condition to solve the equation (14)

$$\int_0^{2\pi}\left(\kappa^2-\sin^2\frac{\theta}{2}\right)^{-1/2}\left[\mathrm{St}(\delta f_u)-\frac{\partial f_{0u}}{\partial t}\right]d\theta=0. \tag{16}$$

Integrating here over θ and neglecting terms quadratic in the small parameters $(V_E/\Theta v_T)$ and $1/\tau_R(\partial\ln V_E/\partial t)$, we have

$$\begin{aligned}&\frac{\nu}{\epsilon S}A(x)\frac{\partial}{\partial\kappa^2}\left[\kappa E\left(\frac{1}{\kappa^2}\right)\frac{\partial}{\partial\kappa^2}\delta f_u\right]\\&=-\frac{4\pi\sigma\dot{V}_E}{\Theta v_T}\sqrt{2x\epsilon S}\left[1-\frac{K(1/\kappa^2)}{2\kappa}\int_1^{\kappa^2}\frac{d\xi}{\sqrt{\xi}E(1/\xi)}\right]f_0.\end{aligned} \tag{17}$$

To obtain the equation for poloidal rotation, we use the condition that the surface-averaged radial electric current must vanish to maintain plasma quasineutrality.[9] The main contribution to the radial electric current comes from the neoclassical diffusion of ions, which is significantly larger than that for electrons during the process of generation of electric field and poloidal rotation. Neglecting the small electron contribution, we require that the radial electric current of ions due to their neoclassical diffusion[1] vanish:

$$\begin{aligned}\langle nV_r\rangle=&-\sum_\sigma\int_0^{2\pi}\int_0^\infty\int_{\sin^2\frac{\theta}{2}}^\infty\frac{\mu B_0\epsilon}{m\omega_B r}\sin\theta(f_1\\&+f_2)\sqrt{\frac{\mu B_0}{m}\epsilon S}\frac{d\kappa^2}{\left(\kappa^2-\sin^2\frac{\theta}{2}\right)^{1/2}}d\frac{\mu B_0}{m}d\theta=0.\end{aligned} \tag{18}$$

Transforming to variables x and κ^2 and integrating this equation over θ by parts, we rewrite it with the help of Eqs. (10) and (14) as

$$\begin{aligned}\langle nV_r\rangle_D=&-\frac{\epsilon v_T^4}{2\omega_B\Theta}\sum_\sigma\int_0^{2\pi}\int_0^\infty\int_{\sin^2\frac{\theta}{2}}^\infty x\left[\mathrm{St}(f_0)-\frac{\partial f_0}{\partial t}\right.\\&\left.+\mathrm{St}(\delta f_u)\right]d\kappa^2dx\,d\theta.\end{aligned} \tag{19}$$

The first term in square brackets describes the neoclassical diffusion of ions:

$$\langle nV_r\rangle_D=-\frac{F_{\|}}{m\omega_B\Theta}\equiv-\int mv_{\|}\mathrm{St}(f_0)d\mathbf{v}\frac{d\theta}{2\pi}\Big/m\omega_B\Theta S, \tag{20}$$

where

$$F_{\|}\equiv(\nabla\pi)_{\|}=0.37mn\nu\sqrt{\frac{\epsilon}{S}}\left[\frac{V_E}{\Theta}+U_{\|}+\frac{1}{\Theta S}(U_p-1.17U_T)\right] \tag{21}$$

is the component of the viscous force parallel to the magnetic field.[2] Here

$U_p=(cT/eB_0)d\ln nT/dr$ and $U_T=(cT/eB_0)d\ln T/dr$

are the pressure and temperature gradient drifts, respectively. The main contribution to the longitudinal viscosity force comes from the boundary layer between trapped and untrapped particles in velocity space, described by the delta function in the expression for the collisional term $\mathrm{St}(f_0)\sim\delta(\kappa^2-1)$ (see details in Ref. 1). Note that due to the substitution of variables (9), both the longitudinal velocity and the collisional term in Eq. (18) depend explicitly on the poloidal angle. This permitted us to explicitly perform orbital averaging of the integrand in (18), integrating over perpendicular and longitudinal adiabatic invariants.

