Interaction of ultrashort relativistically intense laser pulses with matter: conservative models and instabilities of the light field

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We discuss a conservative, three-dimensional model of the nonlinear interaction of ultrashort laser pulses of relativistic intensity with matter, taking into account the generation of oscillations in the electronic component of the induced plasma. Instability of the light field is studied in the framework of the model. © *1996 American Institute of Physics.* [S1063-7761(96)00609-9]

Recently, in connection with the development of superintense lasers, a number of researchers have considered the interaction of ultrashort laser pulses with matter at relativistic intensities of the light field. The optics of ultrashort pulses of nonrelativistic intensities have been completely worked out. The basic results in this field are summarized, for example, in the monographs of Akhmanov, Visloukh, and Chirkin[1] and Sukhorukhov.[2] A number of questions on the physics of the interaction of strong laser radiation with matter have been discussed in the book of Koroteev and Shumaĭ.[3]

When laser radiation of relativistic intensity interacts with matter, the leading edge the pulse produces rapid ionization, and therefore the radiation propagates in an induced plasma. The pioneering papers on the corresponding plasma nonlinearities (relativistic and striction) are those of Akhiezer and Polovin,[4] Askar'yan,[5] Litvak,[6] and others. Recent work on the interaction of laser radiation of relativistic intensities with matter can be summarized as follows. A mathematical model of the interaction of long laser pulses with a plasma was formulated in Ref. 7. A detailed mathematical study of this model[8] showed that the equations given in Ref. 7 have a denumerable set of eigenmodes. In addition, it was established in Ref. 8 by numerical simulation that at large depths, laser pulses of supercritical intensity experience stabilization and their asymptotic transverse profiles are described by the lowest eigenmodes of the problem. This phenomenon is called relativistic-striction self-channeling of laser radiation. The model of Refs. 7 and 8 was extended in Refs. 9 and 10 to take into account the effect of higher-order dispersion on the nature of the propagation, and it was shown that under conditions of sharp self-focusing of light in the plasma there is strong self-modulation of an ultrashort laser pulse.

Physical effects associated with the finite duration of the laser pulse have also been considered in a number of other papers.[11–15] For example, a model of the propagation of infinitely wide laser pulses of finite length was worked out in Ref. 11, and the equations of this model were extended in Ref. 12 to three spatial dimensions. However, these equations do not transform into the equations obtained in Ref. 7 in the limit of an infinitely long pulse. We recall that the nonrelativistic three-dimensional interaction of laser pulses with a plasma was considered in Ref. 13, where a number of dispersion terms in the equations describing the propagation of the radiation were neglected. Equations closest to those derived in the present paper were given in Ref. 14. However, a number of terms associated with the dependence of the Langmuir waves generated by the laser radiation on the transverse coordinates were omitted in Ref. 14, with the result that the problem became nonconservative.

Most theoretical studies of the problem are based on Maxwell's equations and the equations of cold, collisionless, relativistic hydrodynamics of charged particles in an electromagnetic field in the absence of collisions and thermal effects. These equations can be written in relativistically invariant notation or in the usual three-dimensional form and have an energy–momentum tensor of matter and field whose components are conserved quantities (see Ref. 9, for example). Because of the complexity of these equations, in most papers they are averaged over the period of the laser radiation, and other approximations are introduced, leading to a system of simplified equations, which are also required to be conservative.

The present paper is devoted to the derivation of a conservative, time-dependent, three-dimensional model taking the following physical effects into account: diffraction and refraction of the radiation, relativistic and striction nonlinearities in its interaction with the plasma, and the generation of plasma waves by the propagating laser pulses. The Lagrangian formulation of the problem is given below and the general instability of a uniform light field is analyzed.

