Compton production of axions on electrons in a constant external field

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The cross section for the photoproduction of an axion on a relativistic electron ($\gamma + e \rightarrow e + a$) in a constant external electromagnetic field is calculated in a model with tree-level axion-electron coupling. An estimate of the contribution $Q_a^{(C)}$ of this Compton process to the axion luminosity of the magnetized, highly degenerate relativistic electron gas in the outer layers of a neutron star is obtained. The condition $Q_a^{(C)} < Q_\nu$, where Q_ν is the known luminosity due to neutrino synchrotron emission ($e \rightarrow e + \nu + \bar{\nu}$) yields bounds on the axion-electron coupling constant and the axion mass: $g_{ae} \lesssim 1 \times 10^{-13}$, $m_a \lesssim 4 \times 10^{-3}$ eV. These bounds are consistent with those previously found for other conditions and axion processes. © *1996 American Institute of Physics.* [S1063-7761(96)00311-3]

1. Theories which generalize the standard model of the interactions of elementary particles[1] by expanding the Higgs sector allow the appearance of new light (pseudo)scalar bosons associated with the spontaneous breaking of additional global symmetries (see, for example, the reviews in Refs. 2 and 3). The axion is one such pseudo-Goldstone boson, which provides a fairly natural solution to the problem of *a priori* strong CP violation in the standard model.[4,5] According to experimental data,[6] the constants of the possible coupling of an axion with ordinary particles are very small (the "invisible" axion). Therefore, axions can play an appreciable role in astrophysics[2,3] under the conditions of dense stellar matter, high temperatures, and strong external electromagnetic fields.

An analysis of various processes leading to the production of axions and astrophysical methods for obtaining bounds on the parameters of axion models was given in Ref. 3, where, however, the possible influence of strong external fields was not taken into account. In Ref. 7 we investigated a new mechanism for producing axions, viz., the synchrotron emission of axions by relativistic electrons in a magnetic field ($e \rightarrow e + a$), calculated its contribution to the energy losses of a magnetized neutron star, and obtained a new, less stringent bound on the axion–electron coupling constant:

$$g_{ae} \lesssim 5 \times 10^{-14}. \tag{1}$$

We stress that in the absence of an external field this process is forbidden by the energy–momentum conservation law.

Strong external fields have a significant influence on the processes that take place in their absence (see, for example, Ref. 8). These "free" processes include the Compton photoproduction of axions on electrons ($\gamma + e \rightarrow e + a$), which is the main mechanism of axion emission from horizontal branch stars,[3] which should also be taken into account [along with bremsstrahlung emission on nuclei: $e + (Z,A) \rightarrow (Z,A) + e + a$] in red giants.[3,9] The external fields can be neglected under these conditions. On the other hand, in neutron stars[10] strong magnetic fields significantly alter the probabilities of free processes and open up new reaction channels.

In this paper the axion Compton effect ($\gamma e \rightarrow ea$) on relativistic electrons in a constant electromagnetic field is considered on the basis of a model with tree-level axion-electron coupling. The corresponding interaction Lagrangian has the form[3]

$$\mathcal{L}_{ae} = \frac{g_{ae}}{2m}(\bar{\psi}\gamma^\mu \gamma^5 \psi)\partial_\mu a, \tag{2}$$

where m is the electron mass, and $\gamma^5 = -i\gamma^0 \gamma^1 \gamma^2 \gamma^3$, a system of units in which $\hbar = c = 1$, $\alpha = e^2/4\pi \simeq 1/137$, and a metric with the signature $(+ - - -)$ are used. We note that in models in which an axion is coupled only with heavy fermions on the tree level there is an effective low-energy direct interaction of an axion with a photon,[3] which provides for the Primakoff mechanism for the photoproduction of axions. The Primakoff effect on relativistic electrons in an external field was investigated in Ref. 11. In our case [see (2)] this mechanism is a radiation correction to the Compton mechanism (it corresponds to the familiar triangular $\gamma a \gamma$ diagram with a fermion loop[3]).

To simplify the further calculations we take into account that in first-order perturbation theory with respect to the coupling constant g_{ae} the Lagrangian (2) is equivalent to the pseudoscalar interaction Lagrangian[3]

$$\tilde{\mathcal{L}}_{ae} = -i g_{ae}(\bar{\psi}\gamma^5 \psi)a, \tag{3}$$

which was also used in Ref. 7. Equation (3) is derived from (2) using the divergence of the axial current for a free Dirac field, $\partial_\mu(\bar{\psi}\gamma^\mu \gamma^5 \psi) = -2im(\bar{\psi}\gamma^5 \psi)$.

The cross section of the Compton process $\gamma e \rightarrow ea$ in a constant external field is calculated below, and its contribution to the luminosity of a neutron star is evaluated and then compared with the corresponding contribution of the axion synchrotron emission $e \rightarrow ea$ (Ref. 7).

2. The amplitude S_{fi} of the axion Compton effect in an external electromagnetic field to lowest order in the constants e and g_{ae} of the $e\gamma e$ and eae interactions follows from (3) and the known Lagrangian of the electromagnetic interaction:

$$S_{fi}=\frac{eg_{ae}}{2V(\omega\omega')^{1/2}}\int d^4xd^4x'\,\bar\psi_{p'}(x)[\,\gamma^5S(x,x')\hat e$$

$$\times\exp(ik'x-ikx')+\hat eS(x,x')\gamma^5$$

$$\times\exp(ik'x'-ikx)]\psi_p(x'). \tag{4}$$

Here ψ_p ($\psi_{p'}$) and $S(x,x')$ are, respectively, the exact wave function of the initial (finite) electron and the propagator of an electron in a given external field; $k^\mu=(\omega,\mathbf{k})$ and $k'^\mu=(\omega',\mathbf{k}')$ are the four-momenta of the incident photon and the axion emitted; $\hat e=\gamma^\mu e_\mu$; e^μ is the polarization four-vector of the photon $(ek=0)$; and V is the normalization volume.

