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NONDIRECT DETERMINATION OF THE RATIO $R = \sigma_L/\sigma_T$ AT SMALL x FROM HERA DATA

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The values of the deep inelastic scattering cross-sections ratio $R = \sigma_L/\sigma_T$ are found in the range $10^{-4} \leq x \leq 10^{-2}$ from F_2 and $dF_2/d\ln Q^2$ HERA data using very simple relations based on perturbative QCD.

1. ВВЕДЕНИЕ

In recent years the behaviour of deep inelastic lepton–hadron scattering at the small values of Bjorken variable x has been intensively studied. One of the many interesting deep inelastic scattering variables is the ratio of cross-sections of the absorption of a longitudinally and transversely polarized photon by hadron: $R = \sigma_L/\sigma_T$. The ratio R, which may be represented as the combination of the longitudinal $F_L(x, Q^2)$ and transverse $F_2(x, Q^2)$ DIS structure functions:

$$R(x, Q^2) = \frac{F_L(x, Q^2)}{F_2(x, Q^2) - F_L(x, Q^2)}, \tag{1}$$

is a very sensitive QCD characteristic because it vanishes for free quarks. At small values of x, R data are not yet available, as they require a rather cumbersome procedure (see [1], for example) for the extraction from the experiment.

We study the behaviour of $R(x, Q^2)$ at small values of x, using the H1 data [2, 3] and the method [4] of replacement of the Mellin convolution by ordinary products. By analogy with the case of the gluon distribution function (see [3, 5–7]) it is possible to obtain the relation between $F_L(x, Q^2)$, $F_2(x, Q^2)$, and $dF_2(x, Q^2)/d\ln Q^2$ at small x. Thus, the small x behaviour of the ratio $R(x, Q^2)$ can be extracted directly from the measured values of $F_2(x, Q^2)$ and its derivative. These extracted values of R may be well considered as new small x «experimental data»[1]. Moreover, when experimental data for R at small x become available with a good accuracy, a violation of this exactly perturbative relation will be an indication of the importance of other effects as higher twist contribution and/or of nonperturbative QCD dynamics at small x.

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1) Although with the theoretical prejudice contained in the starting QCD relation.

We follow the notation of our previous works [7, 8]. The singlet quark $s(x, Q_0^2)$ and gluon $g(x, Q_0^2)$ parton distributions[2)] at some Q_0^2 are parameterized by (see, for example, [9])

$$p(x, Q_0^2) = A_p x^{-\delta}(1-x)^{\nu_p}(1+\epsilon_p\sqrt{x}+\gamma_p x) \tag{2}$$

(hereafter $p = s, g$).

Further, we restrict the analysis to the case of large δ values (i.e., $x^{-\delta} \gg 1$) following recent H1 data [2]. The more complete analysis concerning to the extraction of the longitudinal structure function $F_L(x, Q^2)$, may be found in [8], where we took into account also the case $\delta \sim 0$ corresponding to the standard pomeron.

Assuming the Regge-like behaviour for the gluon distribution and $F_2(x, Q^2)$ at $x^{-\delta} \gg 1$:

$$g(x, Q^2) = x^{-\delta}\tilde{g}(x, Q^2), \quad F_2(x, Q^2) = x^{-\delta}\tilde{s}(x, Q^2),$$

we obtain the following equation for the Q^2 derivative of the structure function F_2 [3)]:

$$\begin{aligned}
\frac{dF_2(x, Q^2)}{d\ln Q^2} &= -\frac{1}{2}x^{-\delta}\sum_{p=s,g}\left[r_{sp}^{1+\delta}(\alpha)\,\tilde{p}(0, Q^2) + r_{sp}^{\delta}(\alpha)\,x\tilde{p}'(0, Q^2) + O(x^2)\right], \\
F_L(x, Q^2) &= x^{-\delta}\sum_{p=s,g}\left[r_{Lp}^{1+\delta}(\alpha)\,\tilde{p}(0, Q^2) + r_{Lp}^{\delta}(\alpha)\,x\tilde{p}'(0, Q^2) + O(x^2)\right],
\end{aligned} \tag{3}$$

where $r_{sp}^{\eta}(\alpha)$ and $r_{Lp}^{\eta}(\alpha)$ are the combinations of the anomalous dimensions of Wilson operators $\gamma_{sp}^{\eta} = \alpha\gamma_{sp}^{(0),\eta} + \alpha^2\gamma_{sp}^{(1),\eta} + O(\alpha^3)$ and Wilson coefficients[4)] $\alpha B_L^{p,\eta}\left(1 + \alpha R_L^{p,\eta}\right) + O(\alpha^3)$ and $\alpha B_2^{p,\eta} + O(\alpha^2)$ of the η «moment» (i.e., the corresponding variables extended from integer values of argument to noninteger ones):

