Macroscopic electrodynamics of an antiferromagnet with a Goldstone branch in its spectrum

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The permeabilities are computed for a two-sublattice antiferromagnet having Goldstone branches in its magnon spectrum (two in a constant field H, one when $H=0$). The dispersion laws of the magnetic excitations (quasi-static waves and magnetic polaritons) are studied. The solutions of spatially inhomogeneous problems in media of this type are discussed. [S1063-7761(96)02112-9]

1. MAGNETIC SUSCEPTIBILITY

In computing the high-frequency magnetic susceptibility tensor χ_{ik} of magnetic substances, one frequently uses the phenomenological approach: one introduces the magnetizations $\mathbf{M}_i$ of the magnetic sublattices and solves the linearized Landau–Lifshitz equations describing their motion. In solving electrodynamic problems, one can often neglect the spatial dispersion of χ_{ik} while retaining the frequency dispersion. However, in this case, one loses one branch of the oscillations (the Goldstone branch), whose frequency ω equals zero when the wave vector is $\mathbf{k}=0$.

To include the Goldstone mode in the system of constitutive equations of macroscopic electrodynamics, it is necessary when computing the χ_{ik} tensor to take into account its spatial dispersion. Let us consider the very simple case of a two-lattice antiferromagnet of the easy-axis (EA) or easy-plane (EP) type. The magnetic field $\mathbf{H}$ is directed along the axis of the antiferromagnet. In an antiferromagnet of the EP type, the magnetic moments have the configuration shown in Fig. 1. The invariance of the magnetic energy under uniform rotation of the system of magnetic moments around the axis gives rise to the Goldstone branch of the oscillation ($\omega=0$). The same magnetic structure occurs in an antiferromagnet of the EA type for $H>H_{\mathrm{SF}}$, where H_{SF} is the spin-flip field.[1] Naturally, it is necessary in both cases to assume that there is no anisotropy in the basal plane.

The magnetic structure in weak fields (in particular, in an antiferromagnet of the EP or EA type) is determined by the anisotropy constants, whereas the Goldstone mode is weakly sensitive to anisotropy and can be studied using as an example a purely exchange isotropic antiferromagnet in a constant, homogeneous magnetic field.

The magnetic energy density corresponding to our assumptions is

$$W=\delta\mathbf{M}_1\cdot\mathbf{M}_2+\frac{1}{2}\alpha\left(\frac{\partial\mathbf{M}_1}{\partial x_i}\cdot\frac{\partial\mathbf{M}_1}{\partial x_i}+\frac{\partial\mathbf{M}_2}{\partial x_i}\cdot\frac{\partial\mathbf{M}_2}{\partial x_i}\right)+\alpha'\frac{\partial\mathbf{M}_1}{\partial x_i}\cdot\frac{\partial\mathbf{M}_2}{\partial x_i}-(\mathbf{M}_1+\mathbf{M}_2)\cdot\mathbf{H}. \qquad (1)$$

Here we have $\delta>0$, and $|\alpha|$, $|\alpha'|\sim\delta a^2$, where a is the interatomic distance (see below for the signs of α and α'). It follows from this that the magnetic moments $\mathbf{M}_1$ and $\mathbf{M}_2$, for $H<2\delta M\equiv H_E$, are symmetric with respect to the magnetic field (at angle θ, see Fig. 1), with

$$\cos\theta=\frac{H}{2M\delta}=\frac{H}{H_E},\quad H\leqslant 2M\delta=H_E.$$
$$\theta=0,\quad H>2M\delta, \qquad (2)$$

while the effective fields entering into the Landau–Lifshitz equations,

$$\frac{\partial\mathbf{M}_1}{\partial t}=g\mathbf{M}_i\times\mathbf{H}_i,\quad i=1,2, \qquad (3)$$

are the variational derivatives of the energy with respect to the magnetic moments,

$$\mathbf{H}_i=-\frac{\partial W}{\partial\mathbf{M}_i},\quad i=1,2, \qquad (3')$$

and g is the magnetomechanical ratio. We have from Eqs. (1) and (3′) that

$$\mathbf{H}_1=\mathbf{H}-\delta\mathbf{M}_2+\alpha\Delta\mathbf{M}_1+\alpha'\Delta\mathbf{M}_2,$$
$$\mathbf{H}_2=\mathbf{H}-\delta\mathbf{M}_1+\alpha\Delta\mathbf{M}_2+\alpha'\Delta\mathbf{M}_1. \qquad (4)$$

Let us linearize Eqs. (3) and (4), assuming that

$$\mathbf{M}_i=\mathbf{M}_{i0}+\mathbf{m}_i,\quad \mathbf{H}=\mathbf{H}_0+\mathbf{h} \qquad (5)$$

(we shall henceforth omit subscript 0 in the equilibrium, constant, homogeneous values of $\mathbf{M}_{i0}$ and $\mathbf{H}_0$). We set

$$\mathbf{h}=\mathbf{h}_0e^{-i(\omega t-\mathbf{k}\cdot\mathbf{r})}. \qquad (6)$$

It is easy to write a system of linear equations connecting the components of the vectors

$$\mathbf{m}=\mathbf{m}_1+\mathbf{m}_2,\quad \mathbf{l}=\mathbf{m}_1-\mathbf{m}_2 \qquad (7)$$

with the magnetic field $\mathbf{h}$:

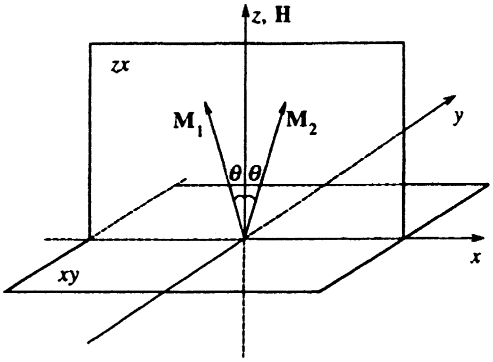

FIG. 1. Geometry of the problem.

$$
\begin{aligned}
&-i\omega m_x = g(H+\alpha_+ k^2 M\cos\theta)m_y - 2gM\cos\theta h_y,\\
&-i\omega m_y = -g(H+\alpha_+ k^2 M\cos\theta)m_x + 2gM\cos\theta h_x\\
&\qquad - g\alpha_- k^2 M\sin\theta\, l_z,\\
&-i\omega m_z = gM\sin\theta\alpha_- k^2 l_y,\\
&-i\omega l_x = gM\cos\theta\alpha_- k^2,\\
&-i\omega l_y = gM\cos\theta\alpha_- k^2 - gM\sin\theta(2\delta+\alpha_+ k^2)m_z\\
&\qquad + 2gM\sin\theta h_z,\\
&-i\omega l_z = gM\sin\theta(2\delta+\alpha_+ k^2)m_y - 2gM\sin\theta\, h_y,
\end{aligned}
\tag{8}
$$

where

$$
\alpha_+ = \alpha + \alpha',
$$

$$
\alpha_- = \alpha - \alpha',
$$

Taking axial anisotropy into account of course somewhat alters the coefficients in the system of Eqs. (8). The third equation is an exception—it does not change. This means that, in the absence of spatial dispersion (for $k=0$), we have $\chi_{zx}=0$.

