Microlensing by neutralino stars

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(Submitted 20 May 1996)
Zh. Éksp. Teor. Fiz. **110**, 1921–1932 (December 1996)

The authors investigate the microlensing of background stars by neutralino stars, which have recently been proposed by Gurevich and Zybina [Phys. Lett. A **208**, 276 (1995)] as a candidate for the role of missing mass. The optics of such a gravitational microlens is analyzed in detail, specifically the lens equation, its solutions, image amplification, and the critical and caustic curves. © *1996 American Institute of Physics.*[S1063-7761(96)00112-6]

1. INTRODUCTION

The first results of microlensing observations, reported in publications by three groups,[1–3] disclosed a new phenomenon predicted earlier by Paczinsky[4] and Griest.[5] It should be noted that microlensing had been discussed previously by several different authors.[6–9] The nature of the gravitational microlenses is so far unknown, but the most commonly accepted hypothesis characterizes them as compact, nonluminous bodies of the brown dwarf type, although other possibilities are not dismissed, including the familiar red dwarf stars.[10] Nevertheless, bodies of an altogether different nature are not ruled out; in particular, the existence of dark-matter bodies consisting of supersymmetric, weakly interacting particles (neutralinos) has been discussed in recent papers.[11,12] The authors have shown that such stars can be formed in the early stages of evolution of the Universe and remain stable over cosmological times.

In the present article we investigate the microlensing of a star behind a gravitational lens; in the form of a neutralino star.

We consider the microlensing of the star in a rather crude model, which is nonetheless simple and enables us to obtain final results in analytical form. A more refined model of the influence of the gravitational field of a neutralino star can certainly be treated,[12] but we can still expect to correctly obtain a qualitative assessment of the phenomenon in question. The model presented below utilizes the geometrical optics approximation, and the effects associated with the diffraction or mutual interference of images and analyzed in Refs. 13–18 and 19 are disregarded.

2. BASIC ASSUMPTIONS AND RELATIONS

We approximate the density of the mass distribution of the neutralino star in the form

$$\rho_{NeS}(r)=\rho_0\frac{a_0^2}{r^2}, \tag{1}$$

where r is the instantaneous distance from the center of the star, ρ_0 is the bulk density of the neutralino star at the distance a_0 from the center, and a_0 is the "radius" of the neutralino star. Our investigated density function is an approximation of the function used in the work of Gurevich and Zybin:[11]

$$\rho_{NeS}(r)=Kr^{-1.8}.$$

Here K is a normalization factor with units chosen to preserve the dimensionality of the equation.

Thus, the surface mass density $\Sigma(\xi)$ is readily calculated on the basis of relation (1):

$$\Sigma(\xi)=2\rho_0\int_0^{\sqrt{a_0^2-\xi^2}}\frac{a_0^2}{\xi^2+h^2}dh=2\rho_0\frac{a_0^2}{\xi}\arctan\frac{\sqrt{a_0^2-\xi^2}}{\xi}. \tag{2}$$

For $a_0 \gg \xi$ the surface density satisfies $\Sigma(\xi)\rightarrow \pi\rho_0 a_0^2/\xi$.

In this case the lens equation, which describes the deflection of a light ray through the angle $\bar{\alpha}_{Nes}(\xi)$ as it passes through the gravitational lens, has the form

$$\boldsymbol{\eta}=\frac{D_s}{D_d}\boldsymbol{\xi}-D_{ds}\hat{\boldsymbol{\alpha}}_{Nes}(\boldsymbol{\xi}). \tag{3}$$

Here D_s is the distance from the source to the observer, D_d is the distance from the gravitational lens (the galaxy) to the observer, D_{ds} is the distance from the source to the gravitational lens, the vectors $\boldsymbol{\eta}$ and $\boldsymbol{\xi}$ characterize the true position of the light source in its plane and the positions of the images in the plane of the lens, respectively, and

$$\hat{\boldsymbol{\alpha}}_{NeS}(\boldsymbol{\xi})=\int_{R^2}d^2\xi'\frac{4G\Sigma(\boldsymbol{\xi}')}{c^2}\frac{\boldsymbol{\xi}-\boldsymbol{\xi}'}{|\boldsymbol{\xi}-\boldsymbol{\xi}'|^2}, \tag{4}$$

where G is the gravitational constant, c is the speed of light, and $\boldsymbol{\xi}'$ is the vector variable of integration over the R^2 plane. We note that the density-distance relation described by Eq. (1) has two major shortcomings: 1) A singularity occurs at $r = 0$ (the infinite density of the neutralino star at $r = 0$ is most likely attributable to some inauspicious aspect of the model), although it is readily apparent that the mass does not become infinite in this case; 2) the neutralino star has infinite mass when the expressions for the density are analyzed up to infinitely large a_0. In the investigation of the gravitational lens effect, however, it is not affected by the mass concentrated at $\xi'>\xi$.

