Magnetic properties of inhomogeneous ferromagnets

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The temperature and magnetic-field dependences of the average magnetization are analyzed on the basis of a generalization of Landau's theory of second-order phase transitions to the case of spatially inhomogeneous ferromagnets (K. P. Belov, *Magnetic Transitions*, Consultants Bureau, New York, 1961). Analytic expressions are obtained for the Belov–Arrott plots, which describe the dependence of the square of the average magnetization on the ratio of the magnetic field strength to the average magnetization. © *1996 American Institute of Physics.* [S1063-7761(96)01805-7]

1. The peculiar behavior of Invar alloys when they pass into the ferromagnetic state, the features of the temperature dependence of the spontaneous magnetization, and the complexity of the determination of the Curie temperature T_c have been known for a long time.[1] Significant divergence from Landau's ordinary thermodynamic theory of the phase transitions[1,2] of spatially homogeneous ferromagnets was observed long ago for Invar alloys over a broad temperature range near the transition to the magnetically ordered state, where the spontaneous magnetization is small. The magnetization "tails" observed near the transition were attributed to "remnants" of the spontaneous magnetization above a temperature that was termed Curie temperature. This temperature was determined from thermodynamic coefficients or curves of equal magnetization.[1] The temperature range over which the spontaneous magnetization tails of Invar alloys are displayed can amount to tens of percent of the entire range in which ferromagnetism exists. Therefore, understanding the phenomena over such a broad temperature range is also of interest from the standpoint of practical applications. Bearing in mind the existence of ferromagnetism in the range of the spontaneous magnetization tails, we shall henceforth assume that this range is located below the true Curie temperature, i.e., the temperature of the transition from the paramagnetic state to the ferromagnetic state.

The basis for our assertion is an idea advanced long ago regarding the nature of the magnetization tails, which were attributed to the inhomogeneity of the alloy. Due to inhomogeneity, different parts of the alloy have different local Curie temperatures. In Ref. 1 not only was this view advanced, but a theoretical model describing the phenomenon of magnetization tails was presented. This physical view was shared by a large group of investigators, including experimentalists (see, or example, Refs. 3–5), who observed anomalies in the magnetically ordered phase transition in Invar alloys, and theoreticians,[6–10] who developed models to interpret such anomalies. It can be claimed that an approach which generalizes the theory of second-order phase transitions to the spatially inhomogeneous case was proposed in Ref. 1.

The spatially inhomogeneous theory was applied in Ref. 10 to the interpretation of the difference between the temperature at which ferromagnetism is established (the Curie temperature T_c) and the temperature $T_c-\Delta T$ of the peaks of the specific heat and the temperature coefficient of thermal expansion of Invar alloys, to which attention was called in Refs. 4 and 5 (see also Ref. 9). A discussion of the features of the magnetic susceptibility not only at the temperature at which ferromagnetism is established T_c, but also at the lower temperatures $T_c-\Delta T<T<T_c$ was launched in Ref. 10.

In the present communication we consider the magnetic properties of a ferromagnet with an inhomogeneous, spatially distributed local Curie temperature $T_c(\mathbf{r})$ in detail. The maximum value $T_c^{\max}(\mathbf{r})=T_c$ corresponds to the temperature at which ferromagnetism appears, and the minimum value $T_c^{\min}(\mathbf{r})=T_c-\Delta T$ corresponds to the specific heat peak.[9,10] Below we shall also discuss Belov–Arrott plots, which are isotherms describing the dependence of the square of the average magnetization on the ratio of the magnetic field strength to the average magnetization. We stress, in particular, that when Belov–Arrott plots are used, the Curie temperature is experimentally determined from the isotherm which passes through the origin of coordinates in the Belov–Arrott plane.

2. In the simplest model of Landau's theory, in which the free energy as a function of the magnetization is taken in the form

$$\frac{\alpha}{2}(T-T_c)M^2+\frac{b}{4}M^4, \tag{2.1}$$

where a and b are parameters which do not depend on M and the temperature T, and the Belov–Arrott plots have the form of parallel straight lines for different T. It has been established experimentally for Invar alloys (see, for example, Refs. 4 and 5) that the Belov–Arrott isotherms are straight lines near the transition to the magnetically ordered state only in a sufficiently strong magnetic field. In weaker fields that are still strong enough to rule out disorder in the orientation of the domains, the Belov–Arrott isotherms are

