Transport of resonance radiation in dense, dispersive media

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A theory is developed for the transport of resonance radiation in highly absorbing media. A system of equations is derived for the generalized spectral "intensity" of the radiation and for the populations of resonance states. The generalized spectral intensity depends on the frequency and the wave vector, which are independent variables and simultaneously satisfy two equations, one in the form of a kinetic equation and the other in the form of a wave equation with a source on the right-hand side. The radiation intensity involved in standard transport theory is obtained from the generalized intensity by appropriate integration with respect to the wave vector and can significantly exceed the Planck intensity in the interior of a dense medium. Boundary conditions are derived for the radiation intensity at the interface of a medium with vacuum, and it is shown that radiation emitted from a slightly inhomogeneous medium has at the center of the spectral line an appreciable residual intensity, which is in fact an order of magnitude higher than the intensity obtained from the standard theory, despite the self-conjugation effect. In optically thick media the spatial distribution of the populations of excited atoms can be determined from the well-known Biberman–Holstein integral equations, because dispersion-related phenomena become all but inconsequential in the wings of the spectral line. A numerical procedure is developed for solving the resulting equations for the generalized spectral intensity of radiation and is illustrated in the example of resonance transition in sodium vapor. [S1063-7761(96)00911-0]

1. INTRODUCTION

The theory of the transport of resonance radiation in gases and plasma has a variety of physical[1,2] and mathematical[3,4] aspects. Depending on the conditions of the problems in regard to transport either in disperse media or in a dense, hot plasma, it is necessary to investigate various mechanisms for the absorption and scattering or resonant photons by atoms or multiply charged ions, taking into account the influence of surrounding particles on the profile of the spectral line and on the photon frequency redistribution function.[2]

The topic investigated in greatest detail is the limit of comparatively low gas densities, for which a state-of-the-art machinery has been developed to derive the fundamental resonance radiation transport equations,[5–10] and good agreement has been attained between theory and experiment.[10,11] Progress is currently witnessed in the theory of spectral line broadening and the frequency redistribution of photons in a dense plasma of multiply charged ions with allowance for the dynamics and nonlinear interference effects in the emission spectra as a result of the mixing of radiator states in the plasma microfield.[12–16,18]

We now examine in greater detail the criteria for the gas density to be low enough to validate the existing resonance radiation transport theory. It is a well-known fact[2,10] that the condition $A\sim\Gamma$ (where A is the probability of spontaneous emission and Γ is the collisional linewidth, which is determined in the neutral gas by the Vlasov–Fursov excitation transport mechanism[19]) separates regions of coherent scattering, $A\gg\Gamma$ (without any change of frequency in the rest frame of an atom) and the regime of complete frequency redistribution, $A\ll\Gamma$ (Ref. 1). Taking into account the relation of A and Γ to the square of the matrix element of the dipole moment operator d (Ref. 19),

$$A=\frac{4}{3}\frac{\omega^3 d^2}{\hbar c^3}, \tag{1}$$

$$\Gamma\approx N\pi\frac{d^2}{\hbar} \tag{2}$$

(here ω is the resonant photon frequency, c is the speed of light, and N is the number density of atoms), we find that $\Gamma>A$ holds if

$$\frac{N\lambda^3}{32\pi}>1. \tag{3}$$

In relation (3) λ is the photon wavelength. For the resonance line of sodium vapor condition (3) is satisfied for $N>3\times10^{14}$ cm^{-3}. If in this case the collisional width satisfies the relation $\Gamma\ll kv_T$, where kv_T is the Doppler linewidth ($k=2\pi/\lambda$, and v_T is the thermal velocity of the emitting particle), the energy absorption coefficient of resonance radiation at the center of the line, k_0, is determined by the Doppler broadening and can be written in the form[19]

$$k_0=\frac{\lambda^2}{4}\frac{A}{\sqrt{\pi}kv_T}\frac{g_2}{g_1}N. \tag{4}$$

Here g_2 and g_1 are the statistical weights of the excited and ground states of the atom. In Eq. (4) the density of excited atoms is assumed to be low ($N_2\ll N_1\approx N$). For sodium vapor

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the condition $\Gamma < kv_T$ implies $N < 10^{17}$ cm^{-3}. The strength of the resonance field decays in the medium according to the law[20]

$$|E| = |E_0| \exp\left(-\frac{\varepsilon''}{2} \frac{\omega}{c} x \right) = |E_0| \exp\left(-\frac{k_0 x}{2} \right), \tag{5}$$

where ε'' is the imaginary part of the permittivity of the medium at the frequency corresponding to the center of the line. Equation (5) has been derived on the assumption that the decay of the field at one wavelength is small, and the real part of the dielectric permittivity is close to unity.

It follows from Eq. (5) that $\varepsilon'' \sim k_0 \lambda$, where $\lambda = \lambda/2\pi$, and the quantity $\varepsilon' - 1$ admits the estimate

$$(\varepsilon' - 1)_{\max} \approx k_0 \lambda / 2. \tag{6}$$

Here the subscript "max" corresponds to maximum departure from resonance, $\Delta = \omega - \omega_0 \approx \max(\Gamma, kv_T)$. The condition that the atomic density be low enough that the existing resonance radiation transport theory holds, is formulated by means of the Biberman criterion,[1] which in the given situation implies $(\varepsilon' - 1)_{\max} \ll 1$ or, from (4) and (6),

$$\frac{g_2}{g_1} \frac{N\lambda^3}{32\pi} \frac{A}{\sqrt{\pi} k v_T} \ll 1. \tag{7}$$

When inequality (7) holds, condition (3) can be satisfied with a considerable margin, because $A < \Gamma < kv_T$. But if $\Gamma > kv_T$ holds, the absorption coefficient at the center of the line is determined by the collisional width Γ:

$$k_0 = \frac{\lambda^2}{4} \frac{A}{\pi \Gamma} \frac{g_2}{g_1} N. \tag{8}$$

In this limit, based on (1) and (2), the parameter $k_0\lambda/2$ assumes the form

$$\frac{k_0 \lambda}{2} \approx \frac{g_2}{g_1} \frac{2}{\pi} \approx 1 \tag{9}$$

and saturates as the density increases. Consequently, for $\Gamma > kv_T$ the influence of the permittivity on the resonance radiation transport process must be taken into account in the theory, because $|\varepsilon' - 1| \sim 1$. Condition (9) implies that the mean free path of the radiation is only 4π times shorter than the wavelength.

We note that the condition $\Gamma > kv_T$ does not conflict with the representation of the medium by the gas approximation,[21] which has the form $Na^3 < 1$, where a is the amplitude of scattering with excitation transport and is associated with Γ by the obvious relation

$$\Gamma \approx N \sigma v_T \approx N \pi a^2 v_T. \tag{10}$$

From (2) and (10) we obtain $a \approx d/\sqrt{\hbar v_T}$, and the condition for the validity of this approximation can be expressed in the form

$$Na^3 \approx \frac{Nd^2}{\hbar v_T k} \frac{kd}{\sqrt{\hbar v_T}} \approx \frac{\Gamma}{\pi k v_T} ka \ll 1. \tag{11}$$

Since the inequality $ka \ll 1$ holds for sodium vapor, condition (11) can also be satisfied for $\Gamma > kv_T$. Numerically the condition for validity of the gas approximation is violated in the given situation for $N > 10^{19}$ cm^{-3}. Here the theory is greatly complicated by the need to take multiple-particle collisions into account. One of the qualitative phenomena that arise in this limit, in principle, is the possibility of nonradiative transport of the excitation. However, if the discussion is limited by the criterion $Na^3 > 1$, as is evident from the foregoing analysis, the resonance radiation transport theory needs to be generalized to the case of a dense medium, i.e., the case $\Gamma > kv_T$, when the permittivity differs significantly from unity. Levin and Rytov[22] have remarked that the notion of the spectral intensity of radiation is ill-defined in highly absorbing media, owing to the violation of the quasiclassicality condition, and they propose that problems associated with the transport of radiant energy be solved in the language of spectral energy fluxes, i.e., the Poynting vector expressed in terms of frequency.

In this article we seek to generalize the theory of resonance radiation transport to the case of dense gaseous media ($\varepsilon' - 1 \sim 1$) but with an upper bound on the density: $Na^3 \leqslant 1$.The equations for the correlation functions of the electromagnetic field are formulated in the language of kinetic Green's functions,[5,8–10,23] which permits us to introduce the generalized radiation intensity $J(\omega, \mathbf{k})$, in which the frequency ω and the wave vector $\mathbf{k}$ are independent variables, as opposed to the conventional theory, in which the relation $\sqrt{\varepsilon'}\omega = c|\mathbf{k}|$ is customarily assumed to hold, along with $\varepsilon' = 1$. In general the function $J(\omega, \mathbf{k})$ is not positive, and certain of its moments, i.e., integrals with respect to k with different weighting functions, have physical significance. The intensity $J(\omega, \mathbf{k})$ satisfies two equations simultaneously, one of which has the form of a kinetic equation, while the other is a wave equation with an appropriate source on the right-hand side. This fact is well known in formal radiation transport theory.[24] However, the discussion is usually confined to the case of weakly absorbing media without spontaneous sources, and the relation $J(\omega, \mathbf{k}) \propto \delta(\omega - ck)$ follows from the homogeneous wave equation for the spectral intensity. For a homogeneous equilibrium medium we derive an equation for the physical spectral intensity J_ω, generalizing the Clausius law[22,25,26] $J_\omega^{Kl} = J_\omega^0 \varepsilon'$, where J_ω^0 is the Planck intensity for blackbody radiation in vacuum, to the case of strong absorption. The excess of J_ω^{Kl} above the Planck intensity J_ω^0 for $\varepsilon' > 1$ is attributable to the decrease in the photon propagation velocity in a dense transparent medium. In an absorbing medium the radiation intensity can exceed both J_ω^0 and J_ω^{Kl} since photons are also strongly absorbed and effectively cluster at a given point in space. This phenomenon takes place both in an unbounded medium and in a bounded medium with dimensions greater than the mean free path of the radiation.

The equations for $J(\omega, k)$ involve the absorption coefficient and spectral intensity of spontaneous emission of radiation, both extended to the case of high densities, where the probability of spontaneous emission in a highly absorbing medium is accurately expressed by the relation

$$A = A_0 \mathrm{Re} \sqrt{\varepsilon}. \tag{12}$$

Here A_0 is the probability (1) of spontaneous emission in vacuum (in Refs. 25 and 26 this relation is derived for trans-

parent media, where $\mathrm{Re}\sqrt{\varepsilon}\approx\sqrt{\varepsilon'}$). It follows from Eq. (12), in particular, that if a radiator with frequency ω is situated in a plasma for which the plasma frequency $\omega_p=\sqrt{4\pi e^2N_e/m}$ is close to ω (e and m are the electron charge and mass, and N_e is the density of electrons in the plasma), the spontaneous emission rate becomes small and even equal to zero (neglecting damping, i.e., if $\varepsilon\approx 1-\omega_p^2/\omega^2$ holds for $\omega<\omega_p$.

In the article we also obtain a generalized expression for the permittivity $\varepsilon(\omega,k)$ of a resonant medium with allowance for the previously determined expressions for the spectral densities of the atomic distributions in the limit of broad spectral lines.[27] This fact is ignored in Ref. 21, in which expressions are derived for $\varepsilon(\omega,k)$ in the limit $\Gamma\gg kv_T$. Since we are interested in the limit $\Gamma>kv_T$ and since condition (3) is definitely satisfied, for resonance radiation transport we have a regime of complete frequency redistribution, and the complete system of equations augments the intensity equations with equations for the populations of the atomic states. If $J(\omega,k)$ is eliminated from the latter equations, equations generalizing the well-known Biberman–Holstein equations[28,29] are obtained for the populations of excited atoms. It is important to note that, since radiation transport in the regime of complete frequency redistribution takes place in the far wings of the lines, i.e., at frequencies for which $|\varepsilon'-1|\ll 1$, in optically dense media, the spatial distribution of the populations of excited atoms in the volume coincides with the distribution deduced from the solution of the standard Biberman–Holstein equations except in near-boundary regions with dimensions of the order of the photon mean free path. A similar statement is made in Lozanskiĭ and Firsov's book,[30] but they do not give generalized equations for the populations of excited atoms.

Below, to formulate the conditions satisfied by the intensity $J(\omega,k)$ at an interface separating two media, we give a rigorous solution of the problem of the emergence of radiation from an absorbing medium into vacuum (cf. the analogous problem in Ref. 22, which is solved by means of the reciprocity theorem). The present study helps to refine the limits of validity of the theory developed thus far and to predict certain new phenomena.

The article is organized as follows. A qualitative analysis of the problem is given in Sec. 2, and expressions are derived for the equilibrium spectral intensity of radiation in a highly absorbing medium. A simple derivation of relation (12) is also given. In Sec. 3 equations are derived for the generalized spectral intensity of the radiation and for the populations of excited atomic states. In Sec. 4 an expression is given for the permittivity of a resonant medium, generalized to the case of broad atomic lines (cf Ref. 27). The solution of the problem of emission of radiation from a heated absorbing medium into vacuum is discussed in Sec. 5. A rigorous solution is given, and approximate boundary conditions for the intensity $J(\omega,k)$ are formulated. In Sec. 6 the results of numerical calculations using the previously derived system of radiation transport equations are given for the case of the resonance line of sodium.

2. QUALITATIVE ANALYSIS

We begin with a brief look at the case of a transparent medium, for which $\varepsilon''\ll 1$, without making any additional assumptions as to the quantity ε'. The vacuum Planck radiation formula can be generalized for the spectral energy density of equilibrium radiation to deduce the following relation in such a medium:[20,22,25]

$$U_\omega d\omega=\int\frac{2\hbar\omega n(\omega)d\Omega}{(2\pi)^3}k^2\frac{dk}{d\omega}d\omega. \tag{13}$$

Here $n(\omega)$ are the equilibrium photon occupation numbers,

$$n(\omega)=\frac{1}{\exp(\hbar\omega/T)-1}. \tag{14}$$

The factor 2 in Eq. (13) accounts for the two polarization directions, and integration over the solid angles Ω produces the factor 4π.