As a starting point we use Maxwell's equations in the Coulomb gauge and the equations of cold, relativistic hydrodynamics for the electronic component of the plasma:

$$\Box \mathbf{A}=\nabla \phi_t+\gamma^{-1} n \mathbf{p}, \tag{1}$$

$$\Delta \phi=n-1, \tag{2}$$

$$\nabla \mathbf{A}=0, \tag{3}$$

$$(\mathbf{p}-\mathbf{A})_t-\gamma^{-1}[\mathbf{p}[\nabla,\mathbf{p}-\mathbf{A}]]=\nabla(\phi-\gamma), \tag{4}$$

$$\gamma=(1+\mathbf{p}^2)^{1/2}. \tag{5}$$

We assume that the duration of the laser pulse is so small that the ions of the plasma can be considered as fixed. In (1)–(5) the vector and scalar potentials $\mathbf{A}$ and ϕ are normalized by mc^2/e, the momentum $\mathbf{p}$ of the electron fluid by mc, the density of electrons n by its unperturbed value n_0, the time by ω_p^{-1} (where ω_p is the unperturbed value of the plasma frequency), and the spatial coordinates by c/ω_p.

The gauge condition (3) is equivalent to the continuity equation

$$n_t+(\nabla,\gamma^{-1}n\mathbf{p})=0. \tag{6}$$

The system of equations (1)–(5) conserves energy and momentum, as spelled out in Ref. 9, for example.

We consider the propagation of laser radiation along the z axis. We make the following assumptions:

A) The vector potential of the laser radiation is transverse and circularly polarized:

$$\mathbf{A}=(1/2)(\mathbf{e}_x+i\mathbf{e}_y)a(\tau,\xi,\mathbf{x}_\perp)\exp[i(kz-\omega t)]+\text{c.c.}, \tag{7}$$

where

$$\tau=\varepsilon t/2, \tag{8}$$

$$\varepsilon=\omega^{-1}, \tag{9}$$

$$\xi=z-v_g t, \tag{10}$$

$$v_g=(1-\omega^{-2})^{1/2}, \tag{11}$$

$$k=(\omega^2-1)^{1/2}. \tag{12}$$

In (7) the frequency ω of the laser radiation is normalized by ω_p. Equation (11) defines the group velocity, and the dispersion relation (12) corresponds to propagation of a weak wave in the unperturbed plasma. We assume that the density of the plasma is subcritical, i.e. $\omega>1$.

B) The momentum of the electron component of the plasma is written in the form

$$\mathbf{p}=\mathbf{A}+\nabla\psi. \tag{13}$$

This representation is equivalent to the assumption that there is no rotational correction to the motion of the electron component of the plasma. Substituting (13) into the vector equation (4), we obtain a single scalar relation

$$\psi_t=\phi-\gamma.$$

C) We neglect the generation of harmonics of the laser radiation, as well as quasistatic magnetic fields. This approximation is valid when $\omega\gg1$, i.e., when the laser frequency is much higher than the plasma frequency.

D) It is not difficult to show that in the normalization used here, the terms in the hydrodynamic equations involving the derivatives n_τ and ψ_τ are proportional to the small parameter ε, and therefore if the laser frequency is much higher than the plasma frequency these terms can be omitted in the first approximation. Physically this means that the response of the electron component of the plasma in the comoving coordinate system can be taken as instantaneous.

The following system of equations results from these assumptions:

$$ia_\tau+\Delta_\perp a+(1-\eta)a+\varepsilon^2 a_{\xi\xi}+\varepsilon a_{\xi\tau}-(\varepsilon^2/4)a_{\tau\tau}=0, \tag{14}$$

$$(\Delta_\perp+\partial^2_{\xi\xi})\phi=(\eta/2\phi)(1+|a|^2+|\nabla_\perp\psi|^2+\phi^2)-1, \tag{15}$$

$$(\eta\phi)_\xi=(\nabla_\perp,\eta\nabla_\perp\psi), \tag{16}$$

$$\psi_\xi=(1/2\phi)(1+|a|^2+|\nabla_\perp\psi|^2-\phi^2). \tag{17}$$

Here we have used the notation $\eta=n/\gamma$, and the function η plays the role of the independent variable.

The system of equations (14)–(17) has not been considered before in the literature. Because of the approximations discussed above it is not equivalent to the original equations (1)–(5), and therefore to justify our model physically it is required, in particular, to show that the equations of the model satisfy the analogs of the basic conservation laws. However these analogs are difficult to obtain directly from the conservation of energy–momentum of the original relativistically invariant system of equations. In the present paper we use another approach. The Lagrangian is chosen such that its variation with respect to the variables a^*, ϕ, ψ, η gives the equations (14)–(17), respectively. We then use Noether's theorem to obtain the conservation laws.