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curved. Corresponding bending was obtained in the theoretical work in Ref. 6, where it was attributed to the inhomogeneity of the local Curie temperature $T_c(\mathbf{r})$. It was theorized that the inhomogeneity can be taken into account within a model of small fluctuations of $T_c(\mathbf{r})$. This produced a picture of approximately symmetric "curving" of the isotherms toward the linear plot corresponding to the Curie temperature. Such a picture is far from that observed experimentally.[4] In addition, the fluctuation approach in Ref. 6 led to agreement between the temperature at which the ferromagnetic state is established and the temperature of the specific heat peak, which contradicts the existence of the tails. The result of a numerical calculation of Belov–Arrott isotherms, which was similar to the experimental dependence, was presented in a later paper.[4] A very complicated dependence of the free energy on the magnetization was used in Ref. 4, precluding the derivation of analytic laws.

To make up for the inadequate development of the theory of the magnetic properties of inhomogeneous ferromagnets near the ferromagnetic transition,[1] we utilize the approach in Ref. 10, in which the treatment was based on a thermodynamic potential (compare Ref. 1):

$$\Phi_M(P,T[M])=\int_0^1 ds W(s)\left\{\frac{\alpha(P)}{2}[T-T_c(P,s)]M^2+\frac{b(P)}{4}M^4\right\}. \tag{2.2}$$

Here P is the pressure, s is a parameter which defines the physical cause of the difference between the local Curie temperatures $T_c(P,s)$, and $W(s)$ is the probability distribution of s. The potential (2.2) is a functional of the local magnetization M, which determines the average magnetization $\langle M\rangle=\int_0^1 ds W(s)M(s)$. The local magnetization is determined from the following relation, which is obtained by varying the potential (2.2):

$$H=\alpha(T-T_c+\Delta Ts)M+bM^3. \tag{2.3}$$

Here it is assumed that $T_c(P,s)=T_c-\Delta Ts$, and H is the strength of the external magnetic field.

On the one hand, the linear dependence of the local Curie temperature on x used below can be regarded as an approximation over a narrow range of $T_c(s)$, and, on the other hand, as in the case of Fe_3Pt, it can describe such variation over a comparatively broad range of T_c due to the variation of the metallurgical order parameter $0\leqslant S^2\leqslant 1$ when $s=S^2$ (Ref. 11) and $T_c(S^2) = (294.3+177.6S^2)$ K.

The thermodynamic potential written with an accuracy to M^4 is probably suitable for describing Permalloy[3] over a very broad temperature range. At the same time, such a description is reasonable for other alloys near T_c. We note that we completely neglect critical fluctuations. The spatial inhomogeneity of a magnet can hamper their elucidation, at least in the case of strongly inhomogeneous magnets, which include, according to Ref. 4, for example, the disordered Invar alloy Fe_3Pt. According to Ref. 4, the temperature range of the magnetization tails for the latter alloy has a width of approximately $0.1T_c$. In accordance with the treatment below, such a broad temperature range for a phase transition does, in fact, correspond to strong inhomogeneity. The material in this paper is intended specifically for strongly inhomogeneous magnets.

3. We begin our treatment in the high-field limit, in which

$$t_0=\frac{3b}{\alpha\Delta T}\left(\frac{H}{2b}\right)^{2/3}\gg|t+s|, \tag{3.1}$$

where

$$t=\frac{T-T_c}{\Delta T}. \tag{3.2}$$

Then we have approximately

$$\langle M\rangle=\left(\frac{H}{b}\right)^{1/3}-\frac{\alpha\Delta T}{3b}\left(\frac{b}{H}\right)^{1/3}(t+\langle s\rangle), \tag{3.3}$$

where $\langle s\rangle=\int_0^1 ds\, sW(s)$. Hence for the magnetic susceptibility we obtain

$$\chi=\frac{\partial\langle M\rangle}{\partial H}=\frac{1}{3(H^2b)^{1/3}}+\frac{\alpha\Delta T}{9(H^2b)^{2/3}}(t+\langle s\rangle), \tag{3.4}$$

and for the Belov–Arrott isotherms we have straight lines:

$$\langle M\rangle^2=\frac{H}{b\langle M\rangle}-\frac{\alpha\Delta T}{b}(t+\langle s\rangle). \tag{3.5}$$

The existence of experimental Belov–Arrott plots in the high-field region enables us to determine b from the slope of the linear segments. Then, the points of intersection of the x axis $\langle M\rangle^2=0$ with the continuations of the linear segments of the Belov–Arrott plots specify

$$\left(\frac{H}{\langle M\rangle}\right)_{\langle M\rangle^2=0}\equiv\chi_{hf}^{-1}=\alpha(T-T_c+\Delta T\langle s\rangle). \tag{3.6}$$