Making use of the relation between k and ω in a transparent medium,

$$k=n\omega/c, \tag{15}$$

where $n^2\approx\varepsilon'>0$, from (13) and (15) we obtain

$$U_\omega=\frac{\hbar\omega^3}{\pi^2c^3}\varepsilon'\frac{d(n\omega)}{d\omega}n(\omega). \tag{16}$$

The quantity $d(n\omega)/d\omega$ is known to be positive in a transparent medium.[20] Transforming from the energy density to the spectral intensity J_ω, which is defined by the expression[19,22,25]

$$U_\omega=\frac{1}{v_g}\int J_\omega d\Omega, \tag{17}$$

where v_g is the group velocity of the wave packet,

$$v_g=\frac{d\omega}{dk}=\frac{c}{d(n\omega)/d\omega}, \tag{18}$$

from (16) and (17) we obtain the Clausius equation[22,25]

$$J_\omega=\frac{\hbar\omega^3\varepsilon'}{4\pi^3c^2}n(\omega). \tag{19}$$

Equation (16) has also been obtained previously[20] from an analysis of electromagnetic fluctuations in a transparent medium. In an unbounded homogeneous medium Kirchhoff's law can be used for the spectral intensity of radiation:[19]

$$J_\omega=\varepsilon_\omega/k_\omega, \tag{20}$$

where ε_ω is the spectral density of spontaneous emission per unit volume[27] in unit intervals of solid angles and frequencies:

$$\varepsilon_\omega=\frac{\tilde{N}_2A\hbar\omega}{4\pi}\varphi(\omega)\exp\left[-\frac{\hbar(\omega-\omega_0)}{T}\right]. \tag{21}$$

Here $\tilde{N}_2$ is the effective population of the excited state of the atom (see Sec. 3 below), A is the probability of spontaneous emission in the medium, $\varphi(\omega)$ describes the profile of the spectral line, normalized to unity, and ω_0 is the atomic tran-

sition frequency. The absorption coefficient k_ω in Eq. (20) can be obtained from a relation generalizing (5) (Ref. 20):

$$k_\omega=\frac{\omega}{c}\frac{\varepsilon''}{n}=\frac{k_\omega^0}{n}, \tag{22}$$

where k_ω^0 is the absorption coefficient at a solitary atom (i.e., without regard for dispersion of the medium: $n=1$); it is written in the form[27]

$$k_\omega=\frac{\lambda_0^2}{4}\frac{A_0\varphi(\omega)g_2/g_1}{n}\left\{\widetilde{N}_1-\frac{g_1}{g_2}\widetilde{N}_2 \times\exp\left[-\frac{\hbar(\omega-\omega_0)}{T}\right]\right\}. \tag{23}$$

In (23) $\lambda_0=2\pi c/\omega_0$ and A_0 are the vacuum values of the wavelength and the probability of spontaneous emission [cf. (1)].

Substituting the values of (21) and (23) into Eq. (20) and invoking the Boltzmann relations between the populations $\widetilde{N}_2$ and $\widetilde{N}_1$ under thermodynamic equilibrium conditions, $\widetilde{N}_2/\widetilde{N}_1=(g_2/g_1)\exp(-\hbar\omega_0/T)$, we obtain

$$J_\omega=\frac{A}{A_0}n\frac{\hbar\omega^3 n(\omega)}{4\pi^3c^2}. \tag{24}$$

It follows from a comparison of (19) and (24) [cf. (12)] that

$$A=A_0 n\approx A_0\sqrt{\varepsilon'}. \tag{25}$$

Equation (25) and the relation $\lambda=\lambda_0/n$ can be used to write the absorption coefficient (23) in the form

$$k_\omega=\frac{\lambda_0^2 A}{4}\varphi(\omega)\frac{g_2}{g_1}\left\{\widetilde{N}_1-\frac{g_1}{g_2}\widetilde{N}_2\exp\left[-\frac{\hbar(\omega-\omega_0)}{T}\right]\right\}. \tag{26}$$

We note that relation (25) can also be obtained by Fermi's golden rule in the first perturbation order with respect to the interaction of an atomic electron with an electromagnetic field in a medium[19] in the gauge $H_{\text{int}}=-\mathbf{d}\cdot\mathbf{E}$ with the vector potential normalized to the field energy in a transparent medium.[20]

To extend relations of the type (19) and (25) to the case of a highly absorbing medium ($\varepsilon''\sim 1$) we can use an equation derived on the basis of the fluctuation–dissipation theorem for the equilibrium spectral density of fluctuations of the strength of a transverse electromagnetic field in the medium (see, e.g., Refs. 26 and 31):

$$\langle E_iE_j\rangle_{\mathbf{k}\omega}=\frac{8\pi\hbar\omega^4 n(\omega)\varepsilon''}{|\omega^2\varepsilon-k^2c^2|^2}\left(\delta_{ij}-\frac{k_ik_j}{k^2}\right) \equiv J(\omega,k)\left(\delta_{ij}-\frac{k_ik_j}{k^2}\right). \tag{27}$$

In Sec. 3 we obtain an analogous relation for the spectral intensity $J(\omega,\mathbf{k})$ from the kinetic Green's function $D_{ij}^{-+}(\mathbf{k},\omega)$ (Refs. 8–10 and 23) in a homogeneous equilibrium medium. Taking into account the definition of the electromagnetic energy flux[20,21]

$$\mathbf{S}=\frac{c}{4\pi}\mathbf{E}\times\mathbf{H} \tag{28}$$

and the relation between the electric and magnetic fields

$$\mathbf{H}=\frac{c}{\omega}\mathbf{k}\times\mathbf{E}, \tag{29}$$

from (27)–(29) we can obtain a generalized expression for the radiation intensity in an equilibrium absorbing medium (see also Ref. 26):

$$J_\omega=\frac{c^2}{\omega}\int_0^\infty kJ(\omega,k)\frac{k^2dk}{2\pi^2}\frac{}{(2\pi)^3}. \tag{30}$$

A definition analogous to (30) allowing for the fact that the spectral Green's functions involve integration over $d\omega/2\pi$ (see Sec. 3 below) and taking into account the two independent directions of polarization of the electric field is obtained in the more formal treatment by means of kinetic Green's functions.

Taking Eq. (27) into account, we write (30) in the form

$$J_\omega=\frac{\hbar\omega n(\omega)c^2}{2\pi^3}\int_0^\infty\frac{\omega^2\varepsilon''(\omega,k)k^3}{|\omega^2\varepsilon-c^2k^2|^2}\frac{dk}{\pi}. \tag{31}$$

In a weakly absorbing medium (i.e., for $\varepsilon''\to 0$) the integrand in (31) reduces to $k^3\delta(\omega^2\varepsilon'-c^2k^2)$, and integration with respect to k transforms expression (31) into (19). In a highly absorbing medium the integral (31) must be evaluated with allowance for the dependence of $\varepsilon=\varepsilon'(\omega,k)+i\varepsilon(\omega,k)$ on the wave number k. If spatial dispersion is ignored because $\Gamma>kv_T$ holds in a dense medium (see Sec. 1), the integral (31) diverges logarithmically. However, for large k, i.e., for $k>\Gamma/v_T=\widetilde{k}_0$, the imaginary part of the permittivity decreases: $\varepsilon''\approx 1/k$ (see below). We can therefore estimate the integral (31) without regard for the dependence of ε on k by cutting off the logarithmically divergent integral at $k\approx\widetilde{k}_0$. In this approximation we obtain

$$J_\omega=\frac{\hbar\omega^3 n(\omega)}{4\pi^3c^2}\frac{\varepsilon'}{2}\left(1+\frac{2}{\pi}\arctan\frac{\varepsilon'}{\varepsilon''}+\frac{4\varepsilon''}{\pi\varepsilon'}\ln 2a_V\right). \tag{32}$$

Here $a_V=\Gamma/2(\omega/c)v_T$ is the Voigt parameter.[19] Equation (32) goes over to (19) in the limit $\varepsilon''\to 0$. The results of numerical calculations of the integrals (31) for sodium vapor with allowance for the dependence of ε on the wave number k are given in Sec. 6, where they show that the accuracy of the approximation (32) is satisfactory. Because expression (32) contains a logarithmic approximation that increases with the density of the gas [since $\Gamma\sim N$; see (2)], the equilibrium intensity in the absorbing medium can be almost an order of magnitude higher than the Planck intensity within the limits of the spectral line profile, i.e., for $\Delta\leqslant\Gamma$.

We now consider the extension of Eq. (25) to a highly absorbing medium. The radiation width of state 2 (see Ref. 27) is determined by the mass operator $\Sigma^{+-}(p)$ according to the relation[23]

$$-i\Sigma_2^{+-}(p)=d_i\int iD_{ij}^{+-}(k)iG_1^{+-}(p-k)\frac{d^4k}{(2\pi)^4}d_j, \tag{33}$$

where the photon Green's function $D_{ij}^{+-}(k)$ is related to the function $D_{ij}^{-+}(k)$ as defined by (27) according to the equation[23]

$$D_{ij}^{+-}(k)=D_{ij}^{-+}(-k). \tag{34}$$

In Eq. (33) G_1^{+-} is the Green's function for atoms in the ground state (see Refs. 23 and 27), and d_i is the ith projection of the matrix element of the dipole moment operator. Using the relation $n(-\omega)\varepsilon''(-\omega)=[1+n(\omega)]\varepsilon''(\omega)$ [with allowance for the fact that $\varepsilon''(\omega)$ is an odd function[20]] and keeping the contribution from unity in this equation to characterize the intrinsic width of the spontaneous emission line from (27), (33), and (34) we obtain

$$A\approx\frac{4}{3}\frac{\omega^2 d^2}{\hbar}\int_0^\infty\frac{2\omega^2\varepsilon''k^2}{|\omega^2\varepsilon-c^2k^2|^2}\frac{dk}{\pi}. \tag{35}$$

The applicability of relation (35) is discussed below in Sec. 3. For now we note that its use rests on the assumption $\Gamma>kv_T$. Evaluating the integral (35) in this approximation by the residue theorem (i.e., disregarding spatial dispersion), we obtain [cf. (12))]

$$A=\frac{4}{3}\frac{\omega^3 d^2}{\hbar c^3}\mathrm{Re}\sqrt{\varepsilon(\omega)}=A_0\ \mathrm{Re}\sqrt{\varepsilon(\omega)}. \tag{36}$$

The results of numerical calculations of integrals of the form (35) for the resonance doublet of sodium show that Eq. (36) is quite accurate and yields a difference (for $N\geqslant 10^{17}$ cm^{-3}) by a factor of approximately 1.3 from the vacuum spontaneous emission probability A_0 in the red wing of the line and by a factor of approximately 0.6 in the blue wing, so that the emission probabilities in these two cases differ more than twofold within the line.

Zemtsov and Starostin[27] have investigated the influence of the density of the medium on the spontaneous emission probability in relation to the previously reported[32–34] experimental observation of this phenomenon. A relation of the form (36) does not offer any new results for the case investigated in Ref. 27, because the plasma density is low under the experimental conditions in Refs. 32–34 ($N_e\sim 10^{19}$ cm^{-3}), and $\mathrm{Re}\sqrt{\varepsilon(\omega)}\approx 1$.

3. TRANSPORT EQUATION FOR RESONANCE RADIATION IN DENSE, DISPERSIVE MEDIA

The system of transport equations for resonance radiation includes equations for the correlation functions of the electromagnetic field $D_{ij}^{\alpha\alpha'}(x,x')$ and the atomic Green's functions $G_k^{\alpha\alpha'}(x,x')$. Here the superscripts $\alpha=\pm$ are the Keldysh variables, which characterize the type of chronological ordering of the field operators,[5,23] $\hat{E}_i$ is the Heisenberg operator of the electric field, and $\hat{\psi}$ is the Heisenberg field operator for atomic particles.

Thus, for the kinetic photon Green's function $D_{ij}^{-+}(x,x')$ we have the quantum statistical average

$$iD_{ij}^{-+}(x,x')=\langle\hat{E}_i(x')\hat{E}_j(x)\rangle, \tag{37}$$

and for the kinetic atomic-particle Green's function[27] we can write, analogously,

$$iG_n^{-+}(x,x')=-\int\varphi_n^*(\xi)\times\langle\hat{\psi}^+(x',\xi')\hat{\psi}(x,\xi)\rangle\varphi_n(\xi')d\xi\,d\xi'. \tag{38}$$

Here $x=\{\mathbf{r},t\}$ is the 4-coordinate of the center of gravity of an atom, and ξ are variables characterizing the internal motion of an atomic electron in the nth state ($n=1,2$ in the two-level case), which is described by the set of atomic wave functions $\varphi_n(\xi)$. The functions D^{--} (G^{--}) and D^{++} (G^{++}) are expressed in terms of the retarded [D^R (G^R)] and advanced [D^A (G^A)] Green's functions and the above-defined functions D^{-+} (G^{-+}) (Ref. 23:

$$D^{--}=D^R+D^{-+},\quad D^{++}=-D^A+D^{-+}. \tag{39}$$

The functions D^{+-} (G^{+-}) are expressed in terms of (37) and (39):

$$D^{+-}=D^R-D^A+D^{-+}. \tag{40}$$

The spectral Green's functions $D_{ij}^{\alpha\alpha'}(\mathbf{r},t,\mathbf{k},\omega)$ and their counterparts $G_k^{\alpha\alpha'}$ are defined as the corresponding Fourier components with respect to the difference coordinate:

$$D_{ij}^{\alpha\alpha'}(\mathbf{r},t,\mathbf{k},\omega)=\int D_{ij}^{\alpha\alpha'}(\mathbf{r}_1,t_1,\mathbf{r}_2,t_2)\times\exp(i\omega-i\mathbf{k}\cdot\boldsymbol{\rho})d\tau\,d^3\rho. \tag{41}$$

Here $\tau=t_1-t_2$, $t=(t_1+t_2)/2$, $\boldsymbol{\rho}=\mathbf{r}_1-\mathbf{r}_2$, and $\mathbf{r}=(\mathbf{r}_1+\mathbf{r}_2)/2$.

