Instantaneous joint distributions of ion microfield and its time derivatives and effects of dynamical friction in plasmas*)

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The generalized joint simultaneous distribution of the electric microfield vector in plasmas and its time derivatives is presented. The results consistently generalize the approach of Chandrasekhar and von Neumann to the temporal fluctuations of the plasma microfield, taking into account the complexity of the plasma composition and screening by plasma electrons, repulsion and correlations between ions, and plasma polarization. Analytic formulas for the universal functions describing the constraint moments of the first time derivative of the ion microfield are obtained on the basis of the Baranger–Moser cluster expansion scheme. These results permit improvement in the theoretical approach to ion dynamics effects based on the time expansion of the microfield. A theoretical basis for the inclusion of microfield fluctuations due to low-frequency collective plasma oscillations is developed. The results modify the notion introduced by Chandrasekhar of the dynamical friction experienced by a test or radiating ion in plasmas over microscopic distances, if it has nonzero velocity. These new contributions come from the totality of the effects considered, specifically the plasma composition, repulsion and correlations of ions, screening by electrons and the neutralizing electron background, and low-frequency plasma oscillations. © *1996 American Institute of Physics.* [S1063-7761(96)01007-4]

1. INTRODUCTION

Temporal variation of the ion microfield in plasmas is found to be important for its spectral characteristics.[1–27] Several models and approaches have been developed to calculate spectra including the influence of the ion dynamics.[1–10,12–28] Here we present the generalization of an approach[27,28] having its origin in the work of Chandrasekhar and von Neumann.[1] The theoretical treatment is based on the generalized Baranger-Moser cluster expansion scheme[29–33] and gives the statistical solution for the instantaneous low-frequency joint microfield distribution function of the electric microfield vector and its time derivatives to within the accuracy of the two-particle correlation functions in the case of a neutral radiator and the three-particle correlations in the case of a charged one. Thus, the accuracy of results depends on the the accuracy of the correlation functions involved.[28,31–36] Formally this approach enables us to treat arbitrary mixtures of ions in plasmas with arbitrary values of the coupling parameters and allows any determination of the electric microfield to be used, and thus makes it possible to incorporate nonlinear electron screening on the basis of the Density Functional Theory (hereafter DFT).[32–34] However, often in practice for strongly coupled plasmas[37–39] this still does not yield good agreement with computer simulations, for example by the Monte Carlo method. That is why it is shown how to obtain solutions similar to those mentioned above by an appropriate development of the APEX (Adjustable Parameter Exponential Approximation[38,39]) scheme, which was proved to be reliable for strongly coupled plasmas.

The general formula for the first constraint moment of the first time derivative of the electric ion microfield strength vector, with the electric ion microfield strength vector fixed, is derived for any arbitrary type of (static) electron screening and correlation functions, and any composition of the ion charges in the plasma, and thus for any value of the plasma coupling parameters. In particular, for a strongly coupled plasma this formula is obtained by elaboration of the APEX approach. This result incorporates so-called polarization effects,[28,36] which result from the finiteness of the plasma coupling parameters, and the "metamorphosis" of ions into quasiparticles that carry the screening cloud of electrons. In the OCP (One-Component Plasma Model[40–42]) limit for ions, these terms represent the influence of the negative uniform neutralizing background of the plasma electrons, which is absent from the Chandrasekhar-von Neumann formulation of the problem for stars.[1] The importance of the influence of the neutralizing background on the moments of the microfield fluctuations in the OCP model was recognized previously for other cases. For example, the moment of the square of the time derivative of the absolute value **F** of the microfield, $\langle \dot{F}^2 \rangle$, was considered in Ref. 16. It is noteworthy that the present consideration concerns distributions and fluctuations at $t=0$ in comparison with studies of the time evolution of the joint microfield distributions.[16–19,23] Thus, the above effects yield new contributions to the dynamical friction,[1] the existence of which is established through analysis in which the first constraint moment of the first time derivative of the electric ion microfield with fixed electric ion microfield is a key quantity.[1]

We distinguish two parts of the low-frequency plasma microfield. The first, which comes from individual ions (or quasiparticles) and was discussed above, is called the "individual'' part of the total microfield, while the second, which comes from the low-frequency plasma oscillations, is called the ''collective'' part of the total microfield. However, one should keep in mind that the individual part of the total microfield has a many-body character, because it is produced by many ion-perturbers due to long-range interaction potentials. It is known that the contribution of low-frequency plasma oscillations to the total (or global) microfield in plasmas may be important.[20] The same is true of fluctuations of the collective part of the total (or global) microfield, which also contribute to the dynamical friction of an ion in plasmas. In the last section of this paper we present an approach that permits the self-consistent treatment of temporal fluctuations of the microfield due to low-frequency plasma oscillations and fluctuations of the microfield from separate particles (or quasiparticles). All this makes it possible to include the influence of the temporal variation of the low-frequency collective part of the total microfield on the dynamical friction of ions in plasmas, in addition to the effects discussed earlier.

2. GENERAL RESULTS FOR JOINT SIMULTANEOUS DISTRIBUTION FUNCTIONS OF ELECTRIC MICROFIELD AND ITS TIME DERIVATIVES

Let a radiator with net charge Z_0 be located in the vicinity of field ions and electrons in plasma. The Hamiltonian of the whole system can be written in the form

$$\hat{H}=\hat{H}_0+\hat{H}_e+\hat{H}_i+\hat{H}_r+\hat{V}_{0e}+\hat{V}_{0i}+\hat{V}_{ei}+\hat{V}_{0r}+\hat{V}_{ir}+\hat{V}_{er}. \tag{1}$$

Here $\hat{H}_0$ is the unperturbed Hamiltonian of the radiator; $\hat{H}_{e,i}$ are Hamiltonians of free plasma electrons and ions respectively; $\hat{H}_r$ is the Hamiltonian of the radiation field; and the symbol $\hat{V}$ designates the pairwise interaction operators between subsystems. We assume here that the radiation field is weak enough to ignore all interactions with it from the outset. It is assumed that the levels of the radiator are rapidly populated, while the plasma is quasistationary and optically thin.

Our aim is to trace the consequences connected with ion microfield time variation due to the thermal motion of particles at the initial time $t=0$. The interaction of the radiator with electrons may be considered either in the impact approximation or in terms of the relaxation theory, or in the one-electron approximation.[6] At this point in the treatment the traditional question of the initial correlations arises.[32–34,37–43] Here we assume that it is possible to introduce effective potentials for the ion–radiator interaction, including the screening effects of plasma electrons. There are several possible ways of dividing this system into almost decoupled subsystems.[32–34,37–44] The only assumption is that the plasma microfield approach is valid.[37–39] This means in particular the neglect of ion perturber configurations penetrating within the bound electron orbits. Although the interaction with such configurations may be treated in the many-body or the binary approach depending on the physical situation, it cannot be expressed in terms of the microfield, and their contribution should usually be suppressed due to the Coulomb repulsion between ions that pass within the bound electron orbits.

Thus, we suppose that $\hat{V}_{0i}$ can be written in the form

$$\hat{V}_{0i}=-\hat{\mathbf{d}}\mathbf{F}+\frac{1}{6}\hat{Q}_{\alpha\beta}\frac{\partial F_\alpha}{\partial x_\beta}-\frac{e^2}{6}\hat{r}^2\nabla\mathbf{F}, \tag{2}$$

$$\hat{r}^2=\sum_l \hat{x}_l^2+\hat{y}_l^2+\hat{z}_l^2, \tag{3}$$

where $\hat{\mathbf{d}}$, $\hat{Q}_{\alpha\beta}$ are the dipole and quadrupole moment operators of the radiator, respectively, with the sign of the electron charge included in their definition; $\hat{x}_l, \hat{y}_l, \hat{z}_l$ are the operators of the Cartesian coordinates of the ''optical electrons'' radiator labelled by l; $\mathbf{F}$ is the ion electric microfield vector, defined at the origin of the coordinate system along with the derivatives of its components. The last term in (2) derives from plasma polarization. Although in this work we shall not use these expressions explicitly, they define the logical basis of our consideration.

Assume that there is an arbitrary set $\{s\}$ of species of ion perturbers in the plasma. Then by virtue of the quasineutrality condition

$$N=N_e=\sum_s Z_sN_s, \tag{4}$$

where N_e is the electron plasma density at infinity and N_s is the partial density of the ion perturbers of species s with effective net charge Z_s. With these definitions, we can start with general expressions for the instantaneous joint simultaneous distribution functions of the individual (but nonbinary) component of the electric ion microfield $\mathbf{F}$ and its time derivatives $\dot{\mathbf{F}}$ at the origin of an arbitrary laboratory system of coordinates,

$$W(\mathbf{F};\dot{\mathbf{F}})=\frac{1}{(2\pi)^6}\int d^3\rho\int d^3\sigma \exp[-i(\boldsymbol{\rho}\mathbf{F}+\boldsymbol{\sigma}\dot{\mathbf{F}})A(\boldsymbol{\rho};\boldsymbol{\sigma}), \tag{5}$$

where $A(\boldsymbol{\rho};\boldsymbol{\sigma})$ denotes the characteristic function of the joint distribution. Equation (5) is derived assuming the additivity of the electric fields from the individual particles and their time derivatives,

$$\mathbf{F}=\sum_j \mathbf{F}_j, \quad \mathbf{F}=\sum_j \dot{\mathbf{F}}_j. \tag{6}$$

The characteristic function of (5) can be expressed in the form

$$A(\boldsymbol{\rho};\boldsymbol{\sigma})=\exp[-NC(\boldsymbol{\rho};\boldsymbol{\sigma})]. \tag{7}$$