The system of Eqs. (8) breaks up into two systems: one includes m_x, m_y, l_z, h_x, h_y, while the other includes m_z, l_x, l_y, and h_z. Each system generates its own branch of the oscillations, the dispersion laws of which can be found by setting $\mathbf{h}=0$ and by setting the corresponding determinants equal to zero.

We call the branch in which m_x and m_y oscillate the transverse ($\perp$) branch and that in which m_z oscillates the longitudinal ($\parallel$) branch.

The dispersion law of the transverse oscillations has the form

$$
\omega_\perp^2(k) = g^2(H + M\alpha_+ k^2\cos\theta)^2 + 2(gM\sin\theta)^2\delta\alpha_- k^2. \tag{9}
$$

We have omitted the term containing $\alpha_-\alpha_+ k^4$, since the Hamiltonian of Eq. (1) includes only terms that are quadratic in the gradients of the moments of the sublattices, and keeping such terms means going to higher order. Naturally, we shall omit such terms in what follows.

The dispersion law of the longitudinal oscillation has the form

$$
\omega_\parallel^2(k) = (gM\sin\theta)^2 2\delta\alpha_- k^2 = 2(gM)^2\left(1 - \frac{H^2}{H_E^2}\right)\delta\alpha_- k^2. \tag{10}
$$

This is the Goldstone branch, in which $\omega_\parallel \to 0$ when $k=0$. When $\mathbf{H}=0$ ($\sin\theta=1$), the transverse oscillations are also described by the Goldstone branch, so that, as is well known, the two branches become degenerate:

$$
\omega_\perp^2(k) = \omega_\parallel^2(k) = \omega_0^2(k) = 2(gM)^2\delta\alpha_- k^2, \quad \mathbf{H}=0. \tag{11}
$$

Equations (10) and (11), along with Eq. (1), show that the condition for the antiferromagnetic state to be stable is that the inequalities

$$
\delta > 0, \quad \alpha - \alpha' > 0. \tag{12}
$$

be satisfied. The existence of two degenerate Goldstone branches for $\mathbf{H}=0$ and one such branch for $\mathbf{H} \neq 0$ has a simple, obvious explanation: a system of magnetic moments coupled into an antiferromagnetic configuration can freely rotate around the magnetic field for $\mathbf{H} \neq 0$, whereas it can freely rotate in three-dimensional space for $\mathbf{H}=0$.

Equations (9) and (10) describe the spin-wave spectrum neglecting dipole–dipole forces and retardation.

As already explained, our problem is to compute the magnetic susceptibility tensor $\chi_{ik}(\omega,k)$. This makes it possible to solve problems involving the macroscopic electrodynamics of an antiferromagnet with a Goldstone branch in the spectrum.

From Eq. (8), we have

$$
m_i = \chi_{ik}(\omega^k)h_k, \tag{13}
$$

with the structure of the χ_{ik} tensor being

$$
\hat{\chi} = \begin{pmatrix} \chi_{xx} & i\chi' & 0 \\ -i\chi' & \chi_{yy} & 0 \\ 0 & 0 & \chi_{zz} \end{pmatrix} \tag{14}
$$

and

$$
\begin{aligned}
&\chi_{zz} = \frac{2g^2M(H + M\cos\theta\alpha_+ k^2)\cos\theta}{\omega_\perp^2(k) - \omega^2},\\
&\chi_{yy} = \chi_{xx} + \frac{1}{\delta}\frac{\omega_\parallel^2(k)}{\omega_\perp^2(k) - \omega^2},\\
&\chi_{zz} = \frac{1}{\delta}\frac{\omega_\parallel(k)}{\omega_\parallel^2(k) - \omega^2}, \quad \chi' = \frac{2\omega gM\cos\theta}{\omega_\perp^2(k) - \omega^2}.
\end{aligned}
\tag{15}
$$

We recall that $\delta \gg 1$ holds in most antiferromagnets.

Let us consider the limiting cases. When $H=0$ holds [see Eq. (11)],

$$
\chi_{xx} = \chi_{xy} = \chi_{yx} = 0, \quad \chi_{yy} = \chi_{zz} = \frac{1}{\delta}\frac{\omega_0^2(k)}{\omega_0^2(k) - \omega^2}. \tag{16}
$$

For $H = H_E = 2\delta M$, the magnetic moments of the sublattices occupy a parallel position relative to $\mathbf{H}$ (they collapse)—a second-order (spin-flip) phase transition into a state similar to a ferromagnetic state occurs. The frequency

TABLE I.

	H_{SF}, kOe	H_E, kOe	Θ_N, K	α^5, kOe·cm^2	v_a, m·s^{-1}
$CsMnFe_3$	2.48	350	53.3	0.95	699
$MnCO_3$	3.04	320	32.5	0.61	432
$FeBO_3$	5.3	3×10^3	348.0	5.67	4.5×10^3

$\omega_\parallel(k)$ of the longitudinal oscillations vanishes identically ($\sin\theta=0$ at $H=H_E$), along with χ_{zz}. The frequency $\omega_\perp(k)$ of the transverse oscillations is determined by the usual equation (for an antiferromagnet),

$$\omega_\perp(k)=g(H+M\alpha_+k^2),\quad H\geqslant H_E, \tag{17}$$

while

$$\chi_{xx}=\chi_{yy}=\frac{2gM\omega_\perp(k)}{\omega_\perp^2(k)-\omega^2},\quad \chi'=\frac{2\omega gM}{\omega_\perp^2(k)-\omega^2},\ \chi_{zz}=0. \tag{18}$$

If the formulation of the problem makes it possible to neglect spatial dispersion, and $0<H\leqslant H_E$ holds,

$$\chi_{xx}=\chi_{yy}=\frac{1}{\delta}\frac{(gH)^2}{(gH)^2-\omega^2},$$

$$\chi_{zz}=0,\quad \chi'=\frac{1}{\delta}\frac{\omega gH}{(gH)^2-\omega^2}. \tag{19}$$

Note that, for $0<H\leqslant H_E$, anisotropy in the xy plane is a consequence of spatial dispersion:

$$\chi_{yy}-\chi_{xx}=\frac{1}{\delta}\frac{\omega_\parallel^2(k)}{\omega_\perp^2(k)-\omega^2}.$$

For $H\geqslant H_E$, after spatial dispersion is neglected,

$$\chi_{xx}=\chi_{yy}=\frac{2g^2MH}{(gH)^2-\omega^2},\quad \chi'=\frac{2\omega gM}{(gH)^2-\omega^2},\quad \chi_{zz}=0. \tag{20}$$

In what follows, we devote most of our attention to various types of oscillations for $H<H_E$. We note in this case that Eqs. (15) allow a limiting transition to $H=H_E$. A comparison of Eqs. (9), (15), (17), and (18) shows that there are two dispersion mechanisms: an antiferromagnetic mechanism, which occurs at $\mathbf{H}=0$ and plays a role for $H<H_E$ (the terms associated with it contain α_- [see Eq. (8)]), and a ferromagnetic mechanism (the terms of ferromagnetic nature contain α_+ [see Eqs. (8), (9), and (15)]).