The retarded Green's function for photons in an unbounded medium, $D_{ij}^R(\mathbf{k},\omega)$, obeys the following well-known expression[35,36] in the transverse gauge (div $\mathbf{A}=0$):

$$D_{ij}^R=\frac{4\pi\hbar\omega^2}{\omega^2\varepsilon(\omega,\mathbf{k})-k^2c^2}\left(\delta_{ij}-\frac{k_ik_j}{k^2}\right). \tag{42}$$

The advanced Green's function D_{ij}^A is obtained from (42) by forming the complex conjugate. A formal derivation of equations describing radiation transport in gaseous media having a low density bounded by the Biberman criterion (7) has been published earlier,[8,10,27] using the approximation $\varepsilon\to 1$ in Eq. (42). We note that Levinson[9] has investigated the transport of resonance phonons in dense media, using an expression similar to (42) for the phonon Green's function, with the appropriate polarization operator (which, however, is not expressed in terms of the permittivity of the medium). For the transport of resonance radiation allowing for the deviation from unity automatically implies that the approximation of complete frequency redistribution is valid, whereas in Ref. 9 the limit of partial frequency redistribution is investigated as an alternative approximation to the Biberman–Holstein theory.

The transport equation for incoherent resonance radiation in the form of a kinetic equation for photons is a special case of the Dyson equation,[5,8–10,23,27] which is written as follows in the quasiclassical approximation for the function D_{ij}^{-+}:

$$-2i\left[\omega\frac{\partial}{\partial t}+c^2k(\boldsymbol{\Omega}\cdot\nabla)\right]D_{ij}^{-+}$$
$$=4\pi\hbar\omega^2(\Pi^{+-}D_{ij}^{-+}-\Pi^{-+}D_{ij}^{+-}). \quad (43)$$

Here $\boldsymbol{\Omega}$ is the unit vector along the direction of $\mathbf{k}$. The polarization operators $\Pi^{\alpha\alpha'}$ are expressed in the resonance approximation in terms of the atomic Green's functions[8,10,27] (it is assumed for simplicity that the states $n=1$, 2 are nondegenerate):

$$\Pi^{+-}(\omega,\mathbf{k})=i\frac{d^2}{3}\int\frac{d\omega_p d\mathbf{p}}{(2\pi)^4}G_2^{+-}(p+k)G_1^{-+}(p), \quad (44)$$

$$\Pi^{-+}(\omega,\mathbf{k})=i\frac{d^2}{3}\int\frac{d\omega_p d\mathbf{p}}{(2\pi)^4}G_2^{-+}(p+k)G_1^{+-}(p). \quad (45)$$

The following expressions have been derived[27] for the Green's function $G_n^{\alpha\alpha'}(p)$ in the limit of complete frequency redistribution:

$$G_n^{-+}(p)=\frac{2\pi i}{\hbar}a_n(\varepsilon)\tilde{N}_n(p), \quad (46)$$

$$G_n^{+-}(p)=-\frac{2\pi i}{\hbar}a_n(\varepsilon)[1-\tilde{N}_n(p)], \quad (47)$$

where $a_n(\varepsilon)$ is the "line profile" of the nth state,

$$a_n(\varepsilon)=\frac{\gamma_n/2\pi}{\varepsilon^2+(\gamma_n/2)^2}, \quad (48)$$

$\hbar\varepsilon=\hbar\omega-\hbar\omega_n-E(p)+\mu$, ω_n is the frequency of the nth state, $E(p)$ is the translational kinetic energy of a moving atom, and μ is the chemical potential. The width γ_n represents the sum of the radiation width A and the collisional width Γ ($\gamma_1\to 0$ for the ground state). For the generalized occupation numbers $\tilde{N}_n(p)$ we have the expression[10,27]

$$\tilde{N}_n(p)=\frac{\tilde{N}_n\lambda_T^3\exp\{-[\hbar\varepsilon+E(p)]/T\}}{1+\tilde{N}_n\lambda_T^3\exp\{-[\hbar\varepsilon+E(p)]/T\}}. \quad (49)$$

Here $\tilde{N}_n$ is the effective, in general nonequilibrium, population of the nth state, and $\lambda_T=\sqrt{2\pi\hbar^2/MT}$ is the thermal de Broglie wavelength of the atom. For narrow spectral lines ($\hbar\varepsilon\approx\gamma_i\ll T$) and a nondegenerate atomic gas ($\tilde{N}_n\lambda_T^3\ll 1$) we obtain from (49)

$$\int\tilde{N}_n(p)\frac{d^3p}{(2\pi)^3}=\tilde{N}_n\approx N_n. \quad (50)$$

Here N_n is the true population obtained by integrating (46) with respect to ε and p. In the case of broad lines ($\hbar\gamma_n\approx T$) the generalization (49) is significant and can be used to obtain the correct Planck formula for the photon occupation numbers,[10,27] whereas in standard radiation transport theory we find in its place $n(\omega)=1/[\exp(\hbar\omega_0/T)-1]$, $\omega_0=\omega_2-\omega_1$, i.e., the exponential function involves the transition frequency ω_0 instead of the instantaneous frequency ω.

From Eq. (43), taking relation (40) into account, we obtain

$$-2i\left[\omega\frac{\partial}{\partial t}+c^2k(\boldsymbol{\Omega}\cdot\nabla)\right]D_{ij}^{-+}$$
$$=4\pi\hbar\omega^2[(\Pi^{+-}-\Pi^{-+})D_{ij}^{-+}-\Pi^{-+}(D_{ij}^R-D_{ij}^A)]. \quad (51)$$

From Eq. (51) in the homogeneous stationary case we deduce

$$D_{ij}^{-+}=\frac{\Pi^{-+}}{\Pi^{+-}-\Pi^{-+}}(D_{ij}^R-D_{ij}^A). \quad (52)$$

Using the expressions for the polarization operators (44) and (45), substituting expressions (46)–(49) therein, and taking into account the fact that $\tilde{N}_2/\tilde{N}_1=\exp(-\hbar\omega_0/T)$ in thermodynamic equilibrium, from Eqs. (52) and (42) we obtain [see Refs. 20, 26, and 31 and Eq. (27)]

$$iD_{ij}^{-+}=\frac{4\pi\hbar\omega^2n(\omega)2\omega^2\varepsilon''}{|\omega^2\varepsilon-c^2k^2|^2}\left(\delta_{ij}-\frac{k_ik_j}{k^2}\right). \quad (53)$$

Consequently, the method of kinetic Green's functions reproduces the results of the fluctuation theory while simultaneously providing a means for extending the results to the nonequilibrium situation.

We now introduce the scalar spectral intensity $J(\omega,\mathbf{k},\mathbf{r},t)$ by means of the relation

$$iD_{ij}^{-+}=J(\omega,\mathbf{k},\mathbf{r},t)\left(\delta_{ij}-\frac{k_ik_j}{k^2}\right). \quad (54)$$

We write Eq. (43) for the stationary case in the form of the standard radiation transport equation[2–4]

$$(\boldsymbol{\Omega}\cdot\nabla)J(\omega,k,\boldsymbol{\Omega},\mathbf{r})=-k_\omega J(\omega,k,\boldsymbol{\Omega},\mathbf{r})+\tilde{\varepsilon}(\omega,k,\boldsymbol{\Omega},\mathbf{r}). \quad (55)$$

Here k_ω is the absorption coefficient and $\tilde{\varepsilon}$ is the generalized spectral (with respect to k and ω) intensity of volume spontaneous emission. From Eqs. (43)–(49) we obtain explicit expressions for k_ω and $\tilde{\varepsilon}$ [cf. (21)–(23)]:

$$k_\omega=\frac{\omega^2\varepsilon''(\omega,k)}{c^2k}=\frac{4}{3}\frac{g_2}{g_1}\frac{\pi^2\omega^2d^2}{\hbar c^2k}\varphi(\omega,k)$$
$$\times\left\{\tilde{N}_1-\frac{g_1}{g_2}\tilde{N}_2\exp\left[-\frac{\hbar(\omega-\omega_0)}{T}\right]\right\}, \quad (56)$$

$$\tilde{\varepsilon}=\frac{4}{3}\frac{\omega^3d^2}{\hbar c^2k}\hbar\omega\frac{\omega^2\varepsilon''}{|\omega^2\varepsilon-c^2k^2|^2}(2\pi)^3\tilde{N}_2\varphi(\omega,k)$$
$$\times\exp\left[-\frac{\hbar(\omega-\omega_0)}{T}\right], \quad (57)$$

which are written with allowance for the possible degeneracy of states 1 and 2 and with the relation between the permittivity ε and the polarization operator Π^R (Refs. 17, 23, and 35):

$$\varepsilon=1-4\pi\hbar\Pi^R,$$

$$\Pi^R=\frac{1}{2}(\Pi^{--}-\Pi^{++})-\frac{1}{2}(\Pi^{+-}-\Pi^{-+}), \quad (58)$$

where the first term in Π^R contributes to ε', and the second term to ε'' [cf. (44)–(49)].

Expressions (56) and (57) involve the generalized line profile $\varphi(\omega,k)$ (Ref. 27):

$$\varphi(\omega,k)=\lambda_T^3\int_{-\infty}^{\infty}d\varepsilon\int\frac{d^3p}{(2\pi\hbar)^3}a_2(\varepsilon+\omega-\omega_0-\mathbf{k}\cdot\mathbf{v})$$
$$\times a_1(\varepsilon)\exp\left[-\frac{\hbar\varepsilon+E(p)}{T}\right]$$
$$\times\left\{1+\tilde{N}_2\lambda_T^3\exp\left[-\frac{\hbar\varepsilon+\hbar(\omega-\omega_0)+E(p)}{T}\right]\right\}^{-1}$$
$$\times\left\{1+\tilde{N}_1\lambda_T^3\exp\left[-\frac{\hbar\varepsilon+E(p)}{T}\right]\right\}^{-1}. \qquad (59)$$

We have derived (59) with allowance for relations (46)–(49), which generalize standard line profile theory to the case of broad spectral lines.[27] If we multiply both sides of Eq. (55) by the factor $(c^2/2\pi^2\omega)\times k^3/(2\pi)^3$ and integrate with respect to k [cf. (30)], we obtain the relation

$$(\mathbf{\Omega}\cdot\nabla)J_\omega(\mathbf{\Omega},\mathbf{r})=-\frac{c^2}{2\pi^2}\int_0^\infty\frac{\omega\varepsilon''(\omega,k)}{c^2k}$$
$$\times J(\omega,k,\mathbf{\Omega},\mathbf{r})\frac{k^3dk}{(2\pi)^3}+\varepsilon_\omega. \qquad (60)$$

We have used the definition (30) for the spectral intensity J_ω encountered in conventional radiation transport theory. The quantity ε_ω characterizes the spectral source of spontaneous emission in the equation for J_ω and has the form [cf. (57)]

$$\varepsilon_\omega=\tilde{N}_2\exp\left[-\frac{\hbar(\omega-\omega_0)}{T}\right]\frac{4}{3}\frac{\omega^2d^2}{2\pi\hbar}\hbar\omega$$
$$\times\int_0^\infty\frac{\omega^2\varepsilon''\varphi(\omega,k)}{|\omega^2\varepsilon-c^2k^2|^2}\frac{k^2dk}{\pi}. \qquad (61)$$

The dependence of the line profile $\varphi(\omega,k)$ on the wave number k is not essential in the case $\Gamma>kv_T$. The spatial dispersion of the permittivity can be disregarded within the same error limits by virtue of the convergence of the integral (61), where it is assumed that $\varepsilon(\omega,k)\approx\varepsilon(\omega)$. Evaluating the integral (61), we obtain [cf (21) and (35)]

$$\varepsilon_\omega=\frac{\hbar\omega}{4\pi}A_0\,\mathrm{Re}(\sqrt{\varepsilon})\tilde{N}_2\varphi(\omega)\exp\left[-\frac{\hbar(\omega-\omega_0)}{T}\right]. \qquad (62)$$

Here A_0 is the probability of spontaneous emission in vacuum, defined by relation (1). Consequently, the spectral intensity of spontaneous emission in a highly absorbing medium has a highly accurate and straightforward form (in the limit of complete frequency redistribution), but contains in addition: 1) renormalization of the spontaneous emission rate in the medium (the factor $\mathrm{Re}\sqrt{\varepsilon}$); 2) the generalized line profile $\varphi(\omega)$, which depends in general on the populations $\tilde{N}_i$ (59); 3) the factor $\exp[-\hbar(\omega-\omega_0/T]$, which generalizes the standard theory to the case of broad spectral lines ($\hbar\Gamma\approx T$; Ref. 27).