The temperature dependence $\chi_{hf}(T)$ thus found makes it possible to determine the Curie constant $C=\alpha^{-1}$ and the high-field paramagnetic Curie temperature:

$$T_c^{hf}=T_c-\Delta T\langle s\rangle. \tag{3.7}$$

The latter value corresponds to the Belov–Arrott plot for which the continuation of the linear segment from the high-field region passes through the origin in the Belov–Arrott plane: $\langle M\rangle^2=0$, $H/\langle M\rangle=0$. The expression (3.7) is lower than the temperature T_c at which the ferromagnetic state is established. At the same time, this expression characterizes the high-field paramagnetic Curie temperature.

If the temperature T_c^m, which corresponds to the specific heat maximum, is determined experimentally, then, according to Ref. 10, this allows us to consider

$$T_c^m=T_c-\Delta T \tag{3.8}$$

determined. Obviously, experimental data on the temperatures (3.7) and (3.8) make it possible to find $\Delta T(1-\langle s\rangle)=T_c^{hf}-T_c^m$. The temperature T_c^m is frequently identified with the Curie temperature. We stress that this temperature is below the high-field paramagnetic Curie temperature (3.7).

4. We now turn to the case of weak magnetic fields $t_0 \ll |t+s|$. At first we restrict ourselves to the region which allows analytical expansion in integer powers of the magnetic field. In our model this is possible in the paramagnetic region, in which $T > T_c$ $(t > 0)$. Then we have approximately

$$\langle M \rangle = \frac{H}{\alpha \Delta T} \left\langle \frac{1}{t+s} \right\rangle - \frac{bH^3}{(\alpha \Delta T)^4} \left\langle \frac{1}{(t+s)^4} \right\rangle, \tag{4.1}$$

where $\langle \phi(s) \rangle = \int_0^1 ds \phi \langle s \rangle W(s)$. For the Belov–Arrott plots Eq. (4.1) gives

$$\langle M \rangle^2 = \left\{ \frac{H}{\langle M \rangle} \frac{1}{\alpha \Delta T} \left\langle \frac{1}{t+s} \right\rangle - 1 \right\} \times \left\{ \frac{H^3}{\langle M \rangle^3} \frac{b}{(\alpha \Delta T)^4} \left\langle \frac{1}{(t+s)^4} \right\rangle \right\}^{-1}. \tag{4.2}$$

Hence, at small magnetization values we obtain

$$\langle M \rangle^2 = \left\{ \frac{H}{\langle M \rangle} \frac{1}{\alpha \Delta T} \left\langle \frac{1}{t+s} \right\rangle - 1 \right\} \frac{\alpha \Delta T}{b} \left\langle \frac{1}{t+s} \right\rangle^3 \left\langle \frac{1}{(t+s)^4} \right\rangle^{-1}. \tag{4.3}$$

For the slope of the Belov–Arrott plots we have

$$(\langle M \rangle^2)' \equiv \frac{d \langle M \rangle^2}{d(H / \langle M \rangle)} = \frac{1}{b} \left\langle \frac{1}{(t+s)} \right\rangle^4 \left\langle \frac{1}{(t+s)^4} \right\rangle^{-1}. \tag{4.4}$$

Well above T_c, we have

$$(\langle M \rangle^2)' = \frac{1}{b} \left\{ 1 - 6 \frac{\langle s^2 \rangle - \langle s \rangle^2}{t^2} \right\}. \tag{4.5}$$

Thus, the inhomogeneity of a magnet causes the slope of the Belov–Arrott plots in the paramagnetic region to be smaller in a weak field than in a strong field. This corresponds to experiment.[4,5]

In the limit $H \to 0$, according to (4.1), for the paramagnetic susceptibility we obtain

$$\chi_p(T) = \frac{1}{\alpha \Delta T} \left\langle \frac{1}{t+s} \right\rangle. \tag{4.6}$$

If $t \gg 1$, we have

$$\chi_p^{-1}(T) = \alpha \left\{ T - T_c + \Delta T \langle s \rangle - \frac{(\Delta T)^2}{T - T_c} [\langle s^2 \rangle - \langle s \rangle^2] \right\}. \tag{4.7}$$

The deviation from Eq. (3.6) clearly decreases with increasing temperature.