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To reduce the lens equation (3) to dimensionless form, we use the characteristic radius a_0 corresponding to the "mass" of the microlens, i.e.,

$$M=4\pi\rho_0 a_0^3. \tag{5}$$

We introduce the dimensionless variables

$$x=\frac{\xi}{a_0},\quad y=\frac{\eta}{\eta_0},$$

where

$$\eta_0=a_0\frac{D_s}{D_d},$$

whereupon

$$\hat{\alpha}(\mathbf{x})=\frac{1}{\pi}\int_{R^2}d^2x' k(\mathbf{x}')\frac{\mathbf{x}-\mathbf{x}'}{|\mathbf{x}-\mathbf{x}'|^2},$$

$$k(\mathbf{x})=\frac{\Sigma(a_0\mathbf{x})}{\Sigma_{\rm cr}},\quad \Sigma_{\rm cr}=\frac{c^2D_s}{4\pi G D_d D_{ds}}.$$

We write the surface density function of the mass of the neutralino star in the form

$$\Sigma(\xi)=\pi\rho_0\frac{a_0^2}{\xi}. \tag{6}$$

Assuming that the surface density is an axisymmetric function, we can write the gravitational lens equation in the scalar form[20]

$$y=x-\alpha(x)=x-\frac{m(x)}{x}, \tag{7}$$

where

$$m(x)=2\int_0^x x'dx'k(x').$$

We recall that the function $k(x)$ has the form

$$k(x)=\frac{k_0}{x}, \tag{8}$$

where

$$k_0=\frac{\pi\rho_0 a_0}{\Sigma_{\rm cr}}=\frac{M}{a_0^2}\frac{4\pi G D}{c^2}, \tag{9}$$

$$D=\frac{D_d D_{ds}}{D_s}. \tag{10}$$

Consequently, for the lens equation we obtain

$$y=x-R_0\frac{x}{|x|}, \tag{11}$$

where $R_0 = 2k_0$. We normalize distances in the plane of the lens and the source to R_0, i.e., we introduce the variables $\hat{y} = y/R_0$ and $\hat{x} = x/R_0$, whereupon the lens equation acquires the simple form

$$\hat{y}=\hat{x}-\frac{\hat{x}}{|\hat{x}|}. \tag{12}$$

We drop the circumflex "^" everywhere from now on. It is readily apparent that the dimensionless form of the lens equation coincides with the analogous equation for the galactic mass distribution model corresponding to an isothermal sphere.[20] The gravitational lens equation corresponding to the isothermal sphere model is discussed in some detail in a survey by Refsdal and Surdej.[22]

We now give certain results that follow from the solutions of the lens equation (12). Without sacrificing generality, we can assume that $y > 0$. In the case $y < 0$, the lens equation has two solutions $x_+ = y + 1$ and $x_- = y - 1$. Only the one root $x_+ = y + 1$ exists in the case $y > 1$. Figure 1 shows a circular source and its image for various distances between the center of the source and the center of the gravitational lens. It is important to note that the images in the radial direction for a noncompact lens are thicker than in the case of a Schwarzschild lens (roughly twice as thick for small values of y). Images of a circular source distorted by a Schwarzschild lens can be found in Ref. 21.