As in Ref. 11, we choose a constant crossed field ($\mathbf{E}\perp\mathbf{H}$, $|\mathbf{E}|=|\mathbf{H}|=F=$const) as the external field. As we know,[8,12] for processes with ultrarelativistic electrons a crossed field simulates an arbitrary constant field with intensity $F\ll H_0=m^2c^3/e\hbar=4.41\times10^{13}$ G. The intensity tensor of a crossed field is

$$F^{\mu\nu}=n^\mu B^\nu-n^\nu B^\mu, \quad nB=0, \quad n^2=0, \tag{5}$$

$$B^2=-F^2,$$

and in the special reference frame in which $n^\mu=(1,\mathbf{n})$ and $B^\mu=(0,-F,0,0)$ the 3-vectors $\mathbf{E}=(F,0,0)$, $\mathbf{H}=(0,F,0)$, and $\mathbf{n}=(0,0,1)$ follow the right-hand rule.

The state of an electron in a crossed field is assigned by the four-quasimomentum $p^\mu=(\varepsilon,\mathbf{p})$, $p^2=m^2$. When the field is removed, it transforms into the four-momentum of a free electron. The electron wave function has the form[12]

$$\psi_p(x)=(2\varepsilon V)^{-1/2}E_p(x)u(p), \tag{6}$$

$$E_p(x)=(1+\zeta_p\hat n\hat B\varphi)\exp[-iS_p(\varphi)], \tag{7}$$

$$S_p(\varphi)=px+\zeta_p(Bp)\varphi^2-\frac{e^2B^2}{6(np)}\varphi^3,$$

$\zeta_p=e/2(np)$, and $\varphi=nx$. The bispinor $u(p)$ is normalized by the condition $\bar uu=2m$ and satisfies the Dirac equation for a free electron: $(\hat p-m)u(p)=0$. For the electron propagator we choose the representation[12]

$$S(x,x')=\int\frac{d^4q}{(2\pi)^4}\frac{E_q(x)(\hat q+m)\bar E_q(x')}{q^2-m^2+i0}. \tag{8}$$

To simplify the calculations we confine ourselves to the following kinematic condition:

$$nk=0,$$

or [see (5)]

$$k_\mu F^{\mu\nu}=0, \tag{9}$$

i.e., in the special reference frame the photon momentum $\mathbf{k}\uparrow\uparrow\mathbf{n}$. The same condition was utilized to investigate the original Compton effect $(\gamma e\to e\gamma)$[13] and the Primakoff effect.[1] We note that one feature of the kinematics (9) is that in this case the photon decay process $\gamma\to e^+e^-$ is forbidden and the Dirac equation allows an exact solution for an electron in an external field having the form of the superposition of the crossed field (5) and a monochromatic plane wave of arbitrary intensity propagating along $\mathbf{n}$ (Ref. 13). We also

neglect the axion mass m_a in the case of high-energy electrons under consideration, i.e., $k'^2=0$ [it is known from astrophysics that $m_a\lesssim10^{-2}$ eV (Ref. 9)].

We substitute (6) and (8) into (4) and integrate over the coordinates x and x', using the Fourier transform of the functions $E_p(x)$ with respect to the phase variable φ [Ref. 12]. Then, integrating over the virtual quasimomentum q^μ [see (8)] with the aid of four-dimensional δ functions that express the quasimomentum conservation law at the vertices x and x', we obtain the amplitude of the process (4) in the form

$$S_{fi}=\frac{eg_{ae}}{4V^2}\frac{1}{\sqrt{\omega\omega'\varepsilon\varepsilon'}}\int_{-\infty}^\infty ds\int_{-\infty}^\infty ds'(2\pi)^4$$

$$\times\delta^{(4)}(p'+k'-p-k-sn)$$

$$\times\bar u(p')\{D(p+k+s'n)\gamma^5\Gamma(s-s')$$

$$\times(\hat p+\hat k+s'\hat n+m)[\hat e\,\delta(s')+2i\zeta_p(eB)\hat n\,\delta'(s')]$$

$$+D(p'-k-s'n)[\hat e\,\delta(s')+2i\zeta_{p'}(eB)\hat n\,\delta'(s')](\hat p'-\hat k$$

$$-s'\hat n+m)\Gamma(s-s')\gamma^5\}u(p). \tag{10}$$

Here

$$D(q)=(q^2-m^2+i0)^{-1}, \quad \Gamma(s)=A_0(s)-i\zeta\hat B\hat nA_1(s),$$

$$\zeta=\zeta_{p'}-\zeta_p=\frac e2\left(\frac1{np'}-\frac1{np}\right). \tag{11}$$