As T_c is approached, the (low-field) paramagnetic susceptibility (4.6) increases. At the Curie point it can be finite only if the mean $\langle s^{-1} \rangle$ is finite. For example, if $W(s) = 2s$, $\langle s^{-1} \rangle = 2$ and $\chi_p(T_c) = 2/(\alpha \Delta T)$.

5. There is another possibility for analytic expansion in powers of H in the ferromagnetic region at $T < T_c^m = T_c - \Delta T$, where Eq. (2.3) gives (when $t < -1$):

$$\langle M \rangle = \sqrt{\frac{\alpha \Delta T}{b}} \langle \sqrt{-t-s} \rangle + \frac{H}{2 \alpha \Delta T} \left\langle \frac{1}{-t-s} \right\rangle. \tag{5.1}$$

The first term in this formula corresponds to the average spontaneous magnetization.[10] The second term specifies the magnetic susceptibility

$$\chi(T) = \frac{1}{2 \alpha \Delta T} \left\langle \frac{1}{-t-s} \right\rangle. \tag{5.2}$$

As T approaches T_c^m, the susceptibility increases and can go to infinity. For the Belov–Arrott plots Eq. (5.1) gives the straight lines

$$\langle M^2 \rangle = \frac{\alpha \Delta T}{b} \langle \sqrt{-t-s} \rangle^2 + \frac{H}{\langle M \rangle} \frac{1}{b} \langle \sqrt{-t-s} \rangle^2 \left\langle \frac{1}{-t-s} \right\rangle. \tag{5.3}$$

However, these straight lines differ from the straight lines for the high-field limit. First, according to (5.3), intersection with the y axis $(H/\langle M \rangle = 0)$ occurs at a finite value of the square of the spontaneous magnetization, while in the high-field case (3.5) the continuation of each linear Belov–Arrott plot intersects the y axis at a larger value, since $|t| - \langle s \rangle$ exceeds $\langle \sqrt{|t| - s} \rangle^2$. When $|t| \gg 1$, the latter expression has the form $|t| - \langle s \rangle - (4|t|)^{-1} [\langle s^2 \rangle - \langle s \rangle^2]$. Second, the slope of the linear Belov–Arrott plots (5.3)

$$(\langle M \rangle^2)' = \frac{1}{b} \langle \sqrt{-t-s} \rangle^2 \left\langle \frac{1}{-t-s} \right\rangle \tag{5.4}$$

is steeper than the slope of the straight line (3.5). Both these differences correspond to experiment.[4,5] We note that at $T = T_c^m$, the slope (5.4) can be infinite if $\langle (1-s)^{-1} \rangle$ diverges.

6. We now turn to the intermediate transitional temperature range

$$T_c - \Delta T < T < T_c (-1 < t < 0), \tag{6.1}$$

which most distinctly distinguishes the inhomogeneous ferromagnet model. The high-field limit in this range has already been considered. In the limit of a magnetic field equal to zero, we have the spontaneous magnetization[10]

$$\langle M_{sp} \rangle = \sqrt{\frac{\alpha \Delta T}{b}} \int_0^{|t|} ds W \sqrt{|t| - s}, \tag{6.2}$$

which specifies the y-intercepts of the Belov–Arrott plots. At the same time, while the Belov–Arrott plots intersect the x axis at a finite value of $H/\langle M \rangle$ above T_c, in the range (6.1) such intersection is possible only at $H/\langle M \rangle = 0$ and only if $T = T_c$. This behavior of the Belov–Arrott plots is due to the fact that the magnetic susceptibility goes to infinity in the limit $H = 0$ in the range (6.1).[10]

Let us consider some consequences of the exact solution of Eq. (2.3). We write the Cardano equations in the following forms:

$$M(s) = 2 \sqrt{\frac{\alpha \Delta T}{3b}} \sqrt{s - |t|} \sinh \left\{ \frac{1}{3} \operatorname{arcsinh} \left(\frac{t_0}{s - |t|} \right)^{3/2} \right\},$$

$$|t| < s < 1, \tag{6.3}$$

$$M(s)=2\sqrt{\frac{\alpha\Delta T}{3b}}\sqrt{|t|-s}\cosh\left\{\frac{1}{3}\text{ arccosh }\left(\frac{t_0}{|t|-s}\right)^{3/2}\right\}, \tag{6.4}$$

$$(|t|-t_0)\theta(|t|-t_0)<s<|t|,$$

$$M(s)=2\sqrt{\frac{\alpha\Delta T}{3b}}\sqrt{|t|-s}\cos\left\{\frac{1}{3}\text{ arccos }\left(\frac{t_0}{|t|-s}\right)^{3/2}\right\}, \tag{6.5}$$

$$0<s<(|t|-t_0)\theta(|t|-t_0),$$

$$\theta(x)=\begin{cases}0, & x<0,\\ 1, & x>0.\end{cases} \tag{6.6}$$

Equation (6.5) corresponds to only one of the three real roots of Eq. (2.3) possible in this range. The other two roots correspond to larger values of the thermodynamic potential (2.2).

