The energy flux and ray trajectories of electromagnetic waves in a plasma with Landau cyclotron damping

A. I. Smirnov and M. D. Tokman

Institute of Applied Physics of the Russian Academy of Sciences 603600, Nizhniĭ Novgorod, Russia
(Submitted 18 January 1996)
Zh. Éksp. Teor. Fiz. **110**, 549–558 (August 1996)

An expression is obtained on the basis of kinetic analysis for the energy flux of electromagnetic waves in a hot magnetized plasma for quasitransverse propagation with respect to the magnetic field. This expression takes account of the effect of electron of kinetic energy transport induced by the hf field in the presence of cyclotron absorption. It is shown that the influence of dissipation on the ray paths near resonance is of the same order as the diffraction corrections. © *1996 American Institute of Physics.* [S1063-7761(96)01408-4]

1. INTRODUCTION

The determination of the energy characteristics of a macroscopic electromagnetic field in a dissipative medium is traditionally considered to be one of the complicated problems of the electrodynamics of continuous media. According to the current view, in analyzing this problem one cannot use a universal phenomenological approach and obtain an expression for the energy flux density in the propagating wave without adducing a specific model of the dissipative processes (see, e.g., Refs. 1–3). In this study the problem of an energy flux of steady electromagnetic radiation is treated to a hot, magnetized plasma in the region of a cyclotron resonance. Its solution is of interest both from the applied and the methodological standpoints.

Energy dissipation and transport of normal waves in a zone of cyclotron absorption is characterized by the following features.[4–6] The Hermitian and anti-Hermitian components of the permitivity $\epsilon_{pm}(\omega,\mathbf{k})$ for the normal mode being studied (ω and $\mathbf{k}$ are its frequency and wave vector) are quantities of the same order of magnitude:

$$\epsilon_{pm}=\epsilon_{pm}^{H}+\epsilon_{pm}^{aH}, \quad |\epsilon_{pm}^{H}|\sim|\epsilon_{pm}^{aH}|. \tag{1}$$

Despite this, the dispersion equation

$$D(\omega,\mathbf{k})=\det\|D_{pm}\|=\det\left\|\delta_{pm}k^2-k_pk_m-\frac{\omega^2}{c^2}\epsilon_{pm}\right\|=0 \tag{2}$$

for real ω has a solution that corresponds to weakly decaying waves[1)] with

$$|\mathrm{Re}\,\mathbf{k}|\gg|\mathrm{Im}\,\mathbf{k}|. \tag{3}$$

Here the imaginary and real parts of the derivatives of the dispersion function $D(\omega,\mathbf{k},\mathbf{r})$ (which in an inhomogeneous plasma depends also on the spatial coordinates $\mathbf{r}$) with respect to $\mathbf{k}$ and $\mathbf{r}$ at the center of the cyclotron absorption line are the same order of magnitude:

$$\left|\mathrm{Re}\left(\frac{\partial D}{\partial\mathbf{r}},\frac{\partial D}{\partial\mathbf{k}}\right)\right|\sim\left|\mathrm{Im}\left(\frac{\partial D}{\partial\mathbf{r}},\frac{\partial D}{\partial\mathbf{k}}\right)\right|. \tag{4}$$

In the case of a Hermitian tensor ϵ_{pm} (nonabsorbing medium), the dispersion function $D(\omega,\mathbf{k},\mathbf{r})$ is the Hamiltonian of ray optics,[5,6] while the vector $\partial D/\partial\mathbf{k}$ is collinear with the energy flux density vector:[2,3,6]

$$\mathbf{S}=\frac{c^2}{16\pi\omega}E_p^*E_m\frac{\partial D_{pm}}{\partial\mathbf{k}}=\Pi_e+\Pi_p, \tag{5}$$

where

$$\Pi_e=\frac{c^2}{8\pi\omega}\,\mathrm{Re}\,[\mathbf{E}^*[\mathbf{kE}]]$$

is the Poynting vector (the purely electromagnetic component of the energy flux density, and

$$\Pi_p=-\frac{\omega}{16\pi}E_p^*E_m\frac{\partial\epsilon_{pm}}{\partial\mathbf{k}}$$

is the energy flux density transported by particles of the plasma (the kinetic component).[1–3,5,6] Here $\mathbf{E}=\{E_m\}$ is the electric field vector.

