Effect of the scattering medium on the production of vector bosons at high energies

A. V. Koshelkin

Moscow State Engineering Physical Institute, 115409 Moscow, Russia
(Submitted January 15, 1996)
Zh. Éksp. Teor. Fiz. **110**, 406–415 (August 1996)

A theory has been developed for the production of massive vector bosons by ultrarelativistic fermions undergoing multiple elastic scattering in a medium. The production cross section of such particles has been found and studied in detail. © *1996 American Institute of Physics.*[S1063-7761(96)00408-8]

1. INTRODUCTION

This paper discusses the production of massive vector bosons by ultrarelativistic fermions undergoing multipe elastic scattering in matter.

It becomes necessary to study such questions when various processes are studied that occur in scattering media at high energies and that proceed via intermediate vector bosons. In particular, this relates to processes that take place during the collision of ultrarelativistic heavy ions. The source of information on the dense nuclear matter that appears in this case is the photons and dileptonic pairs that are formed via intermediate vector bosons during particle scattering in a quark–gluon plasma[1–4] and in a hadronic gas.[5] The investigation of vector-boson formation in matter plays an important role when the strange degrees of freedom of a nuclear medium are studied.[6–8] The annihilation of $K^{\pm}$ mesons during the collision of ultrarelativistic heavy ions is a process that proceeds via intermediate vector bosons. The study of the production of massive vector bosons is of general physical interest in connection with analyzing the processes whereby massive and massless vector fields are produced in matter.

This paper discusses the production of massive vector bosons by ultrarelativistic fermions experiencing multiple elastic collisions in a dense scattering medium. The cross section of the production process of such particles is found. The measured production cross section essentially depends not only on the characteristics of the scattering medium but also on the energy and mass of the generated boson. The production of "massive photons"[9] in matter is studied in detail.

2. FORMULATION OF THE PROBLEM: TRANSITION CURRENT

Let us consider an ultrarelativistic ($\varepsilon \gg \mu$) particle with spin $s=1/2$, which enters a semi-infinite ($z \geqslant 0$) amorphous scattering medium at time $t=0$. Let the energy, momentum, and mass of the particle at time $t=0$ equal ε, $\mathbf{p}_0 = p_0 \mathbf{e}_z$, and μ, respectively ($\mathbf{e}_z$ is the unit vector in the z direction, and $h=c=1$).

The cross section for the creation of a vector boson in matter is determined by

$$d\sigma(E,m)=\frac{\alpha k^2 dk}{8E\pi^3}\int dO_{\mathbf{k}}\,\mathrm{Re}\left\{\int_0^T dt\int_0^{T-t} d\tau\int d^3\mathbf{r}_1\int d^3\mathbf{r}_2 \times\exp(-iE\tau)\exp(i(\mathbf{k}\mathbf{r}_1-\mathbf{k}\mathbf{r}_2)) \times e_\beta^*\Psi_f^+(\mathbf{r}_1,t+\tau)\gamma^0 J^\beta\Psi_i(\mathbf{r}_1,t+\tau)e_\nu\Psi_i^+ \times(\mathbf{r}_2,t)(\gamma^0 J^\nu)^{\pm}\Psi_f(\mathbf{r}_2,t)\right\}, \tag{1}$$

where J^β is the transition current.

In the most general form, the transition current J^β can be represented as

$$J^\beta=F_1\gamma^\beta+F_2\gamma^\beta\gamma^5+F_3\sigma^{\beta\nu}p_\nu+F_4\sigma^{\beta\nu}p_\nu\gamma^5+F_5k^\beta+F_6k^\beta\gamma^5. \tag{2}$$

The following notation has been introduced into Eqs. (1) and (2): E, $\mathbf{k}$, and m are the energy, momentum, and mass of the created boson; $\mathbf{e}_\beta$ is its polarization vector; $\Psi_i(\mathbf{r},t)$ is the fermion wave function; p^β and k^β are the 4-momenta of the fermion and boson, respectively; γ^0, γ^β, and γ^5 are the Dirac matrices;[9] $\sigma^{\beta\nu}=(1/2)(\gamma^\beta\gamma^\nu-\gamma^\nu\gamma^\beta)$; $dO_{\mathbf{k}}$ is the solid angle in the direction of vector $\mathbf{k}$; and α is the interaction constant. The functions F_1, F_2, F_3, F_4, F_5, and F_6 are the fermion form factors. The variables of integration τ and t are the time needed to create a boson and the instant at which it is emitted, and T is the total time the fermion moves in the scattering medium. We should point out that the wave functions of the initial state, $\Psi_i(\mathbf{r},t)$, and the final state, $\Psi_f(\mathbf{r},t)$, of the fermion also depend on the position of the scattering centers in the substance.