Equations (6.3)–(6.5) make it possible to write the average magnetization in the form

$$\langle M\rangle=\frac{H}{3\alpha\Delta T}\{j_1\theta(|t|-t_0)+j_2\theta(t_0-|t|)+j_3\}. \tag{6.7}$$

Here

$$j_{1(2)}=\int_{y_{1(2)}}^{\infty}dy\,W\left\{\left|t\right|-\frac{t_0}{\psi(y)}\right\}\varphi(y),$$

$$j_3=\int_{y_3}^{\infty}dy\,W\left\{\left|t\right|-\frac{t_0}{\psi(-y)}\right\}\varphi(-y), \tag{6.8}$$

where

$$\psi(y)=[y(4y-3)^2]^{1/3},$$

$$y_1=\cos^2[(1/3)\text{ arccos }(t_0/|t|)^{3/2}],$$

$$y_2=\cosh^2[(1/3)\text{ arccosh }(t_0/|t|)^{3/2}],$$

$$y_3=\sinh^2[(1/3)\text{ arcsinh }(t_0/(1-|t|))^{3/2}],$$

and

$$\varphi(y)=\frac{4}{4y-3}-\frac{1}{y}+\frac{24}{(4y-3)^2}. \tag{6.9}$$

The integrals (6.8) can be evaluated in the special case $W(s)=1$, and we obtain

$$\langle M\rangle=\frac{H}{3\alpha\Delta T}\left\{\ln\left\{1+\frac{3}{4}\sinh^{-2}\left(\frac{1}{3}\text{ arcsinh }\left(\frac{t_0}{1-|t|}\right)^{3/2}\right)\right\}+\frac{2}{1+(4/3)\sinh^2[(1/3)\text{ arcsinh }(t_0/(1-|t|))^{3/2}]}-\ln\left\{1-\frac{3}{4}F^{-1}\left(\frac{t_0}{|t|}\right)\right\}-\frac{2}{1-(4/3)F(t_0/|t|)}\right\}, \tag{6.10}$$

where

$$F\left(\frac{t_0}{|t|}\right)=\begin{cases}\cos^2[(1/3)\text{ arccos }(t_0/|t|)^{3/2}], & |t|>t_0,\\ \cosh^2[(1/3)\text{ arccosh }(t_0/|t|)^{3/2}], & |t|<t_0.\end{cases} \tag{6.11}$$

At the Curie temperature ($t=0$) the expression

$$\langle M\rangle=\frac{H}{3\alpha\Delta T}\left\{\ln\left\{1+\frac{3}{4\sinh^2[(1/3)\text{ arcsinh }t_0^{3/2}]}\right\}+\frac{6}{3+4\sinh^2[(1/3)\text{ arcsinh }t_0^{3/2}]}\right\} \tag{6.12}$$

describes the dependence of the average magnetization on the magnetic field. In the low-field limit $t_0\ll 1$,

$$\langle M\rangle=\frac{2H}{3\alpha\Delta T}\left\{1+\ln\frac{(\alpha\Delta T)^{3/2}}{Hb^{1/2}}\right\}. \tag{6.13}$$

The magnetic susceptibility is also characterized by a nonanalytic field dependence

$$\frac{\partial\langle M\rangle}{\partial H}=\chi=\frac{2}{3\alpha\Delta T}\ln\frac{(\alpha\Delta T)^{3/2}}{Hb^{1/2}}. \tag{6.14}$$

The corresponding Belov–Arrott plot is given by the relation

$$\langle M\rangle^2=\frac{(\alpha\Delta T)^3}{b}\left(\frac{\langle M\rangle}{H}\right)^2\exp\left\{2-3\alpha\Delta T\frac{\langle M\rangle}{H}\right\}. \tag{6.15}$$

This expression gives the Belov–Arrott isotherm which approaches the origin $\langle M\rangle^2=0$ and $H/\langle M\rangle=0$ according to the law (6.15).