According to Schneider *et al.*,[20] the amplification of the gravitational lens is defined as the reciprocal of the Jacobian of the transformation described by the gravitational lens equation; specifically, if

$$A(\mathbf{x})=\frac{\partial\mathbf{y}}{\partial\mathbf{x}} \tag{13}$$

or

$$A_{ij}=\frac{\partial y_i}{\partial x_j}, \tag{14}$$

then the amplification is given by the relation

$$\mu(x)=\frac{1}{\det A(x)}. \tag{15}$$

In the case of a symmetric mass distribution the Jacobian obeys the relation

$$\det A(x)=1-\frac{1}{|x|}, \tag{16}$$

so the amplification is equal to

$$\mu=\frac{|x|}{|x|-1}. \tag{17}$$

In this case, clearly, the critical curve (along which the matrix A vanishes at all points of the curve) has the equation $|x| = 1$ (i.e., it is the unit circle). We recall that critical curves which are circles are called tangential.[20] The caustic curve (the transform of the critical curve under the mapping described by the gravitational lens equation) becomes degenerate at the point $y=1$ in this situation. It is now evident how the sources are distorted by a gravitational lens. Clearly, the images are not distorted in the radial direction, but in the tangential direction they are distended in accordance with relation (17). For a Schwarzschild lens in the limit $y \ll 1$ the image is compressed by a factor of two in the radial direction and is similarly distended by a factor $\approx 1/y$ in the tangential

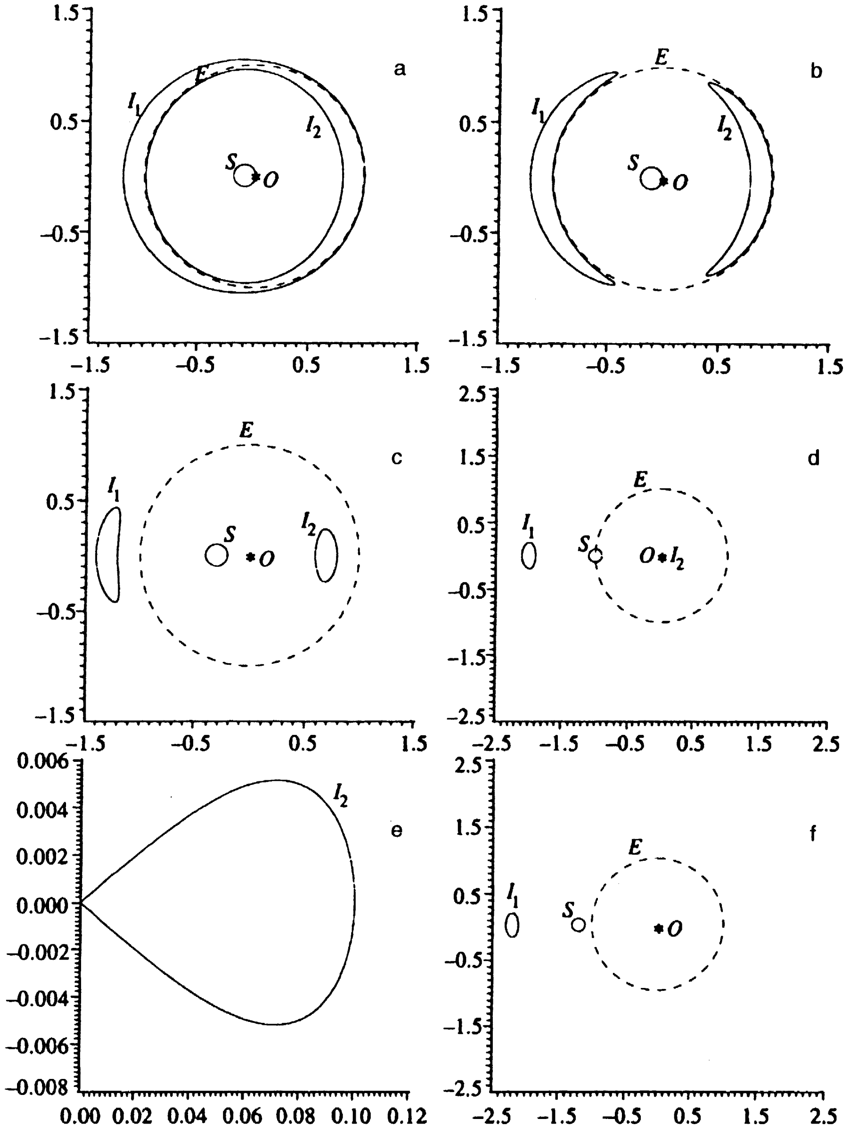

FIG. 1. Boundary of a circular source S with two images I_1 and I_2. The dashed line represents the critical curve, i.e., the Einstein–Hvolson circle. The radius of the source is $r=0.1$. a) Distance between the projection of the center of the source and the projection of the center of the gravitational lens $d=0.09$; b) 0.11; c) 0.3; d) 1; f) 1.2; e) enlarged image I_2 from Fig. 1d; in Fig. 1f the second image (I_2) vanishes. The scales of the coordinate axes give the dimensionless coordinates in the plane of the source (for the image) and in the plane of the gravitational lens (for images).