Using the generalization of the Baranger-Moser cluster expansion approach,[29–32] we obtain

$$C(\boldsymbol{\rho};\boldsymbol{\sigma})=C^{(0)}(\boldsymbol{\rho};\boldsymbol{\sigma};\xi)-\frac{N}{2!}C^{(1)}(\boldsymbol{\rho};\boldsymbol{\sigma}), \tag{8}$$

$$C^{(0)}(\boldsymbol{\rho};\boldsymbol{\sigma}) =\sum_s C_s\int d^3\mathbf{u}_s w_s(\mathbf{u}_s)\int d^3\mathbf{r} g_{sr}(\mathbf{r})f_s(\boldsymbol{\rho};\mathbf{r};\boldsymbol{\sigma}), \tag{9}$$

$$C^{(1)}(\boldsymbol{\rho};\boldsymbol{\sigma})=\sum_{s,s'} C_sC_{s'}\int d^3\mathbf{u}_s w_s(\mathbf{u}_s)\int d^3\mathbf{u}_{s'}w_{s'}(\mathbf{u}_{s'})$$

$$\times\int d^3\mathbf{r}_1\int d^3\mathbf{r}_2;$$

$$f_s(\boldsymbol{\rho};\mathbf{r}_1;\boldsymbol{\sigma})f_{s'}(\boldsymbol{\rho};\mathbf{r}_2;\boldsymbol{\sigma})[g_{ss'}(\mathbf{r}_1;\mathbf{r}_2)-g_{sr}(\mathbf{r}_1)g_{s'r}(\mathbf{r}_2)], \quad (10)$$

$$f_s(\boldsymbol{\rho};\mathbf{r};\boldsymbol{\sigma})=1-\exp[i\varphi_s(\boldsymbol{\rho};\mathbf{r};\boldsymbol{\sigma})], \quad (11)$$

$$\varphi_s(\boldsymbol{\rho};\mathbf{r};\boldsymbol{\sigma};\boldsymbol{\xi})=\boldsymbol{\rho}\mathbf{E}_s(\mathbf{r})+\boldsymbol{\sigma}\dot{\mathbf{E}}_s(\mathbf{r}). \quad (12)$$

Here $w_s(\mathbf{u}_s)$ is the distribution function of the velocities $\mathbf{u}_s$ of perturbers of species s; $C_s\equiv N_s/N$; $g_{sr}(\mathbf{r})$ is the pair-correlation function between a perturber with charge Z_s and a radiator with net charge Z_0 at the origin of the coordinate system, $g_{ss'}(\mathbf{r}_1;\mathbf{r}_2)$ is the pair-correlation function between perturbers with charges Z_s and $Z_{s'}$ in the field of a radiator with charge Z_0 (in other words, this is a triple correlation function of two perturber ions with charges Z_s and $Z_{s'}$ and the ion-radiator with charge Z_0); $\mathbf{E}_s(\mathbf{r})$ is the elementary electric field, produced by any single ion (quasiparticle) of species s at the origin that has the same instantaneous velocity as the radiator at $t=0$. This field is determined by the effective interaction potential for this sort of particle in plasma, and can be expressed in terms of the following equations:

$$\mathbf{E}_s(\mathbf{r})=-eZ_s\frac{\mathbf{r}}{r^3}[1-\kappa_s(r)], \quad (13)$$

$$\nabla\mathbf{E}_s(\mathbf{r})=\frac{eZ_s}{r^2}\frac{\partial\kappa_s(r)}{\partial r}-4\pi eZ_s\delta(\mathbf{r}), \quad (14)$$

$$\oint_{V\to\infty} d^3\mathbf{r}\nabla\mathbf{E}_s(\mathbf{r})=0. \quad (15)$$

The result on the last line follows from the properties of the screening function $\kappa_s(r)$ that come from its definition: $\kappa_s(0)=0$, $\kappa_s(\infty)=1$. Thus, the excess of the free electron density in the accumulation near the ion Z_s is

$$\delta n_e^{(s)}(r)=\frac{1}{4\pi}\frac{Z_s}{r^2}\frac{\partial\kappa_s(r)}{\partial r}. \quad (16)$$

Now it is seen from (13)–(16) that results of Chandrasekhar and von Neumann can be recovered only if one neglects the neutralizing background of electrons, putting N_e equal to zero.

The time derivatives of the elementary ion field are then

$$\dot{\mathbf{E}}_s(\mathbf{r})=\frac{eZ_s}{r^3}[3\mathbf{n}(\mathbf{n}\mathbf{v}_s)-\mathbf{v}_s]\left[1-\kappa_s(r)+\frac{r}{3}\frac{\partial\kappa_s(r)}{\partial r}\right]$$

$$+\frac{\mathbf{v}_s}{3}\frac{eZ_s}{r^2}\frac{\partial\kappa_s(r)}{\partial r}, \quad (17)$$

$$\ddot{\mathbf{E}}_s(\mathbf{r};\mathbf{v}_s;\dot{\mathbf{v}}_s)=\ddot{\mathbf{E}}_s^{(1)}(\mathbf{r};\dot{\mathbf{v}}_s)+\ddot{\mathbf{E}}_s(\mathbf{r};\mathbf{v}_s), \quad (18)$$

$$\ddot{\mathbf{E}}_s^{(1)}(\mathbf{r};\dot{\mathbf{v}}_s)=\frac{eZ_s}{r^3}[3\mathbf{n}(\mathbf{n}\dot{\mathbf{v}}_s)-\dot{\mathbf{v}}_s]\left[1-\kappa_s(r)+\frac{r}{3}\frac{\partial\kappa_s(r)}{\partial r}\right]$$

$$+\frac{eZ_s}{3r^2}\frac{\partial\kappa_s(r)}{\partial r}\dot{\mathbf{v}}_s, \quad (19)$$

$$\ddot{\mathbf{E}}_s(\mathbf{r};\mathbf{v}_s)=\frac{3eZ_s}{r^4}\left\{[2\mathbf{v}_s(\mathbf{n}\mathbf{v}_s)+\mathbf{n}v_s^2]\left[1-\kappa_s(r)+\frac{r}{3}\frac{\partial\kappa_s(r)}{\partial r}\right]-5\mathbf{n}(\mathbf{n}\mathbf{v}_s)^2\left[1-\kappa_s(r)+\frac{7}{15}r\frac{\partial\kappa_s(r)}{\partial r}-\frac{r^2}{15}\frac{\partial^2\kappa_s(r)}{\partial r^2}\right]\right\}, \quad (20)$$

where $\mathbf{n}\equiv\mathbf{r}/r$ and $\mathbf{v}_s=\mathbf{u}_s-\mathbf{u}_r$ is the relative thermal velocity of the particular perturber species s with respect to the velocity of the radiator $\mathbf{u}_r$.

It can be seen from (19) that terms containing $\dot{\mathbf{u}}_s$ appear i.e.

$$\dot{\mathbf{v}}_s=\dot{\mathbf{u}}_s-\dot{\mathbf{u}}_r=\frac{eZ_s}{m_s}\mathbf{F}(\mathbf{r})-\frac{eZ_r}{m_r}\mathbf{F}(0), \quad (21)$$

where $\mathbf{F}(r)$ is the microfield at the location of the field ion species s, and m_s, m_r are the masses of the perturbers and the radiator respectively. These terms result in nonlinearity and the loss of locality of the joint distribution (5) if one attempts to include them in the second time derivative. In particular, the distribution of the microfield at the origin becomes dependent on the values of the microfield over all space. This inconsistency can be avoided by postulating the constancy of the thermal velocities of the field particles, as was done in the work of Chandrasekhar, namely $\dot{\mathbf{u}}_s=0$ for all s. However, the consequences of neglecting the mutual influence of field particles requires special study beyond the scope of the present paper.

As follows from (13)–(16), the screening function has $\kappa_s(\mathrm{r})\geqslant 0$ and can be determined, for example, by means of one of the recent developments of the DFT-approach,[34] which has a more direct connection with spectral features that can actually be observed. Here we will not present in detail the equations exhibiting the connection with the density distribution of the bound electrons, assuming that the perturbers are strictly bare ions. Also we assume that quantum effects in the microfield distribution can be neglected.[33,37–39] The joint distribution (5)–(20), obtained above, gives the "instantaneous" distribution function of the low-frequency (ion) individual (but many-body!) component of the plasma microfield and its time derivatives, which in fact are determined on a time scale τ of order $\omega_{pe}^{-1}\ll\tau\ll(v_iN_i^{1/3})^{-1}$, where ω_{pe} is the electron plasma frequency, v_i is the relative ion thermal velocity with respect to the radiator, and N_i is the ion density.

The basic ideas that support this derivation were put forward by Baranger and Moser are unchanged since then, despite some differences in later approaches,[27–28,30–32] because they are inherent in the plasma microfield formalism.

It should emphasized once more that by virtue of (2) and (13)–(20), the effects of the neutralizing plasma electron background and its polarization (or in other words the appearance of nonuniformity in the plasma electron density distribution) are included properly in this treatment. Convolution of (5) over the $\mathbf{F}$ or $\dot{\mathbf{F}}$ components leads to separate distributions of the field or its time derivatives, and after the appropriate approximations recovers previous results on the subject.

3. FIRST MOMENTS OF JOINT DISTRIBUTION AND SPEED OF MICROFIELD FLUCTUATIONS

One of the most interesting properties of the joint distributions comes from consideration of the constraint moments, obtained by averaging $\dot{\mathbf{F}}$ (or under special assumptions also $\ddot{\mathbf{F}}$) over the joint distribution (5) at a fixed vector value $\mathbf{F}$

$$W(\mathbf{F})\langle\dot{\mathbf{F}}\rangle_{\mathbf{F}}=\frac{N}{(2\pi)^3}\int d^3\boldsymbol{\rho}\,\exp[-i\boldsymbol{\rho}\mathbf{F}]A(\boldsymbol{\rho})\mathbf{D}(\boldsymbol{\rho}), \quad (22)$$

$$\mathbf{D}(\boldsymbol{\rho})=\mathbf{D}^{(0)}(\boldsymbol{\rho})-\mathbf{D}^{(1)}(\boldsymbol{\rho}), \quad (23)$$

$$\mathbf{D}^{(0)}(\boldsymbol{\rho})=\sum_s C_s\mathbf{D}_0^{(s)}(\boldsymbol{\rho}),$$

$$\mathbf{D}^{(1)}(\boldsymbol{\rho})=\frac{N}{2}\sum_{s,s'} C_sC_{s'}\mathbf{D}^{(ss')}(\boldsymbol{\rho}), \quad (24)$$

$$\mathbf{D}^{(s)}(\boldsymbol{\rho})=\int d^3\mathbf{u}_s w_s(\mathbf{u}_s)\times\int d^3\mathbf{r}\, g_{sr}(\mathbf{r})\exp[i\varphi_s(\boldsymbol{\rho};\mathbf{r})]\dot{\mathbf{E}}_s(\mathbf{r}), \quad (25)$$

$$\mathbf{D}^{(ss')}(\boldsymbol{\rho})=\int d^3\mathbf{u}_s w_s(\mathbf{u}_s)\int d^3\mathbf{u}_{s'}w_{s'}(\mathbf{u}_{s'})\int d^3\mathbf{r}_1\times\int d^3\mathbf{r}_2[g_{ss'}(\mathbf{r}_1;\mathbf{r}_2)-g_{sr}(\mathbf{r}_1)g_{s'r}(\mathbf{r}_2)]\times\{\dot{\mathbf{E}}_s(\mathbf{r}_1)\exp[i\varphi_s(\boldsymbol{\rho};\mathbf{r}_1)](1-\exp[i\varphi_{s'}(\boldsymbol{\rho};\mathbf{r}_2)])+\dot{\mathbf{E}}_{s'}(\mathbf{r}_2)\exp[i\varphi_{s'}(\boldsymbol{\rho};\mathbf{r}_2)](1-\exp[i\varphi_s(\boldsymbol{\rho};\mathbf{r}_1)])\}. \quad (26)$$