If we directly extend Eq. (5) to the case of electron-cyclotron waves and treat the dispersion function $D(\omega,\mathbf{k},\mathbf{r})$ as the ray Hamiltonian, then, when we take account of spatial dispersion, the ray paths at the center of the absorption line, according to (4), will be defined in complex space, while the kinetic component of the energy flux density, which was identified in (5) as a separate term, loses its physical meaning, since it becomes a complex quantity. Most often one overcomes these difficulties by using the cold-plasma dispersion equation[4,5] to determine the geometric-optical rays, directing them parallel to the Poynting vector. The correctness of this approach is justified only if the kinetic part of the energy flux density Π_p is small in comparison with the electromagnetic component Π_e ($|\Pi_e|\gg|\Pi_p|$).[5] However, even in this case the "hot" (kinetic) corrections to the "cold" dispersion equation are sometimes important, in particular, when electron-cyclotron waves propagate at small angles to the resonance layer, in which case the relatively small kinetic effects cause appreciable reflection.[7,8]

When the influence of kinetic processes on the path of propagation are described in terms of the ray Hamiltonian, one commonly chooses the real part of the dispersion function $D(\omega,\mathbf{k},\mathbf{r})$ for real values of the wave vector $\mathbf{k}$:[6]

$$\mathrm{Re}\, D = \det\|D^H_{pm}\| = \det\left\| \delta_{pm}k^2 - k_p k_m - \frac{\omega^2}{c^2}\epsilon^H_{pm}\right\|.$$

The correctness of this procedure near the center of a cyclotron absorption line requires justification in view of Eq. (4), despite the condition of weak dissipation, Eq. (3).

In the present study, to calculate the energy flux transported by particles in electron-cyclotron waves, we use quasilinear plasma theory (Sec. 2). We consider quasitransverse (with respect to the constant magnetic field) propagation of normal modes. Moreover, we analyze for a plane stratified geometry the behavior of a quasioptical wave beam in the region of electron-cyclotron resonance (Sec. 3). In the Conclusion we formulate the results of the studies that we have conducted.

2. THE QUASILINEAR THEORY OF KINETIC PROCESSES IN QUASIOPTICAL BEAMS OF ELECTRON-CYCLOTRON WAVES

Consider the cyclotron interaction of electrons in a uniform magnetized plasma with a monochromatic beam of electron-cyclotron waves:

$$\tilde{\mathbf{E}} = \mathbf{E}(\mathbf{r})\exp(i\mathbf{k}_0\mathbf{r} - i\omega t) = \int \mathbf{e}(\mathbf{k})A_k(\mathbf{k})\exp(i\mathbf{k}\mathbf{r} - i\omega t)d^3k, \tag{6}$$

where $\mathbf{k}=\mathbf{k}_0+\Delta\mathbf{k}$, $\mathbf{e}(\mathbf{k})$, and $A_k(\mathbf{k})$ are the wave vector, the polarization unit vector, and the spectral density of the plane electron-cyclotron waves of which the beam is composed. The spatial spectrum of the field of (6) is assumed to be narrow:

$$\Delta k_\perp \ll \max[k_{0\perp}, k_{0\parallel}(v_T/c)^{1/3}], \quad \Delta k_\parallel \ll \max[k_{0\parallel}, (v_T\omega/c^2)]. \tag{7}$$