The functions $A_0(s)$ and $A_1(s)=\partial A_0(s)/\partial s$, which are characteristic of a crossed field,[12] are expressed in terms of the Airy function

$$\Phi(y)=\frac1{\sqrt\pi}\int_0^\infty dt\,\cos\left(yt+\frac{t^3}3\right) \tag{12}$$

according to the relation

$$A_0(s)=\frac1{\sqrt\pi}(4\beta)^{-1/3}\exp\left[i\frac\alpha{8\beta}\left(s-\frac{\alpha^2}{24\beta}\right)\right]\Phi(y), \tag{13}$$

where the argument of the Airy function (12) is

$$y=(4\beta)^{-1/3}\left(s-\frac{\alpha^2}{16\beta}\right),$$

$$\alpha=e\left(\frac{Bp}{np}-\frac{Bp'}{np'}\right), \quad \beta=\frac18e^2B^2\left(\frac1{np}-\frac1{np'}\right). \tag{14}$$

We note that the presence of the δ function $\delta(s')$ and its derivative $\delta'(s')$ in the integrand in Eq. (10) results from the choice of the special kinematics (9), which greatly simplifies the structure of the amplitude. After the trivial integration in (10) over s', for the square of the absolute value of the amplitude averaged (summed) over the spin states of the initial (final) electron, we obtain

$$W_{fi}=\frac{|S_{fi}|^2}T=\frac{2e^2g_{ae}^2}{L_\varphi(2V)^4}\frac{(2\pi)^2}{\omega\omega'\varepsilon\varepsilon'}\int_{-\infty}^\infty ds$$

$$\times\delta^{(4)}(p'+k'-p-k-sn)R, \tag{15}$$

where T is the "observation time," L_φ is the normalization length corresponding to the phase φ in the wave function (6) (Ref. 12), with

$$R = \frac{1}{4}(\text{tr}[(\hat{p}' + m)\gamma^5 Q(\hat{p}+m)\overline{\gamma^5 Q}],$$

$$Q = (a+ib)\Gamma(s) + c\hat{k}\hat{e}A_0(s) + ib_1\Gamma_1(s),$$

$$a = \frac{ep}{kp} - \frac{ep'}{kp'}, \quad b_1 = \omega b = 2e(eB)c, \qquad (16)$$

$$c = \frac{1}{2}\left(\frac{1}{kp} - \frac{1}{kp'}\right),$$

$$\overline{\gamma^5 Q} = \gamma^0(\gamma^5 Q)^+ \gamma^0.$$

The functions $\Gamma(s)$ and $A_0(s)$ were defined in (11) and (13),

$$\Gamma_1(s) = \frac{\partial \Gamma(s)}{\partial s} = A_1(s) - i\zeta\hat{B}\hat{n}A_2(s),$$

$$A_2(s) = \frac{\partial^2 A_0(s)}{\partial s^2}. \qquad (17)$$

The calculation of the trace in (16) gives

$$R = C_0|A_0|^2 + C_1|A_1|^2 + C_2|A_2|^2 + 2\,\text{Re}(C_{01}A_0A_1^* + C_{02}A_0A_2^* + C_{12}A_1A_2^*). \qquad (18)$$

Here

$$C_0 = |z|^2 M - (a^2 + c)(kk'),$$

$$C_1 = b_1^2 M + 2ab_1\zeta F_1 + 2|z|^2\zeta^2 F_2,$$

$$C_2 = 2b_1^2\zeta^2 F_2, \qquad (19)$$

$$iC_{01} = b_1 zM + |z|^2\zeta F_1 - b_1(a - ib/2)(kk'),$$

$$C_{02} = -b_1\zeta(zF_1 + f),$$

$$iC_{12} = b_1\zeta(b_1 F_1 + 2z\zeta F_2),$$

where $z = a + ib$. In (19) we also introduced the notation

$$M = (p'p) - m^2,$$

$$F_1 = (p'Fp) = p'_\mu F^{\mu\nu}p_\nu = (np')(Bp) - (np)(Bp'),$$

$$f = (k'Fe) = (eB)(nk'), \qquad (20)$$

$$F_2 = (p'F^2 p) = -B^2(np)(np').$$

From (15), where W_{fi} is the transition probability per unit time, we find the differential cross section of the process

$$d\sigma = \frac{W_{fi}}{j} \frac{Vd^3p'}{(2\pi)^3} \frac{Vd^3k'}{(2\pi)^3}, \qquad (21)$$

where the flux is $j = (kp)/(V\omega\varepsilon)$. Using the four-dimensional δ function appearing in W_{fi}, we integrate (21) over $\mathbf{p}'$ and s. We transform the remaining phase volume by introducing the following invariant variables instead of the components of the axion momentum $\mathbf{k}'$ (Refs. 12 and 13):

$$u = \frac{\chi}{\chi'} - 1, \quad \tau = \frac{e}{m^4}\frac{(p'\tilde{F}p)}{\chi - \chi'}, \qquad (22)$$

$$\psi = \frac{e}{m^4}\frac{(p'Fp)}{\chi - \chi'}\frac{\kappa}{2\chi},$$

where $\tilde{F}_{\mu\nu} = (1/2)\varepsilon_{\mu\nu\alpha\beta}F^{\alpha\beta}$ is the dual tensor of the field,

$$\chi = \frac{e}{m^3}(pF^2 p)^{1/2}, \quad \kappa = \frac{2(kp)}{m^2} \qquad (23)$$

are invariant parameters, and the variable χ' equals $\chi(p \to p')$. Using (22), we obtain

$$\frac{d^3 k'}{\omega'} = m^2 \frac{2\chi}{\kappa} du\, d\tau\, d\psi. \qquad (24)$$