Because of the 4-transverseness of the boson ($e_\beta^* k^\beta=0$), the contribution of the last two terms (F_5k^β and $F_6k^\beta\gamma^5$) of Eq. (2) to the cross section $d\sigma(E,m)$ equals zero. Note that, to find the boson-production cross section, the matrix elements of the transition current $u^+(p_2)\gamma^0 J^\beta u(p_1)$ must be computed between the fermion states given by the bispinors $u(p_1)$ and $u(p_2)$. Then, taking into account everything mentioned above relative to the terms F_5k^β and $F_6k^\beta\gamma^5$ and using the Dirac equation, we can represent the transition current as

$$J^\beta=f_1\gamma^\beta+f_2\gamma^\beta\gamma^5+F_3(p_1^\beta+p_2^\beta)+F_4\gamma^5(p_1^\beta+p_2^\beta), \tag{3}$$

where $f_1=F_1-2\mu F_3$, $f_2=F_2-2\mu F_4$, and p_1^β and p_2^β are the four-momenta of a fermion in the states between which the transition current is computed.

3. PRODUCTION CROSS SECTION OF VECTOR BOSONS

To compute the observable boson-production cross section $d\Sigma$, we must average Eq. (1) over all possible positions of the scattering centers of the medium.[10–12] To do this, we expand the Ψ functions in Eq. (1) in a complete set of plane waves.[9] Then, neglecting mixing of the spin variables of the fermion as a consequence of the scattering of the latter in a material (which is valid for ultrarelativistic particles[12]), we carry out the summation and averaging over the corresponding spin states of the fermion and boson. As a result, we get from Eq. (1)

$$\frac{d\Sigma(E,m)}{dk}=\frac{\alpha k^2}{8E\pi^3}\int dO_{\mathbf{k}}\,\mathrm{Re}\int_0^T dt\int_0^{T-t}d\tau \times\exp(-iE\tau)\int d^3\mathbf{p}\int d^3\mathbf{p}'|M(\mathbf{p},\mathbf{p}')|^2 \times F_{\mathbf{k}}(\mathbf{p},\mathbf{p}',t,\tau), \tag{4}$$

where $|M(\mathbf{p},\mathbf{p}')|^2$ is the square of the absolute value of the matrix element (averaged and summed over the spin states of the particles) of the boson-production operator $e_\beta^* J^\beta$ by fermion current J^β. The function $F_{\mathbf{k}}(\mathbf{p},\mathbf{p}',t,\tau)$ is the Migdal distribution function for an ultrarelativistic fermion in a scattering medium.[12] The square of the absolute value of the matrix element $|M(\mathbf{p},\mathbf{p}')|^2$ and the function $F_{\mathbf{k}}(\mathbf{p},\mathbf{p}',t,\tau)$ are given in explicit form in Appendix A [see Eqs. (A3) and (A4)].

Integrating in Eq. (4) over $\mathbf{p}$, $\mathbf{p}'$, and $dO_{\mathbf{k}}$ and using Eqs. (A3) and (A4), we get the production cross section of a massive vector boson in a scattering medium:

$$\frac{d\Sigma(E,m)}{dk}=\frac{\alpha\Omega T}{2E\pi^2}\left[\frac{p_0-k}{p_0}\right]^2\left\{\left[\frac{Akp_0}{2\Omega(p_0-k)}-2B\right]\times\left[\frac{\pi}{4}-\int_0^{+\infty}\frac{ds}{\sinh s}\exp\left(-\frac{\Omega s}{b}\right)\sin\frac{\Omega s}{b}\right] +C\left[\int_0^{+\infty}\frac{ds}{\tanh s}\exp\left(-\frac{\Omega s}{b}\right)\sin\frac{\Omega s}{b}-\frac{\pi}{4}\right]\right\}, \tag{5}$$

where $\Omega=E-k+\mu^2k/2p_0(p_0-k)$, $b=[qkp_0/(p_0-k)]^{1/2}$.