When the magnetic field $H\neq 0$ is not too great ($H<H_E$), the leading role in Eqs. (9) and (15) is played by the antiferromagnetic dispersion mechanism. Actually, since $\cos\theta=H/2M\delta$ holds according to Eq. (2), we get

$$\omega_\perp^2=(gH)^2\left(1+\frac{\alpha_+}{\delta}k^2\right)^2+2(gM)^2\left(1-\frac{H^2}{H_E^2}\right)\delta\alpha_-k^2 \tag{21}$$

and the dispersion in the first term has no exchange amplification. Evidently, an expression in which only the antiferromagnetic mechanism is responsible for the dispersion can serve as a good approximation. Thus, if

$$1-\frac{H}{H_E}>\frac{1}{2\delta},$$

we get

$$\omega_\perp^2=(gH)^2+v_a^2(H)k^2,\quad v_a^2=2(gH)^2\left(1-\frac{h^2}{H_E^2}\right)\delta\alpha_-. \tag{22}$$

In the same approximation, we can write the components of the magnetic susceptibility tensor given by Eq. (15) as

$$\chi_{xx}=\frac{1}{\delta}\frac{(gH)^2}{\omega_\perp^2-\omega^2},\quad \chi_{yy}=\frac{1}{\delta}\frac{\omega_\perp^2(k)}{\omega_\perp^2(k)-\omega^2},$$
$$\chi_{zz}=\frac{1}{\delta}\frac{v_a^2(H)k^2}{v_a^2(H)k^2-\omega^2},\quad \chi'=\frac{1}{\delta}\frac{gH\omega}{\omega_\perp^2(k)-\omega^2}. \tag{23}$$

To use the resulting expressions, it is important to know the orders of magnitude of the parameters in them. The parameters for several antiferromagnets are given in Table I. This table also gives the velocity v_a for $H=0$ and the values of the exchange field H_E. The magnetic field varies within wide limits ($0<H\leqslant H_E$). For numerical estimates the following relationships can be used:

$$\delta\sim\frac{\Theta_N}{\mu M},\quad \alpha_-\sim\frac{\Theta_N}{\mu M}a^2,\quad \mu\sim g\hbar,\quad M\sim\frac{\mu}{a^3},$$
$$H_E\sim\frac{\Theta_N}{\mu},\quad v_a(H=0)\sim\frac{\Theta_N}{\hbar}a. \tag{24}$$

If we use the following values: Bohr magneton $\mu\sim10^{-20}$ Oe/G, Néel temperature $\Theta_N\sim10^2$ K, and lattice constant $a\sim10^{-8}$ cm, we get $H_E\sim10^6$ Oe $=10^2$ T and $v_a(H=0)\sim10^5$ cm/sec.

Frequency dispersion usually plays the chief role in the optics of nonmagnetic media, while spatial dispersion is only important close to the resonant frequencies ω_R. This is because the term that contains the wave vector for which $ak\ll1$ (macroscopic electrodynamics) is usually much smaller than the resonant frequency ω_R. In magnets, as is well known, the resonant frequencies have a relativistic origin, while spatial dispersion is caused by exchange (electrostatic) interaction. Therefore, the role of spatial dispersion can be substantial even at frequencies far from resonance.

Our model has $\omega_R=gH$, but $v_a(H=0)k\sim gH$ holds for $ak\sim\mu H/\Theta_N\ll1$. Consequently, it is possible to consider the case of strong spatial dispersion while staying within the framework of macroscopic electrodynamics ($v_ak\gg gH$ if, of course, $H\ll H_E$).

We neglected dissipative processes in computing the components of the magnetic susceptibility tensor, and this naturally limits the possibility of comparing the results with experiment. The main purpose of this article is to demonstrate the role of strong spatial dispersion in solving the problems of the macroscopic electrodynamics of antiferromagnets. On the other hand, it may be thought that using perfect samples when studying the high-frequency properties of antiferromagnets (see, for example, Ref. 2) makes it possible to use the resulting expression to describe the experimental results. It should be kept in mind here that the competition that usually occurs between spatial dispersion and damping is won by spatial dispersion in this case. Actually, the wavevector-dependent term can exceed the resonant frequency (see above), while the imaginary part of the frequency, Im ω, is usually much less than its real part, Re ω.

2. MAGNETOSTATIC WAVES (ALLOWING FOR MAGNETIC-DIPOLE INTERACTION)

By knowing the components of the magnetic susceptibility, one can explain how the magnetic-dipole interaction affects the spin-wave spectrum. To do this, one must use the equations of magnetostatics,[1)] which are valid in the limit $kc \gg \omega$, where c is the speed of light.

$$\operatorname{curl}\mathbf{h}=0, \quad \operatorname{div}(\mathbf{h}+4\pi\mathbf{m})=0, \quad m_i=\chi_{ik}(\omega,k)h_k. \tag{25}$$

The lower-case latin letters designate the variable fields.

From this, we quickly get the dispersion equation relating frequency ω to the wave vector $\mathbf{k}=\mathbf{n}k$:

$$1+4\pi n_i\chi_{ik}(\omega,k)n_k=0 \tag{26}$$

or

$$1+4\pi\{n_x^2\chi_{xx}(\omega,k)+n_y^2\chi_{yy}(\omega,k)+n_z^2\chi_{zz}(\omega,k)\}=0. \tag{26'}$$

At $H=0$, according to Eqs. (16) and (22),

$$\omega^2=v_0^2\left(1+\frac{4\pi}{\delta}\sin^2\psi\right)k^2, \quad v_0=v_a(H=0),$$

$$\sin^2\psi\equiv n_y^2+n_z^2, \tag{27}$$

where ψ is the angle between the vector $\mathbf{n}=\mathbf{k}/k$ and the axis along which the antiparallel magnetic moments of the sublattices are placed. Because of the magnetic-dipole interaction, the spin-wave velocity has acquired a weak ($\delta \gg 1$) dependence on the direction.

For $H \neq 0$, all three diagonal components of $\chi_{ik}(\omega,k)$ are nonzero, and Eq. (26) takes the form [see Eq. (23)]

$$1+\frac{4\pi}{\delta}\left\{\frac{(gH)^2n_x^2}{\omega_\perp^2(k)-\omega^2}+\frac{\omega_\perp^2(k)n_y^2}{\omega_\perp^2(k)-\omega^2}+\frac{v_a^2(H)k^2n_z^2}{v_a^2(H)k^2-\omega^2}\right\}=0,$$

$$\omega_\perp^2=(gH)^2+v_a^2(H)k^2. \tag{28}$$

This biquadratic equation can be solved exactly in radicals, but it is more instructive to show its approximate solution, using $4\pi/\delta \ll 1$. The spin waves have two branches—transverse and longitudinal [see Eqs. (9) and (10)]. From Eq. (28), we have

$$\omega_\perp^2\simeq(gH)^2\left[1+\frac{4\pi}{\delta}(n_y^2+n_x^2)\right]+v_a^2(H)\left(1+\frac{4\pi}{\delta}n_y^2\right)k^2,$$

$$\omega_\parallel^2\simeq v_a^2(H)\left[1+\frac{4\pi}{\delta}n_z^2\right]k^2, \quad 0<H<H_E. \tag{29}$$

Note the difference in the angular dependences of the gap in the transverse mode of the oscillations and the coefficient of k^2. It may seem suspicious that there is no limiting transition from Eq. (29) to Eq. (27). However, this is natural, since we used an approximate method to solve Eq. (26). For the limiting transition, it is necessary to initially set $H=0$ and then to solve the equation, as we did above.