The contribution of the poloidal rotation inertail force to plasma diffusion is described by the last two terms in square brackets in (19). With the help of (15) for $\partial f_0/\partial t$ and (17) for δf, we write it in the form of integrals over the complete elliptic integrals $K(1/\kappa^2)$ and $E(1/\kappa^2)$ of the first and second kind, respectively:

$$\langle nV_r\rangle_I = -\frac{8\sqrt{2}}{\pi^{3/2}}\Gamma\left(\frac{5}{2}\right)\frac{(\epsilon S)^{3/2}}{\omega_B\Theta^2 S}V_E\left[\frac{4}{9}+\int_1^\infty\left\{\kappa E - \frac{\pi^2}{8}\int_1^{\kappa^2}\frac{d\xi}{\sqrt{\xi}E}-\frac{\pi^2}{4}\left(\kappa^2-\frac{1}{2}\right)^{1/2}\frac{\partial}{\partial\kappa^2}\times\left[\left(\kappa^2-\frac{1}{2}\right)^{1/2}\frac{1}{\kappa E}\int_1^{\kappa^2}\left(1-\frac{K}{2\kappa'}\int_1^{\kappa'^2}\frac{d\xi}{\sqrt{\xi}E}\right)d\kappa'^2\right]\right\}d\kappa^2\right], \tag{22}$$

where $\Gamma(5/2)=3\pi^{1/2}/4$ is the gamma function. Here the first and second terms in the outer square brackets represent the contributions to the inertia of poloidal rotation from trapped and untrapped ions. The second term after differentiation with respect to κ^2 takes the form

$$I \equiv \int_1^\infty\left\{\kappa E - \frac{\pi^2}{8}\int_1^{\kappa^2}\frac{d\xi}{\sqrt{\xi}E}-\frac{\pi^2}{4\kappa E}\left(\kappa^2-\frac{1}{2}\right)\left(1-\frac{K}{2\kappa}\int_1^{\kappa^2}\frac{d\xi}{\sqrt{\xi}E}\right)-\frac{\pi^2}{8\kappa E}\int_1^{\kappa^2}\left(1-\frac{K}{2\kappa'}\int_1^{\kappa'^2}\frac{d\xi}{\sqrt{\xi}E}\right)d\kappa'^2\left[1-\left(1-\frac{1}{2\kappa^2}\right)\frac{K}{E}\right]\right\}d\kappa^2. \tag{23}$$

Using the series expansion of the elliptic integrals K and E,[10] we obtain the numerical value $I = 0.12$. Summing the results of calculations for $\langle nV_r\rangle_D$ and $\langle nV_r\rangle_I$, we obtain the equation for the evolution of poloidal rotation in explicit form:

$$1.52nmq^2\sqrt{\frac{S}{\epsilon}}\frac{\partial V_E}{\partial t} = -\frac{F_\parallel}{\Theta}, \tag{24}$$

where $q = \epsilon/\Theta$ is the tokamak safety factor. With the help of the expression (21) for $F_\parallel$, we find the effective inertial mass m_{eff} and the spin-up rate of poloidal rotation ν_{pol}:

$$m_{\text{eff}} = 1.52q^2 m\sqrt{S/\epsilon}, \quad \nu_p = 0.24\nu/\epsilon S. \tag{25}$$

The steady-state velocity of poloidal rotation,[11]

$$V_p \equiv V_E + \Theta U_\parallel + U_p = (1-S^{-1})U_p + 1.17S^{-1}U_T \tag{26}$$

is reached when $F_\parallel = 0$.

4. CONCLUSION

We have shown that intrinsically non-ambipolar neoclassical diffusion of plasma in the banana regime drives poloidal rotation at a spin-up rate (25) three times lower than that calculated in Ref. 5. As a result, the assumption concerning characteristic time scales,

$$\tau_D \gg \nu_p^{-1} \gg \tau_R \gg \tau_B \tag{27}$$

has been justified. We note that in contrast to the results of papers,[12,13] the steady-state velocity of poloidal rotation allowing for orbit squeezing[11] is clearly different from that in the plateau regime. The contrary statement of the paper[13] is based on an incorrect calculation of the longitudinal viscosity force in the paper.[12] (see Eq. (19) for $F_\parallel$).

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