Weak-localization effects in electron billiards

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The magnetoresistance associated with electronic interference effects in antidot lattices with different periods is studied. The distribution of the areas S of the closed trajectories in an antidot lattice is calculated. Unlike "stadium" billiard systems, it exhibits a maximum at S/d^2-1, where d is the lattice period. The interference of these trajectories makes it possible to describe the features of experimental plots of the magnetoresistance, including the nonmonotonic dependence of amplitude of the magnetoresistance oscillations on d. [S1063-7761(96)02408-0]

1. INTRODUCTION

Systems with ballistic electron transport have been actively studied in recent years. Various types of electron billiards have been created on the basis of these systems. The distribution of the electron trajectories differs in billiards of different types. Chaotic stadium-shaped billiards are characterized by an exponential area distribution, while in circular billiards the areas of the closed trajectories are distributed according to a power law.[1] The interference of the electron trajectories makes it possible to experimentally determine features in the distribution of the trajectories according to area. Reproducible fluctuations of the conductance as a function of the magnetic field have been discovered in samples with dimensions smaller than the phase coherence length L_φ (Ref. 2). The mean period and the correlation magnetic field β_c of these oscillations are determined by the quantity Φ_0/S, where $\Phi_0=hc/e$ and S is the area of a characteristic trajectory. Thus, the distribution of areas of the trajectories can be extracted from the frequencies of the magnetoresistance fluctuations.[2,3] The interference of trajectories running in the clockwise and counterclockwise directions leads to weak-localization effects, which cause a negative magnetoresistance in macroscopic samples or magnetoresistance oscillations with period $\Phi_0/2S$ in cylindrical samples and in two-dimensional networks of connected hexagonal loops.[4,5]

One type of electron billiards is a lattice of antidots. Such a system can be obtained by etching holes (antidots) of submicron diameter in a GaAs/AlGaAs heterostructure with a two-dimensional electron gas. The electrons move ballistically in this antidot lattice. In a classically strong magnetic field, in which the cyclotron diameter of the electron is comparable to the lattice period, regular electron orbits appear, and the transport of electrons in the lattice is altered significantly. This leads to the appearance of commensurate magnetoresistance oscillations, which have been detected experimentally.[6–8] However, transport in zero and weak magnetic fields has not been adequately studied. One of our previous papers[9] describes the magnetoresistance features discovered in a sample containing 10^5 antidots, which were associated with interference of the trajectories with backscattering (weak localization) and suppression of this interference by a magnetic field (the Aharonov–Bohm effect). The present work is an experimental and theoretical investigation of the weak-localization effects in a two-dimensional electron gas with an antidot lattice, which is a variant of Sinai electron billiards.

2. EXPERIMENTAL SAMPLES

The experimental samples were Hall bridges fabricated on the basis of GaAs/AlGaAs heterostructures with a two-dimensional electron gas. The distance between the potential probes was 500 μm, and the bridge width was 200 μm. The electron density and mobility in the original heterostructures were equal to $n_S=4\times10^{11}$ to 5×10^{11} and $\mu=2\times10^5$ to 5×10^5 cm^2/V·s, respectively. A square antidot lattice created by electron lithography and reactive plasma etching covered part of the sample between the potential contacts. Samples with lattice periods $d=0.5$, 0.6, 0.7, 0.8, 0.9, 1.0, and 1.3 μm and geometric diameters of the antidots equal to 0.1–0.15 μm were studied. The effective antidot diameter a consists of the lithographic diameter and the thickness of the depletion region. In a previous study[9] we estimated the effective diameter to be $a=0.3$–0.35 μm using the approximation that the mobility does not depend on the lattice period d. Several samples with identical periods, but different mobilities of the original two-dimensional gas were also prepared. After deposition of the antidot lattice, the electron mobilities in the samples became equal, even when the original mobilities differed by a factor of two. The magnetoresistance was measured by means of the four-point technique using an active ac bridge at 70–700 Hz in magnetic fields up to 8 T at 1.3–4.2 K. Commensurate oscillations, that give way to Shubnikov-de Haas oscillations at higher magnetic fields, were observed in all the samples.