At the temperature of the specific heat peak $t=-1$ and $t_0<1$,

$$\langle M\rangle=\frac{H}{3\alpha\Delta T}\left\{-\ln\left\{1-\frac{3}{4\cos^2[(1/3)\text{ arccos }t_0^{3/2}]}\right\}-\frac{6}{3-4\cos^2[(1/3)\text{ arccos }t_0^{3/2}]}\right\}. \tag{6.16}$$

In the low-field limit $t_0\ll 1$,

$$\langle M\rangle=\frac{2}{3}\sqrt{\frac{\alpha\Delta T}{b}}+\frac{H}{3\alpha\Delta T}\left\{1+\ln\frac{(\alpha\Delta T)^{3/2}}{HB^{1/2}}\right\}. \tag{6.17}$$

Here, as in the case of (6.13), we have a nonanalytic dependence on the magnetic field. For the magnetic susceptibility we obtain

$$\chi=\frac{\partial\langle M\rangle}{\partial H}=\frac{1}{3\alpha\Delta T}\ln\frac{(\alpha\Delta T)^{3/2}}{Hb^{1/2}}. \tag{6.18}$$

Here we have a twofold decrease in comparison with (6.14). For the Belov–Arrott isotherm Eq. (6.16) gives

$$\langle M\rangle^2=\frac{4\alpha\Delta T}{9b}+\frac{8}{27b}\frac{H}{\langle M\rangle}\left\{1-\ln\left(\frac{H}{\langle M\rangle}\frac{2}{3\alpha\Delta T}\right)\right\}. \tag{6.19}$$

A comparison with Eq. (6.15) reveals that in the transitional range (6.1) the exponential dependence of $\langle M\rangle^2$ on $H/\langle M\rangle$ is replaced by a new dependence $\sim (H/\langle M\rangle)\ln(H/\langle M\rangle)$, which differs from a linear dependence.

7. Several limiting equations can be obtained in the case of an arbitrary distribution $W(s)$ for the transitional range (6.1). First of all, in the case of weak magnetic fields ($|t|\gg t_0$) and $1-|t|\gg t_0$, we have

$$\langle M\rangle=\langle M_{sp}\rangle+\frac{H}{\alpha\Delta T}\left\{W(|t|)\left\{\frac{1}{2}\ln\left|t\right|+\ln(1-|t|)+1+\ln\frac{(\alpha\Delta T)^{3/2}}{Hb^{1/2}}\right\}+\frac{1}{2}\int_0^{|t|}ds\frac{W(s)-W(|t|)}{|t|-s}+\int_{|t|}^1 ds\frac{W(s)-W(|t|)}{s-|t|}\right\}, \tag{7.1}$$

where the spontaneous magnetization is given by Eq. (6.2). Hence for the magnetic susceptibility we have

$$\chi=\frac{\partial\langle M\rangle}{\partial H}=\frac{1}{\alpha\Delta T}\left\{W(|t|)\left\{\ln\frac{\alpha\Delta T}{H\sqrt{b}}+\frac{1}{2}\ln\left|t\right|+\ln(1-|t|)\right\}+\frac{1}{2}\int_0^{|t|}ds\frac{W(s)-W(|t|)}{|t|-s}+\int_{|t|}^1 ds\frac{W(s)-W(|t|)}{s-|t|}\right\}. \tag{7.2}$$

Extraction of the logarithmic dependence on the magnetic field from experimental data might make it possible to experimentally determine $W(s)$. Under the assumption that the induced magnetization is small compared with the spontaneous, the equation of the Belov–Arrott plots has the form

$$\langle M\rangle^2=\langle M_{sp}\rangle^2+\left\{1+\frac{H}{\langle M\rangle}\frac{2}{\alpha\Delta T}\left\{W(|t|)\left(\frac{1}{2}\ln\left|t\right|+\ln(1-|t|)+1+\ln\frac{(\alpha\Delta T)^3}{\sqrt{b}\langle M_{sp}\rangle}-\ln\frac{H}{\langle M\rangle}\right)+\frac{1}{2}\int_0^{|t|}ds\frac{W(s)-W(|t|)}{|t|-s}+\int_{|t|}^1 ds\frac{W(s)-W(|t|)}{s-|t|}\right\}\right\}. \tag{7.3}$$