It can be shown that the problem (14)–(17) can be obtained by varying the following Lagrangian:

$$\begin{aligned}L&=L'+(\varepsilon^2/4)|a_\tau|^2,\\ L'&=ia^*a_\tau-\varepsilon^2|a_\xi|^2-(\varepsilon/2)(a_\xi a^*_\tau+\text{c.c.})-(\nabla_\perp a,\nabla_\perp a^*)\\ &\quad+|a|^2+\phi_\xi^2+|\nabla_\perp\phi|^2+2\phi(\eta\psi_\xi-1)-\eta(1+|a|^2\\ &\quad+|\nabla_\perp\psi|^2-\phi^2).\end{aligned} \tag{18}$$

According to Noether's theorem, the system (14)–(17) has the following invariants:

The Hamiltonian

$$H=H'+(\varepsilon^2/4)\int d^2x_\perp d\xi|a_\tau|^2,$$

$$\begin{aligned}H'=\int d^2x_\perp d\xi(\varepsilon^2|a_\xi|^2+(\nabla_\perp a,\nabla_\perp a^*)-\phi_\xi^2-|\nabla_\perp\phi|^2\\ -|a|^2+2(\phi-1)),\end{aligned} \tag{19}$$

the number of particles

$$N=N'+(\varepsilon^2/4)\int d^2x_\perp d\xi(a^*_\tau a-\text{c.c.}), \tag{20}$$

$$N'=\int d^2x_\perp d\xi(i|a|^2+(\varepsilon/2)(a^*a_\xi-\text{c.c.})) \tag{21}$$

and the momentum

$$\mathbf{P}_\perp=\mathbf{P}'_\perp+(\varepsilon^2/4)\int d^2x_\perp d\xi(a^*_\tau\nabla_\perp a+\text{c.c.}), \tag{22}$$

$$\mathbf{P}'_\perp=\int d^2x_\perp d\xi(ia^*\nabla_\perp a-(\varepsilon/2)(a^*_\xi\nabla_\perp a+\text{c.c.})), \tag{23}$$

$$P_\parallel=P'_\parallel+(\varepsilon^2/4)\int d^2x_\perp d\xi(a^*_\tau a_\xi+\text{c.c.}), \tag{24}$$

$$P'_\parallel=\int d^2x_\perp d\xi(ia^*a_\xi-\varepsilon|a_\xi|^2). \tag{25}$$

A number of papers assume that the complex amplitude of the vector potential is slowly varying in time. In this approximation the term $(\varepsilon^2/4)a_{\tau\tau}$ can be omitted in (14), while (15)–(17) remain unchanged. Then (14)–(17) has the Lagrangian L' and the invariants H', N', $\mathbf{P}'_\perp$ and $\mathbf{P}'_\parallel$ defined by (18), (19), (21), (23), and (25), respectively. The system of equations (14)–(17) with the omission of the term $(\varepsilon^2/4)a_{\tau\tau}$ has also not been considered before.

The model of Ref. 14 is closest to the equations discussed above. Our equations differ from those of Ref. 14 by the presence of some additional terms which, in particular, guarantee that the system is conservative.

Our model differs from that of Ref. 12 in that it takes into account the effect of the finiteness of the transverse aperture of the laser pulse on the nonlinear response of the plasma. Because of the additional terms, in the limit of a long pulse our model transforms into the problem of relativistic-striction self-channeling of pulses and filamentation considered earlier.[7,8]

The propagation of a laser pulse in a model completely neglecting the generation of plasma oscillations has been considered earlier.[9,10] We show that this case follows from the system of equations (14)–(17). Indeed, in this case we can set the potential correction ψ to the momentum of the electrons equal to zero. Then we obtain from (5), (13), and (17) that $\varphi=\gamma=(1+|a|^2)^{1/2}$. The superfluous equation (16) is then omitted. We finally obtain the nonlinear wave equation

$$ia_\tau+\varepsilon^2 a_{\xi\xi}+\varepsilon a_{\xi t}-(\varepsilon^2/4)a_{\tau\tau}+\Delta_\perp a+(1-(1+\Delta_\perp\gamma+\partial^2_{\xi\xi}\gamma)/\gamma)a=0. \qquad (26)$$