We consider the first term on the right-hand side of Eq. (60). Because of the factor $1/k$ in the integrand, this term has the form of a new moment of the generalized spectral intensity $J(\omega,k,\mathbf{\Omega},\mathbf{r})$, which is not reducible to the customary form $-k_\omega J_\omega(\mathbf{\Omega},\mathbf{r})$, so a closed equation for J_ω does not exist. Only in the case of a weakly absorbing medium, i.e., $\varepsilon''\ll 1$ [satisfying the Biberman criterion (7)] can we assume [cf (57) and (53)] that

$$J(\omega,k)\approx A\,\delta(\omega^2\varepsilon'-c^2k^2),$$

whereupon Eq. (60) assumes the customary form

$$(\mathbf{\Omega}\cdot\nabla)J_\omega=-\frac{\omega\varepsilon''}{c\sqrt{\varepsilon'}}J_\omega+\varepsilon_\omega. \qquad (63)$$

In the homogeneous equilibrium case Eqs. (63), (56), and (62) lead to the Clausius law [cf. (19)] for the spectral intensity in a weakly absorbing medium:

$$J_\omega^{cl}=\frac{\varepsilon_\omega c}{\omega\varepsilon''}\sqrt{\varepsilon'}=J_\omega^0\varepsilon'. \qquad (64)$$

In a highly absorbing medium in the homogeneous equilibrium case we obtain the following relation from (55) and (30):

$$J_\omega=\frac{c^2}{2\pi^2\omega}\int_0^\infty\frac{\tilde{\varepsilon}}{k_\omega}\frac{k^3dk}{(2\pi)^3}=J_\omega^0\varepsilon'(\omega)\psi(\omega), \qquad (65)$$

where the function $\psi(\omega)$ has the approximate form [cf. (32)]

$$\psi(\omega)\approx\frac{1}{2}\left(1+\frac{2}{\pi}\arctan\frac{\varepsilon'}{\varepsilon''}+\frac{4\varepsilon''}{\pi\varepsilon'}\ln 2a_V\right). \qquad (66)$$

The function satisfies $\psi(\omega)\to 1$ in the limit $\varepsilon''\to 0$, whereas for $\varepsilon''\approx 1$ and high densities (since $a_V\sim N$) it $\psi(\omega)$ increases logarithmically, causing J_ω to be an order of magnitude greater than J_ω^0 in a highly absorbing medium, consistent with Planck's law. The question of modifying the Clausius equation, which is usually written in the form $J_\omega^{cl}=J_\omega^0n^2$ with corrections to account for absorption, has been discussed in the literature (see, e.g., Ref. 22). If we introduce the refractive index n and the effective absorption index χ (Ref. 20) by the definitions $n=\mathrm{Re}\sqrt{\varepsilon}$ and $\chi=\mathrm{Im}\sqrt{\varepsilon}$, we can express the quantities ε' and ε'' involved in (65) and (66) in terms of n and χ:

$$\varepsilon''=2n\chi,\quad \varepsilon'=n^2-\chi^2. \qquad (67)$$

We now write the expression for the equilibrium spectral intensity in a highly absorbing medium in these terms:

$$J_\omega=\frac{1}{2}J_\omega^0n^2\left\{\left(1-\frac{\chi^2}{n^2}\right)\left[1+\frac{2}{\pi}\arctan\frac{n(1-\chi^2/n^2)}{2\chi}\right]\right.$$
$$\left.+\frac{4}{\pi}\frac{2\chi}{n}\ln 2a_V\right\}. \qquad (68)$$

For small values of χ/n expression (68) assumes the form

$$J_\omega=J_\omega^0n^2\left(1-\frac{\chi^2}{n^2}-\frac{2\chi}{\pi n}+\frac{4\chi}{\pi n}\ln 2a_V\right). \qquad (69)$$

We have previously written the equation for the kinetic photon Green's function D_{ij}^{-+} in the form (43), which is

obtained[5,23] from the equations for the Green's function $D_{ij}^{-+}(x_1,x_2)$ in coordinate representation by subtracting a Dyson equation of the form $\Box_2 D^{-+}=j_2$ from the analogous equation $\Box_1 D^{-+}=j_1$ and then transforming to the Fourier components with respect to the difference coordinate x_1-x_2. For compactness we use the symbols j_1 and j_2 here to denote the corresponding right-hand sides of the Dyson equations, $\Box$ denotes the d'Alembertian, $\Box=\Delta-c^{-2}\partial^2/\partial t^2$, and the subscript 1 or 2 signifies differentiation with respect to the coordinate x_1 or x_2. This procedure yields a radiation transport equation in the form of a corresponding kinetic equation for photons. To the accuracy with which satisfies Eq. (43), it should satisfy the Green's function, a wave-type equation obtained analogously to (43) but using the sum rather than the difference of equations of the type $\Box_1 D^{-+}=j_1$ and $\Box_2 D^{-+}=j_2$ and with the transformation to Fourier components with respect to x_1-x_2. Like (43), (51), and (55)–(57), an analogous wave equation can thus be obtained for $J(\omega,k,\mathbf{\Omega},\mathbf{r})$:

$$\left[\frac{1}{2}\left(\frac{\partial^2}{\partial t^2}-c^2\Delta\right)+2(c^2k^2-\omega^2\varepsilon')\right]J=\frac{8\pi\hbar\omega^4\varepsilon''}{|\omega^2\varepsilon-c^2k^2|}\times\left\{\frac{\tilde{N}_1}{\tilde{N}_2}\frac{g_2}{g_1}\exp\left[\frac{\hbar(\omega-\omega_0)}{T}\right]-1\right\}^{-1}\times 2(c^2k^2-\omega^2\varepsilon'). \tag{70}$$

In Eq. (70) in the general case, as in (56) and (57), $\tilde{N}_1$ and $\tilde{N}_2$ can be nonequilibrium populations. In thermodynamic equilibrium, on the other hand, the second factor on the right-hand side of Eq. (70) reduces to the Planck occupation numbers $n(\omega)$. In the stationary and homogeneous case Eq. (70) leads to the same result (53) as that obtained form the kinetic equation for J Eqs. (55)–(58).

A transport equation in the form (70) is used in formal radiation transport theory,[24] but with zero on the right-hand side. If we disregard the right-hand side (e.g., in the limit $\varepsilon''\to 0$) and the wave operator on the left-hand side, as is justified for the case of low absorption ($k_\omega \ll k$) and slow processes ($\omega\tau\gg 1$), it follows from (70) that

$$J(\omega,k)=A\,\delta(\omega^2\varepsilon'-c^2k^2). \tag{71}$$

If we use this relation in Eq. (60), we can obtain an equation of the form (63), where $A=J_\omega(2\pi)^5c^2/\omega\varepsilon'$.

Equations (55) and (70) need to be solved simultaneously in the general case. In Sec. 5 we give an example of such a simultaneous application of both forms of the equations for the generalized spectral intensity $J(\omega,k,\mathbf{\Omega},\mathbf{r})$ in the stationary case for the formulation of boundary conditions to be satisfied by the function J at an interface between two media.

To close the system of equations describing the transport of resonance radiation, we need to augment Eqs. (55) and (70) with equations for the populations $\tilde{N}_1$ and $\tilde{N}_2$, which determine the absorption coefficient k_ω and the spectral intensity of spontaneous emission $\tilde{\varepsilon}$. By analogy with Ref. 27 we can derive the corresponding equations using the Dyson equation for the kinetic Green's function G_n^{-+} and relations (46)–(49) in the approximation of complete frequency redistribution. We confine the discussion here to the stationary case, when the density of excited particles is low, $\tilde{N}_2)\ll\tilde{N}_1\approx N$ (the Wien limit). For $\tilde{N}_2$ in this limit we have the equation

$$\frac{2}{3}\frac{d^2}{\hbar^2}\frac{g_2}{g_1}\int\frac{d\omega d\mathbf{\Omega}k^2dk}{(2\pi)^3}J(\omega,k,\mathbf{\Omega},\mathbf{r})\varphi(\omega,k)\tilde{N}_1-\frac{8}{3}\frac{d^2}{\hbar}\int\frac{d\omega k^2dk}{\pi}\frac{\omega^4\varepsilon''(\omega,k)}{|\omega^2\varepsilon-c^2k^2|^2}\varphi(\omega,k)\times\exp\left[-\frac{\hbar(\omega-\omega_0)}{T}\right]\tilde{N}_2-W\left[\tilde{N}_2-\tilde{N}_1\frac{g_2}{g_1}\times\exp\left(-\frac{\hbar\omega_0}{T}\right)\right]=0, \tag{72}$$

where the first term (I_1) corresponds to the photoabsorption of radiation, which leads to the excitation of atoms, the second term (I_2) corresponds to their spontaneous decay, and the last term (I_3) describes the collisional exchange between states 1 and 2 (cf. Ref. 27).

If we use the approximation (71) and substitute this expression into the first term of Eq. (72), we can reduce the photoabsorption contribution to the standard form[1–4] [cf. (23)]

$$I_1=\frac{g_2}{g_1}\int\frac{\lambda_0^2A_0}{4\sqrt{\varepsilon'}}\frac{J_\omega}{\hbar\omega}\varphi(\omega)\tilde{N}_1d\omega d\mathbf{\Omega}. \tag{73}$$

The term corresponding to spontaneous decay (I_2) can be written as follows in the approximation used in the derivation of (62) from expression (61) (Ref. 27):

$$I_2=-\int\frac{4}{3}\frac{\omega^3d^2}{\hbar c^3}\,\mathrm{Re}[\sqrt{\varepsilon(\omega)}]\varphi(\omega)\times\exp\left[-\frac{\hbar(\omega-\omega_0)}{T}\right]d\omega\tilde{N}_2. \tag{74}$$

If we have $\varepsilon'\approx 1$ and the linewidth, which determines the profile $\varphi(\omega)$, is small, $\hbar\Gamma\ll T$, then Eq. (74) is reduced to the standard form $I_2=-A_0\tilde{N}_2$.

In general, the collisional transition rate W exhibits a complicated dependence on the populations $\tilde{N}_n$ and the densities of the particles inducing the decay and excitation of state 2 (Ref. 2).[27] In the unionized gas the rate of extinction of excitations with energy transfer $\hbar\omega_0$ into translational degrees of freedom is small[37] in comparison with the spontaneous decay rate ($W/A_0\ll 1$). The opposite limit $W\gg A_0$ is possible in a gas containing impurity molecules or in a plasma.[37] We note that in optically dense media radiation transport in the volume takes place in the limit of complete frequency redistribution in the far wings of the line ($\Delta\gg\Gamma$). Here all the corrections associated with the deviation of the permittivity from unity become inconsequential ($\varepsilon''\to 0$, $\varepsilon'\to 1$). Radiation transport equations of the form (63) (with $\varepsilon'=1$) and the approximation (71) are valid in the wings, and once the intensity J_ω has been eliminated from (73) the well-known Biberman–Holstein integral equations[28,29] can be obtained for the populations $\tilde{N}_2$, so that the spatial distribution of excited atoms, which is associated

with the emission of radiation and is obtained from the more general case (72), essentially coincides with the distribution obtained in the Biberman–Holstein approximation.[30] In Sec. V we give integral equations in a more general form for the populations of excited atoms. It must be noted here that, besides volume radiation transport, which makes the problem of the spatial distribution of excited particles nonlocal, it is also necessary in dense media to take account of wave reflection from an interface which produces additional contributions to the equation for $\bar{N}_2$, but these are insignificant in optically dense media in the far wings of the spectral line. For broad lines ($\hbar\Gamma \geqslant T$) it is essential to include a factor $\exp[-\hbar(\omega-\omega_0)/T]$, which renders the diffusion approximation valid in the lines for the transport of resonance radiation (see Ref. 27).

The need to include wave reflection from the interface for $|\varepsilon-1|\sim 1$ substantially restricts the applicability of the scalar theory, in which the radiation is described by a single function $J(\omega,\mathbf{k})$, the approximation (54). Inasmuch as the reflection coefficient depends on the polarization of the radiation, strictly speaking, it is necessary to abandon this approximation and work with the Stokes parameters instead of the scalar intensity J. In Sec. 5 we examine this problem in closer detail in the case of radiation from a half-space, where we mention other restrictions on the applicability of the radiation transport theory set forth here. In view of its relative simplicity and relationship to standard radiation transport theory, we show that the approximate approach described in this article is justified in a number of cases for the solution of practical problems by modification of the boundary conditions using Eqs. (55) and (70) simultaneously.

4. PERMITTIVITY OF A RESONANT MEDIUM

Vdovin and Galitskiĭ[21] have discussed the problem of the permittivity of a resonant gas in detail, deriving equations for the collisional widths Γ in the limit $\Gamma \gg kv_T$, but within the gas approximation, i.e., for $Na^3 \ll 1$. In this limit we restrict our scope to the generalization of the equations for $\varepsilon(\omega,k)$ to the case of broad spectral lines, using the relations (46)–(49) obtained in Ref. 27 and assuming that the widths Γ_n are described by expressions given in Ref. 21.

We write in explicit form expressions for the polarization operators $\Pi^{\alpha\alpha'}(k)$ encountered in the definition of the permittivity ε in the single-loop (resonance) approximation.

For example, for the contribution Π^R from the first term in (58) for nondegenerate states 1 and 2 we have [cf. (44) and (45)]

$$\begin{aligned}\operatorname{Re}\Pi^R=-\frac{id^2}{2\times 3}\int\Big[&G_1^{--}(p)G_2^{--}(p+k)\\&+G_1^{--}(p+k)G_2^{--}(p)-G_1^{++}(p)G_2^{++}(p+k)\\&-G_1^{++}(p+k)G_2^{++}(p)\Big]\frac{d^4p}{(2\pi)^4}.\end{aligned}\tag{75}$$

We note that terms containing the Green's functions $G_1^{\alpha\alpha'}$ with the argument $p+k$ are nonresonant terms for $\omega-\omega_0 \ll \omega_0$ and $\omega>0$. We omit them from now on, even though they should be included for $\omega>0$, when $\omega+\omega_0 \ll \omega_0$ holds, to reproduce the correct symmetry properties:[20] $\varepsilon'(\omega)=\varepsilon'(-\omega)$; $\varepsilon''(\omega)=-\varepsilon''(-\omega)$.