Here and below the subscripts "$\perp$" and "$\parallel$" denote the projections perpendicular and parallel to the magnetic field $\mathbf{H}=H_0\mathbf{z}_0$ ($\mathbf{z}_0$ is a unit vector along the z axis, H_0=const); $\Delta k_\perp$ and $\Delta k_\parallel$ are the characteristic widths of the spatial spectrum of the wave beam along the component of the wave vector corresponding to the subscript, and v_T is the thermal velocity of the electrons.[2)] Taking account of Eq. (7), we can represent the field $\mathbf{E}(\mathbf{r})=(E_x, E_y, E_z)$ in Eq. (6) in the following form:

$$\mathbf{E}(\mathbf{r}) = \left[\mathbf{e}\, A(\mathbf{r}) - i\frac{\partial A}{\partial r_j}\frac{\partial \mathbf{e}}{\partial k_j}\right]_{\mathbf{k}=\mathbf{k}_0}, \tag{8}$$

where

$$A(\mathbf{r}) = \int A_k(\mathbf{k}_0+\Delta\mathbf{k})e^{i\Delta\mathbf{k}\mathbf{r}}d^3\Delta k$$

is the slowly varying amplitude of the field of the beam, and the subscripts j and k run through x, y, and z.

We describe the distribution function of the electrons in the magnetoactive plasma by the kinetic equation

$$\frac{\partial f}{\partial t} + \mathbf{v}\frac{\partial f}{\partial \mathbf{r}} + \omega_H\frac{\partial f}{\partial \psi} = -\frac{\mathbf{F}}{m}\frac{\partial f}{\partial \mathbf{v}}, \tag{9}$$

which takes account, in the weakly relativistic approximation, of the dependence of the gyrofrequency ω_H on the velocity v: $\omega_H \simeq \omega_{H0}(1-v^2/2c^2)$, $\omega_{H0}=eH/mc$ (e and m are the charge and rest mass of an electron). In Eq. (9) we have introduced the following notation:

$$\mathbf{F} = -e\left(\mathbf{E} + \left[\frac{\mathbf{v}}{\omega}[(\mathbf{k}_0 - i\nabla)\mathbf{E}]\right]\right)\exp(i\mathbf{k}_0\mathbf{r} - i\omega t) \tag{10}$$

is the Lorentz force acting on the electron from the quasioptical beam of (6); $\psi=\arcsin(v_x/v_\perp)$ is the phase of the electron gyrorotation, and $v_\perp=\sqrt{v_x^2+v_y^2}$.

Using the quasilinear approximation (9), we resolve the distribution function f into stationary and time-oscillating components:

$$f = \Phi + \tilde{f}e^{-i\omega t}, \tag{11}$$

and find the latter by linearizing Eq. (9) in the field $\mathbf{E}$:

$$\tilde{f}(\mathbf{r}) = \int \mathbf{f}_k(\mathbf{k})\mathbf{e}(\mathbf{k})A_k(\mathbf{k})e^{i\mathbf{k}\mathbf{r}}d^3k, \tag{12}$$

where the vector $\mathbf{f}_k$ is defined by the usual relations for the Fourier harmonics (Ref. 9).[3)]

Taking into account in Eq. (12) the inequality (7), we can write $\tilde{f}(\mathbf{r})$ in the form

$$\tilde{f}(\mathbf{r}) = \left[\mathbf{f}_k(\mathbf{k}_0)\mathbf{e}(\mathbf{k}_0)A(\mathbf{r}) - i\frac{\partial}{\partial k_m}\left(\mathbf{f}_k(\mathbf{k}_0)\mathbf{e}(\mathbf{k}_0)\frac{\partial A}{\partial r_m}\right)\right]e^{i\mathbf{k}_0\mathbf{r}}. \tag{13}$$

The energy flux density transverse to the magnetic field transported by electrons, $(\Pi_p)_\perp=\int \mathbf{v}_\perp(mv^2/2)\Phi d^3v$ is obtained from the kinetic equation (9) averaged over time after integrating it over phase space with the weight $\mathbf{v}_\perp(mv^2/2)$. Omitting the small corrections of order r_H/L (r_H is the electron gyroradius, and $L\propto|\partial\ln A/\partial\mathbf{r}|^{-1}$ is the characteristic scale on which the field varies in the electromagnetic beam), after lengthy but uncomplicated mathematical transformations we have

$$(\Pi_p)_x = -\frac{1}{2\omega_H}\mathrm{Re}\int\left(ev_\perp(\mathbf{E}^*\mathbf{v})\sin\psi - F_y^*\frac{v^2}{2}\right)\tilde{f}d^3v,$$
$$(\Pi_p)_y = \frac{1}{2\omega_H}\mathrm{Re}\int\left(ev_\perp(\mathbf{E}^*\mathbf{v})\cos\psi - F_x^*\frac{v^2}{2}\right)\tilde{f}d^3v. \tag{14}$$