Averaging the coefficients (19) over the photon polarizations according to the relation $\overline{e^\mu e^\nu} = -g^{\mu\nu}/2$ and expressing the result in terms of the variables (22), from (21) and (18) with consideration of the relationship between the functions $A_n = \partial^n A_0/\partial s^n$ and the Airy function [see (12) and (13)], we obtain an integral representation for the total cross section of the process:

$$\sigma = \frac{4\alpha\alpha_a}{m^2\kappa^3}\int_0^\infty du \int_{-\infty}^\infty d\tau \frac{u^4}{(1+u)^3}\frac{1}{(2\tilde{\chi})^{1/3}}$$

$$\times [F_0\Phi^2 + F_1\Phi'^2 + F_{01}\Phi\Phi']. \qquad (25)$$

Here

$$\alpha = \frac{e^2}{4\pi}, \quad \alpha_a = \frac{g_{ae}^2}{4\pi},$$

$$F_0 = 1 - \tilde{\kappa} + \tilde{\kappa}^2/2 + \eta^2 + (3 - 2\tilde{\kappa} + \eta^2)\tau^2 + 2\tau^4, \qquad (26)$$

$$F_1 = (2\tilde{\chi})^{2/3}(1 + \eta^2 + 2\tau^2),$$

$$F_{01} = 4\eta(2\tilde{\chi})^{1/3}(1 + \tau^2 - \tilde{\kappa}/4),$$

where $\tilde{\kappa} = \kappa/u$, $\tilde{\chi} = \chi/u$, and the argument of the Airy function is

$$y = \left(\frac{u}{2\chi}\right)^{2/3}\left(1 + \tau^2 - \frac{\kappa}{u}\right). \qquad (27)$$

We have also introduced the parameter

$$\eta = 2\chi/\kappa, \qquad (28)$$

which specifies the influence of the external field on the process[8] (see also Refs. 11 and 13). In (25) we performed the integration over the variable ψ, using the fact that the spectrum $d^3\sigma/dud\tau d\psi$ does not depend on ψ and the relation[12]

$$\int d\psi = L_\varphi \frac{kp}{np}.$$

Therefore, the cross section (25) does not depend on the unphysical quantity L_φ, as should be expected.

3. The integrand in (25) determines the differential cross section of the process $d^2\sigma/dud\tau$ for an ultrarelativistic electron in an arbitrary constant external field of intensity $F \ll H_0$ (for the exact formulation of the applicability conditions see Ref. 8). In the case of a constant magnetic field (**H**

parallel to the z axis), which is of interest for astrophysics, the variables (22) and the parameters (23) take on the form

$$u = \frac{p_\perp}{p'_\perp} - 1, \quad \tau = \frac{\varepsilon' p_z - \varepsilon p'_z}{m(p_\perp - p'_\perp)}, \tag{29}$$

$$\chi = \frac{p_\perp}{m}\frac{H}{H_0}, \quad \kappa = \frac{2\omega}{m^2}(\varepsilon - p_z).$$

Here $\varepsilon = (m^2 + p_\perp^2 + p_z^2)^{1/2}$ is the energy, $p_\perp$ and p_z are the transverse and longitudinal components of the electron momentum with respect to $\mathbf{H}$, and by virtue of (9) the photon momentum satisfies $\mathbf{k}\uparrow\uparrow\mathbf{H}$. The result (25), which was obtained for a crossed field, is also applicable to a magnetic field ($F_{\mu\nu}F^{\mu\nu} > 0$, $F_{\mu\nu}\widetilde{F}^{\mu\nu} = 0$), if the photon moves along the field [see (9)] and the following conditions hold[8,13]

$$f_i \ll (1, \chi, \kappa), \quad i = 1,2, \tag{30}$$

where the invariants f_i equal

$$f_1 = \frac{e}{m^2}\left|\frac{1}{2}F_{\mu\nu}F^{\mu\nu}\right|^{1/2} = \frac{H}{H_0}, \tag{31}$$

$$f_2 = \frac{e}{m^4}|p_\mu \widetilde{F}^{\mu\nu}k_\nu|^{1/2} = \frac{H}{2H_0}\kappa.$$

In the reference frame in which $p_z = 0$ holds the conditions (30) take on the form

$$\varepsilon \simeq p_\perp \gg m, \quad H \ll H_0, \quad \omega \ll m, \quad \omega \gg \omega_H = eH/\varepsilon. \tag{32}$$

4. We perform the integration in (25) over the variable τ using known relations from the theory of Airy functions (see, for example, Ref. 12). We obtain the cross section in the form

$$\sigma = \sqrt{\pi}\frac{r_{ae}^2}{\kappa}\int_0^\infty \frac{du\, u^2}{(1+u)^3}\left[\Phi_1(x) + 2\eta^2\widetilde{\chi}^{-2/3}\left(1 - \frac{2}{\kappa}\right)\Phi(x)\right.$$

$$\left. - \eta^2(1+\eta^2)\widetilde{\chi}^{-4/3}\Phi'(x)\right]. \tag{33}$$

Here

$$\Phi_1(x) = \int_x^\infty dt\,\Phi(t)$$

with the argument

$$x = \left(\frac{u}{\chi}\right)^{2/3}\left(1 - \frac{\kappa}{u}\right), \tag{34}$$

and we have introduced the characteristic radius

$$r_{ae} = \frac{eg_{ae}}{4\pi m}.$$

The dependence of the form of the spectrum $d\sigma/du$ [the integrand in (33)] on the parameter η given in (28) is typical of processes that take place in the absence of an external field [compare this with the ordinary Compton effect $\gamma e \to e\gamma$ (Ref. 8, p. 86)]. The features of the spectrum are directly related to the properties of Airy functions.[12,13]

In the limit $\eta \ll 1$, on the smooth distribution of the free process ($\eta = 0$)

$$\frac{d\sigma_0}{du} = \pi\frac{r_{ae}^2}{\kappa}\frac{u^2}{(1+u)^3}, \quad 0 \leq u \leq \kappa, \tag{35}$$

we impose the characteristic oscillations caused by a weak external field, and the cross section $d\sigma/du$ decreases monotonically in the region $u > \kappa$, which is forbidden in the absence of a field for $u \gg \kappa$ the decrease is exponential).