The expressions (21)–(25) for the first moments can be rewritten in terms of the microfield distribution function, which can be useful for general analysis:

$$W(\mathbf{F})\langle\dot{\mathbf{F}}\rangle_{\mathbf{F}}=N\sum_s C_s\int d^3\mathbf{u}_s w_s(\mathbf{u}_s)\int d^3\mathbf{r}\, g_{sr}(\mathbf{r})\dot{\mathbf{E}}_s(\mathbf{r})\times W(\mathbf{F}-\mathbf{E}_s(\mathbf{r}))-\frac{N}{2}\sum_{ss'} C_sC_{s'}\int d^3\mathbf{u}_s w_s(\mathbf{u}_s)\times\int d^3\mathbf{u}_{s'}w_{s'}(\mathbf{u}_{s'})\int d^3\mathbf{r}_1\int d^3\mathbf{r}_2[g_{ss'}(\mathbf{r}_1;\mathbf{r}_2)-g_{sr}(\mathbf{r}_1)g_{s'r}(\mathbf{r}_2)]\{\dot{\mathbf{E}}_s(\mathbf{r}_1)[W(\mathbf{F}-\mathbf{E}_s(\mathbf{r}_1))-W(\mathbf{F}-\mathbf{E}_s(\mathbf{r}_1)-\mathbf{E}_{s'}(\mathbf{r}_2))]+\dot{\mathbf{E}}_{s'}(\mathbf{r}_2)\times[W(\mathbf{F}-\mathbf{E}_{s'}(\mathbf{r}_2))-W(\mathbf{F}-\mathbf{E}_s(\mathbf{r}_1)-\mathbf{E}_{s'}(\mathbf{r}_2))]\}. \quad (27)$$

The expressions of $\ddot{\mathbf{F}}$ can be obtained by substituting $\ddot{\mathbf{E}}_s(\mathbf{r})$ for $\dot{\mathbf{E}}_s(\mathbf{r})$ on the right hand side of (24)–(26), if we put $\dot{\mathbf{u}}_s=\theta$ for all $\{s\}$ as discussed earlier. The results (1)–(26) have the most general form.

In order to obtain more detailed expressions, one must make reasonable approximations concerning the correlation functions. Here we assume that the pair correlation functions depend only on the magnitude of the difference vector $\mathbf{r}=\mathbf{r}_1-\mathbf{r}_2$. This enables us to identify and analyze the angle dependence explicitly. However the noncentral terms (see the second citation in Ref. 32) are omitted in this approximation and a calculation of their contribution, although also important, is beyond the scope of the present paper.

We use the Kirkwood approximation to factor the triple correlations; that is,[40–44]

$$g_{sr}(\mathbf{r})\equiv g_{sr}(r),$$

$$g_{ss'}(\mathbf{r}_1;\mathbf{r}_2)\simeq g_{ss'}(|\mathbf{r}_1-\mathbf{r}_2|)g_{sr}(r_1)g_{s'r}(r_2), \quad (28)$$

$$h_{ss'}(|\mathbf{r}_1-\mathbf{r}_2|)\equiv g_{ss'}(|\mathbf{r}_1-\mathbf{r}_2|)-1. \quad (29)$$

The Kirkwood approximation is usually thought to be appropriate mostly for weakly correlated plasmas.[40,41] However, here a mixed approximation is proposed: after applying the Kirkwood approximation for factoring the triple correlation function, the HCN (hypernetted chain) approximation should be used for the pair correlation function. The HCN approximation in turn is thought to be reliable for strongly correlated plasmas[40–43] as well. This procedure should be good for calculations of the microfield distribution that depend on correlation functions as "externally defined variables," i.e., correlation functions must be obtained from other special works. Nor do we consider the influence of dynamic screening and retardation effects on the correlation functions or the other quantities involved. One can then find the following general expressions for (28):

$$h_{ss'}(|\mathbf{r}_1-\mathbf{r}_2|)=\sum_{n=0}^{\infty}(2n+1)P_n\times(\cos[\widehat{\mathbf{r}_1\mathbf{r}_2}])h_{ss'}(n;r_1;r_2), \quad (30)$$

$$h_{ss'}(n;r_1;r_2)=\int_0^{\infty}dk\,k^2\,j_n(kr_1)j_n(kr_2)h_{ss'}(k), \quad (31)$$

$$h_{ss'}(k)=\frac{1}{(2\pi)^3}\int d^3\mathbf{r}\,\exp(i\mathbf{k}\mathbf{r})h_{ss'}(r). \quad (32)$$

Here $P_n(z)$ are the Legendre polynomials with the cosine of the angle between $\mathbf{r}_1$ and $\mathbf{r}_2$ as the argument, while $j_n(y)$ are the spherical Bessel functions. Equations (27)–(31) enable us to simplify the general results (1)–(26) and obtain, for example, for the microfield distribution of the reduced microfield value $\beta\equiv F/F_0$, where F_0 is the normal field,[1,6] more general formulas than were usually applied before. Thus, we have as in Refs. 28, 45, and 46

$$W(\mathbf{F})=4\pi F^2W(F), \quad A(\boldsymbol{\rho})=A(\rho),$$

$$F_0=\Lambda eN^{2/3}, \quad \Lambda\equiv2\pi(4/15)^{2/3}, \quad (33)$$

$$W(\beta)=\frac{2\beta}{\pi}\int_0^{\infty}dk\,k\,\sin k\beta\,A(k),$$

$$A(k)=\exp\{-[\Psi_0(k)-\Psi_1(k)]\}, \quad (34)$$

$$\Psi_0(k)=\frac{4\pi}{\Lambda^{3/2}}\sum_s C_sI_s(k),$$

$$\Psi_1(k)=\frac{8\pi^2}{\Lambda^3}\sum_{ss'} C_sC_{s'}I_{ss'}(k),$$

$$r_0\equiv\left(\frac{e}{F_0}\right)^{1/2}, \tag{35}$$

$$I_s(k)=\int_0^\infty dx\ x^2 g_{sr}(r_0x)\left\{1-\frac{\sin k\epsilon_s(x)}{k\epsilon_s(x)}\right\},$$

$$\epsilon_s(x)=\frac{Z_s}{x^2}[1-\kappa_s(r_0x)], \tag{36}$$

$$I_{ss'}(k)=\int_0^\infty dx_1\ x_1^2\int_0^{x_1}dx_2\ x_2^2\ g_{sr}(r_0x_1)g_{s'r}(r_0x_2)$$
$$\times\sum_{n=0}^{\infty}(-1)^n(2n+1)\{j_n[\epsilon_s(x_1)]-\delta_{0n}\}$$
$$\times\{j_n[\epsilon_{s'}(x_2)]-\delta_{0n}\}h_{ss'}(n;r_0x_1;r_0x_2). \tag{37}$$

To proceed with the first moments of the ion microfield time derivative, we use the following relation between the space and time derivatives:

$$\langle\dot{\mathbf{F}}_i\rangle_{\mathbf{F}}=\left\langle\frac{\partial F_i}{\partial x_k}\right\rangle_{\mathbf{F}}\langle\dot{x}_k\rangle_{\mathrm{v}}\mathbf{e}_i, \tag{38}$$

where $\mathbf{e}_i$ are the unit basis vectors of the Cartesian coordinate system, and the symbol $\langle...\rangle_{\mathrm{v}}$ denotes the average over the perturber velocity distribution due to the first integrals in (24), (25). We then substitute in this relation the expressions for the first moments of the components of the microfield nonuniformity tensor (compare with Ref. 45), which can be obtained in the compact form

$$\left\langle\frac{\partial F_i}{\partial x_k}\right\rangle_{\mathbf{F}}=\frac{2\pi eN}{3}\left\{B_D(\beta)\left(\frac{3F_iF_k}{F^2}-\delta_{ik}\right)\right.$$
$$\left.+2\delta_{ik}B_{D0}(\beta)\right\}, \tag{39}$$

where the universal functions $B_D(\beta)$ and $B_{D0}(\beta)$ are defined by

$$B_D(\beta)=B_D^{(0)}(\beta)-B_D^{(1)}(\beta),$$

$$B_{D0}(\beta)=B_{D0}^{(0)}(\beta)-B_{D0}^{(1)}(\beta), \tag{40}$$

$$B_D^{(0)}(\beta)=\frac{12}{\pi}\frac{\beta^2}{W(\beta)}\sum_s C_sZ_sb_s(\beta),$$

$$B_D^{(1)}(\beta)=\frac{12}{\pi}\frac{\beta^2}{W(\beta)}\sum_{ss'} C_sC_{s'}b_{ss'}(\beta), \tag{41}$$

$$B_{D0}^{(0)}(\beta)=\frac{2}{\pi}\frac{\beta^2}{W(\beta)}\sum_s C_sZ_sb_s^{(0)}(\beta),$$

$$B_{D0}^{(1)}(\beta)=\frac{2}{\pi}\frac{\beta^2}{W(\beta)}\sum_{ss'} C_sC_{s'}b_{ss'}^{(0)}(\beta), \tag{42}$$

$$b_s(\beta)=\int_0^\infty dk\ k^2A(k)j_2(k\beta)\Phi_s(k), \tag{43}$$

$$b_s^{(0)}(\beta)=\int_0^\infty dk\ k^2A(k)j_0(k\beta)\Phi_s^{(0)}(k), \tag{44}$$

$$\Phi_s(k)=\int_0^\infty dx\ x^2g_{sr}(r_0x)j_2[k\epsilon_s(x)]\Phi_s(x), \tag{45}$$