For $H \gg H_E$ holds, the role of the magnetic-dipole interaction is the same as in ferromagnets. From Eqs. (26) and (28), we have

$$\omega^2=\omega_\perp(k)[\omega_\perp(k)+8\pi gM(n_x+n_y)],$$

$$\omega_\perp=g(H+M\alpha_+k^2), \quad H \gg H_E=2M\delta. \tag{30}$$

Since $H \gg H_E$ holds, the terms that depend on the wave vector are small, and an expansion can be used. Then

$$\omega=g\left\{\sqrt{H[H+8\pi M(n_x^2+n_y^2)]}+\frac{1}{2}M\alpha_+k^2\left[\sqrt{\frac{H}{H+8\pi M(n_x^2+n_y^2)}}+\sqrt{\frac{H+8\pi M(n_x^2+n_y^2)}{H}}\right]\right\}.$$

The renormalization of the gap of the spin-wave spectrum (or of the ferromagnetic resonance frequency) from gH to $g\sqrt{HB}$ is apparently the most important effect, where $B=H+8\pi M$ is the induction corresponding to a magnetization equal to $2M$. It must be kept in mind, however, that we have taken $H \gg M$ and that the angular dependence both in the value of the gap and in the coefficient of k^2 is a small correction (in ferromagnets, $4\pi M$ can exceed H.)

3. MAGNETIC POLARITONS

The polariton dispersion law is determined by solving Maxwell's equations supplemented by the constitutive equations; it has the form of a plane wave. In the optical range, the magnetic susceptibility is usually assumed to equal unity, while only the frequency dispersion of the permittivity is included in the calculation. In computing the dispersion law of a magnetic polariton, it is natural to consider low frequencies (by comparison with optical frequencies), which allows one to replace the permittivity of the magnet with its static value, i.e., to assume that

$$\mathbf{d}=\varepsilon\mathbf{e}, \tag{31}$$

where ε is a constant greater than unity. A magnetic polariton (in particular, in antiferromagnets) has one more essential difference from an ordinary (dielectric) polariton: there is no basis for neglecting spatial dispersion when one calculates its dispersion law.

It is convenient to reduce the system of Maxwell's equations to one vector equation,

$$\mathbf{k}_x(\mathbf{k}\times\mathbf{h})=-\frac{\omega^2}{c^2}\varepsilon\mathbf{b},\quad \mathbf{b}=\hat{\mu}\cdot\mathbf{h},\tag{32}$$

with the permeability tensor defined in terms of χ_{ik}:

$$\mu_{ik}(\omega,\mathbf{k})=\delta_{ik}+4\pi\chi_{ik}(\omega,\mathbf{k}).\tag{33}$$

The determinant of the system of Eqs. (32) determines the dispersion law of a magnetic polariton. Note that, when the μ_{ik} tensor is anisotropic and gyrotropic, the magnetic field $\mathbf{h}$ is not necessarily transverse (to the wavevector $\mathbf{k}$). The induction vector $\mathbf{b}$ must be transverse. From Eq. (32), $\mathbf{b}\cdot\mathbf{k}=0$.

Before studying the dispersion law of a magnetic polariton, let us comment on the accurate longitudinal magnetic polariton. In the preceding section, we considered quasistatic waves, whose dispersion law is approximate since it neglects electrodynamic retardation (the finiteness of the velocity of light). Electromagnetic waves differ from magnetostatic ones in that an electric field participates in the waves, along with a magnetic field and a magnetic moment. For oscillations of the electric field to be excited, the induction vector $\mathbf{b}$ must have a component perpendicular to $\mathbf{k}$. When the magnetic susceptibility tensor χ_{ik} has the structure of Eq. (14), the condition for the absence of magnetic charges ($\operatorname{div}\mathbf{b}=0$) can be exactly satisfied for a wave propagating along the z axis if

$$\mu_{zz}(\omega,k)\equiv 1+4\pi\chi_{zz}(\omega,k)=0,\quad k_z=k,\quad k_x=k_y=0.\tag{34}$$

Hence, using Eqs. (15), (10), and (22), we get

$$\omega=\sqrt{1+\frac{4\pi}{\delta}}\,v_a(H)k,\quad \mathbf{k}=(0,0,k).\tag{35}$$

Comparing Eq. (35) with the second of Eqs. (29), we see that the approximate (quasistatic) solution is converted to an exact solution when $n_z=1$.

For $\mathbf{H}=0$, the wave can propagate in the yz plane while remaining exact:

$$\omega=\sqrt{1+\frac{4\pi}{\delta}}v_0k,\quad v_0=v_a(H=0),$$

$$\mathbf{k}=(0,k_y,k_z),\quad k=\sqrt{k_y^2+k_z^2}.\tag{36}$$

The existence of (exact) longitudinal magnetic polaritons, as remarked in Ref. 3, can help to distinguish losses of a magnetic and an electrical nature (see also below).

We now turn to the properties of ordinary magnetic polaritons, i.e., to Eqs. (32) and (33). We begin with the case $\mathbf{H}=0$. The permeability given by Eq. (33) has no gyrotropic terms, while the anisotropy is shown by the fact that $\mu_{xx}=1$, while

$$\mu_{yy}=\mu_{zz}=\tilde{\mu},\quad \tilde{\mu}=1+\frac{4\pi}{\delta}\frac{(v_0k)^2}{(v_0k)^2-\omega^2}.\tag{37}$$

Let wave vector $\mathbf{k}$ be at an angle ψ to the x axis. Depending on polarization, two polaritons can propagate in a magnetic material. For $h_z\neq 0$, and $h_x=h_y=0$, the equation for a polariton has the customary form:

$$k^2=\frac{\omega^2}{c^2}\varepsilon\tilde{\mu}(\omega,k).\tag{38}$$

If $h_z=0$ and h_x, $h_y\neq 0$ hold, the equation is somewhat more completed:

$$k^2=\frac{\omega^2}{c^2}\frac{\tilde{\mu}(\omega,k)}{\tilde{\mu}(\omega,k)\sin^2\psi+\cos^2\psi}.\tag{39}$$