As follows from (34), in the limit $\eta \gg 1$ the spectrum $d\sigma/du$ has a maximum at $u \sim \chi \gg \kappa$, and there are no oscillations over a considerable region.

When $u \gg \kappa$, holds the mechanism of axion emission becomes essentially a synchrotron mechanism, and the differential probability $dw = jd\sigma$ takes on the factorized form typical of processes involving soft photons:

$$\frac{dw}{du} = 2\xi^2\left(\frac{\chi}{\kappa}\right)^4\left(\frac{u}{\chi}\right)^2\frac{dw_{SE}}{du}. \tag{36}$$

Here dw_{SE} is the probability of synchrotron axion emission ($e \to ea$):[7]

$$\frac{dw_{SE}}{du} = \frac{\alpha_a}{2\sqrt{\pi}}\frac{m^2}{\varepsilon}\frac{u^2}{(1+u)^3}\frac{1}{x_0}[-\Phi'(x_0)],$$

where $x_0 = (u/\chi)^{2/3} = x(\kappa = 0)$ [see (34)]. In (36) we introduce the wave intensity parameter[12] $\xi = eF_\omega/m\omega$, where F_ω is the amplitude of the field intensity of the wave corresponding to the incident photon: $VF_\omega^2/2 = \omega$.

We note that the result (36) is similar to the corresponding result for the ordinary Compton effect in a constant external field in the same kinematic region[13] (see also Ref. 8, p. 80). Its applicability is restricted by the condition that the wave field be weak:[8,13]

$$\xi^2(\chi/\kappa)^4 \ll 1. \tag{37}$$

The infrared divergence of the probability (36) [and the cross section (33)] is totally eliminated as $\kappa \to 0$, as in the case of the $\gamma e \to e\gamma$ process.[8] In the region just indicated only the total probability of axion synchrotron emission ($e \to ea$), stimulated emission ($e \to ea\gamma$), and Compton scattering ($\gamma e \to ea$) has physical meaning, and a correction $\sim \xi^2$ due to the interference contribution to the amplitude of the $\gamma e \to ea\gamma$ process, where the final photon γ is identical to the photons of the incident wave, must be taken into account in the probability of the $e \to ea$ process. All the diverging (as $\kappa \to 0$) terms in the total probability dw are reduced, and we have $dw \simeq dw_{SE}$. We note that the infrared divergence accompanying the absorption of soft photons was examined for decay processes in a wave field in Ref. 12.

We present the asymptotic forms of the total cross section (33) with respect to the parameter (28). The method used to obtain them is the same as in the case of the cross section of the Primakoff effect.[11]

In the limit $\eta \ll 1$, an external field causes a correction $\sigma_1 \sim \chi^2$ to the cross section σ_0 of the free process [the integral of (35)]:

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$$\sigma = \sigma_0 + \sigma_1 = \pi r_{ae}^2 \left[\frac{1}{\kappa} \ln \kappa_1 - \frac{1}{2\kappa_1^2} (3\kappa_1 - 1) + 2\frac{\chi^2}{\kappa_1^5} \right.$$

$$\left. \times (2 + 5\kappa_1 - 2\kappa_1^2) \right], \tag{38}$$

where $\kappa_1 = \kappa + 1 = (k+p)^2/m^2$ is the normalized Mandelstam variable s.

In the limit $\eta \gg 1$, the influence of the external field dominates. In the region $\kappa \ll \chi \ll 1$, from (36) we obtain the probability estimate

$$w \sim \xi^2 \left(\frac{\chi}{\kappa} \right)^4 w_{SE},$$

where it has been taken into account that the main contribution to the integral over u is made by $u \sim \chi$. The criterion (37) for the applicability of perturbation theory follows from the condition $w \ll w_{SE}$ (compare Ref. 8, p. 81). In the limit $\chi \gg 1$, from (33) we find the asymptotic form of the cross section

$$\sigma = 8\pi \frac{r_{ae}^2}{\kappa} \left(\frac{\chi}{\kappa} \right)^4. \tag{39}$$

The corresponding probability $w = j\sigma$ has the form

$$w = \frac{\xi^2}{2} \left(\frac{\chi}{\kappa} \right)^4 \alpha_a \frac{m^2}{\varepsilon}.$$

We note that the asymptotic form of the total probability of axion synchrotron emission for $\chi \gg 1$ is

$$w_{SE} = \frac{\Gamma(2/3)}{27} \alpha_a \frac{m^2}{\varepsilon} (3\chi)^{2/3}.$$