Substituting the expressions $G_i^{--}=G_i^R+G_i^{-+}$ and $G_i^{++}=-G_i^A+G_i^{-+}$ into (75) and dropping integrals that contain products of the type $G_1^RG_2^R$ and $G_1^AG_2^A$, since their poles lie in the same half-plane, we reduce (75) to the simpler form

$$\begin{aligned}\operatorname{Re}\Pi^R(k)=-\frac{id^2}{2\times 3}\int\Big\{&G_1^{-+}(p)[G_2^R(p+k)+G_2^A(p+k)]\\&+G_2^{-+}(p+k)[G_1^R(p)+G_1^A(p)]\Big\}\frac{d^4p}{(2\pi)^4}.\end{aligned}\tag{76}$$

Proceeding analogously with the contribution to Π^R from the second term $(\Pi^{-+}-\Pi^{+-})/2$ in (58) and summing it with (76), we obtain

$$\begin{aligned}\Pi^R(k)=-\frac{id^2}{3}\int\Big[&G_1^{-+}(p)G_2^R(p+k)+G_2^{-+}(p+k)\\&\times G_1^A(p)\Big]\frac{d^4p}{(2\pi)^4}.\end{aligned}\tag{77}$$

Using the expressions for the Green's functions G_n^{-+} (46)–(49) and the analogous expressions for G_n^R and G_n^A (Ref. 27), from (77) we obtain

$$\begin{aligned}\Pi^R(k)=\frac{d^2}{3\hbar^2}\lambda_T^3\int d\varepsilon\frac{d^3p}{(2\pi)^3}\Bigg\{&\widetilde{N}_1\exp\left(-\frac{\hbar\varepsilon+E_p}{T}\right)\\&\times a_1(\varepsilon)\left[1+\widetilde{N}_1\lambda_T^3\exp\left(-\frac{\hbar\varepsilon+E_p}{T}\right)\right]^{-1}\left(\varepsilon+\omega-\omega_0\right.\\&\left.-\mathbf{k}\cdot\mathbf{v}+i\frac{\gamma_2}{2}\right)^{-1}+\widetilde{N}_2\exp\left(-\frac{\hbar\varepsilon+E_p+\hbar\omega-\hbar\omega_0}{T}\right)\\&\times a_2(\varepsilon+\omega-\omega_0-\mathbf{k}\cdot\mathbf{v})\left[1+\widetilde{N}_2\lambda_T^3\right.\\&\left.\times\exp\left(-\frac{\hbar\varepsilon+E_p+\hbar\omega-\hbar\omega_0}{T}\right)\right]^{-1}\left(\varepsilon-i\frac{\gamma_1}{2}\right)^{-1}\Bigg\}.\end{aligned}\tag{78}$$

If we consider the case of narrow spectral lines ($\hbar\varepsilon,\hbar\Delta \ll T$) and disregard terms of the form $\widetilde{N}_1\lambda_T^3 \ll 1$ in the denominators of Eq. (78), we obtain the Drude equation for Π^R:

$$\Pi^R(k)=\frac{d^2}{3\hbar^2}\lambda_T^3\int\frac{d^3p}{(2\pi)^3}\frac{\exp[-E(p)/T](\widetilde{N}_1-\widetilde{N}_2)}{\omega-\omega_0-\mathbf{k}\cdot\mathbf{v}+i(\gamma_1+\gamma_2)/2}.\tag{79}$$

For broad lines it is necessary to take into account the dependence on ε in exponential functions of the form $\exp(-\hbar\varepsilon/T)$ in expressions (78). For example, if the widths γ_i do not depend on ε [this assumption underlies the derivation of (79) but is invalid in the general case[27]] and if Doppler shifts are ignored, expressions of the following type arise in the evaluation of the integrals with respect to ε:

$$I=\int_{-\infty}^{\infty}\frac{d\varepsilon}{2\pi}\left[\exp\left(\frac{\varepsilon-\omega_1+\mu}{T}\right)+1\right]^{-1}\left[\left(\varepsilon-\omega_1+i\frac{\gamma_1}{2}\right)^{-1}-\left(\varepsilon-\omega_1-i\frac{\gamma_1}{2}\right)^{-1}\right]\left(\varepsilon+\omega-\omega_2+i\frac{\gamma_2}{2}\right)^{-1}. \quad (80)$$

Integrating Eq. (80) with the aid of the residue theorem, we obtain new contributions, not found in (79), from the poles of the first factor [for $\varepsilon-\omega_1+\mu=i\pi T(2n_1)$], which are usually omitted by virtue of their smallness with respect to the parameter $\hbar\gamma_i/T$ or $\hbar\Delta/T\ll 1$.

In general, such terms must be included in calculating the resonance contribution to the permittivity on the basis of the general expression (78). We also note that when the imaginary part of the permittivity

$$\varepsilon=1-4\pi\hbar\Pi^R$$

is calculated, Eq. (56) with the profile $\varphi(\omega,k)$ represented in (59) is obtained from Eq. (78).

In the most interesting case, when the lower state is the ground state, for $a_1(\omega)$ we have $a_1(\omega)=\delta(\varepsilon)$ and, if we disregard the contribution of excited states, for Π^R we obtain an expression of the form (79), in which it is now required to drop terms containing $\tilde{N}_2$. Degeneracy is taken into account by simply multiplying this expression by g_2/g_1. It is also immediately apparent that the Biberman criterion is written in the form $4\pi\hbar\Pi^R\ll 1$ in this case and has the form (7) for $kv_T\gg\Gamma$.

5. EXAMPLE OF AN APPLICATION OF RESONANCE RADIATION TRANSPORT THEORY IN DENSE, DISPERSIVE MEDIA; CONDITIONS OF VALIDITY

Let us consider the elementary problem of radiation from a half-space occupied by a heated, highly absorbing, homogeneous medium. This problem has been solved previously[22] using the reciprocity theorem from electrodynamics and the fluctuation–dissipation theorem. It can be solved formally and rigorously by the method of Green's functions for nonequilibrium processes[5,23,25,36] and by the resonance radiation transport theory developed above in Sec. 3. By comparing these approaches it is possible to formulate boundary conditions for the generalized intensity, to refine the restrictions on the validity of such an approach, and to predict new qualitative phenomena.

Let a half-space filled with a medium of permittivity $\varepsilon(\omega)$ occupy the domain $z\geqslant 0$. Here we confine the problem to the case of zero spatial dispersion and begin with the electric field vector polarized in the x-direction. Transforming to Fourier components with respect to the time difference and the transverse coordinates in the equations for the stationary retarded Green's function $D^R_{xx}(z,z')$ (we drop the subscripts xx from now on),

$$D^R_{q\omega}(z,z')=\int \exp[i\omega\tau-i\mathbf{q}\cdot(\mathbf{r}_\perp-\mathbf{r}'_\perp)]\times D^R(t,z,r_\perp;t',z',r'_\perp)d\tau d^2\boldsymbol{\rho}_\perp, \quad (81)$$

for the function $D^R_{q\omega}(z,z')$ we obtain the equation (an analogous equation holds for the derivative d^2/dz'^2)

$$\left(\omega^2\bar{\varepsilon}-c^2q^2+c^2\frac{d^2}{dz^2}\right)D^R_{q\omega}(z,z')=4\pi\hbar\omega^2\delta(z-z'), \quad (82)$$

in which $\bar{\varepsilon}=\varepsilon_1=\varepsilon$ holds for $z>0$, and $\bar{\varepsilon}=\varepsilon_0=1$ for $z<0$.

The solution of Eq. (82) with allowance for the continuity of the function $D^R_{q\omega}(z,z')$ and its first derivative at $z=0$ has the form

$$D^R_{q\omega}(z,z')=-\frac{2\pi i\hbar\omega^2}{c^2}\times\begin{cases}\frac{1}{k_z^1}\{\exp[ik_z^1|z-z'|]+r\exp[ik_z^1|z+z'|]\}, & z,z'>0,\\ \frac{1}{k_z^1}(1+r)\exp[ik_z^1z-ik_z^0z'], & z>0,\quad z'<0,\\ \frac{1}{k_z^0}(1-r)\exp[-ik_z^0z+ik_z^1z'], & z<0,\quad z'>0,\\ \frac{1}{k_z^0}\{\exp[ik_z^0|z-z'|]-r\exp[-ik_z^0(z+z')]\}, & z,z'<0.\end{cases} \quad (83)$$

In (83) we have introduced the notation

$$k_z^1=\sqrt{\frac{\omega^2}{c^2}\varepsilon-q^2},\quad k_z^0=\sqrt{\frac{\omega^2}{c^2}-q^2},\quad r=\frac{k_z^1-k_z^0}{k_z^1+k_z^0}.$$

If we transform from the solution (83) obtained above to the Fourier components with respect to the difference coordinate $\zeta=z-z'$ for a fixed value of their half-sum $Z=(z+z')/2$, as is the custom in kinetic theory[5,23] and as we have done above [see (41)], we obtain

$$D^R_{q\omega}(Z,k)=\int_{-\infty}^{\infty}D^R_{q\omega}(z,z')\exp[-ik(z-z')]d\zeta. \quad (84)$$

Substituting Eq. (83) into (84) and making use of the fact that for $Z>0$ the domain of integration in (84) extends from $-\infty$ to $-2Z$ ($z<0$, $z'>0$), then from $-2Z$ to $2Z$ ($z,z'>0$), and from $2Z$ to $+\infty$ ($z>0$, $z'<0$), we obtain

$$\begin{aligned}D^R_{q\omega}(Z>0,k)=&\frac{iD_0(1-r)}{k+(k_z^1+k_z^0)/2}\exp[2iZ(k+k_z^1)]\\&-\frac{iD_1}{k+k_z^1}\exp[2iZ(k+k_z^1)]+\frac{iD_1}{k-k_z^1}\\&\times\exp[-2iZ(k-k_z^1)]-\frac{iD_1(1-r)}{k-(k_z^1+k_z^0)/2}\\&\times\exp[-2iZ(k-k_z^1)]-\frac{2ik_z^1}{k^2-(k_z^1)^2}D_1\\&+\frac{2rD_1}{k}\exp(2iZk_z^1)\sin(2kZ).\end{aligned} \quad (85)$$

Here

$$D_1=-\frac{2\pi i\hbar\omega^2}{k_z^1c^2},\quad D_0=-\frac{2\pi i\hbar\omega^2}{k_z^0c^2}.$$

It is evident from (85) that only the fifth term in this expression does not depend on the sum coordinate Z and corresponds to the expression (42) used above. All other terms decay with increasing distance from the boundary at distances $L\approx 1/\mathrm{Im}\ k_z^1$ They also vanish in the limit $r\to 0$, when $k_z^1=k_z^0$ It is also evident from (85) that the transition to Fourier components for multidomain solutions of the type (83) at a fixed value of the sum coordinate has serious problems: the condition $Z>0$ can be satisfied in the integration of (84) not only for $z>0$, $z'<0$, but also for $z<0$, $z'<0$ and for $z>0$, $z'<0$. We shall therefore work with the function $D^R_{q\omega}(z,z')$ below.

Knowing the functions $D^R_{q\omega}$ and $D^A_{q\omega}$ [the latter is obtained from (83) by forming the complex conjugate], we can find the kinetic Green's function $D^{-+}_{q\omega}$ from the relation[5,23]

$$D^{-+}_{q\omega}(z,z')=-\int_0^\infty D^R_{q\omega}(z,z'')\Pi^{-+}(\omega)D^A_{q\omega}(z'',z')dz''. \tag{86}$$

For simplicity we assume here that the polarization operator Π^{-+} [see (45)] does not depend on the coordinates, i.e., excited atoms occupy space uniformly, and spatial dispersion is insignificant.

If radiation of a resonance character with respect to the 1–2 transition, is incident on the medium from the outside, expression (86) must be augmented with the contribution from the solution of the homogeneous equation for the function $D^{-+}_{q\omega}$ subject to the condition of continuity of the function and its first derivatives [of the form (82) with zero right-hand side]. We restrict the present study to aspects of the description of radiation emitted from heated media. For phenomena in the category of Wood's specular reflection of resonance radiation from a dense medium of metal vapor these terms give the required contributions for the given situation, which are not discussed in this article.

We first compute the integral in (86) for z, $z'>0$. Using the solution (83), we obtain

$$D^{-+}_{q\omega}(z,z')=-|D_1|^2J(z,z')\frac{\Pi^{-+}(\omega)}{\varepsilon''(\omega)}. \tag{87}$$

Here $J(z,z')$ has the form

$$\begin{aligned}J(z,z')=&-2\frac{|k_z^1|^2|1-r|^2}{k_z^0}\exp[ik_z^1z-ik_z^{*1}z']\\&+2k_z^1\exp[-ik_z^{*1}|z-z'|]+2k_z^{*1}\exp[ik_z^1|z\\&-z'|]+2r^*k_z^1\exp[-ik_z^{*1}(z+z')]\\&+2rk_z^{*1}\exp[ik_z^1(z+z')].\end{aligned} \tag{88}$$

Transforming from the function $D^{-+}_{q\omega}$ defined in (37) to the spectral Poynting vector [see (28)–(30) and Ref. 22) $S_{z\omega}$, which is equal to

$$S_{z\omega}=\frac{c^2}{4\pi\omega}\frac{1}{\pi}\mathrm{Re}\int\frac{\partial}{\partial z}D^{-+}_{q\omega}(z,z')|_{z'\to z}\frac{d^2q}{(2\pi)^2}, \tag{89}$$

from (87)–(89) for $z=z'>0$ we obtain

$$\begin{aligned}S_{z\omega}=&\frac{\hbar^2\omega}{2}\int\mathrm{Re}\left\{\frac{i\Pi^{-+}(\omega)}{\varepsilon''(\omega)}\left[\frac{8k_z^1\ \mathrm{Re}\ k_z^0}{|k_z^1+k_z^0|^2}\exp[i(k_z^1\right.\right.\\&-k_z^{*1})z]-2r^*\exp(1-2ik_z^{*1}z)\\&\left.\left.+2r\exp(2ik_z^1)\right]\right\}\frac{d^2q}{(2\pi)^2}\\=&\frac{\hbar^2\omega}{2}\int\mathrm{Re}\left[\frac{i\Pi^{-+}(\omega)}{\varepsilon''(\omega)}\frac{8k_z^1\ \mathrm{Re}\ k_z^0}{|k_z^1+k_z^0|^2}\right.\\&\left.\times\exp(-2\ \mathrm{Im}\ k_z^1z)\right]\frac{d^2q}{(2\pi)^2}.\end{aligned} \tag{90}$$

We now consider the flux at the boundary for $z=z'\to+0$. Bearing in mind that the polarization operator $\Pi^{-+}(\omega)$ is purely imaginary [proportional to i; see (45)–(47)], along with the relation $\varepsilon''(\omega)=-2\pi i\hbar(\Pi^{+-}-\Pi^{-+})$ [see (58)] and the fact that in thermodynamic equilibrium [see (52) and (53)]

$$\frac{\Pi^{-+}}{\Pi^{+-}-\Pi^{-+}}=n(\omega),$$

from (90) we obtain

$$S_{z\omega}=-\frac{2\hbar\omega n(\omega)}{\pi}\int\frac{\mathrm{Re}\ k_z^1\ \mathrm{Re}\ k_z^0}{|k_z^1+k_z^0|^2}\frac{d^2q}{(2\pi)^2}. \tag{91}$$

The minus sign in (91) signifies that the energy flux is directed from the medium into vacuum (toward negative z). We introduce the energy reflection coefficient $R_\perp$, defining it as $R_\perp=|r|^2$. We find at once that

$$\frac{\mathrm{Re}\ k_z^1\ \mathrm{Re}\ k_z^0}{|k_z^1+k_z^0|^2}=\frac{1-R_\perp}{4}, \tag{92}$$

where, bearing in mind that $\mathrm{Re}\ k_z^0$ vanishes for $q>\omega/c$ and defining $q=(\omega/c)\sin\theta=(\omega/c)x$, from (91) and (92) we have[22]

$$S_{z\omega}=-\frac{\hbar\omega^3n(\omega)}{4\pi^2c^2}\int_0^1(1-R_\perp)xdx. \tag{93}$$

For $R_\perp=0$, from (93) we obtain half the Planck flux, which is given by the relation[22]

$$P_\omega=\int_{(\cos\theta<0)}J^0_\omega\cos\theta\sin\theta d\varphi d\theta=-\frac{\hbar\omega^3n(\omega)}{4\pi^2c^2}. \tag{94}$$

The halving of the flux implies that we have considered only one of two possible directions of polarization of the electromagnetic field. Equations (83) and (86) can be used to calculate the energy flux in vacuum (for $z<0$, $z'<0$). It is naturally independent of the coordinate z and has the same value (93).