Multiplying the time-averaged kinetic equation by $mv^2/2$ and integrating it over phase space, we obtain the evident relationship

$$\frac{\partial(\Pi_p)_\parallel}{\partial z} + \mathrm{div}(\Pi_p)_\perp = -\frac{1}{2}\mathrm{Re}\int e(\mathbf{E}^*\mathbf{v})\tilde{f}d^3v, \tag{15}$$

where

$$(\Pi_p)_\parallel = \int v_\parallel\frac{mv^2}{2}\Phi_0 d^3v$$

is the energy flux density in the direction of the magnetic field, and $\Phi_0=\int_0^{2\pi}\Phi d\psi$ is the component of the stationary distribution function averaged over the gyroangle.

By using Eqs. (8) and (10), we bring Eq. (15) into the form

$$\frac{\partial(\Pi_p)_\parallel}{\partial z}+\operatorname{div}(\Pi_p)_\perp=Q-\operatorname{div}\frac{\omega}{16\pi}|A|^2e_j^*e_n\frac{\partial\epsilon_{jn}^H}{\partial\mathbf{k}}\bigg|_{\mathbf{k}=\mathbf{k}_0},$$

$$Q=\frac{\omega}{8\pi i}|A|^2\left[\left(1+\frac{\partial\arg A}{\partial r_m}\frac{\partial}{\partial k_m}\right)e_j^*e_n\epsilon_{jn}^{aH}\right]_{\mathbf{k}=\mathbf{k}_0} \tag{16}$$

$$\approx\frac{\omega}{8\pi i}|A|^2e_j^*e_n\epsilon_{jn}^{aH}|_{\mathbf{k}=\mathbf{k}_0+\nabla(\arg A(\mathbf{r}))}.$$

Here ϵ_{jn}^H and ϵ_{jn}^{aH} are the Hermitian and anti-Hermitian components of the permittivity tensor of the plasma,

$$\epsilon_{jn}=\delta_{jn}-\frac{4\pi i}{\omega}e\int v_j(f_k)_nd^3v, \tag{17}$$

and $\mathbf{k}=\mathbf{k}_0+\nabla(\arg A(\mathbf{r}))$ is the local wave vector directed along the gradient of the phase of the field of the wave beam.

As already noted in the Introduction, in principle ϵ_{jn}^H and ϵ_{jn}^{aH} can be quantities of the same order of magnitude. In a two-dimensional wave beam, where we have $\partial A/\partial y=0$, in the case in which the anti-Hermitian part of the dielectric permittivity tensor is small (i.e., when $|\epsilon_{jn}^H|\gg|\epsilon_{jn}^{aH}|$), by using an expansion in the small parameter $(k_0L)^{-1}$, we can obtain a standard expression for $(\Pi_p)_\perp$:

$$(\Pi_p)_x=-\frac{\omega}{16\pi}|A|^2e_j^*e_n\frac{\partial\epsilon_{jn}^H}{\partial k_\perp}\bigg|_{\mathbf{k}=\mathbf{k}_0},\quad(\Pi_p)_y=0. \tag{18}$$

But if $|\epsilon_{jn}^H|\sim|\epsilon_{jn}^{aH}|$ holds, one cannot obtain any single analytical representation of $(\Pi_p)_\perp$ analogous to Eq. (18) for an arbitrary relationship between $k_{0\parallel}$ and $k_{0\perp}$. In what follows we will consider only with cases of quasitransverse propagation[4] ($k_{0\parallel}\ll k_{0\perp}$) of an O-mode with a frequency close to ω_H and an X-mode with frequency close to $2\omega_H$, when the condition for the dipole approximation is satisfied ($k_\perp r_H\ll 1$). For both of these examples, upon taking account of their intrinsic character of the polarization of the field $\mathbf{E}$ ($E_\parallel\gg E_\perp$ for the O-mode and $E_\parallel\ll E_\perp$ for the X-mode), we find from (14)[5] that