Therefore, for $\chi \gg 1$ we have $w \ll w_{SE}$ for

$$\xi^2 (\chi/\kappa)^4 \ll \chi^{2/3}.$$

5. We evaluate the contribution $Q_a^{(C)}$ of the process considered above to the axion luminosity of a magnetized, highly degenerate relativistic electron gas under the conditions of a neutron star.[10] The luminosity $Q_a^{(C)}$, i.e., the rate of energy loss by a unit volume of the gas due to the axion Compton effect $\gamma e \to ea$, is expressed in terms of the cross section $d\sigma/du$ (33) in the form

$$Q_a^{(C)} = \int \frac{2d^3p}{(2\pi)^3} \frac{2d^3k}{(2\pi)^3} \int_0^\infty du \frac{d\sigma}{du} \omega' n_F(\varepsilon)$$

$$\times [1 - n_F(\varepsilon')] n_B(\omega). \tag{40}$$

Here

$$n_F(\varepsilon) = \left[\exp\left(\frac{\varepsilon - \mu}{T} \right) + 1 \right]^{-1},$$

$$n_B(\omega) = \left[\exp\left(\frac{\omega}{T} \right) - 1 \right]^{-1} \tag{41}$$

are, respectively, the Fermi and Bose distribution functions of the initial electrons and photons at the temperature T, $n_F(\varepsilon')$ is the distribution of the final electrons, and μ is the chemical potential of the electron gas. The energy of an emitted axion ω' can be expressed in terms of only the energy of the initial electron ε and the variable u (29) over a significant range of emission angles when the condition (30) holds:

$$\omega' = \frac{u\varepsilon}{1+u}. \tag{42}$$

This permits the use in (40) of the spectrum

$$\frac{d\sigma}{du} = \int d\tau \frac{d^2\sigma}{dud\tau}.$$

Let us examine, as in Refs. 11 and 7, the case of a highly degenerate relativistic gas in a nonquantizing magnetic field, in which the following conditions hold

$$T \ll \mu \simeq \varepsilon_F = (m^2 + p_F^2)^{1/2} \simeq p_F = (3\pi^2 n_e)^{1/3} \gg m,$$

$$p_F \gg \omega_F = eH/\varepsilon_F, \quad \omega_F \ll T, \tag{43}$$

where n_e is the electron density. Under the conditions (43) the quantization of the transverse momentum $p_\perp$ of the electron in the magnetic field can be neglected[8] ($p_\perp^2 = 2eHn$, $n = 0,1,2,\ldots$), and the quasiclassical formula (33) can be used for the cross section $d\sigma$ in (40) after replacing the summation over the electron states by integration over the phase volume.

The further calculations are similar to those performed in Ref. 11 for the Primakoff effect. The Fermi factor

$$F = n_F(\varepsilon)[1 - n_F(\varepsilon')] \tag{44}$$

has a narrow maximum at $p \approx p_F$, which is determined by the overlap of the transition regions of the Fermi "satellites:" $|\varepsilon - \varepsilon_F| \lesssim T$ and $|\varepsilon' - \varepsilon_F| \lesssim T$. Hence, taking into account (41)–(43), we find that in the region $|p - p_F| \lesssim T \ll p_F$, which makes the main contribution to the integral over p, we presumably have

$$\omega' \simeq up_F, \quad u \ll 1, \quad \varepsilon' = \varepsilon - \omega - \omega' \simeq \varepsilon - \omega', \tag{45}$$

since $\omega \lesssim T \ll \varepsilon_F \sim \varepsilon \sim \varepsilon'$.

We restrict ourselves to the case

$$T \ll T_c = \gamma_F^3 \omega_F, \tag{46}$$

where $\gamma_F = p_F/m \gg 1$. This condition holds over a broad range of values of n_e and T for neutron stars.[10,14] With consideration of (43), (45), and (46) it is not difficult to obtain estimates of the effective values of the variable u, the kinematic parameter κ, and the ratio u/χ (Ref. 11):

$$u \sim \kappa \sim \frac{T}{p_F} \ll 1, \quad \frac{u}{\chi} \sim \frac{T}{T_c} \ll 1. \tag{47}$$

Therefore, the effective values of the argument x (34) of the Airy functions in (33) are

$$x \lesssim (u/\chi)^{2/3} \sim (T/T_c)^{2/3} \ll 1. \tag{48}$$

Taking into account (48), from (33) we obtain the following approximate expression for the differential cross section in (40):

$$\frac{d\sigma}{du} = 8 \times 3^{1/6} \Gamma\left(\frac{2}{3} \right) \frac{\alpha \alpha_a}{m^2} \frac{\chi^{8/3}}{\kappa^5} u^{10/3}, \tag{49}$$

where we have used $\chi/\kappa \sim T_c T/T \gg 1$ and taken

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$$\Phi'(x) \simeq \Phi'(0) = -\frac{3^{1/6}}{2\sqrt{\pi}}\Gamma\left(\frac{2}{3}\right).$$

We perform the integration over p in (40) by setting $p = p_F$ everywhere, except in F given by (44). The remaining integral is calculated in a first approximation with respect to $T/p_F \ll 1$:

$$\int_0^\infty dp\, F \simeq \omega'\left[\exp\left(\frac{\omega'}{T}\right) - 1\right]^{-1}. \qquad (50)$$

We substitute into (40) the cross section (49), where we should set

$$\chi = \chi_F\sin\theta, \qquad \kappa = \frac{2\omega}{m^2}p_F(1 - \mathbf{n}\cdot\mathbf{v}),$$

$$\chi_F = \gamma_F\frac{H}{H_0}, \qquad \mathbf{n} = \frac{\mathbf{k}}{\omega}, \qquad \mathbf{v} = \frac{\mathbf{p}}{\varepsilon}, \qquad |\mathbf{v}| = \frac{p_F}{\varepsilon_F}.$$