It can be shown that the problem (26) has the Lagrangian

$$L=ia^*a_\tau-\varepsilon^2|a_\xi|^2-(\varepsilon/2)(a_\xi a^*_\tau+\text{c.c.})-(\nabla_\perp a,\nabla_\perp a^*)+|a|^2+|\nabla_\perp\gamma|^2-2\gamma+(\varepsilon^2/4)|a_\tau|^2,$$

and its invariants are

$$H=\int d^2x_\perp d\xi(\varepsilon^2|a_\xi|^2+(\nabla_\perp a,\nabla_\perp a^*)-|\nabla_\perp\gamma|^2-|a|^2+2(\gamma-1)+(\varepsilon^2/4)|a_\tau|^2)$$

and the integrals defined by (20), (22), and (24). The wave equation (26) has been studied both numerically and analytically.[9,10]

The propagation of a tapered pulse, in which the variation of the cross section of the pulse leads to important effects,[7,8] as well as the propagation in a plasma of pulses from a very wide transverse aperture,[11,15] can be derived from the system of equations (14)–(17).

For sufficiently long pulses one can put $\partial_\xi=0$ in (14)–(17). In addition, neglecting the term $(\varepsilon^2/4)a_{\tau\tau}$, we obtain the following nonlinear Schrödinger equation

$$ia_\tau+\Delta_\perp a+(1-(1+\Delta_\perp\gamma)/\gamma)a=0.$$

It has the Lagrangian

$$L=ia^*a_\tau-(\nabla_\perp a,\nabla_\perp a^*)+|a|^2-2\gamma+|\nabla_\perp\gamma|^2$$

and the invariants[8,16]

$$H=\int d^2x_\perp d\xi((\nabla_\perp a,\nabla_\perp a^*)+2(\gamma-1)-|\nabla_\perp\gamma|^2),$$

$$N=i\int d^2x_\perp|a|^2,$$

$$\mathbf{P}_\perp=i\int d^2x_\perp a^*\nabla_\perp a.$$

This problem has been thoroughly studied. Its denumerable eigenmodes has been given and the relativistic-striction self-channeling of strong, ultrashort pulses in a plasma has been considered numerically.[8] The essence of this phenomenon is that at large propagation lengths of the laser radiation into the plasma, the transverse profiles of the momentum and density amplitudes of the electronic component of the plasma cease to depend upon the longitudinal coordinate and approach the lowest eigenmodes of the problem. It has been shown numerically[8] that to obtain self-channeling it is sufficient that the laser power exceed a certain threshold, called the critical power for relativistic-striction self-focusing. The critical power has been calculated analytically.[8] In addition, the condition $H<0$ is sufficient for relativistic-striction self-channeling of a laser pulse in a plasma.[16]

We consider further the case of large transverse aperture of the pulse. In this limit one can put $\nabla_\perp=0$ and then it follows from (16) that $\eta=\phi^{-1}$. In addition, we omit the final term in (14). Then (14) and (15) transform to

$$ia_\tau+(1-\phi^{-1})a+\varepsilon^2 a_{\xi\xi}+\varepsilon a_{\xi t}-(\varepsilon^2/4)a_{\tau\tau}=0, \qquad (27)$$

$$\phi_{\xi\xi}=(1/2)((1+|a|^2)/\phi^2-1). \qquad (28)$$

This problem was considered in Ref. 11. Equation (28) was also given in Ref. 15.