$$\Phi_s^{(0)}(k)=4\pi\int_0^\infty dx\ x^2\ g_{sr}(r_0x)j_0[k\epsilon_s(x)]$$
$$\times\{r_0^3\delta n_e^{(s)}(r_0x)/Z_s\}, \tag{46}$$

$$\Phi_s(x)\equiv\frac{1}{x^3}\left[1-\kappa_s(r_0x)+\frac{x}{3}\frac{\partial\kappa_s(r_0x)}{\partial x}\right], \tag{47}$$

$$b_{ss'}(\beta)=\int_0^\infty dk\ k^2A(k)j_2(k\beta)b_{ss'}(k), \tag{48}$$

$$b_{ss'}^{(0)}(\beta)=\int_0^\infty dkk^2A(k)j_0(k\beta)b_{ss'}^{(0)}(k), \tag{49}$$

$$b_{ss'}(k)=\int_0^\infty dx_1\ x_1^2\int_0^{x_1}dx_2\ x_2^2g_{sr}(r_0x_1)$$
$$\times g_{s'r}(r_0x_2)b_{ss'}(k;x_1;x_2), \tag{50}$$

$$b_{ss'}(k;x_1;x_2)$$
$$=Z_s\Phi_s(x_1)\left\{j_2[k\epsilon_s(x_1)]h_{ss'}(0;r_0x_1;r_0x_2)\right.$$
$$-\sum_{n=0}^{\infty}(-1)^n(2n+1)\left[\left(\frac{3n(n-1)}{2k^2\epsilon_s^2(x_1)}-1\right)j_n[k\epsilon_s(x_1)]\right.$$
$$\left.+\frac{3}{k\epsilon_s(x_1)}j_{n+1}[k\epsilon_s(x_1)]\right]j_n[k\epsilon_{s'}(x_2)]$$
$$\left.\times h_{ss'}(n;r_0x_1;r_0x_2)\right\}, \tag{51}$$

$$b_{ss'}^{(0)}(k)=\int_0^\infty dx_1\ x_1^2\int_0^{x_1}dx_2\ x_2^2\ g_{sr}(r_0x_1)$$
$$\times g_{s'r}(r_0x_2)b_{ss'}^{(0)}(k;x_1;x_2), \tag{52}$$

$$b_{ss'}^{(0)}(k;x_1;x_2)=4\pi r_0^3\delta n_e(r_0x_1)$$
$$\times\left\{j_0[k\epsilon_s(x_1)]h_{ss'}(0;r_0x_1;r_0x_2)\right.$$
$$-\sum_{n=0}^{\infty}(-1)^n(2n+1)j_n[k\epsilon_s(x_1)]$$
$$\left.\times j_n[k\epsilon_{s'}(x_2)]h_{ss'}(n;r_0x_1;r_0x_2)\right\}. \tag{53}$$

Substituting (38) into (37) and cycling through the components of $\mathbf{F}$ and $\mathbf{u}_r$, the desired result is

$$\langle \dot{\mathbf{F}} \rangle_{\mathbf{F}} = -\frac{2\pi e N}{3} \{B_D(\beta)(3(\mathbf{n}_F\mathbf{u}_r)\mathbf{n}_F - \mathbf{u}_r) + 2B_{D0}(\beta)\mathbf{u}_r\},$$

$$n_F \equiv \frac{\mathbf{F}}{F}. \qquad (54)$$

The contribution of the field variation due to ion perturbers vanishes as a consequence of the assumed spherical symmetry of the perturber velocity distributions. It can also be seen from this result that the contributions of polarization effects, coming from $B_D(\beta)$ and $B_{D0}(\beta)$, have different signs in the expression for the vector directed along $\mathbf{u}_r$, but do not cancel each other due to the different symmetry of the interactions (quadrupole and scalar respectively) that give rise to these terms. This is also the case in the coefficient associated with $\mathbf{F}$. Using the relations

$$\dot{\mathbf{u}}_r = \frac{eZ_r}{m_r} \mathbf{F}(0), \qquad (55)$$

where m_r is the mass of the radiator, it is possible to write an expression for the part of the second time derivative of the field, assuming $\mathbf{u}_s=0$ for all $\{s\}$, due to the time independence of the ion perturber ensemble:

$$\langle \ddot{\mathbf{F}}^{(1)} \rangle_{\mathbf{F}} = -\frac{4\pi e^2 Z_r N}{3} [B_D(\beta) + B_{D0}(\beta)] \frac{\mathbf{F}(0)}{m_r}. \qquad (56)$$

This expression with the opposite sign is proportional to the zz component of the microfield nonuniformity tensor in the coordinate system in which $\mathbf{F}$ points in the direction of $\mathbf{Z}$:

$$\langle \ddot{\mathbf{F}}^{(1)} \rangle_{\mathbf{F}} = -eZ_r \left\langle \frac{\partial F_z}{\partial Z} \right\rangle_{\mathbf{F} \parallel \mathbf{Z}} \frac{\mathbf{F}(0)}{m_r}. \qquad (57)$$

The results (54) and (56) show a new physical effect in the theory of dynamical friction: the friction induced by the electric field of the electron neutralizing background. This is the physical sense of the introduced polarization terms. These terms do not vanish if one takes the OCP limit for ions, when this background has a constant uniform density in the space. That is why the transition to the previous results of Chandrasekhar and von Neumann evidently is possible only if we artificially set this density equal to zero.

4. MODIFICATION OF JOINT DISTRIBUTIONS IN LIMITS OF WEAK AND STRONG PLASMA COUPLING

The validity of the generalized Baranger–Moser approach, strictly speaking, cannot be proved for strongly coupled plasma, although at the same time there are no rigorous objections against its use for nonideal plasmas. That is why we consider here two additional approximations which traditionally are thought to correspond to weak and strong plasma coupling. In the limit of weak plasma coupling, when the ion–ion and the ion–electron coupling parameters are at most of order unity, it is possible to use the Debye-Hückel approximation with linearization of the perturber–perturber correlation function.[28] The results in this approximation could be obtained from the general expressions written above by the following substitutions

$$\kappa_s(r_0 x) \to \kappa_s^D(x) \equiv 1 - \exp[-ax](1+ax),$$

$$a \equiv \frac{r_0}{r_D}, \quad r_D \equiv \sqrt{\frac{T_e}{4\pi e^2 N_e}}, \qquad (58)$$

$$g_{sr}(r_0 x) \to \exp\left[-Z_0 Z_S \Theta a^2 \frac{\Lambda^{3/2}}{4\pi} \frac{\exp[-ax]}{x}\right],$$

$$\Theta \equiv \frac{T_e}{T_i}, \qquad (59)$$

$$h(n;x_1;x_2) \to -\Theta a^3 \frac{\Lambda^{3/2}}{4\pi} f_n^{>}(ax_1) f_n^{<}(ax_2), \qquad (60)$$

$$f_n^{>}(z) \equiv (-1)^n z^n \left(\frac{d}{zdz}\right)^n \frac{e^{-z}}{z},$$

$$f_n^{<}(z) \equiv z^n \left(\frac{d}{zdz}\right)^n \frac{\sinh(z)}{z}, \qquad (61)$$

where in (60) we used the traditional notation from Ref. 30, and T_1, T_e are the ion and electron temperatures in plasmas respectively.

For large values of the ion–ion coupling parameter it is convenient to exploit the Adjustable Parameter Exponential (APEX) Approximation developed in Refs. 47, 48. Attempts to use APEX[47,48] to treat the microfield time dependence[17–19] and its spatial derivatives[35–36,49] in strongly coupled plasmas were made recently by several groups of researchers. We prefer to follow here the general approach from Refs. 35, 36, 45 and the original work on the APEX approach.[47,48] The APEX field distribution for the microfield is the distribution with specified constraints on the second moment of F with modified Debye screening of the elementary electric field.

In the present case in the construction of the joint distribution function it is necessary to use an expression for the time derivative of the elementary electric field not related directly to the APEX field expression, but which in principle should be obtained from the initial field expression (13), which comes from physical considerations and satisfies Eq. (15).[45,46] In other words in order to obtain the true joint distribution, one must have the freedom to convolve over some of the independent variables to get the independent distribution over the other part also of the independent variables entering in the definition of the joint distribution. However in the APEX approach, as we see below, these requirements may be fulfilled only partly. Indeed the generalization of the APEX approach in our notation is obtained by the following changes in the formulas of the preceding section

$$C^{(1)}(\boldsymbol{\rho}) \equiv 0, \quad k_s^{\mathrm{APEX}}(r) = 1 - \exp[-a\alpha_s r](1 + a\alpha_s r), \qquad (62)$$

$$g_{sr}^{\mathrm{APEX}}(r) = g_{sr}(r) \frac{E_s(r)}{E_s^{\mathrm{APEX}}(r)}, \qquad (63)$$

$$\varphi_s^{\mathrm{APEX}}(\boldsymbol{\rho};\mathbf{r};\boldsymbol{\sigma};\boldsymbol{\xi}) = \boldsymbol{\rho}\mathbf{E}_s^{\mathrm{APEX}}(\mathbf{r}) + \boldsymbol{\sigma}\dot{\mathbf{E}}_s(\mathbf{r}), \qquad (64)$$

where α_s is the fitting APEX parameter for perturber ions of species s.[57] On the other hand, the virtue of the APEX approximation, which is formally confined to this substitution, is that it inevitably renormalizes the pair distributions for the other variables. This is the price paid for restricting the space of the independent variables by the conditions at $Z_0 \neq 0$

$$\langle F^2\rangle = \int dF\, W(F)F^2 = \frac{4\pi N T_i}{Z_0}\sum_s Z_s C_s \mathcal{T}_s, \qquad (65)$$

$$\mathcal{T}_s = \int_0^\infty dx\, g_{sr}(r_0 x)\,\frac{\partial \kappa_s(r_0 x)}{\partial x}, \quad a \equiv \frac{r_0}{r_D},$$

$$r_D \equiv \sqrt{\frac{T_e}{4\pi e^2 N_e}}, \qquad (66)$$

$$\sum_s C_s \int_0^\infty dr\, r^2\, g_{sr}(r) E_s(r) E_s^{\mathrm{APEX}}(r)$$

$$= \frac{T_i}{Z_0}\sum_s Z_s C_s \mathcal{T}_s, \qquad (67)$$

$$\mathcal{T}_s = \frac{\Lambda^{3/2}}{4\pi}\,\Theta a^2 Z_0 Z_s \int_0^\infty dx\, \frac{g_{sr}(r_0 x)}{x^2}$$

$$\times (1-\kappa_s(r_0 x))(1 + a\alpha_s x)\exp(-a\alpha_s x), \qquad (68)$$

where $\{\alpha_s\}$ is the set of parameters determined from (64)–(67).