direction.[20,21] This assertion is easily verified on the basis of simple geometrical considerations. Looking at the case $y > 1$, we have

$$\mu(y)=\mu(x_+)=\frac{y+1}{y}=1+\frac{1}{y}. \tag{18}$$

For the case $0 < y < 1$ we have

$$\mu(x_+)=\frac{|x_+|}{|x_+|-1}=\frac{y+1}{y}=1+\frac{1}{y}, \tag{19}$$

$$\mu(x_-)=\frac{|x_-|}{|x_-|-1}=\frac{y-1}{y}. \tag{20}$$

Since we have $\mu(x_-) < 0$, the total amplification with allowance for the amplifications of the two images is given by the relation

$$\mu(y)=\mu(x_+)+|\mu(x_-)|=\frac{2}{y}. \tag{21}$$

When the gravitational microlens is a point gravitating body (i.e., a Schwarzschild lens), the amplification can be written in the form[20,21]

$$\mu(y)=\frac{y^2+2}{y\sqrt{y^2+4}}. \tag{22}$$

Consequently, the difference between the amplifications of the Schwarzschild lens and the neutralino star is a significant factor in distinguishing between these two objects.

We now consider the two asymptotic limits corresponding to small and large values of y, so as to graphically illustrate the difference between the amplifications in the two cases. The asymptotic forms of $\mu(y)$ for neutralino stars are given by relations (18) and (21). From Eq. (22) we obtain analogous relations for the amplifications of compact bodies:

$$\mu(y)=\frac{1}{y}, \quad y\ll 1, \tag{23}$$

$$\mu(y)=1+\frac{2}{y^4}, \quad y\gg 1. \tag{24}$$

We analyze separately the asymptotic behavior of the amplifications of neutralino stars and compact bodies. In the case $y<1$ [see relations (18) and (21)] and also in the case $y>1$ [relations (21) and (24)] the brightness curve, i.e., the time dependence of the observed luminance of the background star, differs for neutralino stars and compact bodies. In the analysis of the brightness curves for these two objects, when y (which is proportional to the minimum angular diameter distance of the microlensing event) is identical for the neutralino star and the compact body, it is evident that the brightness curve has a higher maximum and broader wings in the first case, thereby providing a test for the discrimination of these objects.

It is also evident from these asymptotic forms that the difference in the microlensing is particularly conspicuous at amplifications close to unity.

In the next section we discuss the possibilities of identifying these two objects on the basis of observational data and the difference in the dependence of the amplifications on the angular diameter distances.

3. PROBABILITY OF MICROLENSING FOR STARS COMPOSED OF NEUTRALINOS

We now consider the probability of microlensing in the case of neutralino stars. We have previously derived equations[5] for this purpose. We choose the distribution of neutralino stars as in Ref. 5, but with the awareness that their distribution near the galactic center can differ considerably from the distribution of ordinary stars. We therefore approximate the probability density function of the mass distribution in the Galaxy by the function[5,23,24]

$$\rho_G(r)=\rho_0\frac{r_c^2+r_0^2}{r^2+r_0^2}, \tag{25}$$

where r_c is the distance from the galactic center to the Sun (r_c=8.5 kpc), and ρ_0 is the bulk density of galactic matter in the vicinity of the Sun ($\rho_0=0.008 M\odot \mathrm{pc}^{-3}$). The parameter r_0 varies in the interval from 2 kps to 8 kpc.