When a polariton propagates along the x axis ($\psi=0$), degeneracy occurs: both polaritons have identical dispersion laws. Since Eq. (38) is a particular case of Eq. (39), we first analyze Eq. (39). We introduce the velocity $v_c=c/\sqrt{\varepsilon}$, the velocity of light in a medium that has the same permittivity as does our magnetic substance but possesses no magnetic properties. It can be seen from Eqs. (37) and (39) that, at some polarization, there exist two polaritons: the dispersion laws of each are linear:

$$\omega=v_{1,2}k,\tag{40}$$

while the velocities are the roots of the following biquadratic equation:

$$\frac{v_c^2}{v^2}=\frac{v_0^2(1+4\pi/\delta)-v^2}{v_0^2(1+(4\pi/\delta)\sin^2\psi)-v^2}.\tag{41}$$

Solving this equation, we get

$$v_{1,2}^2=\frac{1}{2}\left[v_c^2+\left(1+\frac{4\pi}{\delta}\right)v_0^2\right]\pm\sqrt{\frac{1}{4}\left[v_c^2+\left(1+\frac{4\pi}{\delta}\right)v_0^2\right]^2-\left(1+\frac{4\pi}{\delta}\sin^2\psi\right)v_c^2v_0^2}.\tag{42}$$

Since velocity v_c is close to the speed of light, according to Eq. (24),

$$v_0\ll v_c$$

and, consequently,

$$v_{1,2}^2\simeq\begin{cases} v_c^2+\dfrac{4\pi}{\delta}\cos^2\psi\, v_0^2 \\ v_0^2\left(1+\dfrac{4\pi}{\delta}\sin^2\psi\right)\left[1+\dfrac{4\pi}{\delta}\cos^2\psi\,\dfrac{v_0^2}{v_c^2}\right]\end{cases}.\tag{42$'$}$$

For $\psi=\pi/2$ ($k_y=k$, $k_x=k_z=0$), the velocity of one branch is independent of the magnetic properties ($v_1=v_2$), while the velocity of the other coincides with $v_0\sqrt{1+4\pi/\delta}$, the velocity of the exact longitudinal polariton given by Eq. (36).

It is interesting that two Goldstone modes that interact remain Goldstone modes, with the velocity of the fast Goldstone mode increasing and that of the slow Goldstone mode decreasing. This result is in agreement with the principle of collision of terms (when $k=0$, the terms and the frequencies coincide). It should be noted that Eq. (42), obtained using Maxwell's equations, contains no restrictions on the wave velocity (the velocity v_1 can exceed the velocity of light in vacuum). This is apparently associated with the nonrelativistic derivation of the magnetic susceptibility tensor containing spatial dispersion (spatial dispersion means that the excitation can move). A relativistic derivation, one might think, would not so much change the value of $\mu_{ik}(\omega,\mathbf{k})$ as cause the permittivity ε to be renormalized. However, we know of no such derivation.

The increase of the photon velocity due to spatial dispersion of the permeability recalls Leontovich's 1961 paper,[4] in which he explains how the Kramers–Kronig relations are modified because of spatial dispersion of the permittivity. The existence of a limiting velocity of signal propagation imposes limitations on the real and imaginary parts of permittivity with spatial dispersion. It is shown in the cited paper that a limitation must exist, but the models are not evaluated in the sense of how correctly they describe the properties of the medium. Therefore, Ref. 4 is apparently inadequate to eliminate the misunderstanding that can in principle appear.

The magnon velocity v_a also contains no literal restrictions. What is the basis for our confidence that $v_a \ll c$? The estimates of Eqs. (24) provide such a basis, which can be made even more obvious. If $v_a = c$ holds, then

$$\Theta_N = \frac{c\hbar}{e^2}\frac{e^2}{a} = 137\frac{e^2}{a}$$

should exceed the Coulomb energy by a factor of 137. Taking into account the character of the exchange interaction, we see that the magnon velocity cannot be comparable with the velocity of light!

Let us proceed to the case $H \neq 0, H < H_E$, for which the χ_{ik} tensor is given by Eqs. (22) and (23). It is inconvenient to consider a polariton propagating in an arbitrary direction. We shall derive the dispersion law for polaritons propagating along the coordinate axes.

I. $0 \neq H < H_E$, $k_x = k$, $k_y = k_z = 0$. In this case, the polaritons are of two types (two polarizations[2)]). The polariton dispersion law of the first type ($h_x = h_y = 0$, $h_z \neq 0$) coincides with the dispersion law of a polariton for $\mathbf{H}=0$ and $\psi = 0$. It is only necessary to replace v_0 with $v_a = v_0\sqrt{1 - H^2/H_E^2}$ [see Eqs. (40)–(42)].

The dispersion equation of the second type ($h_x \neq 0$, $h_y \neq 0$, $h_z = 0$) is a solution of

$$k^2 = \frac{\omega^2}{c^2}\varepsilon\mu_{\text{eff}}^{\text{I}}, \quad \mu_{\text{eff}}^{\text{I}} = \mu_{yy} - \frac{(\mu')^2}{\mu_{xx}},$$

$$\mu' = \frac{4\pi}{\delta}\frac{gH\omega}{\omega_\perp^2(k) - \omega^2}, \quad \mu_{xx} = 1 + \frac{4\pi}{\delta}\frac{(gH)^2}{\omega_\perp^2(k) - \omega^2},$$

$$\mu_{yy} = 1 + \frac{4\pi}{\delta}\frac{\omega_\perp^2(k)}{\omega_\perp^2(k) - \omega^2}, \quad \omega_\perp^2(k) = (gH)^2 + (v_a)^2. \tag{43}$$

Hence

$$\mu_{\text{eff}}^{\text{I}} = 1 + \frac{4\pi}{\delta}\frac{(\omega_\perp^{\text{I}}(k))^2}{(\omega_\perp^{\text{I}}(k))^2 - \omega^2},$$

$$k^2 = \frac{\omega^2}{c^2}\varepsilon\left\{1 + \frac{4\pi}{\delta}\frac{(\omega_\perp^{\text{I}}(k))^2}{(\omega_\perp^{\text{I}}(k))^2 - \omega^2}\right\}, \tag{44}$$

$$\omega_\perp^{\text{I}}(k) = \sqrt{\left(1 + \frac{4\pi}{\delta}\right)(gH)^2 + (v_a k)^2}.$$

Writing this equation in the form

$$[\omega^2 - (v_c k)^2]\{\omega^2 - [\omega_\perp^{\text{I}}(k)]^2\} = \frac{4\pi}{\delta}(\omega_\perp^{\text{I}}(k))^2\omega^2 \tag{44'}$$

emphasizes that a polariton is a result of the mixing of a spin wave with a photon (the magnon dispersion law takes magnetic-dipole interactions into account: the gap in the spin-wave spectrum is renormalized). Frequently, $4\pi/\delta \ll 1$ holds, and one can speak of resonance between two waves of the spectrum. Since Eq. (44) can be solved exactly, there is no need for qualitative analysis. From Eq. (44'), we have

$$\omega_{1,2}^2 = \frac{1}{2}\left\{(v_c k)^2 + \left(1 + \frac{4\pi}{\delta}\right)(\omega_\perp^{\text{I}}(k))^2\right\} \pm \sqrt{\frac{1}{4}\left\{(v_c k)^2 + \left(1 + \frac{4\pi}{\delta}\right)(\omega_\perp^{\text{I}}(k))^2\right\}^2 - (v_c k)^2(\omega_\perp^{\text{I}}(k))^2}. \tag{45}$$