It can easily be shown that the results (1)–(56) of the preceding sections can be reexpressed in terms of the modified APEX, using (61)–(67). On the other hand they differ from them by the absence of the correlation terms that stem from $C^{(1)}(\rho)$, and by the renormalized perturber–radiator pair distribution function (62). In principle, the correlation terms can be recovered in APEX also, by using its renormalized version[48] in the OCP model.[41] Thus, the results outlined in this section are quite sufficient to exploit the APEX approach together with the results of the preceding one, so we will not present here the lengthy formulas that are now evident from our consideration.

Let us take the case of a single species s_0 of ion perturbers and rewrite the expression for the first moment taking into account (21)–(26) and (61)–(67) in the form

$$W(\mathbf{F})\langle \dot{\mathbf{F}}\rangle_{\mathbf{F}} = \int d^3\mathbf{r}\, \dot{\mathbf{E}}_{s_0}(\mathbf{r}) g_{s_0 r}(r)\,\frac{E_{s_0}(r)}{E_{s_0}^{\mathrm{APEX}}(r)}$$

$$\times W(\mathbf{F} - \mathbf{E}_{s_0}^{\mathrm{APEX}}(\mathbf{r})). \qquad (69)$$

It is obvious from the above expression that our results include renormalization and plasma polarization effects. Concerning the nearest-neighbor approximation limit (NNA) from (68), it is easily seen that the renormalization factor goes to unity at small r, and thus the NNA limit should be recovered. On the other hand, if we convolve the joint distribution constructed in the APEX scheme over **F**, we get the distribution over the first (or second) time derivative of the microfield with the renormalized correlation function. So although there is no reason to use the time derivatives of the APEX electric field instead of the time derivatives of the initial elementary ion electric field in the construction of the joint distribution, one cannot restore the free distribution for the time derivative, which should follow from the starting formulas after the convolution over **F**.

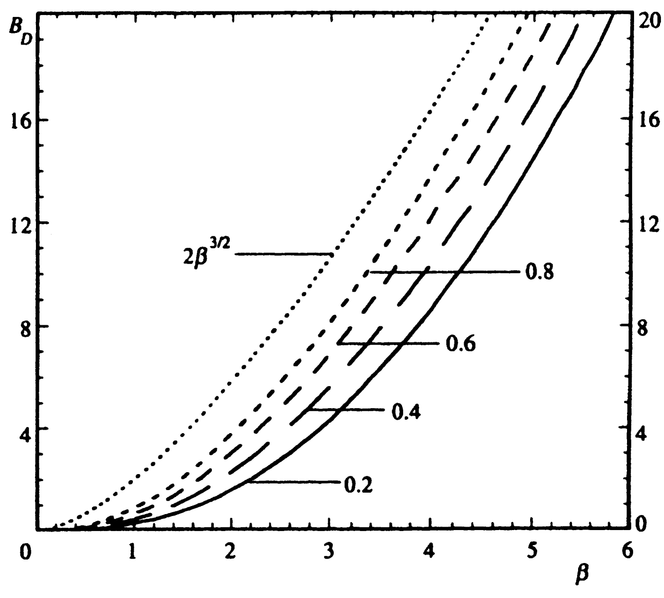

FIG. 1. Function $B_D(\beta)$ versus β for several values of the ratio $a \equiv r_0/r_{De}$ at charged point.

The case of the neutral radiator can be treated in a slightly different manner in accordance with the original work on APEX.[39] The reasoning concerning the choice of the expression for the time derivatives of the elementary electric field presented above is confirmed in the case of the neutral point by the corresponding results from Ref. 18.

As stated by APEX's authors,[39] its applicability was checked up to $\Gamma_i \sim 100$. On the other hand, from general considerations for larger Γ_i one should get a Gaussian for the microfield distribution function, corresponding to the physical picture of very low particle kinetic energy compared to their potential energy and connected with their oscillations near the equilibrium sites. In this case such a simple approximation as NNA fails, and one should apply the ideas of several nearest neighbors that are so familiar in the theory of the intercrystal electric field.

In part these trends come to the APEX approximation through the HCN correlation functions. It is interesting that when Γ_i is increasing, the plasma ion frequency is less than ω_{pe} for any real value of Z_s, but may become larger than $(v_i N_i^{1/3})^{-1}$, and we can have $r_{Di} \ll N_i^{-1/3}$ and $r_{Di} \ll r_D \equiv r_{De}$. Thus, while at small Γ_i the plasma ion modes are slower than the individual particle motion, at large $\Gamma_i \gg 1$ this relation is reversed. That is why for large Γ it is also necessary to consider the statistical subsystem of the collective plasma oscillations[20] and the electric field produced by them, which is considered in this work after the derivation of the asymptotic expansions for $W(\beta), B_D(\beta), B_{D0}(\beta)$.

The functions $W(\beta), B_D(\beta), B_{D0}(\beta)$ were compared in detail for various values of the plasma parameters[45,46] in the Baranger–Moser.[28,45] Monte-Carlo (D. Gilles) and APEX[47,48] approaches.[35–36,49] These studies show the main trends of their dependence on the plasma coupling and other parameters.

The characteristic behavior of the $B_D(\beta), B_{D0}(\beta)$ universal functions is illustrated in Figs. 1 and 2. The results

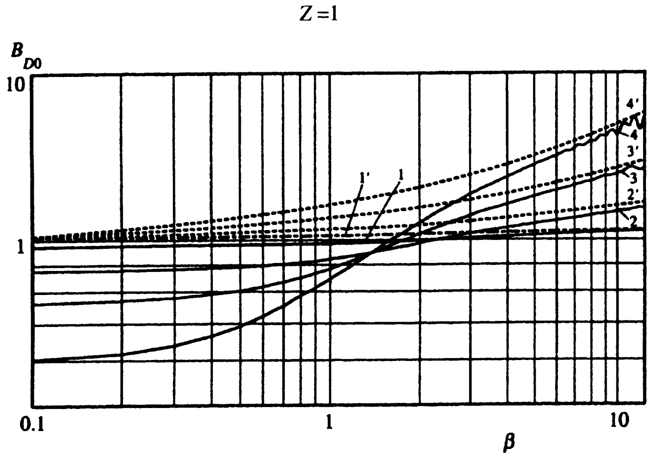

FIG. 2. Function $B_{D0}(\beta)$ versus β for several values of the ratio $a \equiv r_0/r_{De}$ at charged point $Z=1$: *1)* $a=0.2$, *1')* $1+(0.2)^2\sqrt{\beta/3}$, *2)* $a=0.4$, *2')* $0.992+(0.4)^2\sqrt{\beta/3}$, *3)* $a=0.6$, *3')* $0.976+(0.6)^2\sqrt{\beta/3}$, *4)* $a=0.8$, *4')* $0.949+(0.8)^2\sqrt{\beta/3}$.

were obtained by Chantal Stehlé in the linearized Debye–Hückel approximation for the pair (ion–ion) correlation function, and depend on the parameter a, which is the ratio of the characteristic length scale coming from the definition of the normal field F_0 to the electron Debye radius.

In conclusion it should be noted that the algorithm of the APEX application considered is still incomplete.

First of all, as was discussed earlier, we failed to construct a joint distribution with the help of the APEX ideas that would give after convolution over the microfield a free distribution over the time derivatives of the electric microfield, i.e., not limited by any constraints. Therefore this distribution cannot serve as an initial value for the problem of the time evolution of the joint distributions.[18]

Secondly, if we take $E_s(\mathrm{r})$ in the Debye form, then when the APEX screening in strongly correlated plasmas becomes larger than the screening of $E_s(r), g_{sr}^{\mathrm{APEX}}(r)$ loses its physical sense. Specifically, $\int d^3 r g_{sr}^{\mathrm{APEX}}(r)$ becomes infinite, although the APEX microfield distribution gives correct results. However, here we proposed using the DFT screening function $\kappa(r)$, which can have an effective screening parameter enough larger than the APEX one for such cases. If this is not true, the definition of $B_{D0}^{\mathrm{APEX}}(\beta)$ given here will lead to divergence for cases when the APEX screening exceeds twice the value of the DFT screening. Then the diagonal components of $\langle \partial F_i / \partial x_s \rangle_{\mathrm{F}}$ and thus $\langle \mathbf{F} \rangle_{\mathrm{F}}$ cannot be determined by the method outlined above. The same is true of corresponding results in Refs. 35, 36, p. 89, 45, 46 as well.[1)]

The simplest way out of this difficulty consists in adopting $\partial(\mathbf{E}_s(\mathbf{r}))_i/\partial x_k = \partial(\mathbf{E}_s^{\mathrm{APEX}}(\mathbf{r}))_i/\partial x_k$, thus making the joint distribution more restricted. Obviously it is now not so important, because one cannot obtain the joint distribution of the completely independent variables in APEX. Therefore this step could not make the situation more controversial than before.

Another way, perhaps less rigorous, is connected with the fact that the function $B_{D0}(\beta)$ has the meaning of a neutralizing charge, and in order to compensate the drawbacks of $g_{sr}^{\mathrm{APEX}}(r)$ it is natural to use the renormalized version of $B_{D0}^{\mathrm{APEX}}(\beta)$ for these situations, when formally the integral over the APEX charge distribution diverges, inserting $j_0[k\epsilon_s(x)]-1$ in the integrand of the Eq. (46) instead of only $j_0[k\epsilon_s(x)]$.

Anyway all this shows that the question of the APEX application still needs additional study.