The Lagrangian of the above problem has the form

$$L=ia^*a_\tau-\varepsilon^2|a_\xi|^2-(\varepsilon/2)(a_\xi a^*_\tau+\text{c.c.})+|a|^2+\phi^2_\xi-((1+|a|^2)/\phi+\phi),$$

and its invariants are

$$H=\int d\xi\{\varepsilon^2|a_\xi|^2-\phi^2_\xi-|a|^2+[(1+|a|^2)/\phi+\phi]-2\},$$

$$N=\int d\xi[i|a|^2+(\varepsilon/2)(a^*a_\xi-\text{c.c.})],$$

$$P=\int d\xi(ia^*a_\xi-\varepsilon|a_\xi|^2).$$

If the laser pulse length is of order unity (in the normalization used here, i.e., the unnormalized length is of the order of the width of the plasma skin layer) and the function a does not have rapid spatial oscillations in ξ, then the last two terms in (27) can be neglected. In this case

$$a=a_0(\xi)\exp(i(1-\phi_0^{-1}(\xi))\tau),$$

where $\phi_0(\xi)$ is the solution of the ordinary differential equation

$$(\phi^2_{0\xi}+\phi_0+\phi_0^{-1})_\xi=(|a_0|^2/\phi_0^2)\phi_{0\xi},$$

and $a_0(\xi)$ is an arbitrary, sufficiently smooth function. It follows from the equation that in this approximation dissipation of the energy of the pulse by generation of plasma oscillations is not taken into account, i.e., we have the approximation of steady-state pumping.

In this approximation it turns out that the frequency shift of the propagating pulse is equal to the local value of the electron plasma frequency, which varies because of the relativistic increase in the mass of the electrons oscillating in the field of the laser radiation, and because of the modification of the spatial density of the electrons under the influence of the pondermotive force. This effect can be used to perform diagnostics on the irradiated plasma.

We consider, in the framework of the problem (14)–(17), the instability of a uniform radiation field. In this case we omit the term $(\varepsilon^2/4)a_{\tau\tau}$.

The exact solution of (14)–(17) corresponding to propagation of a plane wave has the form

$$a=a_0 \exp[i(1-\gamma^{-1})\tau],$$

$$\gamma=(1+a_0^2)^{1/2},$$

$$\phi=\gamma, \quad \psi=0, \quad \eta=\gamma^{-1}.$$

To study the instability of the light field we represent the solution in the form

$$a=(a_0+a_1)\exp[i(1-\gamma^{-1})\tau],$$

$$\phi=\gamma+\phi_1, \quad \psi=\psi_1, \quad \eta=\gamma^{-1}+\eta_1.$$

The linearized equations for the perturbations (labeled by the 1 subscript) have the form

$$ia_{1\tau}+\Delta_\perp a_1-a_0\eta_1+\varepsilon a_{1\xi\tau}+i(1-\gamma^{-1})\varepsilon a_{1\xi}+\varepsilon^2 a_{1\xi\xi}=0,$$

$$(\Delta_\perp+\partial_\xi^2)\phi_1=(a_0/2\gamma^2)(a_1+a_1^*)+\gamma\eta_1,$$

$$\psi_{1\xi}=(a_0/2\gamma)(a_1+a_1^*)-\phi_1,$$

$$\gamma^{-1}\Delta_\perp\psi_1=\gamma\eta_{1\xi}+\gamma^{-1}\phi_{1\xi}.$$

Assuming a plane-wave solution of the form

$$(a_1,\phi_1,\psi_1,\eta_1)^T=(a_1^0,\phi_1^0,\psi_1^0,\eta_1^0)^T \times\exp[i((\mathbf{k},\mathbf{x}_\perp)+\chi\xi-\Omega\tau)],$$

where $\mathbf{k}$, χ and Ω are related to one another through the dispersion relation, which is obtained by requiring a nontrivial solution for the amplitudes labeled by the 0 superscript.