The coefficients A, B, and C in Eq. (5) depend on the form factor F_i, as well as on the energy and mass of the fermion and the boson and have the following form

$$A=\sum_{n=1}^{6}A_n,\quad B=\sum_{n=1}^{6}B_n,\quad C=\sum_{n=1}^{6}C_n,$$

$$A_1=|f_1|^2\left[\frac{(E-k)^2}{m^2}\left[1-\frac{\mu^2(p_0^2+(p_0-k)^2)}{p_0^2(p_0-k)^2}\right]-\frac{k(E-k)}{m^2}\frac{\mu^2[(2p_0-k)^2+k^2]}{4p_0^2(p_0-k)^2}+\frac{3\mu^2k^2}{4p_0^2(p_0-k)^2}\right],$$

$$A_2=|f_2|^2\left[\frac{A_1}{|f_1|^2}+\frac{3m^2}{p_0(p_0-k)}\right],$$

$$A_3=\frac{(E-k)^2}{m^2}\frac{\mu^2(2p_0-k)^2}{(p_0-k)^2}|F_3|^2,$$

$$A_4=A_3\frac{k^2|F_4|^2}{|F_3|^2(2p_0-k)^2},$$

$$A_5=A_3\frac{(p_0-k)(f_1F_3^*+f_1^*F_3)}{\mu(2p_0-k)|F_3|^2},$$

$$A_6=A_4\frac{(p_0-k)(f_2F_4^*+f_2^*F_4)}{\mu k|F_4|^2},$$

$$B_1=|f_1|^2\left[\frac{k^2}{2m^2}\left(\frac{E}{k}-1\right)-\frac{kp_0(E-k)^2}{2m^2(p_0-k)^2}-\frac{4p_0^2-6p_0k+2k^2}{8(p_0-k)^2}\right], \tag{6}$$

$$B_2=|f_2|^2\left[\frac{B_1}{|f_1|^2}-\frac{p_0}{2(p_0-k)}+\frac{k^2p_0}{(p_0-k)m^2}\left(\frac{E}{k}-1\right)\right],$$

$$B_3=B_4=0,$$

$$B_5=A_3\frac{p_0k^2(f_1F_3^*+f_1^*F_3)}{4\mu^2(2p_0-k)^2|F_3|^2},$$

$$B_6=A_4\frac{(2p_0-k)p_0k(f_2F_4^*+f_2^*F_4)}{4\mu^2k^2|F_4|^2},$$

$$C_1=|f_1|^2\left[\frac{k^4}{2m^2(p_0-k)^2}+\frac{4p_0^2-4kp_0+3k^2}{4(p_0-k)^2}-\frac{Ek^3}{2m^2(p_0-k)^2}\right],$$

$$C_2=|f_2|^2\left[\frac{C_1}{|f_1|^2}+\frac{p_0}{(p_0-k)}+\frac{2k^2p_0}{(p_0-k)m^2}\left(1-\frac{E}{k}\right)\right],$$

$$C_3=\frac{p_0^2k^2(E-k)^2}{m^2(p_0-k)^2}|F_3|^2,\quad C_4=C_3\frac{|F_4|^2}{|F_3|^2},$$

$$C_5=-2B_5,\quad C_6=-2B_6.$$

The parameter q in Eqs. (5) and (6) is the mean square of the angle of multiple scattering of a particle per unit path.[12] Equations (5) and (6) solve the problem of finding the cross section for creation of a vector boson by an ultrarelativistic fermion undergoing multiple elastic scattering in a substance by determining the desired value of $d\Sigma(E,m)/dk$ in terms of the parameters that characterize the scattering medium (q, T), the original fermion (p_0, μ), and the created boson (E, m).

Next let us investigate the resulting production cross section $d\Sigma/dk$ in various regions of variation of the parameters of the problem. In the case of extremely small boson momenta, $k\ll\min\{m;m^2q^{-1}\}$, $m\ll p_0$, letting k tend to zero in Eqs. (5) and (6), we get

$$\frac{d\Sigma}{dk}=\frac{\alpha kqT}{12\pi^2m^2}\left\{\frac{3}{2}|f_1|^2+\frac{7}{2}|f_2|^2\right\}. \tag{7}$$

It follows from Eq. (7) that, in the region of very small momenta of the emitted boson, the process of producing the latter is strongly suppressed, while the production cross section tends to zero according to the law $d\Sigma/dk \propto k$.

However, if the relationship between the parameters of the problem is such that $k \ll m$, $m \ll p_0$, but $k \gg m^2/q$, the production cross section is given by

$$\frac{d\Sigma}{dk} = \frac{\alpha T\sqrt{kq}}{4\pi^2 m}\{|f_1|^2 + 2|f_2|^2\}. \tag{8}$$

Thus, in the region of small $k \ll m$, as the boson momentum increases, the dependence of $d\Sigma/dk$ on k becomes more gradual, changing over from $d\Sigma/dk \propto k$ to $d\Sigma/dk \propto \sqrt{k}$.

Let us examine the production cross section $d\Sigma/dk$ in the region of intermediate values of the boson momentum, when $m \ll k \ll p_0$. In this case, it is expedient to consider two cases:

small E:

$$E \ll q\left[\left(\frac{m}{E}\right)^2 + \left(\frac{\mu}{p_0}\right)^2\right]^{-2}, \quad m \ll k \ll p_0,$$

and large E:

$$E \gg q\left[\left(\frac{m}{E}\right)^2 + \left(\frac{\mu}{p_0}\right)^2\right]^{-2}, \quad m \ll k \ll p_0,$$

(E is the energy of the created boson).