It can be seen from the result that both branches (both polaritons) exist for all wave vectors. In the limit $k \to 0$ we have $\omega \to 0$ for one wave (a photonlike polariton) and

$$\omega_1 \simeq \frac{kc}{\sqrt{\epsilon\mu_0}}\left[1 - \frac{(\mu_0 - 1)c^2k^2}{2\mu_0^3\varepsilon(gH)^2}\right]. \tag{46}$$

The interaction of a photon with a magnon results in renormalization and dispersion of the velocity. For the other branch, as $k \to 0$,

$$\omega_2^2 \simeq \left(1 + \frac{4\pi}{\delta}\right)^2(gH)^2 + \left(1 + \frac{4\pi}{\delta}\right)(v_a k)^2 + \frac{4\pi}{\delta}\frac{(v_c k)^2}{1 + 4\pi/\delta}. \tag{47}$$

An evaluation of the role of spatial dispersion (see the end of Section I) shows that it is possible to consider the limit of large frequencies ω and large wave vectors k simultaneously: $\omega \gg gH$, $v_a k \gg gH$. In this limit,

$$\lim_{\omega \to \infty, k \to \infty} \mu_{\text{eff}}^{\text{I}}(\omega,k) = 1 + \frac{4\pi}{\delta}\frac{v_a^2}{v_a^2 - v^2}, \quad v = \frac{\omega}{k}. \tag{48}$$

This means that the polariton dispersion law at large frequencies and wave vectors asymptotically approaches the dispersion law of a polariton with polarization of the first type (see Fig. 2).

If there were no spatial dispersion, the wave vector $\mathbf{k}$ would go to infinity at a frequency of $\omega = gH\sqrt{1 + 4\pi/\delta}$ (antiferromagnetic resonance). When spatial dispersion is in-

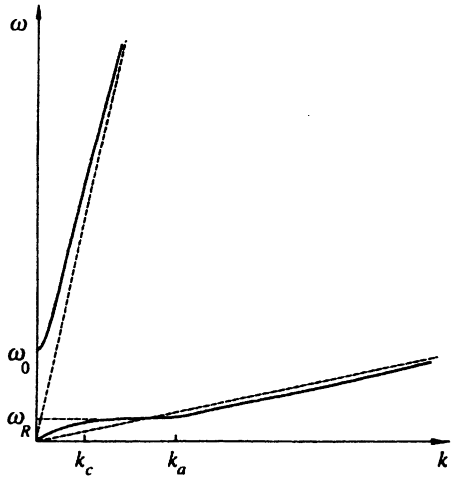

FIG. 2. Schematic diagram of the dispersion law of the ($\mathbf{k}\|\mathbf{x}$) polariton: dashed curves—first polarization, $h_z \neq 0$, $h_x = h_y = 0$; solid curve—second polarization, $h_x, h_y \neq 0$, $h_z = 0$, $k_c = gH/\bar{c}$, $\bar{c} = c/\sqrt{\varepsilon\mu_0}$, $k_a = gH/v_a$, $\omega_0 = \mu_0 gH$, $\omega_R = \sqrt{\mu_0} gH$.

cluded, the frequency $\omega_R \sim gH\sqrt{1+4\pi/\delta}$ shows up as an appreciable decrease of the group velocity of the polariton:

$$\left.\frac{d\omega}{dk}\right|_{\omega_R} \ll v_a \ll v_c$$

(this is indicated in Fig. 2). Note that the limit $k \to \infty$, $\omega \mapsto \infty$, in accordance with Eq. (43), is absent.

II. $0 \neq H < H_E$, $k_y = k$, $k_x = k_z = 0$. The polaritons also possess different dispersion laws in this case, depending on the polarization. For $h_z \neq 0$, $h_x = h_y = 0$ the polariton has a dispersion law identical to that of a polariton with the same polarization with $k_x = k$, $k_y = k_z = 0$ (see above and Fig. 2). In general, since the permeability tensor has only two gyrotropic components ($\mu_{yx} = \mu_{xy}^*$), a polariton with the polarization under consideration is isotropic in the xy plane.

A polariton with polarization $h_z = 0$, $h_x, h_y \neq 0$ for $k_y \neq 0$, $k_x, k_y = 0$ differs from a polariton with $k_x \neq 0$, $k_y = k_z = 0$. In fact, its dispersion law is

$$k^2 = \frac{\omega^2}{c^2}\varepsilon\mu_{\text{eff}}^{\text{II}}, \quad \mu_{\text{eff}}^{\text{II}} = \mu_{xx} - \frac{(\mu')^2}{\mu_{yy}}. \tag{49}$$

After the appropriate substitution,

$$\mu_{\text{eff}}^{\text{II}} = 1 + \frac{4\pi}{\delta}\left(1 + \frac{4\pi}{\delta}\right)\frac{(gH)^2}{(1+4\pi/\delta)\omega_\perp^2(k) - \omega^2}. \tag{49$'$}$$

In the limit $\omega \to 0$ and $k \to 0$, we again have Eq. (46). Moreover,

$$\mu_{\text{eff}}^{\text{II}}(\omega,0) = \mu_{\text{eff}}^{\text{I}}(\omega,0). \tag{50}$$

However, as $k \to \infty$ and $\omega \to \infty$,

$$\lim_{\omega \to \infty, k \to \infty} \mu_{\text{eff}}^{\text{II}}(\omega,k) = 1.$$

It can be shown that as $k \to \infty$ there is only one linear branch: $\omega = v_c k$. To analyze the polariton dispersion law in more detail, we write the solution of Eq. (46):

$$\omega^2 = \frac{1}{2}\left[(v_c k)^2 + \left(1 + \frac{4\pi}{\delta}\right)\left(\omega_\perp^2 + \frac{4\pi}{\delta}(gH)^2\right)\right] \pm \sqrt{\frac{1}{2}\left(\omega_\perp^2 + \frac{4\pi}{\delta}(gH)^2\right)^2 - \left(1 + \frac{4\pi}{\delta}\right)(v_c k)^2 \omega_\perp^2(k)}. \tag{51}$$

Whence we have, as $k \to \infty$,

$$\omega_1 \simeq v_c k, \quad \omega_2 \simeq \sqrt{1 + \frac{4\pi}{\delta}}\, v_a k. \tag{52}$$

In going to the limit in Eq. (49$'$), the equation becomes shorter, and one solution is lost. It is interesting to note that Eqs. (51) and (52) demonstrate asymptotic freedom; i.e., in the limit $k \to \infty$ and $\omega \to \infty$, the photon and magnon are independent! Equations (45) and (51) are virtually indistinguishable in a schematic image of the dispersion law (see Fig. 2). It follows from Eq. (49) that the former resonant frequency, i.e., the frequency for which the group velocity is anomalously small, is the same in both cases.