First we express ϕ_1^0, ψ_1^0, and η_1^0 in terms of a_1^0 with the help of the last three equations of the linearized system:

$$\psi_1^0=a_0(i\chi/(1-\chi^2\gamma))(a_1^0+a_1^{*0})/2, \tag{29}$$

$$\phi_1^0=a_0(\gamma^{-1}/(1-\chi^2\gamma))(a_1^0+a_1^{*0})/2, \tag{30}$$

$$\eta_1^0=-(Q(a_0^2,k^2,\chi^2)/2a_0)(a_1^0+a_1^{*0}). \tag{31}$$

We have used the notation

$$Q(a_0^2,k^2,\chi^2)=(a_0/\gamma)^2((\gamma^{-1}+k^2)/(1-\chi^2\gamma)). \tag{32}$$

It is evident from (29)–(32) that the interaction of the radiation with the plasma is resonant in nature. The resonant spatial frequency

$$\chi_{\text{res}}=\gamma^{-1/2}$$

is an eigenfrequency of the plasma, whose electrons suffer an increase in mass as a result of oscillating with relativistic velocities in the field of the wave of intensity a_0^2.

We return to the study of the dispersion relation for small perturbations. With the help of the first of the equations of the linearized system and the above relations, it is not difficult to obtain the following expression:

$$\Omega=\{\varepsilon\chi[2(1-\gamma^{-1}-k^2-\varepsilon^2\chi^2)+Q(a_0^2,k^2,\chi^2)] \pm D^{1/2}\}\frac{1}{2(1-\varepsilon^2\chi^2)}, \tag{33}$$

where

$$D=D_0+(\varepsilon\chi)^2D_1+(\varepsilon\chi)^4D_2, \tag{34}$$

$$D_0=4k^2[k^2-Q(a_0^2,k^2,\chi^2)], \tag{35}$$

$$D_1=8k^2/\gamma+Q(a_0^2,k^2,\chi^2)[Q(a_0^2,k^2,\chi^2)-4/\gamma], \tag{36}$$

$$D_2=4/\gamma^2. \tag{37}$$

We consider some special cases.

A) Filamentation instability, $\chi=0$. We obtain

$$\Omega_\perp=\pm(k/\gamma)(k^2-a_0^2/\gamma)^{1/2}.$$

The condition for filamentation instability is given by the inequality

$$k^2<a_0^2/\gamma. \tag{38}$$

This result was obtained in Ref. 17 and used to estimate the radiative power in one filament.

B) Modulation instability, $k=0$. With the help of the general expressions (33)–(37) it is not difficult to show that

$$\Omega_\parallel=\frac{\varepsilon\chi}{2(1-\varepsilon^2\chi^2)}(2(1-\gamma^{-1}-\varepsilon^2\chi^2)+Q(a_0^2,0,\chi^2))\pm D_\parallel^{1/2},$$

where

$$D_\parallel=Q(a_0^2,0,\chi^2)(Q(a_0^2,0,\chi^2)-4/\gamma)+(2/\gamma)^2(\varepsilon\chi)^2. \tag{39}$$

The condition for modulation instability is given by the inequality $D_\parallel<0$. It can be shown from (39) that the region of the instability is bounded by $\chi<\chi'$. When $\varepsilon\ll1$ we obtain $\chi'\simeq\gamma^{-1}[1-a_0^2/4\gamma^2]$. For arbitrary ε we have $\chi'<\gamma^{-1}$.

C) General instability. The most interesting case is the general filamentation-modulation instability. It follows from the general expression for the instability increment

$$\Omega=\pm k(k^2-Q(a_0^2,k^2,\chi^2))^{1/2}+O(\varepsilon^2) \tag{40}$$

that the conditions for the instability are

$$0<\gamma\chi^2<1 \quad \text{for } k^2<a_0^2/\gamma \tag{41}$$

and

$$\gamma^{-2}[1-a_0^2/\gamma k^2]<\gamma\chi^2<1 \quad \text{for } k^2>a_0^2/\gamma. \tag{42}$$

We note that the first of these conditions applies to the range of the wave vector k for which the problem describes unstable propagation, independent of the longitudinal coordinates, while the second condition applies to the range of k for which the solution is stable in the absence of longitudinal dependence (see Eq. 38). Therefore when the perturbation depends on the longitudinal coordinate the filamentation in-