If the energy of the created boson is such that

$$E \ll q\left[\left(\frac{m}{E}\right)^2 + \left(\frac{\mu}{p_0}\right)^2\right]^{-2},$$

the characteristic values of s in the integrals in Eq. (5) are large. Then, for such values of E and $m \ll k \ll p_0$, we get from Eqs. (5) and (6)

$$\frac{d\Sigma(E,m)}{dk} = \frac{\alpha\sqrt{q}T}{4\pi^2\sqrt{k}}\left(|f_1|^2 + |f_2|^2 + \frac{m^2}{16p_0^2}|f_1 - 2p_0F_3^*|^2 + \frac{m^2}{4k^2}|f_2 - kF_4^*|^2\right). \tag{9}$$

For $m \ll k \ll p_0$ and $E \gg q[(m/E)^2 + (\mu/p_0)^2]^{-2}$, however, expanding the integrands in Eq. (5) for $s \ll 1$, we find

$$\frac{d\Sigma(E,m)}{dk} = \frac{\alpha qT}{((m/k)^2 + (\mu/p_0)^2)12k\pi^2}\left[\frac{5}{2}|f_2|^2 + \frac{m^2}{2k^2}\left|\frac{3}{2}f_2 - kF_4^*\right|^2 + \frac{m^2}{8p_0^2}\left|\frac{3}{2}f_1 - 2p_0F_3^*\right|^2 + \left(\left(\frac{m}{k}\right)^2 + \left(\frac{\mu}{p_0}\right)^2\right)^{-1}\left[\frac{m^2}{4k^2}|f_1 + 2\mu F_3^*|^2 + \left(\frac{5m^2}{2k^2} + \frac{2\mu^2}{p_0^2}\right)|f_1|^2 + \frac{5\mu^2}{2p_0^2}|f_2|^2 + \frac{m^2}{4k^2}\left|f_2 + \frac{k\mu}{p_0}F_4^*\right|^2\right]\right]. \tag{10}$$

An analysis of Eqs. (9) and (10) shows that, when ultrarelativistic ($E \gg m$) bosons are emitted, the form of the production cross section is mainly determined by the value of the mean square of the angle of the multiple fermion scattering in matter. In this case, as the boson energy increases, the value of $d\Sigma(E,m)/dk$ begins to depend substantially on the ratio of the Lorentz factors of the boson and the fermion [see Eq. (9)].

In the region of extremely large boson momenta $k \simeq p_0 - \mu$, the production cross section is determined by the more general Eqs. (5) and (6).

Next let us consider the case of boson production when there is a single collision of a fermion with a scattering center. This situation corresponds to the limiting transition $q \to 0$, $qT \equiv \langle\theta_0\rangle$ in Eqs. (5) and (6), where $\langle\theta_0\rangle \neq 0$ is the mean square of the single-scattering angle. In this case, for subsequent calculation of the boson-production cross section, it is necessary to take into account the effects that appear when a fermion passes through the boundary of the scattering medium.[13] To do this, the integration over the variable t in Eq. (4) should be extended to the region $-T \leq t \leq 0$. However, the structure of the terms that appear in this case is such that, for $q \to 0$ and $qT \equiv \langle\theta_0\rangle$, they cancel out, and the contribution to the boson-production cross section from the motion of the particle corresponding to integration over t from $t = -T$ to $t = 0$ turns out to be identically equal to zero. Therefore, in the case of a single collision of a fermion with a scattering center, setting $q \to 0$ and $qT \equiv \langle\theta_0\rangle$ in Eqs. (5) and (6), we get

$$\frac{d\Sigma(E,m)}{dk} = \frac{\alpha kqT}{12E\Omega\pi^2}\left[\frac{p_0 - k}{p_0}\right]\left\{\frac{Akp_0}{4\Omega(p_0 - k)} - B + C\right\} \tag{11}$$

for the boson-production cross section, where A, B, and C are determined by Eqs. (6).