$$\begin{aligned}
r_{Ls}^{\eta}(\alpha) &= \alpha B_L^{s,\eta}\left[1 + \alpha\left(R_L^{s,\eta} - B_2^{s,\eta}\right)\right] + O(\alpha^3), \\
r_{Lg}^{\eta}(\alpha) &= \frac{e}{f}\alpha B_L^{g,\eta}\left[1 + \alpha\left(R_L^{g,\eta} - \frac{B_2^{g,\eta}B_L^{s,\eta}}{B_L^{g,\eta}}\right)\right] + O(\alpha^3), \\
r_{ss}^{\eta}(\alpha) &= \alpha\gamma_{ss}^{(0),\eta} + \alpha^2\left(\gamma_{ss}^{(1),\eta} + B_2^{g,\eta}\gamma_{gs}^{(0),\eta} + 2\beta_0 B_2^{s,\eta}\right) + O(\alpha^3), \\
r_{sg}^{\eta}(\alpha) &= \frac{e}{f}\left\{\alpha\gamma_{sg}^{(0),\eta} + \alpha^2\left[\gamma_{sg}^{(1),\eta} + B_2^{s,\eta}\gamma_{sg}^{(0),\eta} + B_2^{g,\eta}\left(2\beta_0 + \gamma_{gg}^{(0),\eta} - \gamma_{ss}^{(0),\eta}\right]\right\}\right] + O(\alpha^3),
\end{aligned} \tag{4}$$

and

$$\tilde{p}'(0, Q^2) \equiv \frac{d}{dx}\tilde{p}(x, Q^2) \quad \text{at} \quad x = 0,$$

where $e = \sum_i^f e_i^2$ is the sum of squares of quark charges.

With accuracy of $O(x^{2-\delta})$, we have for Eq. (3)

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2) We use parton distributions multiplied by x and neglect the nonsinglet quark distribution at small x.

3) Hereafter we use $\alpha(Q^2) = \alpha_s(Q^2)/4\pi$.

4) Because we consider here $F_2(x, Q^2)$ but not the singlet quark distribution.

$$\frac{dF_2(x,Q^2)}{d\ln Q^2} = -\frac{1}{2}\left[r_{sg}^{1+\delta}(\xi_{sg})^{-\delta} g\left(\frac{x}{\xi_{sg}},Q^2\right) + r_{ss}^{1+\delta}F_2(x,Q^2) + (r_{ss}^{\delta} - r_{ss}^{1+\delta})x^{1-\delta}\tilde{s}'(x,Q^2)\right] + \\ + O(x^{2-\delta}) \tag{5}$$

$$F_L(x,Q^2) = r_{Lg}^{1+\delta}(\xi_{Lg})^{-\delta} g\left(\frac{x}{\xi_{Lg}},Q^2\right) + r_{Ls}^{1+\delta}F_2(x,Q^2) + (r_{Ls}^{\delta} - r_{Ls}^{1+\delta})x^{1-\delta}\tilde{s}'(x,Q^2) + \\ + O(x^{2-\delta}) \tag{6}$$

with $\xi_{sg} = r_{sg}^{1+\delta}/r_{sg}^{\delta}$ and $\xi_{Lg} = r_{Lg}^{1+\delta}/r_{Lg}^{\delta}$. From Eqs. (5) and (6) one can obtain F_L as a function both of F_2 and the derivative:

$$F_L(x,Q^2) = -\xi^{\delta}\left[2\frac{r_{Lg}^{1+\delta}}{r_{sg}^{1+\delta}}\frac{dF_2(x\xi,Q^2)}{d\ln Q^2} + \left(r_{Ls}^{1+\delta} - \frac{r_{Lg}^{1+\delta}}{r_{sg}^{1+\delta}}r_{ss}^{1+\delta}\right)F_2(x\xi,Q^2) + O(x^{2-\delta},\alpha x^{1-\delta})\right], \tag{7}$$

where the result is restricted to $O(x^{2-\delta},\alpha x^{1-\delta})$. To arrive to the above equation we have performed the substitution

$$\xi_{sg}/\xi_{Lg} \to \xi = \gamma_{sg}^{(0),1+\delta}B_L^{g,\delta}/\gamma_{sg}^{(0),\delta}B_L^{g,1+\delta}$$

and neglected the term $\sim \tilde{s}'(x\xi_{sg},Q^2)$.

This replacement is very useful. The anomalous dimensions $\gamma_{sp}^{(1),n}$ in the next-to-leading order approximation (NLO) are singular in both points, $n = 1$ and $n = 0$, and their presence in the arguments of $\tilde{p}(x,Q^2)$ makes the numerical agreement between this approximate formula and the exact calculation worse (we have checked this point using some MRS sets [9] of parton distributions).