Near the upper boundary of the transitional range, where

$$|t|\ll t_0\ll 1-|t|\sim 1, \tag{7.4}$$

we have

$$\langle M\rangle=\frac{H}{\alpha\Delta T}\left\{W(|t|)\left[\ln(1-|t|)+\frac{2}{3}\ln\frac{(\alpha\Delta T)^{3/2}}{Hb^{1/2}}+\frac{2}{3}\right]+\int_{|t|}^1 ds\frac{W(s)-W(|t|)}{s-|t|}\right\}+\left(\frac{H}{b}\right)^{1/3}\int_0^{|t|}dsW(s). \tag{7.5}$$

If the last term in Eq. (7.5) is small, the corresponding portion of the Belov–Arrott plot will be described by the expression

$$\langle M\rangle^2=\frac{[\alpha\Delta T(1-|t|)]^3}{b}\left(\frac{\langle M\rangle}{H}\right)^2\exp\left\{2+3\int_{|t|}^1 ds\frac{W(s)-W(|t|)}{W(|t|)(s-|t|)}-\frac{3\alpha\Delta T}{W(|t|)}\frac{\langle M\rangle}{H}\right\}. \tag{7.6}$$

This is a generalization of Eq. (6.15), but only in the portion of the Belov–Arrott plot in the range (7.4).

Near the end of the transitional range, for weak magnetic fields such that

$$1-|t|\ll t_0\ll 1, \tag{7.7}$$

we have

$$\langle M\rangle=\langle M_{sp}\rangle+\frac{H}{\alpha\Delta T}\left\{\frac{1}{3}\ln\frac{(\alpha\Delta T)^{3/2}}{Hb^{1/2}}+\frac{1}{2}\int_0^{|t|}ds\frac{W(s)-W(|t|)}{|t|-s}\right\}+\left(\frac{H}{b}\right)^{1/3}\int_{|t|}^1 dsW(s). \tag{7.8}$$

If the last term in this equation is small, for the Belov–Arrott plot we have

$$\langle M\rangle^2=\langle M_{sp}\rangle^2\frac{\langle M_{sp}\rangle^2}{\alpha\Delta T}\frac{H}{\langle M\rangle}\left\{\frac{2}{3}\ln\left(\frac{(\alpha\Delta T)^{3/2}}{b^{1/2}\langle M_{sp}\rangle}\frac{\langle M\rangle}{H}\right)+\frac{2}{3}+\int_0^{|t|}ds\frac{W(s)-W(|t|)}{|t|-s}\right\}. \tag{7.9}$$

The limiting equations obtained here demonstrate the peculiarity of the dependence of the average magnetization of an inhomogeneous ferromagnet in the temperature range where ferromagnetic order is established. We note that the last terms in Eqs. (7.5) and (7.8) correspond to the presence of a comparatively strong magnetic field. Therefore, Eqs. (7.6) and (7.9), which were obtained with neglect of these terms, correspond to the low-field case.

8. To discuss results, we shall use, in particular, Fig. 1, which presents Belov–Arrott plots that were constructed in accordance with Eq. (6.7) and correspond to the distribution $W(s)=1$ and the condition $\alpha\Delta T=60$. When $T_c=360$ K and $\alpha^{-1}=0.6$ K, the latter corresponds to the suggestion advanced in Ref. 4 that the transitional range has a width which corresponds to $\Delta T=0.1T_c$ in our treatment. The different Belov–Arrott plots in Fig. 1 correspond to different values of t. The linear portions of the curves are observed in the high-field case, according to Sec. 3. In the paramagnetic region ($t>0$) flattening of the curves appears in the figure as $H/\langle M\rangle$ decreases, in agreement with the theory in Sec. 4.

Conversely, below T_c^m ($t<-1$) the Belov–Arrott plots descend more rapidly with decreasing $H/\langle M\rangle$ than in the high-field region, i.e., they become steeper, as was shown in Sec. 5. At the point $T=T_c$, at which $t=0$ we have a violation of the power-law dependence (observed at $t>0$), and the Belov–Arrott isotherm tends exponentially to the origin of the Belov–Arrott coordinates in accordance with Eq. (6.15). This, in particular, qualitatively distinguishes the results of our treatment from the results of the fluctuational treatment of the influence of inhomogeneity in Ref. 6, in which the Belov–Arrott isotherm for $T=T_c$ was a straight line.