If $\tilde{N}_2$ and $\tilde{N}_1$ are not equilibrium populations, as for example in the case when the medium contains excitation sources as a result of external particle or radiation beams or

when an electric current flows through the medium, then in place of $n(\omega)$ in Eqs. (91), (93), (94), etc. [see (97)] we must write the expression [cf. (70)]

$$\{(\tilde{N}_1 g_2/g_1\tilde{N}_2)\exp[\hbar(\omega-\omega_0)/T]-1\}^{-1}.$$

If we consider the problem of radiation emerging from a heated medium with the electric vector polarized in the plane of incidence and if we determine the Green's functions D_{yy} and D_{yz} from equations analogous to (82) (Refs. 35 and 36), we obtain an expression for the energy flux analogous to (93), but now with the reflection coefficient $R_\parallel$, which is equal to[20,22]

$$R_\parallel=\left|\frac{k_z^1-\varepsilon(\omega)k_z^0}{k_z^1+\varepsilon(\omega)k_z^0}\right|^2. \tag{95}$$

Introducing the total reflection coefficient

$$R=\frac{R_\parallel+R_\perp}{2}, \tag{96}$$

we obtain the total energy flux into vacuum from waves of both polarizations in the form[22]

$$S_{z\omega}=-\frac{\hbar\omega^3 n(\omega)}{2\pi^2c^2}\int_0^1(1-R)x\,dx. \tag{97}$$

If we introduce the spectral intensity of radiation emitted from a heated medium into vacuum by analogy with (94), we arrive at Kirchhoff's law[22]

$$J_\omega^{ex}(\theta)=J_\omega^0(1-R), \tag{98}$$

which follows in an obvious way purely from energy considerations and, of course, its derivation does not require the apparatus of Green's functions for nonequilibrium processes. Condition (98) implies that the Planck equation holds for the spectral intensity of radiation J_ω^0 in a vacuum cavity in the interior of any set of media existing in thermodynamic equilibrium.

We note that the radiation intensity in the interior of an equilibrium medium is given by Eq. (65). In the case of weak absorption ($\varepsilon''\to 0$) the transition from the expression $J_\omega^{cl}=J_\omega^0 n^2$ (inside the medium) to radiation at the exit from it into vacuum (98) is well known.[22,38]

The loss of the factor n^2 in this case is attributable to the transition from solid angles for a wave propagating in the medium to solid angles for the refracted wave in vacuum and to the existence of an invariant that follows from Snell's law:[20,25,38]

$$n^2\sin\theta\cdot d\sin\theta=\text{const}. \tag{99}$$

Since we have not made any assumption as to the weakness of absorption in the derivation of expressions (97) and (98), the more general law relating the spectral intensity J_ω^{in} of radiation in the medium as determined in (65) to its value upon exit into vacuum [see (98)] has the form

$$J_\omega^{ex}(\theta)=\frac{J_\omega^{in}}{\varepsilon'\psi(\omega)}(1-R)=J_\omega^0(1-R). \tag{100}$$

We now investigate the same problem by means of the equations for the generalized spectral intensity $J(\omega,k,\mu,z)$ (55) and (70). Here we have the notation $\mu=\cos\theta$, and θ is the angle between the vector $\mathbf{k}$ and the z axis.

In the case of a homogeneous medium we obtain from Eq. (55)

$$\begin{aligned} J^-(\omega,k,\mu,z)&=\frac{\tilde{\varepsilon}}{k_\omega}, \quad \mu<0,\\ J^+(\omega,k,\mu,z)&=\frac{\tilde{\varepsilon}}{k_\omega}+B\exp\left(-\frac{k_\omega z}{\mu}\right), \quad \mu>0. \end{aligned} \tag{101}$$

Substituting this solution into the wave equation (70), dropping the constant terms $\tilde{\varepsilon}/k_\omega$ from the right-hand side, and taking the relation between k_ω and ε'' (56) into account, we obtain

$$B=\tilde{b}\,\delta\left[c^2k^2-\omega^2\varepsilon'-\left(\frac{\omega^2\varepsilon''}{2ck\mu}\right)^2\right]. \tag{102}$$

The unknown field $\tilde{b}$ can be found from the boundary condition, which is naturally formulated in the present situation so that the spectral flux of the radiation into vacuum as determined previously in (97) will coincide with the flux obtained from the solution of Eqs. (101) and (102):

$$\begin{aligned} S_{z\omega}&=\int_{(\mu<0)}\mu J_\omega^0(1-R)d\mu d\varphi=\int_{(\mu>0)}\mu J_\omega^+ d\mu d\varphi\\ &\quad+\int_{(\mu<0)}\mu J_\omega^- d\mu d\varphi. \end{aligned} \tag{103}$$

In (103) we have introduced the definition [cf. (30)]

$$J_\omega^\pm(\mu,z)=\frac{2c^2}{\omega(2\pi)^5}\int J^\pm(\omega,k,\mu,z)k^3dk. \tag{104}$$

Making use of the fact that the constants terms $\tilde{\varepsilon}/k_\omega$ in (103) drop out (the isotropic intensity does not generate a radiation flux), at $z=0$ we obtain

$$\begin{aligned} \int_{(\mu<0)}\mu J_\omega^0(1-R)d\mu&=\frac{2c^2}{\omega(2\pi)^5}\int_{(\mu>0)}\tilde{b}\,\delta\left[c^2k^2-\omega^2\varepsilon'\right.\\ &\quad\left.-\left(\frac{\omega^2\varepsilon''}{2ck\mu}\right)^2\right]k^3dk\,\mu d\mu. \end{aligned} \tag{105}$$

Since the reflection coefficient R (96) depends on the absolute value of the cosine of the angle θ, we can require [see (105)] satisfaction of the relation

$$\begin{aligned} \tilde{b}&=-J_\omega^0(1-R)\left\{\frac{2c^2}{\omega(2\pi)^5}\int\delta\left[c^2k^2\right.\right.\\ &\quad\left.\left.-\omega^2\varepsilon'-\left(\frac{\omega^2\varepsilon''}{2ck\mu}\right)^2\right]k^3dk\right\}^{-1}\\ &=-\frac{(2\pi)^3\hbar\omega^4}{\pi c^3k_0^2}(1-R)\\ &\quad\times\left[1+\left(\frac{\omega^2\varepsilon''}{2c^2k_0^2\mu}\right)^2\right]n(\omega). \end{aligned} \tag{106}$$

Here k_0 is determined from the biquadratic equation formed when the argument of the δ-function in expression (106) is equal to zero:

$$k_0=\frac{\omega}{c}\sqrt{\varepsilon'}\sqrt{\frac{1}{2}\left(1+\sqrt{1+\left(\frac{\varepsilon''}{\varepsilon'\mu}\right)^2}\right)}. \tag{107}$$

From these relations we deduce

$$J_\omega^-(\mu,z)=J_\omega^{\text{in}}=J_\omega^0\varepsilon'(\omega)\psi(\omega),$$
$$J_\omega^+(\mu,z)=J_\omega^{\text{in}}\left[1-\frac{1-R}{\varepsilon'(\omega)\psi(\omega)}\exp\left(-\frac{\omega^2\varepsilon''z}{c^2k_0\mu}\right)\right]. \tag{108}$$

Note that if a boundary condition of the type

$$J^+(\omega,k,\mu,0)=RJ^-(\omega,k,\mu,0) \tag{109}$$

is imposed on the solution (101), it follows from (109) that $B=-(\tilde{\varepsilon}/k_\omega)(1-R)$, but then the solution (101) would not satisfy the wave equation (70), and for the intensity of the radiation emitted into vacuum, instead of J_ω^{ex} [see (98)] we would have $J_\omega^{\text{in}}(1-R)$. Analogously, it is evident from (108) that the solution (101), (102), (106), which satisfies the kinetic equation (55) and the wave equation (70) for photons, does not satisfy the boundary condition $J_\omega^+(\mu,0)=RJ_\omega^-(\mu,0)$ by virtue of the factor $\varepsilon'\psi$ in the expression for J_ω^{in}.

In the wings of the line ($\Delta>\Gamma$) we have $\varepsilon'\to1$, $\psi\to1$, $R\to0$, and the resulting solution gives $J_\omega^{\text{ex}}=J_\omega^0$ for the intensity of the emitted radiation. We also obtain $J_\omega^-=J_\omega^0$ and $J_\omega^+=J_\omega^0[1-\exp(-\omega\varepsilon''z/c\mu)]$, i.e., the standard results in the conventional radiation transport theory.

An analysis of the spectral energy fluxes yields correct results without any reliance on the intensity concept,[22] but fails to solve a number of problems that arise in radiation transport theory. For example, in the equations for the populations of excited atoms (72) the photoabsorption rate is not determined by the energy flux $S_{z\omega}$, but by the intensity J or, in the more general case [due to the impracticality of transforming to Fourier components with respect to the difference coordinate in multidomain problems; cf. (85)] expressions of the form[10]

$$\int\tilde{K}(z,z'')D_{ij}^{-+}(z'',z,\omega,\mathbf{q})dz''d\omega d^2q,$$

where $\tilde{K}(z,z'')$ is the corresponding kernel describing the process of nonlocal absorption of radiation near the boundary of a half-space.[10]

The solution of problems in the presence of an inhomogeneous particle distribution in space in the language of kinetic Green's functions $D_{ij}^{-+}(x,x')$ presents major difficulties. The use of the generalized intensity helps to circumvent some of them, because equations of the type (55) and (70) are obtained in the quasiclassical approximation,[5,23] i.e., the characteristic lengths of variation of the physical quantities are assumed to be large in comparison with the photon wavelength. This approximation is satisfactory if the distribution of particles in space is sufficiently smooth (see Refs. 1–4), but generally speaking, one of the characteristic lengths for the radiation intensity itself is the photon mean free path $L\sim1/k_\omega$, which is commensurate with the wavelength for $\varepsilon''\sim1$. Near boundaries, therefore, the solutions found from the transport equations (55) and (70) [see (101) and (108)], strictly speaking, are invalid. This is evident, for example, from expressions of the type (85) and (88), since only at distances $L>1/\text{Im}\,k_z^1$ do the corresponding contributions to functions of the type D^R and D^{-+} coincide with those used in the derivation of Eqs. (55) and (70). We note that the characteristic dependence of the intensity on the distance near a boundary has the form (101), (108), i.e., is described by the exponential

$$\exp\left(-\frac{\omega^2\varepsilon''z}{c^2k_0\mu}\right)=\exp\left\{-\frac{\omega}{c}\varepsilon''z\times\left[\sqrt{\frac{\varepsilon'\mu^2+\sqrt{(\varepsilon'\mu^2)^2+(\varepsilon''\mu^2)^2}}{2}}\right]^{-1}\right\}, \tag{110}$$

where k_0 is defined in (107). If we compare this expression with the exact solution (90), we see right away that the corresponding arguments of the exponentials (110) and (90) do not coincide, because the quantity $2\,\text{Im}\,k_z^1$ can be written in the form [denoting $q=(\omega/c)\sqrt{\varepsilon'(1-\mu^2)}$]

$$2\,\text{Im}\sqrt{\frac{\omega^2}{c^2}(\varepsilon'+i\varepsilon'')-q^2}=\frac{\omega}{c}\varepsilon''\left[\sqrt{\frac{\varepsilon'\mu^2+\sqrt{(\varepsilon'\mu^2)^2+\varepsilon''^2}}{2}}\right]^{-1}. \tag{111}$$

The greatest difference occurs for small values of μ, i.e., at small angles with the interface. At distances greater than L, i.e., for $z\gg1/\text{Im}\,k_z^1$ the energy flux is equal to zero, since the radiation in the volume is essentially isotropic.

If we transform to Fourier components with respect to the difference coordinate for the function $D_{q\omega}^{-+}$ [see (87) and (88)], we may find that the function $iD^{-+}(z,k,q,\omega)$ at distances $z\gg L$ yields (101). On the other hand, the solution (101) is "hemmed" just beneath the true value of the radiation flux from the medium (105), (106). Consequently, the approximate solution of radiation transport problems using the scalar intensity $J(\omega,\mathbf{k})$ with "correct" boundary conditions [of the type (103), (97)] and satisfying two equations simultaneously — the kinetic equation (55) and the wave equation (70) — can be justified both by its comparative simplicity and by the reducibility of this approach to the conventional procedure[1–4] in the limit $\varepsilon''\to0$, $\varepsilon'\to1$.