Here θ is the angle between $\mathbf{p}$ and $\mathbf{H}$. Then, taking into account (50) and (45), we obtain the luminosity in the form

$$Q_a^{(C)} = \frac{3^{1/6}\Gamma(2/3)}{2(2\pi)^7}\alpha g_{ae}^2 m^5\gamma_F^{-14/3}\left(\frac{H}{H_0}\right)^{8/3}\left(\frac{T}{m}\right)^{13/3}\int_0^\infty\frac{x^{16/3}dx}{e^x - 1}$$

$$\times\int_{y_0}^\infty\frac{dy}{y^3}\frac{1}{e^y - 1}\int d\Omega_{\mathbf{p}}\sin^{8/3}\theta\int\frac{d\Omega_{\mathbf{k}}}{(1 - \mathbf{n}\cdot\mathbf{v})^5}. \qquad (51)$$

Here $d\Omega_{\mathbf{q}}$ is an element of a solid angle in the direction of $\mathbf{q}$, $x = up_F/T$, and $y = \omega/T$. The lower limit of the range for y is associated with the effective mass m_γ of a photon in the medium:

$$y_0 = m_\gamma/T. \qquad (52)$$

We stress that the expression (51) has an approximate character. It was derived with the use of the cross section (33) of the elementary process $\gamma e \to ea$, which was obtained for a fixed direction of the incident photon [see (9)]. In the general case, in which the angle between $\mathbf{k}$ and $\mathbf{H}$ is arbitrary, the cross section of the process depends not only on the parameters χ and κ (23) and on f_1 and f_2 (31), but also on three additional parameters:[8]

$$f_3 = \frac{e}{m^4}|p_\mu F^{\mu\nu}k_\nu|,$$

$$f_4 = \frac{e}{m^3}|(F^{\mu\nu}k_\nu)^2|^{1/2},$$

$$f_5 = \frac{e}{m^6}|p_\mu F^{\mu\nu}F_{\nu\lambda}k^\lambda|.$$

In the calculation of $Q_a^{(C)}$ we neglected the dependence of the cross section of the process on the parameters f_i:

$$\sigma(\chi,\kappa,f_1,\ldots,f_5) \to \sigma(\chi,\kappa,0,\ldots,0).$$

This is possible, if the following conditions hold (compare Ref. 8, p. 84):

$$f_i \ll (1,\chi,\kappa), \qquad i = 1,\ldots,6.$$

In the case under consideration [see (23) and (47), as well as (60) below] we obtain the following estimates of the parameters:

$$\chi \sim \gamma_F H/H_0 \sim 1, \qquad \kappa \sim T/p_F \ll 1,$$

$$f_1 = H/H_0 \ll 1, \qquad f_2 \sim (H/H_0)\kappa \ll \kappa,$$

$$f_3 \sim \chi T/p_F \sim \kappa \ll \chi, \qquad f_4 \sim f_3, \qquad f_5 \sim \chi f_4 \sim f_4.$$

We have noted that the main contribution to $Q_a^{(C)}$ given by (51) is made by angles between $\mathbf{k}$ and $\mathbf{p}$ that are smaller than or of the order of $m/p_F \ll 1$. Consequently, $f_i \sim \kappa \ll \chi$ for $i = 1,3,4,5$. Therefore, the use of the asymptotic form of the spectrum $d\sigma/du$ (49) following from (33) in (40) gives only an estimate of $Q_a^{(C)}$

We took into account the influence of the medium (a dense, highly degenerate electron gas) on the propagating photon fairly roughly by introducing a photon mass in (52), but we neglected the variation of the dispersion law, as well as of the electron propagator. We note that the motion of a photon in a magnetized gas was investigated in detail in Ref. 15. To obtain an estimate we identify m_γ with the plasma frequency ω_p in a highly degenerate relativistic gas (see, for example, Ref. 16):

$$m_\gamma \simeq \omega_p = \left(\frac{4\alpha}{3\pi}\right)^{1/2}p_F. \qquad (53)$$

Then, for the integral over y in (51) we obtain the estimate (under the assumption $y_0 \ll 1$)

$$J_y \simeq \frac{1}{3}\left(\frac{T}{\omega_p}\right)^3. \qquad (54)$$

The remaining integrals reduce to tabulated expressions:

$$J_x = \zeta\left(\frac{19}{3}\right)\Gamma\left(\frac{19}{3}\right), \qquad J_{\mathbf{p}} = 2\pi^{3/2}\frac{\Gamma(7/3)}{\Gamma(17/6)}, \qquad J_{\mathbf{k}} \simeq 8\pi\gamma_F^8.$$

$$Q_a^{(C)} = A g_{ae}^2\alpha^{-1/2}m^5\gamma_F^{1/3}\left(\frac{H}{H_0}\right)^{8/3}\left(\frac{T}{m}\right)^{22/3}, \qquad (56)$$

where the numerical coefficient A is

$$A = \frac{3^{2/3}\Gamma(2/3)}{2^7\pi^3}\zeta\left(\frac{19}{3}\right)\Gamma\left(\frac{19}{3}\right)\frac{\Gamma(7/3)}{\Gamma(17/6)} \simeq 0.106.$$

For applications we represent the result (56) in the "astrophysical" form:

$$Q_a^{(C)} = 5.6\times10^{38}\cdot g_{ae}^2\gamma_F^{1/3}H_{13}^{8/3}T_9^{22/3}\text{erg/cm}^3\cdot\text{s}, \qquad (57)$$

where $H_{13} = H(10^{13}\text{ G})^{-1}$ and $T_9 = T(10^9\text{ K})^{-1}$.