5. ASYMPTOTIC EXPANSIONS FOR $W(\beta), B_D(\beta), B_{D0}(\beta)$

From these equations it follows that the functions $B_D(\beta)$ and $B_{D0}(\beta)$ have the following asymptotic dependence when β is small:

$$W(\beta) \approx \frac{4}{3\pi}\nu\beta^2\left[1-\frac{1}{6}\frac{\nu_2}{\nu}\beta^2\right], \quad \beta \ll 1, \tag{70}$$

$$\nu = \frac{3}{2}\int_0^\infty dk\, k^2 A(k), \quad \nu_2 = \frac{3}{2}\int_0^\infty dk\, k^4 A(k), \tag{71}$$

$$B_D(\beta) \approx \beta^2 \frac{3}{5\nu}\left[\sum_s C_s Z_s J_s^{(2)} + \sum_{ss'} C_s C_{s'} J_{ss'}^{(2)}\right],$$

$$\beta \ll 1, \tag{72}$$

$$J_s^{(2)} = \int_0^\infty dk\, k^4\, A(k)\Phi_s(k),$$

$$J_{ss'}^{(2)} = \int_0^\infty dk\, k^4\, A(k) b_{ss'}(k), \tag{73}$$

$$\begin{aligned} B_{D0}(\beta) \approx \frac{3}{2\nu}\Bigg[&\left(\sum_s C_s Z_s \mathcal{J}_s^{(0)} + \sum_{ss'} C_s C_{s'} \mathcal{J}_{ss'}^{(0)}\right) \\ &- \frac{\beta^2}{6}\left(\sum_s C_s Z_s\left[\mathcal{J}_s^{(2)} - \frac{\nu_2}{\nu}\mathcal{J}_s^{(0)}\right]\right. \\ &\left.+ \sum_{ss'} C_s C_{s'}\left[\mathcal{J}_{ss'}^{(2)} - \frac{\nu_2}{\nu}\mathcal{J}_{ss'}^{(0)}\right]\right)\Bigg], \quad \beta \ll 1, \end{aligned} \tag{74}$$

$$\mathcal{J}_s^{(0)} = \int_0^\infty dk\, k^2\, A(k)\Phi_s^{(0)}(k),$$

$$\mathcal{J}_{ss'}^{(0)} = \int_0^\infty dk\, k^2\, A(k) b_{ss'}^{(0)}(k), \tag{75}$$

$$\mathcal{J}_s^{(2)} = \int_0^\infty dk\, k^4\, A(k)\Phi_s^{(0)}(k),$$

$$\mathcal{J}_{ss'}^{(2)} = \int_0^\infty dk\, k^4\, A(k) b_{ss'}^{(0)}(k). \tag{76}$$

It is clear from (71), (73) that the asymptotic functional dependence on β of the B-functions for $\beta \ll 1$ (the difference in coefficients notwithstanding) is not changed by the screening, correlations, or repulsion, with respect to the pure Coulomb case.[1] But it is changed by polarization effects.[28,36,45,46] The asymptotic behavior of (55) can be simply determined from (69)–(75) by multiplying by β. The contributions of the correlations enter (71) and (73) through the double sums.

These results in the case of the Debye–Hückel approximation for the correlation functions largely agree with corresponding results from.[27,28,36]

In the case of large β one can use implicit variables to obtain an asymptotic expansion in a way similar to Ref. 31:

$$W(\beta)\sim W_{AS}(\beta)\equiv\frac{15}{4\sqrt{2\pi}}\sum_s \frac{C_s y_s^5}{Z_s}\frac{\exp[-V(y_s)]}{Dn(y_s)},$$

$$\beta\gg 1, \tag{77}$$

$$V(y_s)\equiv-\ln[g_{sr}(y_s)]+\frac{15}{2\sqrt{2\pi}}C_s\int_0^{y_s}dx\, x^2\, g_{sr}(x), \tag{78}$$

$$Dn(y_s)\equiv\left|1-\kappa_s(y_s)+\frac{y_s}{2}\frac{\partial\kappa_s(y_s)}{\partial y_s}\right|,$$

$$\beta=\frac{Z_s}{y_s^2}[1-\kappa_s(y_s)], \tag{79}$$

$$B_D(\beta)\sim\frac{3}{W_{AS}(\beta)}\sum_s C_s y_s^2 Nm(y_s)\frac{\exp[-V(y_s)]}{Dn(y_s)},$$

$$\beta\gg 1, \tag{80}$$

$$Nm(y_s)\equiv\left[1-\kappa_s(y_s)+\frac{y_s}{3}\frac{\partial\kappa_s(y_s)}{\partial y_s}\right], \tag{81}$$

$$B_{D0}(\beta)\sim\frac{1}{2W_{AS}(\beta)}\sum_s C_s y_s^3\frac{\partial\kappa_s(y_s)}{\partial y_s}\frac{\exp[-V(y_s)]}{Dn(y_s)},$$

$$\beta\gg 1. \tag{82}$$

The asymptotic expressions in the Debye–Hückel approximation or for the APEX approach can be readily obtained from Eqs. (69)–(81) by the replacements outlined above in the previous two sections. These results in the case of the Debye–Hückel approximation for the correlation functions for the most part agree with corresponding results of Refs. 27, 28, 36, and 46.

6. JOINT DISTRIBUTIONS INCLUDING COLLECTIVE LOW-FREQUENCY PLASMA OSCILLATIONS

These results allow us to consider with much more generality the problem of including the low-frequency collective plasma oscillations in the joint distribution function.[50] It is now possible because the problem of dividing the microfield in plasmas into individual and collective parts was treated in detail in several outstanding publications[51–53] following the idea of Bohm and Pines of collective variables.[54] That is why we shall take the possibility of this separation for granted. Evidently, when there is a significant level of low-frequency oscillations in plasmas the joint distribution should take into account its contribution. Assuming that the statistical subsystems of the individual and collective motions are independent, the full or global joint distribution function $W_G(\mathbf{F}_G;\dot{\mathbf{F}}_G)$ is the convolution of the joint distributions $W(\mathbf{F};\dot{\mathbf{F}})$ from the sections above with the joint distribution $W_c(\mathbf{F}_c;\dot{\mathbf{F}}_c)$ of the collective field $\mathbf{F}_c$ and its time derivatives $\dot{\mathbf{F}}_c$;

$$\mathbf{F}_G=\mathbf{F}+\mathbf{F}_c,\quad \dot{\mathbf{F}}_G=\dot{\mathbf{F}}+\dot{\mathbf{F}}_c, \tag{83}$$

$$W_G(\mathbf{F}_G;\dot{\mathbf{F}}_G)=\frac{1}{(2\pi)^6}\int d^3\boldsymbol{\rho}\int d^3\boldsymbol{\sigma}\exp[-i(\boldsymbol{\rho}\mathbf{F}_G+\boldsymbol{\sigma}\dot{\mathbf{F}}_G)]A_G(\boldsymbol{\rho};\boldsymbol{\sigma}), \tag{84}$$

$$A_G(\boldsymbol{\rho};\boldsymbol{\sigma})=A(\boldsymbol{\rho};\boldsymbol{\sigma})A_c(\boldsymbol{\rho};\boldsymbol{\sigma}). \tag{85}$$

The last equation is due to the convolution structure of the global distribution. It follows immediately that the separate distributions of $\mathbf{F}_G$ and $\dot{\mathbf{F}}_G$ are defined by

$$W_G(\mathbf{F}_G)=\frac{1}{(2\pi)^3}\int d^3\boldsymbol{\rho}\int d^3\boldsymbol{\sigma}\exp[-i\boldsymbol{\rho}\mathbf{F}_G]A_G(\boldsymbol{\rho};0), \tag{86}$$

$$A_G(\boldsymbol{\rho};0)=A(\boldsymbol{\rho};0)A_c(\boldsymbol{\rho};0), \tag{87}$$

$$W_G(\dot{\mathbf{F}}_G)=\frac{1}{(2\pi)^3}\int d^3\boldsymbol{\rho}\int d^3\boldsymbol{\sigma}\exp[-i\boldsymbol{\sigma}\dot{\mathbf{F}}_G]A_G(0;\boldsymbol{\sigma}), \tag{88}$$

$$A_G(0;\boldsymbol{\sigma})=A(0;\boldsymbol{\sigma})A_c(0;\boldsymbol{\sigma}). \tag{89}$$

Consider a multimode oscillatory field. Then

$$\mathbf{F}_c=\sum_{\kappa,j}\mathbf{E}_{\kappa,j}\cos(\omega_\kappa t-\boldsymbol{\kappa}\mathbf{r}_a+\Psi_{\kappa,j}),$$

$$\dot{\mathbf{F}}_c=-\sum_{\kappa,j}\mathbf{E}_{\kappa,j}(\omega_\kappa-\boldsymbol{\kappa}\mathbf{v}_a)\sin(\omega_\kappa t-\boldsymbol{\kappa}\mathbf{r}_a+\Psi_{\kappa,j}), \tag{90}$$

where $\mathbf{E}_{\kappa,j}$ is the vectorial amplitude of the oscillations characterized by the wave vector $\boldsymbol{\kappa}$, the mode index j, and the random phase $\Psi_{\kappa,j}$ uniformly distributed over the interval $(0,2\pi)$; $\mathbf{r}_a$, u_a are the position of a radiator and its velocity respectively. In accordance with the spirit of the quasistatic approximation in the theory of Stark broadening in plasmas, we are interested in the determination of a simultaneous stationary distribution that can be observed on time scales much less than $\omega_\kappa^{-1}\sim\omega_{pi}^{-1}$. That is why without loss of generality we can put $t=0$ and $\mathbf{r}_a=0$.

In the three-dimensional case of isotropic distributions for amplitudes $W(\mathbf{E}_\kappa)=W(E_\kappa)/4\pi E_\kappa^2$ and wave vectors $w(\boldsymbol{\kappa})=w(\kappa)/4\pi\kappa^2$, applying methods from the well known review of Chandrasekhar (see the second citation in Ref. 1, p. 14, Eq. (91)) we arrive at the following remarkable result, if the total number of modes $\mathcal{N}$ tends to infinity, while the energy density of plasma oscillations is finite (compare with Ref. 53)

$$W_c(\mathbf{F}_c;\dot{\mathbf{F}}_c)=W_c(\mathbf{F})W_c(\dot{\mathbf{F}}). \tag{91}$$

The factorization of these distributions should exist in a space of any integer dimension, i.e., for one-, two-, and three-dimensional turbulence. This means that the distributions of the collective microfield and its time derivative are independent in the multimode case. This independence results from the opposite phases in the time evolution of the field and its time derivative.