The microlensing optical thickness in the direction defined by the galactic coordinates b (galactic latitude) and l (galactic longitude) depends on the basic parameters of the distribution as

$$\tau=\tau_0 y^2\left\{-x_h+\frac{1}{2}(1+B)\ln\frac{a+Bx_h+x_h^2}{a}-\frac{B+B^2-2a}{q}\times\left[\arctan\left(\frac{B+x_h}{q}\right)-\arctan\frac{B}{q}\right]\right\}. \tag{26}$$

Here $q=\sqrt{4a-B^2}$, $a=(r_c^2+r_0^2)/L^2$, and $B=-2(r_0/L)\cos b\cos l$ are quantities determined by the structure of the Galaxy and the distribution of stars in it, L is the distance between the observer and the background star, and x_h can be set equal to unity in most cases. Note that the microlensing optical thickness does not depend on the total mass of dark-matter bodies, but depends only on the parameter y, which determines the amplification at the maximum.

To determine the difference in the optical thicknesses (or in the microlensing probabilities) in lensing by compact bodies and neutralino stars, we calculate the amplification dependence of y. In the case of neutralino stars it follows from (18) that the dependence of y on μ has the form,

$$y=\frac{1}{\mu-1}, \quad y>1, \tag{27}$$

and in the case of compact bodies [see (25)] we have

$$y=\sqrt{2[\mu(\mu^2-1)^{-1/2}-1]}. \tag{28}$$

If μ differs only slightly from unity, the latter equation has the form

$$y=\left(\frac{2}{\mu-1}\right)^{1/4}, \quad y>1. \tag{29}$$

We can now determine the ratio of the low-amplification microlensing probability τ_{NeS} for neutralino stars to the microlensing probability τ_{BD} for compact bodies:

$$\frac{\tau_{NeS}}{\tau_{BD}}=\frac{1}{\sqrt{2}(\mu-1)^{3/2}}, \quad y>1. \tag{30}$$

It is important to note here that the grazing distance (x_+) of the background star is less than the radius of the neutralino star in the given equation. It is evident from relation (30) that for low amplifications the probability of microlensing by neutralino stars is much higher than the probability of microlensing by compact bodies. In the case $\mu-1\sim0.1$, for example, the probabilities differ by a factor $\sim$20.

Microlensing is determined not only by the probability, but also by the average frequency of events and the average time of a single event. We now calculate these characteristics for neutralino stars.

Griest[5] denotes the average frequency of events by Γ:

$$\Gamma=yP_G, \tag{31}$$

where P_G is a quantity that depends only on the parameters of the Galaxy and the distribution of dark matter in it. Analogously, the average single-event time can be written in the form[5]

$$t_e=yP'_G, \tag{32}$$

where P'_G is also determined entirely by the parameters of the Galaxy.

Substituting the expressions for y into Eqs. (31) and (32), we find, as should be expected, that the event frequency and the single-event time increases approximately by the factor $1/[2(\mu-1)^3]^{1/4}$ in the case of microlensing by neutralino stars at low amplifications.

Observations of the microlensing effect at low amplifications can therefore serve as a critical test to discriminate between events at neutralino stars and events at compact bodies.

4. DIFFERENCE BETWEEN THE BRIGHTNESS CURVE OF A NEUTRALINO STAR AND THE BRIGHTNESS CURVE OF A COMPACT STAR

Here we discuss the form exhibited by the brightness curve in observations when the gravitational microlens is a neutralino star and how this brightness curve must differ from the brightness curve when the microlens is a compact body.

Clearly, the maximum variation of the luminosity due to microlensing is given by the relation for neutralino stars [see (18)]

$$\Delta\mu_{\max}(y)=\mu_{\max}-1=\frac{1}{y_{\min}}, \tag{33}$$

where $\mu_{\max}$ is the maximum amplification, and $y_{\min}$ is the dimensionless form of the minimum deflection (corresponding to this amplification) of the source from the straight line drawn through the microlens and the observer. The instantaneous value $\Delta\mu_T$ is then smaller than $\Delta\mu_{\max}$, or