The solution (101) of Eq. (55) can be generalized to the case of nonequilibrium and inhomogeneous (but sufficiently smooth) distributions of the density of atomic particles in the form

$$J^-(\omega,k,\mu,z)=\int_z^\infty\tilde{\varepsilon}(z')\left[\exp\left(-\int_z^{z'}\frac{k_\omega dz''}{|\mu|}\right)\right]\frac{dz'}{|\mu|},\quad\mu<0,$$
$$J^+(\omega,k,\mu,z)=B\exp\left(-\int_0^z\frac{k_\omega dz'}{\mu}\right)+\left(\frac{\tilde{\varepsilon}}{k_\omega}\right)_0\times\exp\left(-\int_0^z\frac{k_\omega dz'}{\mu}\right)+\int_0^z\tilde{\varepsilon}(z')\times\left[\exp\left(-\int_{z'}^z\frac{k_\omega dz''}{\mu}\right)\right]\frac{dz'}{\mu},\quad\mu>0. \tag{112}$$

We have written the right-hand sides of expressions (112) so that they will go over to (101) in the homogeneous case

($\tilde{\varepsilon}, k_\omega = \text{const}$). In the case of a smooth distribution of $\tilde{\varepsilon}$ and k (e.g., in a nonuniformly heated body) on the scale of the mean free path $L \sim l/k_\omega$ expressions (122) can be approximately written in the form (101) with $\bar{\varepsilon}$ and k_ω depending adiabatically on z [e.g., by integrating (112) by parts and discarding the "leftover" derivatives]. In this form the principal terms of the solution (112) drop out to the same accuracy on the right-hand side of the wave equation (70), and for the function B we obtain an expression analogous to (102).

Calculating the spectral intensities $J_\omega^\pm(\mu,z)$ described by (104) with the use of the solutions (112), we obtain the condition for the radiation flux emanating from the medium (103) at $z=0$, where now instead of (100) in the inhomogeneous case we have

$$J_\omega^{\text{ex}}(\mu) = \frac{J_\omega^-(\mu,0)}{\varepsilon_0'(\omega)\psi_0(\omega)}[1-R_0(\omega,\mu)]. \tag{113}$$

Here the subscript 0 refers to the values of the corresponding quantities at the boundary $z=0$. Inasmuch as $J_\omega^-(\mu,0)$, like $J^-(\omega,k,\mu,0)$, is now determined by an integral of the type (112), for a nonuniformly heated medium the radiation at the exit into vacuum can exceed the Kirchhoff level (98) at the boundary temperature T_0 because of the deeper heated levels, where the intensity can be considerably higher than the Planck value at the local temperature of the layer [since $\psi \gg 1$ holds in a highly absorbing, dense media within the limits of the line profile ($\Delta \leqslant \Gamma$)].

For the quantity $\tilde{b}$ in (102), taking (113) into account, we obtain

$$\langle\delta\rangle\tilde{b} = -\frac{J_\omega^{(-)}(\mu,0)(1-R_0)}{\varepsilon_0'\psi_0} + J_\omega^{(-)}(\mu,0) - \left\langle\frac{\tilde{\varepsilon}}{k_\omega}\right\rangle_0. \tag{114}$$

Here the angle brackets signify integration with respect to k with the weighting factor $[2c^2/\omega(2\pi)^5]k^3$, as defined in (104). An expression of the form $\langle\delta\rangle$ occurs in the denominator of Eq. (106):

$$\langle\delta\rangle = \frac{k_0^2}{\omega(2\pi)^5}\left[1+\left(\frac{\omega^2\varepsilon''}{2c^2k_0^2\mu}\right)^2\right]^{-1}, \tag{115}$$

where k_0 is given by expression (107).

The populations $\tilde{N}_2$ involved in expression for $\bar{\varepsilon}$ given by (57) are nonequilibrium in general. If the radiation intensity $J_\omega^-(\mu,0)$ is close to $\langle\tilde{\varepsilon}/k_\omega\rangle_0$ (as happens if the integral in (112) is calculated asymptotically by parts for $\mu<0$), the last two terms in (114) essentially drop out. This also occurs in the homogeneous case, and the expression for $\tilde{b}$ goes over to (106).

The solution (112) can be written in the form $J = J_s + J_v$, where J_s is the contribution from the surface term:

$$J_s^+ = \left[B+\left(\frac{\tilde{\varepsilon}}{k_\omega}\right)_0\right]\exp\left(-\int_0^z\frac{k_\omega dz'}{\mu}\right), \quad J_s^- = 0, \tag{116}$$

and the subscript v refers to the volume contributions represented by the corresponding integrals in (112). If we substitute this expression into Eq. (72) for the populations $\tilde{N}_2$, the contribution from the volume terms reduces to a generalized integral operator of the Biberman–Holstein type,[28,29] which in the given situation has the form

$$I_1^v = \int_v d\mathbf{r}' K(\mathbf{r},\mathbf{r}')\tilde{N}_2(\mathbf{r}') \tag{117}$$

and in the case $k_\omega = \text{const}$ (throughout space) can be written

$$I_1^v = \frac{4}{3}\frac{d^2}{\hbar}\int_v d\mathbf{r}'\int\frac{d\omega k^2 dk}{\pi}k_\omega\frac{2\omega^4\varepsilon''}{|\omega^2\varepsilon - c^2k^2|^2} \times\exp\left[-\frac{\hbar(\omega-\omega_0)}{T}\right]\varphi(\omega,k)\frac{\exp[-k_\omega|\mathbf{r}-\mathbf{r}'|]}{4\pi|\mathbf{r}-\mathbf{r}'|^2}\tilde{N}_2(\mathbf{r}'). \tag{118}$$

The Biberman–Holstein operator (118) differs from the usual form by the additional integration with respect to k, where the absorption coefficient k_ω, according to (56), depends on k. For $\varepsilon'' \ll 1$ (i.e., either if the Biberman criterion is satisfied or in the wings of the line) a factor of the form $(1/\pi)\omega^2\varepsilon''/(|\omega^2\varepsilon - c^2k^2|^2)$ goes over to $\delta(\omega^2\varepsilon' - c^2k^2)$, and after integration with respect to k expression (118) assumes the form

$$I_1^v = A_0\int_v d\mathbf{r}'\int d\omega\frac{k_\omega^0\varphi(\omega)}{4\pi|\mathbf{r}-\mathbf{r}'|^2}\exp(-k_\omega|\mathbf{r}-\mathbf{r}'|) \times\exp\left[-\frac{\hbar(\omega-\omega_0)}{T}\right]\tilde{N}_2(\mathbf{r}'). \tag{119}$$

Here A_0 is the probability of spontaneous emission in vacuum (1). We note that a factor of the form $\text{Re}\sqrt{\varepsilon} \approx \sqrt{\varepsilon'}$ in the expression for A [see (12), (25), and (36)] cancels out with the analogous factor in the expression for $k_\omega = k_\omega^0/\sqrt{\varepsilon'}$ [see (22)]. For narrow lines ($\hbar\Gamma \ll T$) the expression $\exp[-\hbar(\omega-\omega_0)/T$ can be dropped from (119) and replaced with unity. In this case (119) goes over to the standard expression.[1,28,29]

However, Eq. (72) also contains an added contribution from surface terms. It has a local rather than a global form and can be written explicitly, for example, in the one-dimensional case:

$$I_1^s = \frac{2}{3}\frac{d^2}{\hbar^2}\frac{g_2}{g_1}\int\frac{d\omega d\Omega k^2 dk}{(2\pi)^3}\exp\left(-\frac{k_\omega z}{\mu}\right) \times\left[\tilde{b}\delta(\cdots)+\left(\frac{\tilde{\varepsilon}}{k_\omega}\right)_0\right]\varphi(\omega,k)\tilde{N}_1(z). \tag{120}$$

For brevity we have introduced the abbreviation $\delta(\cdots)$ to denote the corresponding δ-function shown explicitly in (102). The quantity $\tilde{b}$ is given by expressions (114) and (115). Owing to the exponential in (120), the wings of the spectral line ($\Delta \gg \Gamma$) are important in the integral with re-

spect to ω far from the boundary $[L \gg 1/k_\omega(\Delta=0)]$. In the wings we have $R_0 \to 0$ and $\varepsilon_0' \to \psi_0 \to 1$. On the right-hand side of Eq. (114) the first two terms drop out in this case. For the other terms in the brackets of relation (120) we obtain

$$\tilde{b}\,\delta(\cdots)+\left(\frac{\bar{\varepsilon}}{k_\omega}\right)_0 = 8\pi^2\frac{\tilde{N}_2}{\tilde{N}_1}\hbar\omega^2\left\{-\frac{1}{\langle\delta\rangle}\left\langle\frac{\omega^2\varepsilon''}{\pi|\omega^2\varepsilon-c^2k^2|^2}\right\rangle \times\delta\left[\omega^2\varepsilon'+\left(\frac{\omega^2\varepsilon''}{2ck\mu}\right)^2-c^2k^2\right]+\frac{\omega^2\varepsilon''}{\pi|\omega^2\varepsilon-c^2k^2|^2}\right\}. \tag{121}$$

It is readily apparent from (121) that the two terms in the brackets drop out in the wings of the line $(d\Delta \gg \Gamma, \varepsilon'' \ll 1, \varepsilon' \to 1)$. Consequently, the contribution from the surface source in the equations for the populations of excited atoms in optically thick media can be omitted, and the volume contribution goes over, within small error limits, to the Biberman–Holstein integral operator. For the Biberman–Holstein equation the limit of large optical thickness the transition can be accompanied by transition to a large-scale (hydrodynamic) approximation[39] consisting in the integration of the operator (119) with respect to frequencies, provided that asymptotic expressions are used for k_ω^0 and $\varphi(\omega)$ in the wings of the spectral line.

We note that in unbounded space the distribution of populations is not necessarily determined by radiation processes, owing to the confinement of radiation. In this limit there is no contribution from surface terms, and the volume term I_1^v is exactly cancelled by the term I_2 in (72) if we set $\tilde{N}_2 = \text{const}$ in (118). If $\tilde{N}_2(\mathbf{r}')$ is taken outside the integral sign in (118) at $r'=r$, i.e., if the probability of emission of radiation θ from a bounded volume is used,[1] then for $\theta(r)$ we can obtain $I_1^v + I_2 = I_2\theta(r)$,

$$\theta(\mathbf{r}) = \int\frac{d\mathbf{\Omega}}{4\pi}\int\frac{d\omega k^2dk}{\pi}\frac{2\omega^4\varepsilon''\varphi(\omega,k)}{|\omega^2\varepsilon-c^2k^2|^2} \times\exp(-k_\omega|\mathbf{r}-\mathbf{r}_s(\mathbf{\Omega})|)\exp\left[-\frac{\hbar(\omega-\omega_0)}{T}\right] \times\left\{\int\frac{d\omega k^2dk}{\pi}\frac{2\omega^4\varepsilon''}{|\omega^2\varepsilon-c^2k^2|^2} \times\varphi(\omega,k)\exp\left[-\frac{\hbar(\omega-\omega_0)}{T}\right]\right\}^{-1}. \tag{122}$$

Here $\mathbf{r}_s(\mathbf{\Omega})$ denotes the radius vector of a point on the surface (bounding the volume occupied by the medium), observed from the point of emission $\mathbf{r}$ in the direction $\mathbf{\Omega}$. In an optically thin medium $(k_\omega r \ll 1)$ θ becomes equal to unity. In disperse media $(\varepsilon' \to 1, \varepsilon'' \ll 1)$ and for narrow spectral lines $(\hbar\Gamma \ll T)$ we obtain the well-known expression for $\theta(\mathbf{r})$ (Ref. 1) from (122)):

$$\theta(\mathbf{r}) = \int\frac{d\Omega}{4\pi}\int d\omega\varphi(\omega)\exp[-k_\omega^0|\mathbf{r}-\mathbf{r}_s(\mathbf{\Omega})|]. \tag{123}$$

Here $\varphi(\omega)$ is the line profile, normalized to unity, whose derivation requires that $\hbar\varepsilon$ be neglected in comparison with T in (59), that contributions $\sim N_n\lambda_T^3$ be omitted in the denominator, and that the wave number k be replaced by ω/c. Expression (122) goes over to (123) in the wings of the spectral line.

In problems of the propagation of a transient radiation pulse, as in problems where spatial dispersion is significant, it is necessary to abandon approximations that lead to equations of the type (43) and (70) (the quasiclassical approximation[5,23]) and to work directly with the Dyson equations, in which the polarization operators $\Pi^{\alpha\alpha'}(x_1,x_2)$ incorporate delay and nonlocality effects. For quasistationary problems and for $\Gamma > kv_T$ the above equations (55), (70) (the time derivatives need to be discarded from the latter), and (72) generalize the standard transport equations to the case of dense, dispersive media and can be used to solve a wide range of physical significant problems.

6. NUMERICAL MODELING OF RESONANCE RADIATION TRANSPORT IN HIGHLY ABSORBING MEDIA

To illustrate the above transport theory for resonance radiation, we carry out numerical calculations in the case of the resonance doublet of the sodium atom: the $3s_{1/2}-3p_{1/2}$ transition, $\lambda_1 = 5895.92$ Å (D_1 line); the $3s_{1/2}-3p_{3/2}$ transition, $\lambda_1 = 5889.95$ Å (D_2 line). The atomic constants for these transitions are well known. The oscillator strengths (in absorption) are $f_1 = 0.327$ and $f_2 = 0.653$ (the subscripts 1 and 2 refer everywhere to the D_1 and D_2 lines, respectively), and the spontaneous emission probability is $A = 6.25\times 010^7$ s^{-1} (see, e.g., Ref. 40). The collisional widths under our stated conditions are related to the resonance transfer of excitation and, according to published data,[41] are equal to $\nu_1 = 4.68\times10^{-8}N$ and $\nu_2 = 7.62\times10^{-8}N$, where N [cm^{-3}] is the density of sodium atoms, and ν_i are expressed in hertz.