III. $0 < H \leqslant H_E$, $k_x = k_y = 0$, $k_z \neq 0$. In polaritons of both polarizations, $h_x, h_y \neq 0$, $h_z = 0$. The polariton dispersion law is a solution of Maxwell's equations,

$$k^2 = \frac{\omega^2}{c^2}\varepsilon\mu_{\text{eff}}^{\pm}, \quad \mu_{\text{eff}}^{\pm} = \frac{1}{2}(\mu_{xx} + \mu_{yy}) \pm \sqrt{\frac{1}{4}(\mu_{xx} - \mu_{yy})^2 + \mu'^2}. \tag{53}$$

The sign in front of the square root gives the polarization. When $\mu_{xx} \neq \mu_{yy}$, the waves (the polaritons) are elliptically polarized (the signs correspond to the rotation direction). The μ_{xx}, μ_{yy}, and μ' components are functions of wave vector $\mathbf{k}$ and frequency ω. Because of this, two polaritons correspond to the fixed plus polarization, and one corresponds to the minus polarization.

We should point out that case III under consideration is more complex to analyze than the preceding case (because of the square root in the definition of the permeability $\mu_{\text{eff}}^{\text{III}}$).

Let us consider the plus polarization.[3)] To start out, we explain how the polaritons behave as $k \to 0$ and as $k \to \infty$. In the limit $k \to 0$, as always, we get Eq. (46) for one of the polaritons (similar to a photon). Besides this, a nonzero frequency exists for which $\mu_{\text{eff}}^{\text{III}} = 0$. From Eq. (53) and the permeability values, we have

$$\mu_{\text{eff}}^{+}(\omega,0) = \mu + \mu', \quad \mu = 1 + \frac{4\pi}{\delta}\frac{(gH)^2}{(gH)^2 - \omega^2},$$

$$\mu' = \frac{4\pi}{\delta}\frac{gH\omega}{(gH)^2 - \omega^2}, \tag{54}$$

i.e.,

$$\mu_{\text{eff}}^{+} = \frac{(1+4\pi/\delta)gH - \omega}{gH - \omega} = 1 + \frac{4\pi}{\delta}\frac{gH}{gH - \omega}. \tag{54$'$}$$

As in the preceding cases, we have $\mu_{\text{eff}}^{\text{III}} = 0$ for $\omega = (1+4\pi/\delta)gH$. As $k \to \infty$, $\omega \to \infty$, the limit μ_{eff}^{+} coincides with the limit $\mu_{\text{eff}}^{\text{I}}$ under the same conditions. Consequently, the asymptotic behavior of the dispersion law does not differ from the case considered earlier (see Fig. 2). The resonant frequency (i.e., the frequency at which the group velocity of one of the polariton branches is anomalously small) equals gH [see Eq. (54')].

Let us now consider the minus polarization. After substituting the values of the components, we write the effective permeability as

$$\mu_{\mathrm{eff}}^{-}=1+\frac{1}{2}\frac{4\pi}{\delta}\frac{(gH)^2+\omega_\perp^2-\sqrt{(v_a k)^4+4\omega^2(gH)^2}}{\omega_\perp^2-\omega^2},$$

$$\omega_\perp^2=(gH)^2+(v_a k)^2. \tag{55}$$

We multiply the numerator and the denominator by

$$(gH)^2+\omega_\perp^2+\sqrt{(v_a k)^4+4\omega^2(gH)^2}.$$

As a result, we get

$$\mu_{\mathrm{eff}}^{-}=1+\frac{8\pi}{\delta}\frac{(gH)^2}{(gH)^2+\omega_\perp^2+\sqrt{(v_a k)^4+4\omega^2(gH)^2}}. \tag{56}$$

Substituting μ_{eff}^{-} into Eq. (53), we see that the dispersion equation has one solution (for k^2), of the photon type. For $k\to 0$, as always, we have $\omega\simeq\tilde{c}k$, $\tilde{c}=c/\sqrt{\varepsilon\mu_0}$, whereas, for $k\to\infty$, the frequency is $\omega\simeq v_c k$. Equation (56) makes it possible to compute the dispersion of the velocity of "light" in the limit $k\to 0$ and $k\to\infty$:

$$k^2=\begin{cases}\dfrac{\omega^2}{c^2}\left(1-\dfrac{\mu_0-1}{\mu_0}\dfrac{\omega}{\omega+gH}\right), & \omega\ll 2gH\left(\dfrac{\tilde{c}}{v_a}\right)^2,\\[2ex] \tilde{c}=\dfrac{c}{\sqrt{\varepsilon\mu_0}}, & \\[2ex] \dfrac{\omega^2}{v_c^2}\left[1+(\mu_0-1)\dfrac{v_c^2}{v_a^2}\dfrac{(gH)^2}{\omega^2}\right], & \omega\ll 2gH\left(\dfrac{v_c}{v_a}\right)^2.\end{cases} \tag{57}$$

We recall that $v_a\ll v_c$. Since $\mu_0\sim 1$ holds in most cases, the dispersion of the velocity is small. A distinctive feature of this case (as for the plus polarization) is that the dispersion of the velocity of light is linear in frequency as a consequence of the gyrotropy of the μ_{ik} tensor.

4. SPATIALLY INHOMOGENEOUS PROBLEMS: EXCITATION OF THE POLARITONS

To solve spatially inhomogeneous problems, the constitutive equations, Eqs. (13), should be rewritten in the **r** representation, keeping in mind that the square of the wave vector, k^2, corresponds to the operator $\Delta=-\partial^2/\partial x_i^2$. A characteristic of Eqs. (13) and (23) is that the numerator of the χ_{yy} and χ_{zz} components contains the expression $v_a^2(H)k^2\to -v_a^2\Delta$, signifying that the corresponding constitutive equation contains not the magnetic field component (on the right-hand side), but the derivatives of the magnetic field. For example, to determine the z component of the magnetic moment, m_z, it is necessary to solve

$$v_a^2(H)m_z+\omega^2 m_z=\frac{v_a^2(H)}{\delta}\Delta H-z. \tag{58}$$

The presence of spatial derivatives of the components of the **m** vector [in equations of the type of Eq. (58)] makes it necessary to formulate supplementary boundary conditions (besides the electrodynamic conditions, which are consequences of Maxwell's equations). The formulation of the supplementary boundary conditions is a special problem that requires one to consider the behavior of the magnetic moment densities at the boundary (we will not deal with this question here, but will restrict ourselves to the simplest phenomenological assumptions—see below).