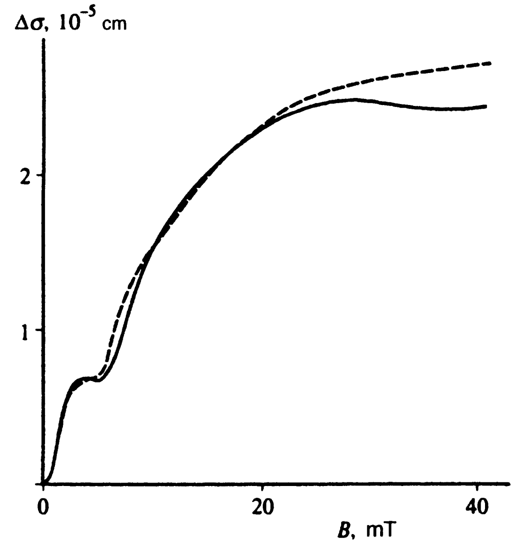

FIG. 1. Magnetoconductance of a sample containing an antidot lattice with period $d = 0.6\ \mu$m. Dashed line – experimental curve at $T = 1.7$ K; solid line – curve calculated from Eq. (3) $A = 0.75$.

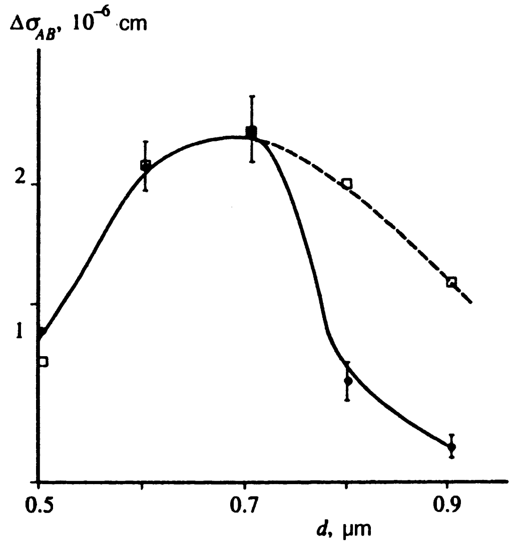

FIG. 2. Amplitude of the magnetoconductance minimum as a function of the lattice period. Circles – experimental values; squares – theoretical values.

3. MAGNETORESISTANCE IN AN ANTIDOT LATTICE

Figure 1 shows how the magnetoconductance of a sample with period $d = 0.6\ \mu$m depends on the magnetic field B. The positive magnetoconductance (negative magnetoresistance) is marked by a minimum, whose position is given by the relation

$$B_c = \Phi_0/2d^2. \tag{1}$$

Similar minima were observed for the samples with $d = 0.5$, 0.6, 0.7, 0.8, and 0.9 μm. Table I presents the positions of the minima for all the periods and values of B_c calculated from Eq. (1). The experiment and the calculation are in good agreement. Figure 2 presents the dependence of the amplitude of the minimum on d after the monotonic component was subtracted. The oscillations observed can be associated with the Aharonov–Bohm effect, which appears in closed electron trajectories with backscattering.[4,5] However, our case is essentially different. The Aharonov–Bohm effect is usually considered for loops, where the electron trajectories can be divided into two groups: left-hand and right-hand trajectories. In a weak-localization process electrons moving in opposite directions interfere, and the period is determined by the relation $B = \Phi_0/2\pi R^2$, where R is the mean radius of the loop. In our case, especially for $d/a \gg 1$, the electrons move chaotically, and the features of the magnetoconductance can be associated only with the portion of trajectories that embrace an area $S \sim d^2$. Since the trajectories are distributed according to area in the general case, there is a nonzero probability of finding trajectories with any area.

TABLE I. Positions of the magnetoconductance minima for samples with different periods.

d, μm	$B^{\exp}$ mT	$B_c = \Phi_0/2d^2$ mT
0.5	8.4	8.24
0.6	5.6	5.7
0.7	3.9	4.2
0.8	3.2	3.2
0.9	2.2	2.5

We assumed that the distribution of the trajectory areas in a billiard system with antidots is not described by the exponential law observed in the case of stadium billiards. To test this hypothesis, we performed a numerical calculation of the distribution of the trajectory areas. The ballistic motion of electrons in an infinite antidot lattice with random initial conditions was considered. The calculation was carried out for a zero magnetic field. We also assumed that the antidots have a circular shape and an infinite potential barrier, on which perfect scattering of the electrons occurs. This assumption was tested experimentally at higher magnetic fields, where commensurate oscillations are observed.[9] We computed the number of closed trajectories after ten collisions with antidots from the onset of motion. If a trajectory had the form of a closed loop with another smaller loop inside, we subtracted the area of the smaller loop, since the flux was reduced in that case.[10] After performing this procedure, we took other initial conditions and performed the calculation again. The total number of closed trajectories was equal to 10^5. It is noteworthy that one of our previous investigations[11] showed that the anomalous behavior of the mesoscopic conductance fluctuations in an antidot lattice with period $d = 0.5\ \mu$m can be explained on the basis of the area distribution obtained. Figure 3 presents the distribution of the electron trajectories as a function of S/d^2, where S is the area of the loop, for three different lattice periods. It is seen that for $S/d^2 < 0.5$ this distribution corresponds to the law

$$P \propto e^{-\alpha S} \tag{2}$$

with characteristic reciprocal area α, which differs for different trajectories due to the decrease in the area between