The temperature dependence of the density N of the sodium vapor is determined from data in Ref. 42. The collisional width dominates the radiation width beginning with densities $N \approx 10^{14}$ cm^{-3} ($T > 550$ K). The Doppler width k_0v_T in the investigated temperature interval is $k_0v_T \sim 10^{11}$ s^{-1} (corresponding to $\Delta\lambda \sim 10^{-2}$ Å) and is greater than the collisional width up to densities $N \sim 10^{17}$ cm^{-3} ($T \sim 800$ K).

Since we are interested in the qualitative aspects of the problem, we restrict the numerical analysis to an approximation in which fine structure is ignored, and the main calculations are carried out for the sodium D_2 line. This approximation becomes formally valid at high densities (for which the most interesting phenomena are observed), when the collisional width Γ exceeds the fine structure of the resonance doublet $\Delta\lambda \approx 6$ Å (this is the case at a vapor temperature $T > 1100$ K).

In the equations for the spontaneous emission probability (35) and the spectral intensity of the radiation (31) we distinguish the integrals

$$I_n = \frac{2}{\pi}\int_0^\infty\frac{\varepsilon''\xi^n}{(\varepsilon'-\xi^2)^2+(\varepsilon'')^2}d\xi. \tag{124}$$

The subscript $n=2$ corresponds to A, and $n=3$ corresponds to J_ω. We have introduced the dimensionless variable $\xi = kc/\omega$ in the integrand of (124). If the permittivity ε does

not depend on the wave number k, the integral I_2 corresponding to the spontaneous emission probability A is computed exactly [cf. (36)]:

$$I_2 = \mathrm{Re}\ \sqrt{\varepsilon}. \tag{125}$$

The integral I_3 corresponding to the intensity J_ω diverges in this approximation, but for a "narrow" line such that the δ-function

$$\delta(\varepsilon' - \xi^2) \approx \frac{1}{\pi} \frac{\varepsilon''}{(\varepsilon' - \xi^2)^2 + (\varepsilon'')^2}, \tag{126}$$

can be distinguished in the integrand, the integral I_3 is computed approximately [cf. (64)]:

$$I_3 \approx \varepsilon'. \tag{127}$$

For I_2 in this approximation we have [cf. (25)]

$$I_2 \approx \sqrt{\varepsilon'}. \tag{128}$$

The results of numerical calculations of the integrals I_2 and I_3 are shown in Fig. 1. It is evident from Fig. 1a that I_2 differs little from unity at low densities $N < 10^{15}$ cm^{-3} ($T < 600$ K), for which the spontaneous emission probability is therefore close to its value in vacuum. At high densities $N > 10^{17}$ cm^{-3} ($T > 800$ K) the integral I_2 differs considerably (by $\sim 30\%$) from unity for a frequency separation (from resonance) $\Delta \approx \pm \gamma/2$, tending to a limit (saturating) as the density increases. Consequently, at high pressures the line profile for frequency separations of the order of the collisional width Γ can differ locally by an appreciable amount from the vacuum value. This effect disappears in the wings of the line. The amount of deviation of the integral I_2 computed by the exact integration of (124) for $n = 2$ from the approximate value (125) is shown in Fig. 1b.

The results of calculations of the equilibrium spectral intensity J_ω [integral I_3, Eq. (124)] are shown in Fig. 1c. At low densities ($N < 10^{15}$ cm^{-3}) we have $I_3 \approx 1$ at all frequencies. As N increases, I_3 increases at the center of the line, attaining a value $I_3 \approx 10$ for $N \sim 10^{20}$ cm^{-3}. Consequently, the local value of the intensity J_ω in the spectral range $|\Delta| \leqslant \Gamma$ can differ substantially (by an order of magnitude) from the Planck value.

We also perform numerical calculations of the integral operator (118) [evaluating the integrals with resect to k in (118)] with a view toward comparing its kernel with the kernel of the Biberman–Holstein operator (119). It follows from these calculations that for frequency separations greater than the linewidth Γ and for virtually any optical thickness the two kernels of the integral operators agree within very small error limits. If we bear in mind that the main contribution in the integrals with respect to ω (118) and (119) for large thicknesses is from the far wings of the line ($\Delta \gg \Gamma$), we find that these calculations provide an opportunity for greatly simplifying the problem of the spatial distribution of populations of excited atoms by solving, instead of Eq. (72), the Biberman–Holstein equations[1,4,30] or the simpler Abramov–Dykhne–Napartovich integral equations in the large-scale approximation.[39]

Figure 2 shows the results of calculations of the spectral intensity J_ω according to (30), based on the numerical solu-

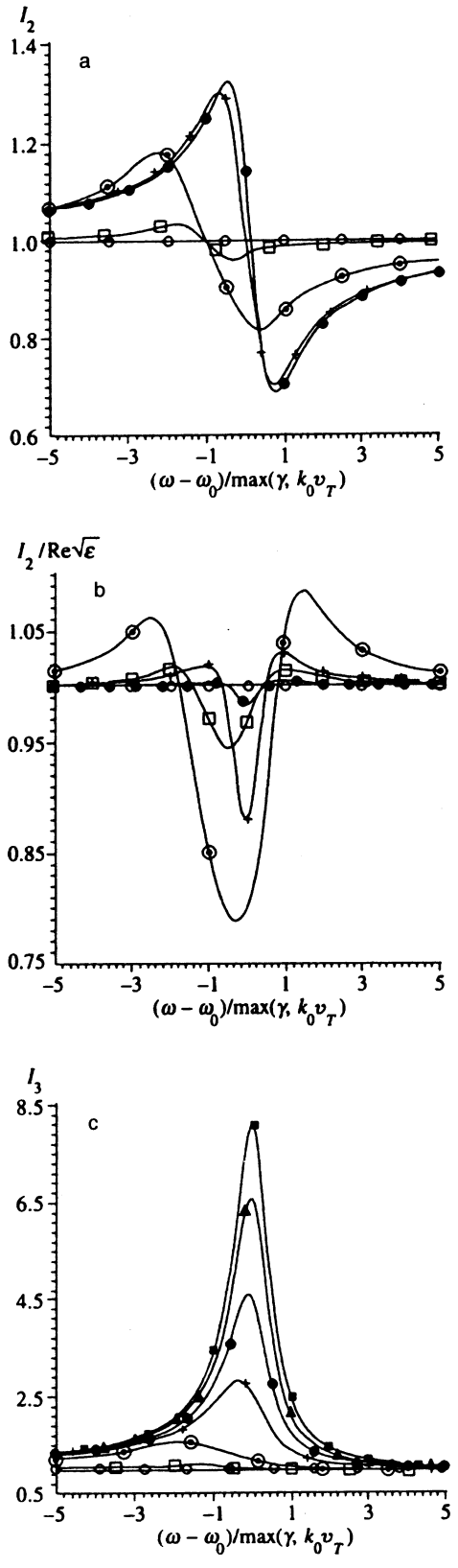

FIG. 1. a) Ratio of the spontaneous emission probability to its vacuum value, $I_2 = A(\omega)/A_0$ (124); b) ratio $I_2/\mathrm{Re}\sqrt{\varepsilon}$ (125); c) ratio of the spectral intensity to its Planck value, $I_3 = J_\omega/J_\omega^0$ (124). Sodium line $3s_{1/2}-3p_{3/2}$. Legend: ○) $T = 500$ K, $N = 1.8\times10^{13}$ cm^{-3}; □) $T = 610$ K, $N = 1.1\times10^{15}$ cm^{-3}; ⊙) $T = 700$ K, $N = 1.2\times10^{16}$ cm^{-3}; ×) $T = 825$ K, $N = 1.4\times10^{17}$ cm^{-3}; ●) $T = 1000$ K, $N = 1.5\times10^{18}$ cm^{-3}; ▲) $T = 1300$ K, $N = 1.8\times10^{19}$ cm^{-3}; $T = 1700$ K, $N = 1.2\times10^{20}$ cm^{-3}.

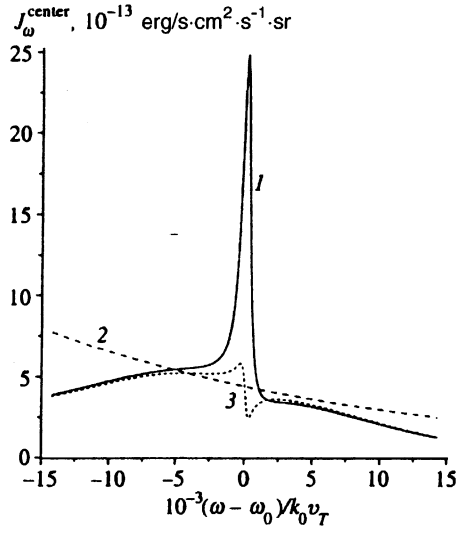

FIG. 2. Radiation profile at the center of a plane layer (0.1 cm). Parabolic temperature profile ($T=600-1200$ K). *1*) Solution of Eqs. (55), (70), (72), and (114); *2*) Planck intensity; *3*) approximation (63). The abscissa represents the ratio of the frequency separation to the Doppler width at $T=600$ K.

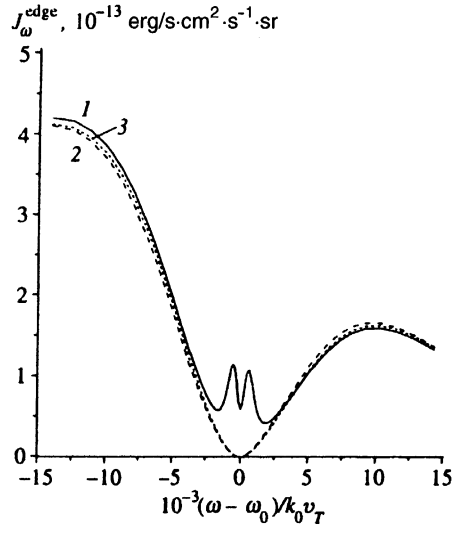

FIG. 3. Radiation emitted from a plane layer (0.1 cm). Parabolic temperature profile ($T=600-1200$ K). *1*) Solution of Eqs. (55), (70), (72), and (114); *2*) classical theory; *3*) approximation (63).

tion of the combined system of equations (55), (70), (72) subject to the boundary condition (114) for a homogeneous, heated, plane layer of sodium vapor. Clearly, the intensity J_ω^{center} ($\mu=0$) at the center of the layer (curve *1*) corresponds to the equilibrium intensity in an absorbing medium at $T=T^{center}$ and exhibits good agreement with the calculations represented in Fig. 1c and obtained from the approximate analytical equations (65) and (66). Also shown in this figure for comparison is the Planck intensity J_ω^0 (curve *2*) and the intensity obtained by the numerical solution of Eq. (63) (curve *3*). At the center of the layer the radiation intensity is practically isotropic and is more than three times the Planck value. The approximation (63) is inapplicable under these conditions.

Figure 3 shows the results of calculations of the intensity J_ω^{edge} of radiation emitted from the layer along the normal to the boundary. We see that in the vicinity of the center of the line the dip in the intensity curve is not as deep as that obtained by solving the equation of standard radiation transport theory, even in spite of self-reversal effects. The intensity of the emitted radiation near the center of the line is more than seven orders of magnitude greater than the value obtained from the conventional theory, and it has a typical double-humped structure mirroring the specific nature of radiation emanating from warmer layers, where the radiation intensity exceeds the Planck level at the corresponding local temperature. We note that the vapor density at the cold boundary of the layer is rather small, and phenomena associated with boundary reflection do not play a significant part ($R\approx 1$). In the wings of the line and in the vicinity of the intensity maximum the results obtained from the solution of the generalized theory and by numerical integration of both Eq. (63) and the standard transport equations virtually coincide. The asymmetry in the intensity of radiation emitted from the layer at the principal maxima in Fig. 3 is attributable to the inclusion of the factor $\exp(-\hbar\Delta/T)$ in Eqs. (55) and (57), and its experimental observation is important for testing the validity of the earlier theory[10,27] and the theory set forth here. The local thermodynamic equilibrium approximation is quite accurately satisfied under the given conditions for calculations of the populations of excited atoms. The calculations of the intensity of radiation emitted from a homogeneous heated layer are in good agreement with Eq. (100). The comparatively high residual intensity obtained from these calculations for radiation emitted from an inhomogeneous heated layer with its characteristic spectral dependence should be an interesting effect to observe experimentally in regard to verifying the validity of the theory. The appreciable excess of the intensity in the interior of the layer over the Planck level could be confirmed by injecting the interior of the medium with a small amount of impurities for which the frequency of transition from its ground state exhibits accidental coincidence with the resonance transition frequency in Na and for which the optical thickness of the layer is small in comparison with unity, i.e., the cumulative radiation in the layer could be used for resonant impurity pumping.

Here we call attention to the complexity of the numerical solution of the combined system of equations (55), (70), (72) with the boundary condition (114). The problem is that the kernels of the integrals in (72) contain sharply varying functions with properties close to those of a delta function. To correctly include the integral terms, we use nonequilibrium grids with allowance for the singularities of the integrand. For example, in the calculations for Figs. 2 and 3 the number of nodes of the k-grid is roughly 400 at the center of the line and more than 16000 in the wings. The computing time for one set of input data on a computer using a Pentium 100 processor is 40–80 min.

In closing, we reiterate that the transport equations obtained in the present study for resonance radiation in dense, dispersive media are approximate. The application of the scalar radiation intensity in inhomogeneous layered media

can be inadequate if polarization effects are at issue. Under these conditions it is necessary to analyze Green's functions $D_{ij}^{-+}(\omega,z,z')$ of the type (86) and (82) directly, which makes it significantly harder to get final results. Since this theory predicts new qualitative results, such as the considerable excess of the spectral intensity J_ω in the interior of a highly absorbing medium over the Planck level J_ω^0 and the high residual intensity level of radiation emitted from an inhomogeneous heated layer, the most acceptable verification of the validity of the new theory would be the experimental corroboration of these effects.

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