$$(\Pi_p)_\perp=-\frac{\omega}{16\pi}|A|^2\left(\mathbf{x}_0e_j^*e_n\frac{\partial\epsilon_{jn}^H}{\partial k_\perp}-\mathbf{y}_0i^{-1}e_j^*e_n\frac{\partial\epsilon_{jn}^{aH}}{\partial k_\perp}\right). \tag{19}$$

The distribution of the energy contribution in the plane perpendicular to the magnetic field is conveniently characterized by the following function:

$$\Gamma(x,y)=(\Pi_p)_\parallel\bigg|_{z\to-\infty}^{z\to\infty}. \tag{20}$$

Using Eqs. (19) and (16) in (20), we obtain the following expression:

$$\Gamma(x,y)=\int_{-\infty}^{\infty}\left[Q-\frac{\omega}{16\pi i}i^{-1}e_j^*e_n\frac{\partial\epsilon_{jn}^{aH}}{\partial k_\perp}\bigg|_{\mathbf{k}=\mathbf{k}_0}\frac{\partial|A|^2}{\partial y}\right]dz. \tag{21}$$

We see from Eq. (21) that the transport of particle energy along the axis perpendicular to the plane of the vectors $\mathbf{k}_0$ and $\mathbf{H}_0$ shifts the profile of the energy contribution with respect to the intensity profile of the hf field. The physical cause of the appearance of this additional transport of particle kinetic energy is that the electrons are bunched in phase with a rotating component of the force exerted by the wave field. It is just this bunching which creates the anti-Hermitian component of the permitivity tensor ϵ_{jn}^{aH} in a collisionless plasma; when we take account of the spatial variation of the hf field (for finite $k_\perp$), the oscillations of the electron bunches give lead rist to an additional energy flux. Interestingly, for this case one can write the Poynting theorem in standard form with an arbitrary relationship between ϵ_{jn}^H and ϵ_{jn}^{aH}. Using Eq. (16) and the conservation of total energy flux of the plasma and of the electromagnetic field, we can easily derive the following relationship:

$$\operatorname{div}\mathbf{S}+Q=0, \tag{22}$$

where

$$\mathbf{S}=\frac{c^2}{16\pi\omega}\left(e_j^*e_n\frac{\partial}{\partial\mathbf{k}}D_{jn}^H\right)\bigg|_{\mathbf{k}=\mathbf{k}_0}|A|^2 \tag{23}$$

is the total energy flux density; in the expression for the density of ohmic losses, Q takes account of their dependence on the local wave vector (see Eq. (16)).[6]

The direction in which the hf field is transported coincides with that of the vector $\mathbf{S}$. The vector $\mathbf{S}$ is collinear with the $\mathbf{k}$ derivative of the real part of the ray Hamiltonian if we define the polarization vector that enters into (23) by the approximate relationship

$$D_{jn}^He_n=0$$

instead of the exact relationship

$$D_{jn}e_n=0.$$

This approximation is justified if the correction to the polarization of the normal fields associated with the thermal motion of the particles alters the energy flux much less than does the nonelectromagnetic (due to motion of the particles) component of the energy flux. In the case of perpendicular propagation of the waves in a weakly relativistic plasma near a cyclotron resonance line, we can easily prove this statement and, despite Eq. (4), we can use the standard scheme of geometric optics, while defining the trajectories of transport of the intensity of radiation by using the real ray Hamiltonian

$$\operatorname{Re}D=\det\|D_{jn}^H\|.$$

To find where the energy is contributed, however, we must take account of the additional (stimulated by the field of the wave) transport of kinetic energy by particles of the plasma associated with the anti-Hermitian component of the dielectric permittivity tensor ϵ_{jn}^{aH}.