The functional dependence of these distributions evidently has a Gaussian form, and depends on the amplitude of the oscillations

$$W_c(\mathbf{F}_c)=3\left(\frac{6}{\pi}\right)^{1/2}\frac{F_c^2}{\langle F_{0c}^2\rangle^{3/2}}\exp\left[-\frac{3}{2}\frac{F_c^2}{\langle F_{0c}^2\rangle}\right], \tag{92}$$

$$W_c(\dot{\mathbf{F}})=3\left(\frac{6}{\pi}\right)^{1/2}\frac{\dot{\mathbf{F}}_c^2}{\langle\dot{\mathbf{F}}_{0c}^2\rangle^{3/2}}\exp\left[-\frac{3}{2}\frac{\dot{\mathbf{F}}_c^2}{\langle\dot{\mathbf{F}}_{0c}^2\rangle}\right], \tag{93}$$

$$\langle F_{0c}^2\rangle=\frac{1}{2}\int_0^\infty d\kappa\langle E_\kappa^2\rangle w(\kappa),$$

$$\langle E_\kappa^2\rangle=\int_0^\infty dE_\kappa E_\kappa^2 W(E_\kappa),$$

$$\langle\dot{\mathbf{F}}_{0c}^2\rangle=\frac{1}{2}\int_0^\infty d\kappa\left(\omega_\kappa^2+\frac{1}{3}\kappa^2 v_a^2\right)\langle E_\kappa^2\rangle w(\kappa).$$

Here we defined the probability for a given pair E_κ and κ by $W(E_\kappa)w(\kappa)/\mathcal{N}$. Thus $\mathcal{N}$ is canceled in the expressions for the mean squares listed above. However, for $\mathbf{E}_{\kappa,j}\|\kappa$ the factorization exists only if a radiator is at rest, $v_a=0$. But for our treatment (see below) of the second constraint moment of the global microfield time derivative, it is sufficient that $A_c(\rho;\sigma)$ be a quadratic function of ρ and σ that is provided by the average over the random phase $\Psi_{\kappa,j}$.

It is important that this type of function can be applied to the nonequilibrium distributions over degrees of freedom in plasmas.

The second moments of the global joint distribution are defined by

$$\langle(\mathbf{F}_G)_\alpha(\mathbf{F}_G)_\beta\rangle=\lim_{\rho_j\to 0}\frac{\partial^2 A_G(\boldsymbol{\rho};0)}{\partial\rho_\alpha\partial\rho_\beta}, \tag{94}$$

$$\langle(\dot{\mathbf{F}}_G)_\alpha(\dot{\mathbf{F}}_G)_\beta\rangle=\lim_{\sigma_j\to 0}\frac{\partial^2 A_G(0;\boldsymbol{\sigma})}{\partial\sigma_\alpha\partial\sigma_\beta}. \tag{95}$$

As can be seen from these results and Eq. (85), they can be expressed in terms of the analogous quantities defined for the individual and collective subsystems. For an individual subsystem, in general

$$\begin{aligned}\langle F_\alpha F_\beta\rangle=&N\sum_s C_s\int d^3\mathbf{r}\, g_{rs}(\mathbf{r})(\mathbf{E}_s(\mathbf{r}))_\alpha(\mathbf{E}_s(\mathbf{r}))_\beta\\&+N^2\sum_{s,s'}C_sC_{s'}\int d^3\mathbf{r}_1\int d^3\mathbf{r}_2\, g_{rs}(\mathbf{r}_1)g_{rs'}(\mathbf{r}_2)\\&\times[g_{ss'}(\mathbf{r}_1;\mathbf{r}_2)-g_{rs}(\mathbf{r}_1)g_{rs'}(\mathbf{r}_2)]\\&\times(\mathbf{E}_s(\mathbf{r}_1))_\alpha(\mathbf{E}_{s'}(\mathbf{r}_2))_\beta,\end{aligned} \tag{96}$$

$$\begin{aligned}\langle\dot{\mathbf{F}}_\alpha\dot{\mathbf{F}}_\beta\rangle=&N\sum_s C_s\int d^3\mathbf{r}\, g_{rs}(\mathbf{r})(\dot{\mathbf{E}}_s(\mathbf{r}))_\alpha(\dot{\mathbf{E}}_s(\mathbf{r}))_\beta\\&+N^2\sum_{s,s'}C_sC_{s'}\int d^3\mathbf{r}_1\int d^3\mathbf{r}_2\, g_{rs}(\mathbf{r}_1)g_{rs'}(\mathbf{r}_2)\\&\times[g_{ss'}(\mathbf{r}_1;\mathbf{r}_2)-g_{rs}(\mathbf{r}_1)g_{rs'}(\mathbf{r}_2)]\\&\times(\dot{\mathbf{E}}_s(\mathbf{r}_1))_\alpha(\dot{\mathbf{E}}_{s'}(\mathbf{r}_2))_\beta.\end{aligned} \tag{97}$$

At a neutral point, these expressions diverge due to the terms in the single-index sums. This can be eliminated only by incorporating quantum effects at small distances. These results are exact for real plasmas (compare with Ref. 38); we wrote these equations together to show the remarkable coincidence in their structure.

The other interesting quantities in which nontrivial features might show up in this approach are the second constraint moments of the global microfield time derivative

$$W(\mathbf{F}_G)\langle(\dot{\mathbf{F}}_G)_\alpha(\dot{\mathbf{F}}_G)_\beta\rangle_{\mathbf{F}_G}=\frac{1}{(2\pi)^3}\int d^3\boldsymbol{\rho}\exp[-i\boldsymbol{\rho}\mathbf{F}_G]\times\lim_{\sigma_j\to 0}\frac{\partial^2 A_G(\boldsymbol{\rho};\boldsymbol{\sigma})}{\partial\sigma_\alpha\partial\sigma_\beta}, \tag{98}$$

$$\begin{aligned}\lim_{\sigma_j\to 0}\left[\frac{\partial^2 A_G(\boldsymbol{\rho};\boldsymbol{\sigma})}{\partial\sigma_\alpha\partial\sigma_\beta}\right.&=\frac{\partial^2 A(\boldsymbol{\rho};\boldsymbol{\sigma})}{\partial\sigma_\alpha\partial\sigma_\beta}A_c(\boldsymbol{\rho};\boldsymbol{\sigma})+\frac{\partial A(\boldsymbol{\rho};\boldsymbol{\sigma})}{\partial\sigma_\alpha}\frac{\partial A_c(\boldsymbol{\rho};\boldsymbol{\sigma})}{\partial\sigma_\beta}\\&\left.+\frac{\partial A(\boldsymbol{\rho};\boldsymbol{\sigma})}{\partial\sigma_\beta}\frac{\partial A_c(\boldsymbol{\rho};\boldsymbol{\sigma})}{\partial\sigma_\alpha}+A(\boldsymbol{\rho};\boldsymbol{\sigma})\frac{\partial^2 A_c(\boldsymbol{\rho};\boldsymbol{\sigma})}{\partial\sigma_\alpha\partial\sigma_\beta}\right],\end{aligned} \tag{99}$$

$$\begin{aligned}\lim_{\sigma_j\to 0}\frac{\partial^2 A(\boldsymbol{\rho};\boldsymbol{\sigma})}{\partial\sigma_\alpha\partial\sigma_\beta}&=A(\boldsymbol{\rho})\Bigg[N\sum_s C_s\int d^3\mathbf{r}\, g_{rs}(\mathbf{r})(\dot{\mathbf{E}}_s(\mathbf{r}))_\alpha(\dot{\mathbf{E}}_s(\mathbf{r}))_\beta\\&\times\exp[i\boldsymbol{\rho}\mathbf{E}_s(\mathbf{r})]-\frac{N^2}{2}\sum_{s,s'}C_sC_{s'}\\&\times\int d^3\mathbf{r}_1\int d^3\mathbf{r}_2\, g_{rs}(\mathbf{r}_1)g_{rs'}(\mathbf{r}_2)[g_{ss'}(\mathbf{r}_1;\mathbf{r}_2)-g_{rs}\\&\times(\mathbf{r}_1)g_{rs'}(\mathbf{r}_2)]\{(1-\exp[i\boldsymbol{\rho}\mathbf{E}_{s'}(\mathbf{r}_2)])\exp[i\boldsymbol{\rho}\mathbf{E}_s\\&\times(\mathbf{r}_1)](\dot{\mathbf{E}}_s(\mathbf{r}_1))_\alpha(\dot{\mathbf{E}}_s(\mathbf{r}_1))_\beta+(1-\exp[i\boldsymbol{\rho}\mathbf{E}_s(\mathbf{r}_1)])\\&\times\exp[i\boldsymbol{\rho}\mathbf{E}_{s'}(\mathbf{r}_2)](\dot{\mathbf{E}}_{s'}(\mathbf{r}_2))_\alpha(\dot{\mathbf{E}}_{s'}(\mathbf{r}_2))_\beta\\&-\exp[i\boldsymbol{\rho}\mathbf{E}_s(\mathbf{r}_1)]\exp[i\boldsymbol{\rho}\mathbf{E}_{s'}(\mathbf{r}_2)]\\&\times[(\dot{\mathbf{E}}_s(\mathbf{r}_1))_\alpha(\dot{\mathbf{E}}_{s'}(\mathbf{r}_2))_\beta+(\dot{\mathbf{E}}_s(\mathbf{r}_1))_\beta(\dot{\mathbf{E}}_{s'}(\mathbf{r}_2))_\alpha]\}\Bigg].\end{aligned} \tag{100}$$

Although it is difficult to analyze these expressions in general form, one can infer that they have the same block structure of the time derivatives of the characteristic function of the individual microfield distribution as in Ref. 1. On the other hand, in a multimode collective subsystem, only the second derivatives ($\alpha=\beta$) of its characteristic function are nonvanishing in (98), which is why the cross terms in (98) vanish in this case.

Thus, we can conclude that the contributions to the temporal fluctuation rate of the global field due to the individual and collective subsystems are nearly independent.

It is now possible to analyze the asymptotic behavior of the second moments of the global distribution at small $\beta_G=F_G/F_0$. Let us consider the moment $M_\perp(\beta_G)$

$= \langle(\dot{\mathbf{F}}_G)^2_\perp\rangle_{\mathbf{F}_G}/F^2_G$. Here as in Ref. 1 the symbol $\perp$ signifies the component of $\dot{\mathbf{F}}_G$ perpendicular to $\mathbf{F}_G$. Clearly, at small β_G the last term in (98) has the same dependence on β_G as $W(\beta_G)$, and so after entering in $M_\perp$ will give the same dependence $\propto\beta_G^{-2}$ as was obtained in Ref. 1 for the expression analogous to the first term in (98) after the neglect of the correlation, screening, and polarization effects. The first term in (98) will make a similar contribution at small β_G. Thus, we have shown that in agreement with Ref. 50, the inclusion of three-dimensional low-frequency multimode plasma oscillations does not eliminate the divergent terms, which are proportional to β_G^{-2}, in the asymptotic behavior of the second moment of the global microfield time derivative, divided by F^2_G, at small values of β_G.