The transitional range $T_c-\Delta T<T<T_c$, which corresponds to the range where the magnetization tails exist, also contains Belov–Arrott isotherms characterized by passage from a creeping exponential dependence on $H/\langle M\rangle$ near the

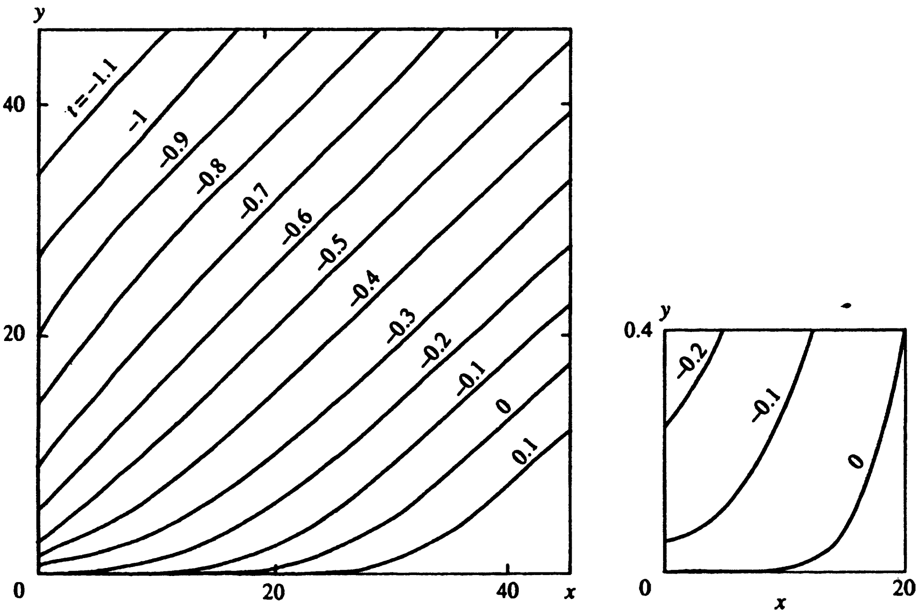

FIG. 1. Plots of the dependence of $y=b\langle M\rangle^2$ on $x=H/\langle M\rangle$ for various values of t; $\alpha\Delta T=60$.

beginning of the establishment of ferromagnetism ($|t|\ll 1$) to steepening of the Belov–Arrott plots, in accordance with Fig. 1 and our analytic treatment.

To summarize, we can state that the temperature range $T_c-\Delta T \leqslant T \leqslant T_c$, which we have termed the transitional range, is the range where ferromagnetism is established. This means that the range of the magnetization tails[1] is, in fact, the range where ferromagnetic order appears in the magnet. Since ferromagnetism has a significant influence on many properties (the specific heat, elasticity, thermal expansion, etc.), the interpretation of the anomalous properties of Invar alloys over the broad range of the magnetization tails should be based on data on the average magnetization. The laws obtained above, which describe the dependences of $\langle M\rangle$ on the magnetic field and the temperature, can serve as a basis for understanding numerous phenomena in Invar alloys, as well as in other inhomogeneous ferromagnets.

In conclusion, we stress that in our work we not only obtained a series of new analytic laws, but we also demonstrated the qualitative difference between our description of the magnetic properties of a spatially inhomogeneous ferromagnet and Shtrikman and Wohlfarth's fluctuational model.[6] This difference is most strikingly displayed over the temperature range in which the magnetization tails correspond to comparatively small remnants of the spontaneous magnetization. In Fig. 1 this difference is illustrated by the Belov–Arrott isotherm for $T=T_c$, as well as the isotherms for $T<T_c$ at which $|t|\ll 1$. The reason for this qualitative peculiarity is the small value of the average magnetization in this temperature range, in which Shtrikman and Wohlfarth's assumption[6] that the magnetization fluctuations are small compared with the average magnetization is violated.

Concluding our discussion, we note that instead of the linear temperature dependence adopted by us in Eq. (2.2), which corresponds, for example, to the experimental data for Permalloy, we can use a different dependence of the coefficient of M^2 in (2.2):

$$\frac{1}{2}\beta(P)[T^{4/3}-T_c^{4/3}-s\Delta T^{4/3}]. \tag{8.1}$$

This dependence corresponds to the spin-fluctuation theory of weak ferromagnets.[12–14] In this case everything presented above can be adapted to Eq. (8.1) using the following replacements of notation: $\alpha\Delta T$ by $\beta\Delta T^{4/3}$, ΔT by $\Delta T^{4/3}$, T by $T^{4/3}$, and T_c by $T_c^{4/3}$. The treatment described above can thereby be transferred completely to the spin-fluctuation model of weak ferromagnets.

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