By comparing the resulting Eq. (11) with Eqs. (7) and (10), we observe that they differ by the factor $qT = N\langle\theta_0\rangle$, where N is the number of scattering centers in the medium. Thus, for very small ($k \ll \min\{m; m^2q^{-1}\}$, $m \ll p_0$) and extremely large ($k \gg \max\{q[(m/k)^2 + (\mu/p_0)^2]^{-2}; m\}$) boson momenta, the production cross section is determined by single collisions of a fermion with the scattering centers of the substance. This circumstance is associated with the fact that, for such k values, the boson coherence length[14] l_{coh} is determined by

$$l_{coh} \sim \lambda_c = 1/m \quad \text{for } k \ll \min\{m; m^2q^{-1}\}, \quad m \ll p_0,$$

$$l_{coh} \sim 1/k((m/k)^2 + (\mu/p_0)^2) \quad \text{for } k \gg \max\{q[(m/k)^2 + (\mu/p_0)^2]^{-2}; m\},$$

as well as by the fact that, at distances of the order of l_{coh}, the mean square of the angle of multiple fermion scattering in the medium ql_{coh} is small by comparison with $(\mu/\varepsilon)^2$.

4. SCATTERING OF "MASSIVE" PHOTONS

Let us consider the production of massive vector bosons by an electromagnetic fermion current (the massive photons of Ref. 9) in a scattering medium. An analysis of this problem plays a paramount role in the study of the production processes of dileptonic pairs and photons during the collision of ultrarelativistic heavy ions, where the processes proceed via intermediate vector bosons produced as a consequence of the electromagnetic interaction of quarks in nuclear matter.[1-4] In this case, the boson mass ($m\sim 0.1-1$ GeV) and the density of the substance ($n\sim\Theta^3$, where $\Theta\sim 0.1-1$ GeV is the temperature of the scattering medium) are such that the parameter Ω/b [see Eq. (5)] can be either small $\Omega/b\ll 1$ (multiple scattering of fermions in the medium is substantial) or large, $\Omega/b\gg 1$ (the effect of multiple scattering on boson production is negligible).

Moreover, the study of the production of massive vector bosons as a consequence of the electromagnetic interaction of fermions in a scattering medium is of independent interest for the analysis of the effect of the mass (and, consequently, the third polarization) of the boson in the process of producing vector particles in matter.

Setting $F_1=1$, $F_2=F_3=F_4=F_5=F_6=0$ in Eqs. (6), we get

$$A=\left[\frac{(E-k)^2}{m^2}\left[1-\frac{\mu^2(p_0^2+(p_0-k)^2)}{p_0^2(p_0-k)^2}\right]-\frac{k(E-k)}{m^2}\frac{\mu^2[(2p_0-k)^2+k^2]}{4p_0^2(p_0-k)^2}+\frac{3\mu^2k^2}{4p_0^2(p_0-k)^2}\right],$$

$$B=\left[\frac{k^2}{2m^2}\left(\frac{E}{k}-1\right)-\frac{kp_0(E-k)^2}{2m^2(p_0-k)^2}-\frac{4p_0^2-6p_0k+2k^2}{8(p_0-k)^2}\right],$$

$$C=\left[\frac{k^4}{2m^2(p_0-k)^2}+\frac{4p_0^2-4kp_0+3k^2}{4(p_0-k)^2}-\frac{Ek^3}{2m^2(p_0-k)^2}\right], \tag{12}$$

$$\Omega=E-k+\frac{\mu^2k}{2p_0(p_0-k)},\quad b=\sqrt{q\frac{kp_0}{p_0-k}},$$

$$\tilde{k}=\frac{kp_0}{p_0-k},\quad a=\sqrt{iq\tilde{k}/2}$$

for the coefficients A, B, and C that appear in Eq. (5) for the boson-production cross section.

In the case of extremely small momenta $k\ll\min\{m;m^2q^{-1}\};m\ll p_0$, we find from Eqs. (5), (12), and (7) that

$$\frac{d\Sigma}{dk}=\frac{\alpha kqT}{8\pi^2m^2}. \tag{13}$$

It follows from the last expression that, for $k\to 0$, the boson-production cross section $d\Sigma/dk$ decreases in proportion to k:

$$\frac{d\Sigma}{dk}\propto k,$$

whereas, when actual photons are produced,[10-12] it increases with decreasing photon momentum as $k\to 0$:

$$\frac{d\Sigma}{dk}\propto k^{-1/2}.$$

In this case, the ratio of the production cross sections of massive and actual photons has the following order of magnitude:

$$\frac{d\Sigma_{\rm bos}/dk}{d\Sigma_{\rm phot}/dk}\sim\frac{q^{1/2}k^{3/2}}{m^2}\ll 1,$$

$$k\ll\min\{m;m^2q^{-1}\},\quad m\ll p_0.$$

Thus, in the region of momenta k close to zero, for arbitrarily small but finite boson masses, we have

$$\frac{d\Sigma_{\rm bos}}{dk}\ll\frac{d\Sigma_{\rm phot}}{dk},$$

with $d\Sigma_{\rm bos}/dk$ being an increasing function of k, while $d\Sigma_{\rm phot}/dk$ is a decreasing function of k.