This result does not agree with corresponding one from preceding work,[55,56] based on much simpler ideas of the derivation of $M_\perp$. There the finiteness of $M_\perp$ at small values of the individual fields was obtained as if induced by low-frequency plasma oscillations. As one can see, this is not confirmed by the more consistent and sophisticated theory presented here. It seems that the statistical average in the first paper of this series[55] was inappropriate to the physical formulation of the problem. Specifically, in Ref. 55 the average was taken over low-frequency plasma oscillations at fixed values of the individual part of the microfield despite the fact that for the weakly coupled plasmas parameters considered in Ref. 55, the low-frequency plasma oscillations have even lower-frequency variation than the individual component of the microfield. Neglecting the need to treat these two parts of the total microfield on the same footing led to the unrealistic behavior of the moment considered and made the results of further calculations, which, were based significantly on it, questionable.[55,56]

Using results obtained in this section, we now are able to derive an expression for the rate of fluctuation of the global microfield $\langle\dot{\mathbf{F}}_G\rangle_{\mathbf{F}_G}$, which is the key quantity for dynamical friction.[1] First of all, for the first derivative of the joint characteristic function over $\boldsymbol{\sigma}$ at $\boldsymbol{\sigma}=0$ in the integrand of the expression for the first moment of $\dot{\mathbf{F}}_G$, one will evidently get

$$\left.\frac{\partial A_G(\boldsymbol{\rho};\boldsymbol{\sigma})}{\partial\boldsymbol{\sigma}}\right|_{\boldsymbol{\sigma}=0}=\left.\frac{\partial A(\boldsymbol{\rho};\boldsymbol{\sigma})}{\partial\boldsymbol{\sigma}}\right|_{\boldsymbol{\sigma}=0}A_c(\boldsymbol{\rho};0)+\left.\frac{\partial A_c(\boldsymbol{\rho};\boldsymbol{\sigma})}{\partial\boldsymbol{\sigma}}\right|_{\boldsymbol{\sigma}=0}A(\boldsymbol{\rho};0). \qquad (101)$$

As follows from (92), (93), the second term in the above expression is zero. That is why the results (53)–(68) for $\langle\dot{\mathbf{F}}_G\rangle_{\mathbf{F}_G}$ are generalized by the substitution in (42),(43),(47)–(49) instead of $A(k)\rightarrow A(k)A_c(k)$, where $A_c(k)=\exp[-\kappa k^2]$, $\kappa=\langle F^2_{0_c}\rangle/6F^2_0$. The same is true as well for the APEX generalization, but it should be written in terms of $A_{\mathrm{APEX}}(k)A_c(k)$.

Thus, we see that low-frequency plasma oscillations lead only to some modulation of the integrand for the **B**-universal functions generalized to include the low-frequency plasma oscillations that enter particularly into the expression for $\langle\dot{\mathbf{F}}\rangle_{\mathbf{F}}$. This provides the key to understanding the relative importance of the individual or collective part of the microfield in line broadening and transport phenomena, if some resonance processes and mode interactions don't take place. If one considers very large coupling parameters then it is physically obvious that the contribution of individual motion will be suppressed in comparison with collective motion. However, it is insufficient to make any definite conclusion about the value of parameter κ, for example, even at thermal equilibrium. Nevertheless, for $\kappa\gg1$, which signifies that the energy in collective degrees of freedom greatly exceeds the energy in individual degrees of freedom, the contribution of the individual part to the global microfield and its temporal fluctuations will be suppressed due to the exponential modulation of the integrand mentioned above. As a consequence, one will get a Gaussian for the global microfield distribution. If simultaneously $\Gamma_i\gg1$, then $\omega_{pi}\gg v_iN_i^{-1/3}$, as noted above, which signifies that the influence of the fluctuations of the collective part of the microfield is more important for a strongly coupled system than the fluctuations of the individual part, because the latter are slower. In the opposite limit $\kappa\ll1$, we recover the results controlled predominantly by individual degrees of freedom.

In spite of some uncertainty in the convolution model used here, and based on the artificial assumption that the individual and collective degrees of freedom are independent, which is very difficult to justify for large coupling parameters,[41] the model provides physically reasonable results. For example, one can deduce from our expressions in a way similar to Ref. 1 that a charged test particle should gain energy under influence of the fluctuating electric fields in plasmas, i.e., the inclusion of all the effects considered in this article does not qualitatively change this general well-known result.[1]

Although calculating the values of the diffusion coefficients due to dynamical friction is outside the scope of this article, it is worth noting that for large coupling parameters, the procedure used for these calculations in current books on plasma physics is still valid if the kinetic energy of the test particle considerably exceeds the characteristic potential energy of the field particles. When these quantities have comparable values, another approach should be used, for example, the so-called liquid approximation.[16,18,23,41]

7. DISCUSSION

The main achievements of the present study are confined to the conceptualization of a way to construct the joint distributions of the microfield and its temporal derivatives in classical plasmas on the basis of the current progress and understanding in the theory of the microfield distribution and the joint distributions of the microfield and its spatial derivatives.

They may be enumerated as follows.

1. The general expressions for the joint distribution of the individual part of the ion plasma electric microfield and its time derivatives, and the first constraint moments of microfield time derivatives are obtained for an arbitrary plasma composition:

in the generalized Baranger–Moser cluster expansion scheme in terms of pair and triple correlation functions, which is reliable at least for plasma coupling constants at most of order unity;

in the generalized APEX approximation in terms of pair correlations for plasma coupling constants that may be much greater than unity.

2. In the Kirkwood approximation neglecting noncentral correlations, the angle dependences are separated and explicit analytic expressions are obtained for the joint distributions and the first constraint moments of microfield time derivatives in terms of Fourier components of pair or triple correlation functions.

3. It is shown that in general these distributions and moments incorporate the effects of screening by plasma electrons and ion–ion correlations. The appearance of polarization effects induced by the nonuniformity of the electron charge distribution around ions is important here. Analytic results are expressed in terms of universal functions of the reduced microfield.

4. Assuming statistical independence of the collective and individual parts of the ion plasma electric microfield (the convolution model), the joint distribution of the collective part of the microfield and its time derivatives is explicitly obtained for a multimode low-frequency plasma oscillation model.

5. On the basis of this convolution model, the joint distribution of the total plasma low-frequency electric microfield and its time derivatives is constructed and the asymptotic forms of the first and the second constraint moments of the microfield first time derivative are analyzed versus the reduced value of the total low-frequency plasma microfield. This gives new insight into the influence of the collective part of the microfield on dynamical friction.

As explained above, the accuracy of the results depends on the accuracy of the available correlation functions, which should be determined specifically in HCN and DFT or other approaches. The general scheme will also not be changed if it includes noncentral terms with triple correlations (see the second work in Ref. 32), but the results will become more cumbersome. We have also shown that it is important to complete this scheme by the appropriate determination of the elementary electric field of an ion with account of screening by electrons, and by the appropriate definition of the time derivative of the elementary electric field. There are several ways of doing so, discussed above. We pointed out the capabilities of DFT theory in the proper determination of the electron density around plasma ions.

The explicit analytic results determine the initial conditions at $t=0$ versus the plasma coupling parameters, its composition, and the radiator species for the various approaches to studying and constructing the probable time-dependent microfield distributions.[2,8,12,17–19] They also have fundamental importance in connection with the behavior of the microfield time fluctuations. The new important and interesting physical feature of these results is the presence of polarization terms, which change the asymptotic dependence versus the microfield value of the first constraint moment of the microfield time derivative for small reduced values of the microfield. Thus, they qualitatively and quantitatively influence dynamical friction.[1,16,23] In the OCP limit for ions, these terms are due to the friction induced by the uniform electron background.

It is also proved that, in comparison with results of Chandrasekhar and von Neumann,[1] three-dimensional multimode collective oscillations in plasmas cannot change the asymptotic behavior of the constraint second moments of the microfield first time derivative at small values of the total electric microfield in plasmas. The boundary value of the second moment of the microfield time derivative sought in Ref. 55 might come from high-frequency electron plasma oscillations, but at present no reliable way to treat the time evolution of the electron and the ion parts of a plasma electric field on the same footing is known in the context of Stark broadening in plasmas.

These results assume static electron screening. Eliminating the static approximation for the electron screening may need some dynamic generalization of the DFT-approach and proper accounting for retardation effects. Moreover, as the joint distribution is an instantaneous one, one should take care of the fact that it may in principle differ from the Monte Carlo simulation, which yields time-averaged results due to its usual settings in the case of weakly correlated plasmas. This results from ion–ion screening, which should not be effective on the time scales under consideration for weakly correlated plasmas, but should be effective for strongly correlated plasmas, because collective modes become faster in the latter case. Thus, these and other approaches fall short through the absence of an automatic and self-consistent adjustment to the appropriate time scales relevant to the physical problems under consideration.

These difficulties also apply to calculations in terms of correlation functions, which cannot distinguish among various time scales, expressions for the initial elementary electric field defined in terms of the pseudopotential, etc.

Finally, in the context of the time evolution of plasma electric fields, it is worthwhile to underline the intuitiveness of the microfield approach, based from the beginning more on physical images than on rigorous mathematical derivations of statistical mechanics and the theory of plasma interactions with a radiator. One of its obvious shortcomings is inability to provide correct calculations of interactions at short distances between a perturber and a radiator, as well as artificiality of the separation into the collective and individual parts of the microfield and into the low-(ion) and high-frequency (electron) parts of the microfield, which reflects rather crude attempts to use the hierarchy of time scales to simplify the solution. But on the other hand, it is important that the same things were successful for deriving a physically correct zero-order approximation for such a complex system as plasmas with strong coupling between its parts that in contrast to much more sophisticated models and approaches, much less tractable, supplied a visual basis for the sufficiently reliable treatment of the spectral observations in experiments.

However, attempts to overcome these difficulties are a task for future work.

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