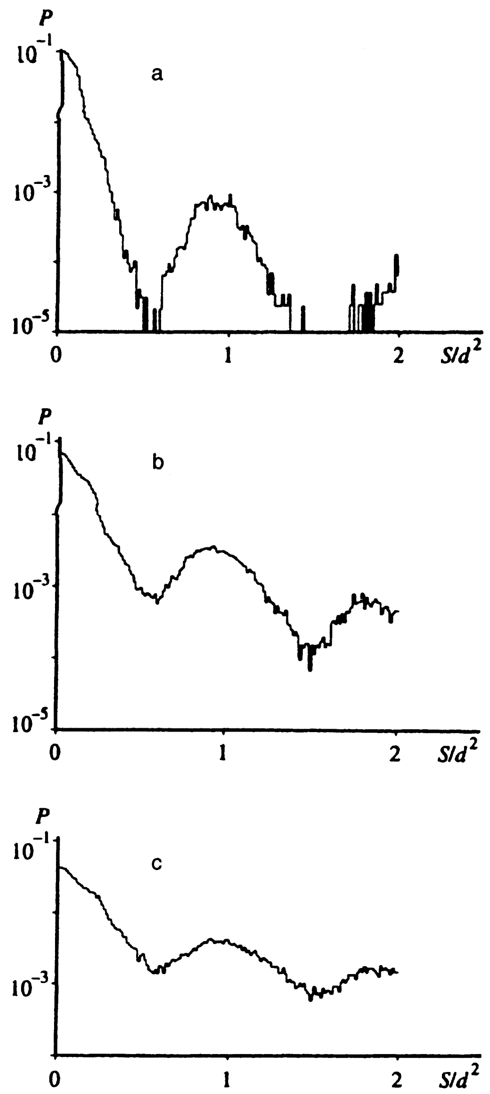

FIG. 3. Distribution of the areas of closed trajectories in lattices with different periods d: a — 0.5; b — 0.7; c — 0.9 μm.

antidots with decreasing d. At $S/d^2=0.5$ the distribution probability of the trajectories decreases, in contrast to the case of stadium billiards. This probability reaches a maximum at $S/d^2=1$ and then decreases. At $S/d^2=1.8$ a second maximum appears for lattices with $d=0.7$ and 0.9 μm. The probability for a closed trajectory with area $S=d^2$ is one or two orders of magnitude higher than for a stadium billiard. We note that the capture of electrons with trajectory areas $S\sim d^2$ by an antidot lattice can account for the observed decrease in mobility from the expected value. For example, in a simple model for the scattering of two-dimensional electrons on disks, the mean free path is $l=d^2/a$, which is approximately three times greater than the measured value.[9] The dependence of the peak amplitude on d is nonmonotonic: its value is approximately equal to 10^{-3} for $d=0.5$ μm, then increases to 6×10^{-2} for $d=0.7$ μm, and finally decreases again for $d=0.9$ μm. The magnetoconductance is specified by the Fourier transform of the distribution probability of the trajectories:[12,13]

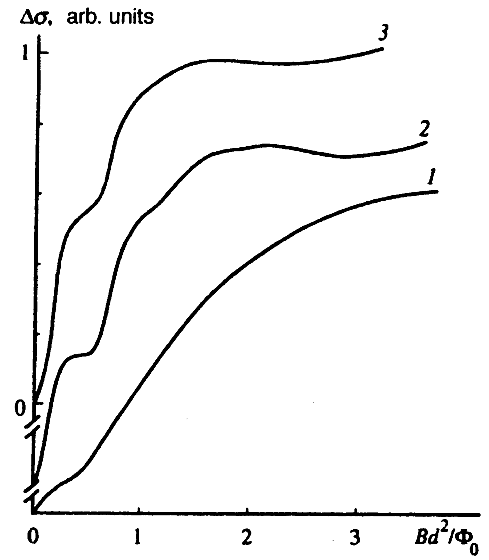

FIG. 4. Magnetoconductance calculated from Eq. (3) for lattices with different periods d: *1* - 0.5; *2* - 0.7; *3* - 0.9 μm.

$$\Delta\sigma=A\frac{e^2}{h}\int P(S)\cos\left(\frac{BS}{\Phi_0}\right)dS. \tag{3}$$