The standard expression for the field intensity in the ray tube[4-6] has the form

$$I=I_0\exp\left[-\int_0^l\frac{\operatorname{Im}k^2}{\operatorname{Re}k}\cos\alpha\,dl\right]$$

(α is the angle between the vectors $\mathbf{S}$ and $\operatorname{Re}\mathbf{k}$, while the differential of the path dl is taken along the vector $\mathbf{S}$). It is valid for a wave beam only in the absence of appreciable distortions of the phase front caused by the dependence of the optical losses on the propagation wave vector $\operatorname{Re}\mathbf{k}$ (see

Eq. (16)). Yet one can estimate the distortions of the phase front in the beam at the center of the electron-cyclotron resonance line only by taking account of absorption. Let us analyze these problems for the following example.

3. QUASIOPTICAL BEAMS OF ELECTRON-CYCLOTRON WAVES IN A PLANE-LAYER PLASMA

Consider the quasi-transverse propagation of a beam of electron-cyclotron waves with ordinary polarization in a plane-layer plasma. The magnetic field is along the z axis, and the field strength varies along the x axis. In the dipole approximation ($k_\perp r_H \ll 1$), the dispersion equation for the O-mode can be represented in the following form:

$$D(\omega,\mathbf{k})=\frac{c^2k^2}{\omega^2}-\left\{1-\frac{\omega_p^2}{\omega^2}\left[1+\left(1-\frac{\omega_p^2}{\omega^2}\right)\cos^2\theta\right]^{-1}+\epsilon_\parallel^{\text{hot}}\right\}=0, \tag{24}$$

where $k^2=k_\perp^2+k_z^2$, $\cos\theta=k_z/k$, and

$$\epsilon_\parallel^{\text{hot}}=\frac{\omega_p^2}{4\omega^2}\frac{c^2k_\perp^2}{\omega^2}\int\frac{v_\parallel^2 v_\perp}{c^2}\frac{\partial\Phi_0}{\partial v_\perp}\left(\frac{\omega}{\omega-k_z v_\parallel-\omega_H}\right)d^3v. \tag{25}$$

In Eq. (25) the distribution function Φ_0 averaged over time and over the gyroangle is normalized to unity, while the singularity in the integrand is traversed according to the Landau rule.[9]

We assume a Gaussian field distribution in the plane $x=0$:

$$E_\parallel|_{x=0}=E_{\parallel 0}\exp\left(-\frac{|\mathbf{a}|^2}{L^2}+i\mathbf{k}_0\mathbf{a}-i\omega t\right). \tag{26}$$

where $\mathbf{a}=\mathbf{y}_0 y+\mathbf{z}_0 z$, and $\mathbf{k}_0=\mathbf{y}_0 k_y+\mathbf{z}_0 k_z$.

We represent the solution of the dispersion equation (24) in the form

$$k_x=k_x(k_{0z}+\Delta k_z, k_{0y}+\Delta k_y, \omega_p^2(x), H_0(x)). \tag{27}$$

In the limit $|\Delta\mathbf{k}|\ll|\mathbf{k}_0|$ we can expand Eq. (27) in powers of $\Delta\mathbf{k}$. Within the projector zone where diffraction effects are insubstantial, it suffices to retain the first two terms of this series:

$$k_x=k_x(\mathbf{k}_0,x)+\Delta\mathbf{k}\frac{\partial}{\partial\mathbf{k}_0}k_x(\mathbf{k}_0,x). \tag{28}$$

Note that, in the resonance region where $|\text{Re}\,\epsilon_\parallel^{\text{hot}}||\text{Im}\,\epsilon_\parallel^{\text{hot}}|$ holds, owing to the narrowness of the cyclotron absorption line, usually varies the quantity $k_x(\mathbf{k}_0,x)$. Upon using the WKB solution for each Fourier harmonic, in the case of the Gaussian beam of (26) we have

$$E_\parallel=E_{\parallel 0}\exp\left(i\mathbf{k}_0\mathbf{a}-i\omega t+i\int_0^x k_x(x)dx+G-i\varphi\right)\times\exp\left(-\frac{1}{L^2}\left(\mathbf{a}+\int_0^x\frac{\partial}{\partial\mathbf{k}_0}\text{Re}\,k_x(x)dx\right)^2\right), \tag{29}$$