The simplest spatially inhomogeneous problem is the excitation of an electromagnetic wave in the half-space $x>0$ occupied by a magnetic substance. Here we choose case I ($0\neq H<H_E$, $k_y=k_z=0$, $h_x\neq 0$, $h_y\neq 0$; i.e., the magnetic field **H** is parallel to the surface, $h_z=0$). The surface impedance

$$Z=-\left.\frac{e_z}{h_y}\right|_{x=0}. \tag{59}$$

is convenient for characterizing the electromagnetic properties of the half-space. Let the frequency be such that two waves ($\omega>\mu_0 gH$, see Fig. 2) can propagate in the magnetic substance. Then

$$h_y=A_1e^{ik_1x}+A_2e^{ik_2x},$$

$$e_z=-\frac{k_1c}{\omega\varepsilon}\left(A_1e^{ik_1x}+\frac{k_2}{k_1}A_2e^{ik_2x}\right).$$

Whence

$$Z=\frac{ck}{\omega\varepsilon}\frac{1+(k_2/k_1)Q}{1+Q},\quad Q=\frac{A_2}{A_1}. \tag{60}$$

The parameter Q, which is the ratio of the amplitudes of the two waves, must be determined from the supplementary boundary conditions. We shall consider two cases (without discussing their physical meaning):

$$1)\ m_y|_{x=0}=0,\quad Q_1=-\frac{\mu_{\mathrm{eff}}^{\mathrm{I}}(k_1)-1}{\mu_{\mathrm{eff}}^{\mathrm{I}}(k_2)-1},$$
$$2)\ \left.\frac{dm_y}{dx}\right|_{x=0}=0,\quad Q_2=-\frac{k_1[\mu_{\mathrm{eff}}^{\mathrm{I}}(k_1)-1]}{k_2[\mu_{\mathrm{eff}}^{\mathrm{I}}(k_2)-1]}. \tag{61}$$

If the frequency is so large that it is possible to use Eq. (48), we get

$$\mu_{\mathrm{eff}}\simeq 1+\frac{4\pi}{\delta}\frac{v_a^2}{v_a^2-v^2},\quad v=\frac{\omega}{k},\quad k_1=\frac{\omega}{v_1},\quad k_2=\frac{\omega}{v_2},$$

$$v_1^2\simeq v_c^2+\frac{4\pi}{\delta}v_a^2,\quad v_2^2\simeq v_a^2-\frac{4\pi}{\delta}\frac{v_a^4}{v_c^2}. \tag{62}$$

Then, since $v_a\ll v_c$,

$$\frac{\mu_{\mathrm{eff}}^{\mathrm{I}}(k_1)-1}{\mu_{\mathrm{eff}}^{\mathrm{II}}(k_2)-1}=\frac{v_a^2-v_2^2}{v_a^2-v_1^2}\simeq\frac{4\pi}{\delta}\left(\frac{v_a}{v_c}\right)^4. \tag{63}$$

Consequently,

$$Q_1\simeq\frac{4\pi}{\delta}\frac{v_a^4}{v_c^4},\quad Q_2\simeq\frac{4\pi}{\delta}\left(\frac{v_a}{v_c}\right)^5. \tag{64}$$

This shows that, in the range under consideration, the impedance[4)] (and consequently, the reflection coefficient from the half-space) is virtually insensitive to the existence of the magnon branch ($\omega = v_1 k$, $v_1 \simeq v_c$). In this case, it is probably simplest to use light scattering to experimentally detect the magnon branch.

The impedance is more informative in another frequency range. Let us consider frequencies close to the "resonant" frequency $\omega = gH_0\sqrt{\mu_0}$. When we have $\omega < \mu_0 g H_0$ (with the polarization under consideration), only one wave propagates, so that, for $\omega < gH_0\sqrt{\mu_0}$, this is a photon-type wave, whereas, for $gH_0\mu_0 > \omega > gH_0\sqrt{\mu_0}$, it is magnon type.

To exactly calculate the fields and the impedance, it is of course necessary to allow for the existence of an exponentially damped field in the body.

Let $\omega = \mu_0 g H_0$ be the resonance frequency (in the absence of spatial dispersion; i.e., for $v_a = 0$). Then

$$k_{\binom{1}{2}} \simeq \binom{1}{i} \frac{\mu_0 g H}{\sqrt{v_a v_c}} \sqrt{\frac{4\pi}{\delta}}, \tag{65}$$

and the impedance is

$$Z \simeq \frac{c}{\varepsilon} \frac{1}{\sqrt{v_a v_c}} \sqrt[4]{\frac{4\pi}{\delta}} (1+i) \binom{1/2}{1}. \tag{66}$$

The upper row in the last equation corresponds to a fixed angular momentum at the boundary ($m_y|_{x=0} = 0$), and the lower row corresponds to a free angular momentum ($dm_y/dx|_{x=0} = 0$). We recall that, for $v_a = 0$, the impedance $Z = \sqrt{\mu/\varepsilon}$ goes to infinity even at resonance ($\omega = \sqrt{\mu_0} gH$), whereas, for a permittivity of $\varepsilon(\omega_R) = \infty$, the impedance goes to zero.

A study of the impedance (the shape of the resonance curve) can give much information concerning the electrodynamics of a polariton in a magnetic substance.

Knowing the permeability tensor makes it possible to solve various electrodynamic problems involving the study of surface magnetic polaritons at the magnet–vacuum boundary and/or in a plate (similar to what is done for a ferromagnet in Ref. 5).

Finally, the existence of a gapless (Goldstone) mode in a wide range of magnetic fields should be manifested in the static thermodynamics of antiferromagnets of the type under consideration. For example, there should be a magnon part of the heat capacity C_M of an antiferromagnet, proportional to T^3 (T is the temperature), with a coefficient that depends on the magnetic field [see Eq. (10)]. The effective Debye temperature Θ_H decreases (along with the velocity v_a) as the magnetic field increases [see the dispersion law of the magnetic vibrations given by Eq. (10)]; this results in an anomalous increase of the magnetic heat capacity C_M.

5. CONCLUDING REMARKS

The entire treatment has been carried out neglecting dissipative processes. Naturally, if we introduce dissipative terms into the Landau–Lifshitz equations, Eqs. (3), it is not especially hard to compute the (now non-Hermitian) components of the permeability tensor, and these can be used to compute the damping of those elementary excitations that we have considered. We have not done this for two reasons: first, we wanted to keep the exposition clear, and, second, we tried to show that it was of interest to study the dispersion laws under "clean" conditions, when dissipative processes could be neglected.

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[1)]Of course, it is quasimagnetostatics that is involved here, since the components of the χ_{ik} tensor contain a dependence on frequency ω.

[2)]Two polaritons can exist for a given polarization (because of spatial dispersion of the permeability). We emphasize that we regard as polaritons only undamped waves in a magnetic substance (naturally, in the absence of dissipative processes). See the remark in section IV on the number of solutions of Maxwell's equations.

[3)]Strictly speaking, we shall use plus polarization to mean the polarization that occurs in the limit ω, $k \to 0$. The functions μ_{xx}, μ_{yy}, and μ', in which both the numerator and the denominator change sign, are under a square root sign. For true plus polarization, it would be necessary to require that the expression under the square root be positive. In this case, different expressions would have to be used in different parts of the ωk plane. However, we shall use one expression.

[4)]Equations (64) show that the contribution of the magnon branch to the electromagnetic field is very small when $v_1 \ll v_2$. This result is not surprising and is associated with the wavelength difference at a fixed frequency.

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