In the case of sufficiently large boson momenta ($k\gg m$), we get from Eqs. (5) and (12)

$$\frac{d\Sigma(E,m)}{dk}=\frac{\alpha\Omega T(m^2/k^2+\mu^2/p_0^2)}{4\pi^2}\times\left\{\frac{3m^2/4k^2}{(m/k)^2+(\mu/p_0)^2}\left[\frac{\pi}{4}-\int_0^{+\infty}\frac{ds}{\sinh s}\times\exp\left(-\frac{\Omega s}{b}\right)\sin\frac{\Omega s}{b}\right]+\left[\int_0^{+\infty}\frac{ds}{\tanh s}\times\exp\left(-\frac{\Omega s}{b}\right)\sin\left(\frac{\Omega s}{b}-\frac{\pi}{4}\right)\right]\right\}. \tag{14}$$

The resulting Eq. (14) for the boson-production cross section substantially differs from the formulas found earlier[4] for $d\Sigma(E,m)/dk$, in which the first term in braces in Eq. (14) is absent altogether. This is because, in Ref. 5, the boson-production cross section was computed with the same accuracy as in Refs. 11 and 12, taking into account only the variations of the dispersion law of the created particles. However, this is not quite correct, since the presence of mass not only changes the dispersion law but also causes the appearance of an additional third polarization in the generated bosons. The existence of the latter evidently implies the presence of additional possibilities when bosons are produced, and, as a consequence, causes an additional term (by comparison with the case of photon production[10-12]) to appear in the formula for the particle-production cross section.

For boson momenta such that $k\gg m$, $k\gg q[(m/k)^2+(\mu/p_0)^2]^{-2}$, by expanding the preexponential factors in the expressions under the integrals in Eq. (14) in small $s\ll 1$, we get

$$\frac{d\Sigma(E,m)}{dk}=\frac{\alpha qT}{48k\pi^2}\frac{11m^2/k^2+8\mu^2/p_0^2}{((m/k)^2+(\mu/p_0)^2)^2}. \tag{15}$$

It follows from Eq. (15) that, for large enough boson masses $m\gg k(\mu/p_0)$, the production cross section (as in the case of small momenta) is an increasing function of k.

On the other hand, as $m\to 0$, Eqs. (5) and (6) transform into the corresponding expressions for the photon-production

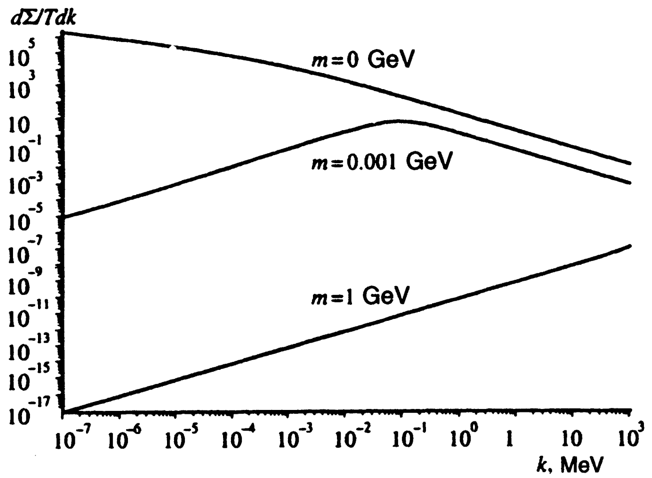

FIG. 1. Production cross section of massive bosons vs momentum in the case $p_0=3.0\times 10^4$ MeV, $\mu=3.0\times 10^2$ MeV, $q=3.0\times 10^4$ sec^{-1}, and $\alpha=10^{-2}$.

cross section[11,12] (which is associated with gradient invariance of the interaction of a fermion and a boson, when the latter is electromagnetic). However, the photon-production cross section ($m=0$) is a strictly decreasing function of the momentum of the particle, while the boson-production cross section for large m increases with k. This means that the character of the momentum dependence of the production cross section of bosons substantially depends on the boson mass. Namely, as the boson mass increases, the dependence of $d\Sigma(E,m)/dk$ on k changes from decreasing to increasing, so that the cross section $d\Sigma(E,m)/dk$ as a function of boson momentum should have an extremum for some intermediate m. Figure 1 shows the dependence of $d\Sigma(E,m)/dk$ on k for various values of the boson mass.