$$\Delta\mu_T=\Delta\mu_{\max}\sin\phi=\frac{\sin\phi}{y_{\min}} \tag{34}$$

(ϕ is a certain auxiliary angle). Consequently, it is readily inferred from an inspection of the right triangle with legs $y_{\min}$ and $L/2$ that the distance traversed by the source (in dimensionless variables in the plane of the source) and corresponding to $\Delta\mu>\Delta\mu_T$ is equal to $L=2y_{\min}\cot\phi$, i.e., this distance is a linear function of $y_{\min}$. This fact can be disclosed, in principle, by analyzing the brightness curve. For example, if (dropping the subscript T from the variable $\Delta\mu$) we choose the angle ϕ_1 so that $\Delta\mu_1=\sqrt{2}/2y_{\min}$ (for $\phi_1=\pi/4$), then $L_1=2y_{\min}$, but if we choose the angle ϕ_2 so that $\Delta\mu_2=1/2y_{\min}$ (for $\phi_2=\pi/6$), then $L_2=2\sqrt{3}y_{\min}$. Consequently, if we denote by Δt_1 the time interval in which the visible luminosity of the background star corresponds to amplifications $\Delta\mu\geqslant\Delta\mu_1$, and by Δt_2 the time interval in which the visible luminosity of the background star corresponds to amplifications $\Delta\mu\geqslant\Delta\mu_2$, then the ratio of these time intervals is equal to the ratio of the corresponding distances, i.e., $\Delta t_2/\Delta t_1=L_2L_1=\sqrt{3}$.

If the microlens is a compact body and the amplification of the gravitational microlens is sufficiently small, it follows from (24) that

$$\Delta\mu_{\max}=\mu_{\max}-1=\frac{2}{y_{\min}^4},$$

but if we introduce the notation

$$\delta\mu_{\max}=\left(\frac{\Delta\mu_{\max}}{2}\right)^{1/4},$$

the arguments set forth in this section can be repeated for the case of a Schwarzschild gravitational microlens with the symbol Δ replaced by δ, since $\delta\mu_{\max}=1/y_{\min}$.

We have thus described how the difference between the brightness curves for neutralino stars and compact bodies can be used, in principle at least, to discriminate these two microlenses on the basis of an analysis of the brightness curve. It is important to note that the same arguments preserve their validity only when the grazing distance (x_+) is smaller than the radius of the neutralino star. It should therefore be noted that the discussion of this section is valid only upon satisfaction of the inequality

$$\frac{1}{R_0}\geqslant 2, \tag{35}$$

and the inequality

$$1\geqslant y\geqslant\frac{1}{R_0}-1, \tag{36}$$

so that the gravitational lens equation has only one solution, which corresponds to an angular diameter distance smaller than the radius of the neutralino star.

5. DISCUSSION AND CONCLUSIONS

We have shown how the brightness curve of a neutralino star can differ from that of a Schwarzschild microlens. It is important to note that throughout the article the values of the grazing distances corresponding to the solutions of the gravitational lens equation have been assumed to be such that these values are smaller than the radius of the neutralino star. Consequently, there are certain restrictions on the parameters of the problem. Specifically, for values of the parameters of a neutralino star $a_0=10^{14}$ cm and $M=5\times10^{-2}M_\odot$ we have $R_0=0.23$, so that inequality (35) is satisfied, and the second inequality (36) can be satisfied. In the case $y>1/R_0-1$ the gravitational lens equation has only one solution, which coincides with the solution of the Schwarzschild lens equation

$$x_+^S=\frac{y+\sqrt{y^2+4/R_0}}{2}.$$

This amplification corresponding to this solution is

$$\mu_+^S=\frac{1}{4}\left(\frac{y}{\sqrt{y^2+4/R_0}}+\frac{\sqrt{y^2+4/R_0}}{y}+2\right).$$

We call attention to the fact that the dependence of the amplification on the grazing distance for a neutralino star differs from the amplification for a Schwarzschild lens for all values of the parameter y [when inequality (35) is satisfied], and in principle these two objects can be distinguished from the differences in the brightness curve in each of these three intervals. In particular, for small values of y the amplification for neutralino stars is twice the amplification for a Schwarzschild lens, whereas for large values of y the deviation of the amplification from unity for a neutralino star is half the deviation for a Schwarzschild lens.

This model has been used in the processing of OGLE data,[1)] which shows that microlensing can be produced by a noncompact body in two of the six events considered.[26]

A more detailed discussion of all possible values of the parameter R_0 for the lens equation of a neutralino star and the corresponding brightness curves are given in Ref. 27.

This work has received partial financial support from the Russian Fund for Fundamental Research (Grant No. 96-02-17434). One of the authors, A. Zakharov, is grateful to the European Southern Observatory for partial support of the work (Grant B-07-036).

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Translated by James S. Wood