where

$$G=\frac{1}{L^2}\left[\int_0^x\frac{\partial}{\partial\mathbf{k}_0}\text{Im}\,k_x(x)dx\right]^2,$$

$$\varphi=2iL^{-2}\left(\mathbf{a}+\int_0^x\frac{\partial}{\partial\mathbf{k}_0}\text{Re}\,k_x(x)dx\right)\times\int_0^x\frac{\partial}{\partial\mathbf{k}_0}\text{Im}\,k_x(x)dx.$$

The representation (29) is suitable only under the condition that the diffraction parameter d is small:

$$d=\frac{2\pi}{L^2}\int_0^x\left(\left|\frac{\partial^2k_x}{\partial k_{0y}^2}\right|+2\left|\frac{\partial^2k_x}{\partial k_{0y}\partial k_{0z}}\right|+\left|\frac{\partial^2k_x}{\partial k_{0z}^2}\right|\right)dx\ll 1, \tag{30}$$

i.e., for propagation over moderate distances.

By analyzing the dispersion relationship (24), we can derive the following estimation formulas:

$$\left|\frac{\partial k_x}{\partial k_{0y}}\right|\simeq\left|\frac{k_{0y}^2}{k_x^2}\right|,\quad\left|\frac{\partial^2 k_x}{\partial k_{0y}^2}\right|\simeq\left|\frac{k_{0y}}{k_x^2}\right|, \tag{31}$$

$$\left|\frac{\partial k_x}{\partial k_{0z}}\right|\simeq 0\quad\text{as}\quad\left|\frac{k_{0z}c^2}{\omega v_T}\right|\to 0, \tag{32}$$

$$\left|\frac{\partial k_x}{\partial k_{0z}}\right|\simeq\left|\frac{\epsilon_\parallel^{\text{hot}}}{k_\perp k_{0z}}\right|\frac{\omega^2}{c^2}\quad\text{when}\quad\left|\frac{k_{0z}c^2}{\omega v_T}\right|\gtrsim 1, \tag{33}$$

$$\left|\frac{\partial^2 k_x}{\partial k_{0z}^2}\right|\simeq 0\quad\text{as}\quad\left|\frac{k_{0z}c^2}{\omega v_T}\right|\to 0, \tag{34}$$

$$\left|\frac{\partial k_x^2}{\partial k_{0z}^2}\right|\simeq\left|\frac{\epsilon_\parallel^{\text{hot}}}{k_\perp k_{0z}^2}\right|\frac{\omega^2}{c^2}\quad\text{when}\quad\left|\frac{k_{0z}c^2}{\omega v_T}\right|\gtrsim 1. \tag{35}$$

We see from (31)–(35) that the parameter d reaches its maximum value when

$$\left|\frac{k_{0z}c^2}{\omega v_T}\right|\gtrsim 1. \tag{36}$$

Here the quantity $\epsilon_\parallel^{\text{hot}}$ equals

$$\epsilon_\parallel^{\text{hot}}\simeq\frac{k_\perp^2c^2}{\omega^2}\frac{\omega_p^2}{\omega^2}\left|\frac{\omega v_T}{k_{0z}c^2}\right|. \tag{37}$$

Using (31)–(35), we find that, as the beam propagates through the gyroresonance layer, the diffraction parameter d satisfies the inequality

$$d\lesssim\frac{2\pi\tau}{(k_{0z}L)^2}, \tag{38}$$

where τ is the optical thickness of the absorbing layer.

In comparison with the standard solution obtained in the geometric-optics approximation, in which $|\text{Im}\,\epsilon_\parallel^{\text{hot}}|\ll|\text{Re}\,\epsilon_\parallel^{\text{hot}}|$ holds, Eq. (29) contains additional factors responsible for the change in both the amplitude and the phase of the field, described by the coefficient $\exp(G-i\varphi)$. For the case in which the electron-cyclotron waves pass through the zone of an absorption line where Eq. (36) holds, we can easily derive the estimates

$$G\simeq\frac{d\tau}{2\pi},\quad\varphi_z'\simeq\frac{k_{0z}d}{2\pi}. \tag{39}$$

In other words, the inaccuracies in the standard solution due to the anti-Hermitian component of the permitivity tensor, like the diffraction corrections, are of the first order in d. They can be substantial only when the beam passes through a resonance layer with a sufficiently large optical thickness in which practically all the incident hf power is absorbed.