5. CONCLUSION

This paper has developed a theory for the production of massive vector bosons by ultrarelativistic fermions undergoing multiple elastic scattering in matter. The production cross section of such particles has been found. The resulting cross section substantially depends on not only the characteristics of the scattering medium but also on the parameters that characterize the initial and the created particles. It has been shown that the boson-production process is strongly inhibited in the region of small boson momenta $k\ll\min\{m;m^2q^{-1}\}$, $m\ll p_0$, while the production cross section tends to zero as $k\to 0$ according to the law $d\Sigma(E,m)/dk \propto k$.

The production of massive photons by ultrarelativistic fermions in a scattering medium has been studied in detail. In the case of extremely small $k\ll\min\{m;m^2q^{-1}\}$, $m\ll p_0$, the resulting production cross section is a linearly increasing function of the boson momentum k, whereas $d\Sigma(E,m)_{\rm phot}/dk$ decreases with increasing k according to a $k^{-1/2}$ law.[10–12] However, in the region of large k, the dependence of the production cross section on the boson momentum is determined by the relationship between all the parameters of the problem (the energies and masses of the fermion and the boson and the mean square of the angle of multiple scattering per unit path), so that $d\Sigma(E,m)/dk$ can be either a decreasing or an increasing function of k or have an extremum (see Fig. 1).

The author is grateful to D. N. Voskresenskiĭ for discussion of the results.

APPENDIX A

The square of the absolute value of the matrix element is determined by

$$|M(\mathbf{p},\mathbf{p}')|^2=6\langle[\bar{u}(p_2)J^\beta u(p_1)]\times[\bar{u}(p_4)\gamma^0[J^+]^\nu\gamma^0 u(p_3)]\rho_{\beta\nu}\rangle. \qquad (A1)$$

where $u(p_i)$ are the Dirac bispinors,[9] $\rho_{\beta\nu}$ is the polarization density matrix of a massive boson,[9] and the angle brackets denote averaging over all the spin states of the particles. The vectors $\mathbf{p}_1$, $\mathbf{p}_2$, $\mathbf{p}_3$, and $\mathbf{p}_4$ are associated with vectors $\mathbf{p}$, $\mathbf{p}'$, and $\mathbf{k}$ by the relationships

$$\mathbf{p}_1=\mathbf{p}-\mathbf{k},\quad \mathbf{p}_2=\mathbf{p},\quad \mathbf{p}_3=\mathbf{p}',\quad \mathbf{p}_4=\mathbf{p}'-\mathbf{k}. \qquad (A2)$$

To compute $|M(\mathbf{p},\mathbf{p}')|^2$, it is necessary to transform from the Dirac bispinors $u(p_i)$ to ordinary two-component spinors. After this, neglecting mixing of the spin states of the fermions as a consequence of collisions in the medium (which is valid for ultrarelativistic particles[12]), we average $|M(\mathbf{p},\mathbf{p}')|^2$ over all the spin states of the particles. As a result, we get an expression for $|M(\mathbf{p},\mathbf{p}')|^2$ that is a combination of the scalar products of the vectors $\mathbf{p}_1$, $\mathbf{p}_2$, $\mathbf{p}_3$, $\mathbf{p}_4$, and $\mathbf{k}$ of Eq. (A2):

$$|M(\mathbf{p},\mathbf{p}')|^2=\Phi((\mathbf{p}_1,\mathbf{p}_2);(\mathbf{p}_1,\mathbf{p}_3);(\mathbf{p}_1,\mathbf{p}_4);(\mathbf{p}_2,\mathbf{p}_3);\times(\mathbf{p}_2,\mathbf{p}_4);(\mathbf{p}_3,\mathbf{p}_4),\ldots\times(\mathbf{p}_1,\mathbf{k});(\mathbf{p}_2,\mathbf{k});(\mathbf{p}_3,\mathbf{k});(\mathbf{p}_4,\mathbf{k})). \qquad (A3)$$

Let us consider the situation in which elastic scattering of the fermions takes place in the medium. Since elastic scattering of ultrarelativistic particles mainly occurs at small angles, it is convenient below to introduce angular vectors $\boldsymbol{\eta}$, $\boldsymbol{\zeta}$, $\boldsymbol{\theta}$:

$$\mathbf{p}=p_0\mathbf{e}_z\left(1-\frac{\boldsymbol{\eta}^2}{2}\right)+p_0\boldsymbol{\eta},\quad \boldsymbol{\eta}\perp\mathbf{e}_z,\quad |\boldsymbol{\eta}|\ll 1,$$

$$\mathbf{p}'=p_0\mathbf{e}_z\left(1-\frac{\boldsymbol{\zeta}^2}{2}\right)+p_0\boldsymbol{\zeta},\quad \boldsymbol{\zeta}\perp\mathbf{e}_z,\quad |\boldsymbol{\zeta}|\ll 1, \qquad (A4)$$

$$\mathbf{k}=k\mathbf{e}_z\left(1-\frac{\boldsymbol{\theta}^2}{2}\right)+k\boldsymbol{\theta},\quad \boldsymbol{\theta}\perp\mathbf{e}_z,\quad |\boldsymbol{\theta}|\ll 1.$$