Using NLO approximation of $r_{sp}^{1+\delta}$ and $r_{Lp}^{1+\delta}$ for the specific value $\delta = 0.3$, we obtain (for $f = 4$ and $\overline{\rm MS}$ scheme)

$$F_L(x,Q^2) = \frac{0.84}{1+59.3\alpha}\left[\frac{dF_2(0.48x,Q^2)}{d\ln Q^2} + 3.59\alpha F_2(0.48x,Q^2)\right] + O(\alpha^2,x^{2-\delta},\alpha x^{1-\delta}). \tag{8}$$

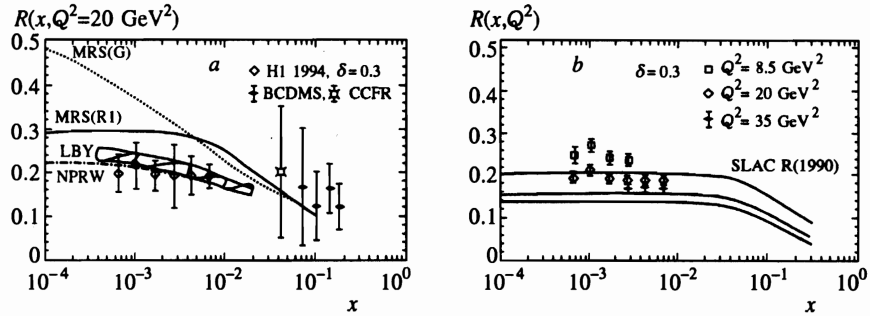
Notice that the α_s correction in the denominator of the factor in the right-hand side of Eq. (8) gives a large contribution. For example, at $Q^2 = 20$ GeV^2 the denominator is $1 + 0.92$.

With the help of Eqs. (1) and (8) we have extracted the ratio $R(x,Q^2)$ from H1 1994 data [2], determining the slopes $dF_2/d\ln Q^2$ from straight line fits as in [3, 5]. In the present calculation, only statistical errors from that measurements have been taken into account. To estimate the systematic errors that have been added in quadrature, we have used those from an early analysis performed by H1 [3]. In the calculation of the running coupling constant $\alpha_s(Q^2)$ at two loops we have taken $\Lambda_{\overline{\rm MS}}^{(4)} = 225$ MeV.

Figure *a* shows the extracted ratio R at $Q^2 = 20$ GeV^2 using the above formula for $\delta = 0.3$. This value of δ is very close to those obtained by various groups from QCD fits to H1 data [2, 10]. Fig. 1a also shows BCDMS [11] and preliminary CCFR (see [12]) data points with much larger errors.

For comparison we have also plotted various predictions for R using QCD formulas at $O(\alpha_s^2)$ [13–15][5] and parton densities extracted from fits to HERA data. The large difference

[5] The quark singlet and nonsinglet kernels in $\overline{\rm MS}$ scheme are taken from [13]. It was noted in [16] that the gluon kernel given in [13] is erroneous. We use the correct result given in [14, 15].



The ratio $R = \sigma_L/\sigma_T$ at small x. The points were extracted from Eqs. (1) and (8) using H1 [2, 3] data. The dashed-dotted line (NPRW) is the prediction of Saclay group [19] based on the dipole picture of BFKL dynamics. The band represents the uncertainty from the DGLAP analysis of HERA data in [18]. It is also shown BCDMS data [11] points at high x and the preliminary CCFR data point from [12]. The solid lines (in Fig. *b*) are the SLAC R(1990) parametrization [20] at $Q^2 = 8.5, 20$ and 35 GeV^2 (lower curve corresponds to lower Q^2 value)

between the result from MRS(G) and the latest set MRS(R1) [17] shows the sensitivity of R to the update of these parton densities to new HERA data. One can also notice that all these predictions remains higher than our extracted points.

By other part, recent theoretical calculations on R based on the conventional NLO Dokshitzher–Gribov–Lipatov–Altarelli–Parisi (DGLAP) evolution analysis of HERA data (LBY) [18] and on the dipole picture of the Balitskii–Fadin–Kuraev–Lipatov (BFKL) dynamics (NPRW) [19] are in a very good agreement with our points obtained with Eq. (8).

Finally, Fig. *b* shows the extracted R with $\delta = 0.3$ at three different Q^2 values showing only statistical errors (to avoid the strong overlap between the data points at different Q^2 values), in comparison with the SLAC R(1990) parametrization [20] based on larger x data. A relatively good agreement at $x \le 10^{-2}$ is achieved when the systematical errors are taken into account. Notice that the points at the same x and different Q^2 are correlated by the form in which the derivative term $dF_2/d\ln Q^2$ is determined.

In summary, we have extracted the ratio $R = \sigma_L/\sigma_T$ at small x from the structure function F_2 and its Q^2 derivative with the help of Eqs. (1), (7), (8). These equations provide the possibility of the nondirect determination of R. This is important since the direct extraction of R from experimental data is a cumbersome procedure (see [1]). Moreover, the fulfillment of Eqs. (1), (7), (8) by deep inelastic scattering experimental data is a cross-check of perturbative QCD at small values of x. Our formulas can also be used as a parametrization of R as a function of the most widely used phenomenological F_2.

The results depend on the specific value of the slope δ. In the case $\delta = 0.3$, which is very close to the values obtained by H1 group [2] at the considered Q^2 interval, we found a relatively good agreement with the SLAC parametrization [20] and also a very good agreement with the studies based on NLO DGLAP and BFKL dynamics (see [18] and [19], respectively). However the calculation performed with the latest sets of HERA parton densities using perturbative QCD at second order (see MRS(R1) curve in Fig. *a*) predicts a slightly higher value of R.

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