6. We compare the axion luminosity (57) with the neutrino synchrotron luminosity $(e \to e + \nu + \bar\nu)$ obtained in Ref. 14 for the same conditions:

$$Q_\nu = 8.97\times10^{14}\cdot H_{13}^2 T_9^5 \text{ erg/cm}^3\cdot\text{s}. \qquad (58)$$

Assuming $Q_a^{(C)} < Q_\nu$, from (57) and (58) we find the upper bound on the axion-electron coupling constant:

$$g_{ae} < 1.27\times10^{-12}\cdot\gamma_F^{-1/6}H_{13}^{-1/3}T_9^{-7/6}. \qquad (59)$$

For the outer layers of a neutron star we have[10,14] $n_e \sim 10^{30}-10^{37}$ cm^{-3}, $T \sim 10^8-10^{10}$ K, and $H \sim 10^{12}-10^{14}$ G. As in Ref. 11, we assume

$$n_e = 10^{34} \text{cm}^{-3}, \quad T = 10^{10} \text{ K}, \quad H = 10^{12} \text{ G}. \quad (60)$$

Then the basic parameters [see (43), (46), and (53)] take the following values:

$$\gamma_F = 25.7, \quad \omega_F = 5.2 \times 10^6 \text{ K}, \quad T_c = 8.9 \times 10^{10} \text{ K},$$

$$\omega_p = 8.5 \times 10^9 \text{ K}.$$

The conditions for the applicability of Eq. (57) are now satisfied. With consideration of (60), we obtain the following upper bound from (59):

$$g_{ae} \lesssim 1 \times 10^{-13}, \quad (61)$$

which is consistent in order of magnitude with the bound (1) obtained in Ref. 7 from an analysis of axion synchrotron radiation, as well as with the bound $\alpha_a \lesssim 0.5 \times 10^{-26}$, or $g_{ae} = (4\pi\alpha_a)^{1/2} \lesssim 2.5 \times 10^{-13}$, which was found in Ref. 9 for the conditions of red giants.

We compare the efficiencies of the Compton and synchrotron mechanisms of axion emission in neutron stars. In Ref. 7 for the axion synchrotron luminosity we obtained

$$Q_a^{(SE)} = 1.59 \times 10^{40} g_{ae}^2 \gamma_F^{-2/3} H_{13}^{2/3} T_9^{13/3} \text{ erg/cm}^3 \cdot \text{s}.$$

Using this value and (57), we find

$$\frac{Q_a^{(C)}}{Q_a^{(SE)}} \approx 3.5 \times 10^{-2} \gamma_F H_{13}^2 T_9^3 \approx 9$$

for the values of the parameters in (60), i.e., under these conditions the Compton mechanism is an order of magnitude more efficient than the synchrotron mechanism. We stress that although the process $\gamma e \to ea$ also takes place in the absence of a field, under the conditions of a neutron star the influence of the magnetic field is decisive: the spectrum $d\sigma/du$ given by (49) differs significantly from the free spectrum (35), leading to a strong dependence of $Q_a^{(C)}$ in (57) on the field intensity.

On the other hand, the bound (1) was obtained from $n_e = 10^{33}$ cm^{-3}, $T = 10^8$ K, $H = 10^{12}$ G [compare (60)]. Under such conditions we have $\omega_p = 4 \times 10^9$ K, i.e., $\omega_p/T \gg 1$. Therefore, at sufficiently low temperatures the Compton contribution is suppressed relative to the synchrotron contribution by the exponentially small factor [see (51)–(53)] $\exp(-\omega_p/T)$.

In conclusion, we obtain a bound on the axion mass m_a within the Dine–Fishler–Srednicki–Zhitnitsky (DFSZ) model,[3] which relates m_a to g_{ae} by the expression[3,9]

$$m_a = g_{ae}(2.8 \times 10^{-11} \cdot \cos^2\beta)^{-1} \text{eV}, \quad (62)$$

where $\cos\beta$ is a model-dependent parameter ($\cos\beta = 1$ is usually assumed for estimates).

Substituting the estimate (61) into (62), we find the upper bound on the axion mass

$$m_a \lesssim 4 \times 10^{-3} \text{eV}, \quad (63)$$

which is consistent with the bound found in Ref. 9 $m_a \lesssim 9 \times 10^{-3}\cos^2\beta$ eV, as well as with the estimate $m_a \lesssim 10^{-2}$ eV obtained in Ref. 11 from an analysis of the Primakoff effect.

We note that the result (1) (Ref. 7) corresponds to a more stringent bound on the mass:

$$m_a \lesssim 2 \times 10^{-3} \text{eV}. \quad (64)$$

Thus, under the conditions of neutron stars the synchrotron and Compton mechanisms of axion emission compete with one another: at sufficiently low temperatures ($T \ll \omega_p$) the synchrotron mechanism predominates, while at $T \gtrsim \omega_p$ the Compton mechanism prevails. The bounds on the axion-electron coupling constant and the axion mass obtained from an analysis of the contributions of these processes to the axion luminosity are consistent with the bounds found for other conditions.

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