4. CONCLUSION

Let us formulate the conclusions from this study.

a) At the center of the cyclotron resonance line, the hf field stimulates additional energy transport by particles of the plasma perpendicular to the plane of the vectors of the external magnetic field and the phase velocity of the propagating electron-cyclotron wave. This transport is absent in the approximation of a purely Hermitian dielectric permittivity tensor.

b) One can apply the real ray Hamiltonian to find the trajectory of the wave beam in the region of a cyclotron absorption line only for propagation distances that are not too long. The corrections to the field due to the anti-Hermitian components of the permitivity tensor calculated in the ray-optics approximation are of the same order of magnitude as the diffraction effects.

The authors are grateful to the Russian Foundation for Fundamental Research for financial support (Grants No. 95-02-04999-a and No. 96-02-17473).

1) This involves polarization of normal electromagnetic waves for propagation perpendicular to the magnetic field.[4–6]

2) The dependence of the conditions of (7) on the parameter v_T/c arises from the specifics of the propagation of waves in a magnetoactive plasma.[9]

3) In Ref. 9 the corresponding expressions contain errors; the corrected formulas can be found, e.g., in Ref. 10.

4) These cases are most important for the theory of electron-cyclotron heating of a thermonuclear plasma.[4–6]

5) We assume, $e_{x,y}0$, for an 0-mode, and e_x0, for an X-mode.

6) A relation analogous to (22) was also obtained in Ref. 5. However, by using an expansion in the normal modes, we were able to substantially simplify the expression for the density of ohmic losses Q.

1 V. L. Ginzburg, *Propagation of Electromagnetic Waves in Plasmas*, 2nd ed. (in Russian), Nauka, Moscow (1967) (Engl. transl., Pergamon Press, Oxford, 1970).

2 V. M. Agranovich and V. L. Ginzburg, *Crystal Optics with Account Taken of Spatial Dispersion and Exciton Theory* (in Russian), Nauka, Moscow (1965) (Engl. transl., *Spatial Dispersion in Crystal Optics and the Theory of Excitons*, Wiley, New York, 1967).

3 V. L. Ginzburg, *Theoretical Physics and Astrophysics* Pergamon, Oxford, (1979).

4 V. V. Alikaev, A. G. Litvak, E. V. Suvorov, and A. A. Fraiman, in *High-Frequency Plasma Heating*, ed. A. G. Litvak, American Institute of Physics, New York (1992), pp. 1–64.

5 A. D. Piliya and V. A. Fedorov, in *Reviews of Plasma Physics* (in Russian), No. 13, ed. B. B. Kadomtsev, Consultants Bureau, New York (1987).

6 E. Westerhof, in *Proc. of the Joint Varenna-Lausanne Int. Workshop "Theory of Fusion Plasmas"*, Italy (1994), pp. 115–130.

7 B. Bindslev, in *Contr. Papers of 9th Joint Workshop on Electron Cyclotron Emission (ECE) and Electron Cyclotron Heating (ECH)*, ed. J. Lohr, World Scientific Publishing, Singapore (1995), pp. 585–607.

8 C. R. Smith, L. D. Pearlstein, A. H. Kritz *et al.*, Preprint of the Livermore National Laboratory, UCRL-IC-11 9843 (1995).

9 A. I. Akhiezer, I. A. Akhiezer, R. V. Polovin *et al.*, *Plasma Electrodynamics*, Pergamon, Oxford, (1975).

10 A. I. Smirnov and M. D. Tokman, Preprint of the Institute of Applied Physics of the Russian Academy of Sciences No. 384, Nizhniĭ, Novgorod (1995).

Translated by M. V. King