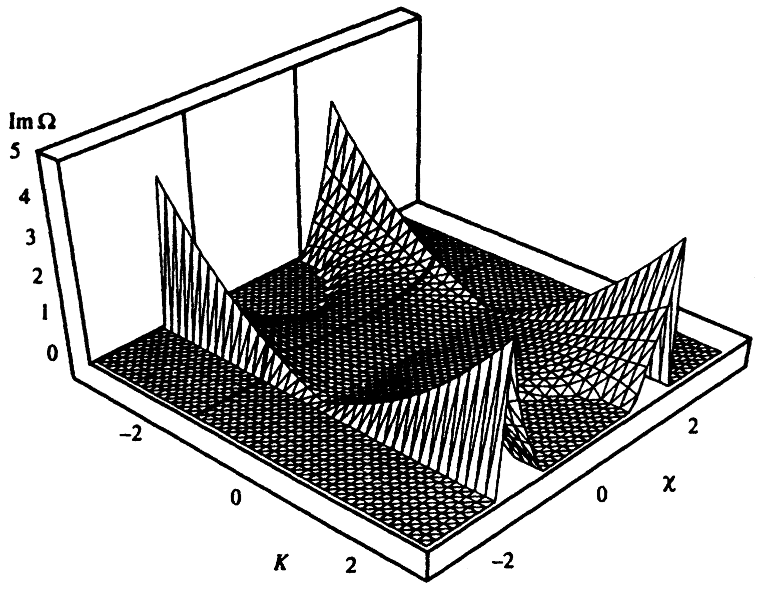

FIG. 1. Dependence of the instability increment of the light field in a plasma Im Ω, where Ω is determined by (33)–(37) of the text, on the wave vectors of the perturbation for the parameters of the problem $\varepsilon=0.2$, $a_0=1$.

stability is neutralized for $\chi^2>\chi^2_{\rm res}=\gamma^{-1}$ and when $k^2>a_0^2/\gamma$ there is an additional instability mechanism of the field.

Figure 1 shows the dependence of the increment of the general instability of the light field on the wave vectors k and χ of the spatial perturbations, for the typical values of the parameters of the problem $\varepsilon=0.2$ and $a_0=1$.

We next consider the following interesting effect. It follows from the inequalities (41) and (42) that a plane wave is stable when $\chi^2>\gamma^{-1}=(1+a_0^2)^{-1/2}$. Because $\gamma^{-1}\rightarrow 0$ when $a_0^2\rightarrow\infty$, a uniform field of large amplitude is stable. This is explained by the fact that in this limit the electrons become so heavy that the nonlinearity "turns off."

In summary, we have discussed a new model of the three-dimensional, nonlinear propagation of relativistically intense, ultra-short, circularly polarized laser pulses in a cold plasma of sub-critical density. The model takes into account the diffraction and refraction of the propagating laser radiation, the relativistic and striction mechanisms of its self-action, which result from, respectively, the mass increase of the free electrons oscillating in the light field with velocities comparable to the speed of light, and the density variation of the electron component of the plasma because of striction, and also wave generation in the electron component of the plasma. The system of equations obtained here differs from those studied previously by the presence of several additional terms in the equation for the amplitude of the vector potential of the laser field, and in the equations describing the oscillations of the electron component of the plasma moving together with the propagation of the radiation.

We have discussed the Lagrangian formulation of the new problem and its four scalar invariants.

We have shown that if the generation of plasma oscillations is neglected, the equations reduce to the nonlinear wave equation, and in the limit of long pulses they reduce to the nonlinear Schrödinger equation, which has been used in a number of papers to study relativistic striction self-channeling, filamentation, and self-modulation of strong, ultrashort laser pulses in matter. In the approximation of a wide transverse aperture we obtain the equations of the earlier one-dimensional theory.

We have shown that our system of equations has an exact solution corresponding to the propagation of a uniform light field in a plasma, and we have considered the stability of this solution to small perturbations. The instability is general, i.e., it is a nonlinear combination of the filamentation and modulation instabilities. In the limit of large longitudinal wavelength of the perturbation, the calculated increment is equal to the increment of the filamentation instability obtained in Ref. 17. We have also shown that a plane wave is stable against perturbations whose longitudinal component of the wave vector exceeds the electron plasma frequency, modified by the increase in the mass of the electrons oscillating in the field of the unperturbed wave.

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