Expanding the scalar products of vectors $\mathbf{p}_1$, $\mathbf{p}_2$, $\mathbf{p}_3$, $\mathbf{p}_4$, and $\mathbf{k}$ and all the coefficients in Eq. (A3) for $|M(\mathbf{p},\mathbf{p}')|^2$ in powers of $|\boldsymbol{\eta}|$, $|\boldsymbol{\zeta}|$, $|\boldsymbol{\theta}|$, and $(\mu/p_0)\ll 1$, we get

$$|M(\mathbf{p},\mathbf{p}')|^2=A+B[(\boldsymbol{\eta}-\boldsymbol{\theta})^2+(\boldsymbol{\zeta}-\boldsymbol{\theta})^2]+C(\boldsymbol{\eta}-\boldsymbol{\theta})\cdot(\boldsymbol{\zeta}-\boldsymbol{\theta})+D[(\boldsymbol{\eta}-\boldsymbol{\theta})^2-(\boldsymbol{\zeta}-\boldsymbol{\theta})^2], \qquad (A5)$$

where A, B, and C are defined by Eqs. (6). The coefficient D is not represented explicitly, since the contribution of the last term in Eq. (A5) in the boson production cross section equals zero.

Integrating Eq. (A5) with a distribution function $F_{\mathbf{k}}(\mathbf{p},\mathbf{p}',t,\tau)$, of the form[12]

$$F_{\mathbf{k}}(\mathbf{p},\mathbf{p}',t,\tau)=\delta(p-p_0)\delta(p'-p_0) \times\frac{\exp\{-\zeta^2/qt\}}{\pi qt}\frac{a}{\pi q\sinh(a\tau)} \times\exp\left\{-\frac{a}{q}\coth(a\tau)(\boldsymbol{\eta}-\boldsymbol{\zeta})^2+\frac{2a}{q}\tanh\left(\frac{a\tau}{2}\right) \times(\boldsymbol{\eta}-\boldsymbol{\zeta})(\boldsymbol{\theta}-\boldsymbol{\zeta})-\frac{2a}{q}\tanh\left(\frac{a\tau}{2}\right)(\boldsymbol{\theta}-\boldsymbol{\zeta})^2 +i\tilde{k}\tau\right\}, \quad \text{(A6)}$$

$$\tilde{k}=kp_0(p_0-k)^{-1},\quad a=(iq\tilde{k}/2)^{1/2},$$

we get the boson-production cross section determined by Eqs. (5) and (6).

[1] G. Domokos, Phys. Rev. D **28**, 123 (1983).
[2] L. D. McLerran and T. Toimela, Phys. Rev. D **31**, 545 (1985).
[3] K. Kajantie, J. Kaputsa, L. D. McLerran, and A. Mekjian, Phys. Rev. D **34**, 2746 (1986).
[4] J. Cleymans, V. V. Goloviznin, and K. Redlich, Phys. Rev. D **47**, 989 (1993).
[5] J. Cleymans, V. V. Goloviznin, and K. Redlich, Phys. Rev. D **47**, 173 (1993).
[6] D. B. Kaplan and A. E. Nelson, Phys. Lett. B **175**, 57 (1986).
[7] H. Fujii, T. Muto, T. Tatsumi, and R. Tamagaki, Phys. Rev. C **50**, 3140 (1994).
[8] G. Q. Liu, C. M. Ko, and Bao-An Li, Phys. Rev. Lett. **74**, 235 (1995).
[9] V. B. Berestetskiĭ, E. M. Lifshitz, and L. P. Pitaevskiĭ, *Quantum Electrodynamics* (Nauka, Moscow, 1980; Pergamon P., Oxford, 1971).
[10] L. D. Landau and I. Ya. Pomeranchuk, Dokl. Akad. Nauk SSSR **92**, 535 (1953).
[11] A. B. Migdal, Dokl. Akad. Nauk SSSR **96**, 49 (1954).
[12] A. B. Migdal, Phys. Rev. **103**, 1811 (1956).
[13] I. I. Gol'dman, Zh. Éksp. Teor. Fiz. **38**, 1866 (1960) [Sov. Phys. JETP **11**, 1341 (1960)].
[14] V. M. Galitskiĭ and V. V. Yakimets, Zh. Éksp. Teor. Fiz. **46**, 1066 (1964) [Sov. Phys. JETP **19**, 723 (1964)].

Translated by W. J. Manthey