Figure 4 presents plots of the dependence of the magnetoconductance on B calculated from Eq. (3) for three different periods. It is seen that a minimum appears at $B=\Phi_0/2d^2$ superposed on the positive magnetoconductance. The amplitude of this minimum depends monotonically on the period: it increases up to $d=0.7$ μm and then decreases again. The additional minima observed for the lattice with $d=0.7$ μm at $B\approx1.2\Phi_0/d^2$ and $3\Phi_0/d^2$, as well as for the lattice with $d=0.9$ μm at $B\approx2\Phi_0/d^2$, are determined by features in the area distribution shown in Fig. 3. It is difficult to discern these minima experimentally, because the contribution of the commensurate oscillations, which lead to the appearance of a positive magnetoconductance, begins to dominate in this range of magnetic fields. The commensurate oscillations also influence the resistance at weaker magnetic fields. However, in the samples with a short period these oscillations, which are associated with electron trajectories that run along rows of the lattice,[8] are smeared because of the disappearance of the corresponding trajectories due to obscuring by the antidots in other rows. In this case, in weak magnetic fields (up to ≈ 20 mT for the samples investigated) we can compare the experimental plot of the weak-localization magnetoconductance with our calculations. Figure 1 shows experimental and theoretical plots for the sample with $d=0.6$ μm. We see good agreement between these two curves with the fitting parameter $A=0.7$. The first minimum on the experimental curve has a smaller amplitude and a more symmetric shape in comparison to the calculation. In stronger magnetic fields no minimum is observed on the experimental plot of the magnetoconductance. The coherence length L_φ was assumed to be infinite in our calculations. The form of the magnetoconductance curve, excluding the Aharonov–Bohm minimum, is described by a Lorentzian dependence

$$\Delta\sigma = A\frac{e^2}{h}\frac{1}{1+(B/\alpha\Phi_0)^2}, \quad (4)$$

which can be obtained by substituting the relation (2) for the area distribution in stadium billiards[12,13] into (3). Figure 2 shows the amplitudes of the minima as a function of d, which were obtained by subtracting the Lorentzian plots from curves *1*, *2*, and *3* (Fig. 4). We also see good agreement between theory and experiment for the antidot lattices with $d<0.7\ \mu$m when $A=0.45$. This parameter is smaller than the value obtained from fitting the complete magnetoconductance curves in the theory and the experiment (Fig. 1). This means that the distinctive closed trajectories with area $S=d^2$ are more sensitive to nonideal conditions than are the other chaotic paths. We did not calculate A, because such a calculation requires knowledge of the transmission probability in our system. (Such a problem has been solved only for a cavity with two leads.[12,13])

The calculated amplitudes for lattices with $d>0.7\ \mu$m decrease, but more slowly than do the experimentally obtained values. This can be explained by the following arguments. First, a finite coherence length must be introduced into the calculations, since we assumed that in our case contributions to the interference are made by trajectories with a length less than the ballistic phase coherence length $v_F\tau_\varphi$, where v_F is the Fermi velocity. However, in Ref. 14 the phase coherence time τ_φ was measured in stadium billiards in samples with the same mobility. Even if we postulate the dependence $\tau_\varphi \propto T^{-1}$, using the data in Ref. 14 we obtain $L_\varphi=5-10\ \mu\mathrm{m} \gg d$ at $T=1.7$ K. Second, we neglected the finiteness of the elastic mean free path l. In our case it is $l=5\ \mu$m, and the electrons in a lattice with $d>0.8\ \mu$m can undergo scattering by an impurity after 10–15 collisions with antidots, which can alter the area distribution. The diffuse nature of the scattering on the surface of an antidot must also be taken into account. Finally, A depends on the period, since the probability of transmission through the lattice varies. It is also shown in Fig. 4 that the slope of the magnetoconductance curve increases with the period and reaches saturation in weak magnetic fields. This means that the critical magnetic field $B_0=\alpha\Phi_0$, which is determined from the Lorentzian dependence of the magnetoconductance curve, decreases as the period increases. The value of the critical field B_0 is determined by the characteristic area of a billiard. We note that to study the transition from a billiard system on antidots to the geometry of connected hexagonal loops, the period must be decreased further at the same antidot diameter.

4. CONCLUSIONS

Thus, it has been shown in this work that the distribution of the closed trajectories in a lattice of artificial scatterers depends on the shape of the scatterers and the ratio between their diameter and the lattice period. The negative magnetoresistance caused by weak-localization effects is described by a Lorentzian curve, in agreement with experiments on antidot lattices with small periods. As the period increases, a maximum appears on the area distribution at $S/d^2=1$. The probability of the appearance of a trajectory with such an area is one to two orders of magnitude greater than in the case of stadium billiards. The interference of these closed trajectories produces features of the electron transport that are in good agreement with our measurements of the negative magnetoresistance in antidot lattices with different periods.

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