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OSCILLATORY DISINTEGRATION OF NONEVOLUTIONARY MAGNETOHYDRODYNAMIC DISCONTINUITIES

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Trans-Alfvénic shock waves are considered in the approximation of small amplitude and almost parallel propagation to the magnetic field. Such shocks are nonevolutionary, since the problem of time evolution of their small perturbation does not have a unique solution. Therefore, they cannot exist as stationary configurations and must disintegrate or transform to some more general, nonsteady flow. The disintegration configuration necessarily includes an Alfvén discontinuity that is also nonevolutionary. It is shown that the contradiction inherent in the nonevolutionary configuration is removed if its time evolution has the form of oscillatory disintegration, i.e., reversible transformation of one type of the discontinuity to the other. In this process fast and slow shock or rarefaction waves as well as contact discontinuities are emitted.

1. INTRODUCTION

The problem of disintegration of hydrodynamic discontinuities has a long history since the publication of the paper by Kotchine [1]. He considered the disintegration of an arbitrary discontinuity into a set of other discontinuities and rarefaction waves in the framework of nonmagnetic hydrodynamics. Some time later, Bethe [2] studied the disintegration of a shock wave. Magnetic field complicates the situation, enlarging the number of possible disintegration configurations. For a small-amplitude arbitrary discontinuity such configurations were obtained by Lyubarskii and Polovin [3]. In general, the problem cannot be solved in an analytical form. Gogosov [4] has given a quantitative solution that determines the type of the configuration, depending on the flow parameters.

The disintegration of a shock wave is closely related to the problem of its evolutionarity, formulated in Refs. [5–7]. It is suggested that small perturbations should be imposed on the discontinuity surface to study the question of its disintegration. In this case small-amplitude waves occur on both sides of the surface. The amplitudes of these waves are related by the linearized boundary conditions obtained from conservation laws at the discontinuity. If the amplitudes of the outgoing waves cannot be determined unambiguously from these conditions by the amplitudes of the incident waves, then the problem of the time evolution of the infinitesimal perturbations does not have a unique solution, and the discontinuity is called nonevolutionary. This problem is encountered when the number of unknown parameters (the amplitudes of the outgoing waves and the discontinuity displacement) is incompatible with the number of independent equations.

Since a physical problem must have a unique solution, it is not correct to assume that the perturbation of a nonevolutionary discontinuity is infinitesimal. Such a discontinuity cannot exist in a real medium as a stationary configuration, because the infinitesimal perturbation leads to a finite variation of the initial flow. This variation is the disintegration of the discontinuity into other discontinuities, which move away from the place of their formation, or a transition to a more general nonsteady flow. In the ideal medium the disintegration is instantaneous in the sense that the secondary discontinuities become separated in the beginning of the disintegration process. In a dissipative medium the spatial profiles of the magnetohydrodynamic (MHD) properties are continuous. Nevertheless, the principal result remains the same, the flow is rearranged toward a nonsteady state, and after a large enough period of time the disintegration manifests itself.

The evolutionarity requirement gives additional (compared to the Zemplen theorem) restrictions on the flow parameters at the shock surface. They follow from the fact that the direction of wave propagation (toward the discontinuity surface or away from it) and hence the number of the outgoing waves depends on the flow velocity at the surface. If it is large enough, then the given wave may be carried down by the flow. Therefore, at an evolutionary discontinuity the flow velocity must be such that it provides the compatibility of the set of boundary equations. This form of evolutionarity condition was applied to MHD shock waves in Refs. [8–10].

As a result, the fast (I $\rightarrow$ II) and slow (III $\rightarrow$ IV) shocks, for which the flow velocity both upstream and downstream is larger and smaller than the Alfvén velocity, respectively, are evolutionary, while the trans-Alfvénic shock waves (TASWs) are not. Here the Roman numbers indicate the states upstream and downstream of the shock, in which the values of the normal flow velocity fall into the intervals separated by the three phase velocities: fast magnetosonic, Alfvén, and slow magnetosonic velocities. These states are arranged in order of increasing entropy.

The important fact that favors the nonexistence of nonevolutionary shock transitions is that they can (while the evolutionary ones cannot) be realized also through a set of several shock and rarefaction waves [11–13]. One more argument for the nonexistence of nonevolutionary shocks is that they are isolated solutions of the Rankine–Hugoniot problem that do not have neighboring solutions corresponding to small deviation of boundary states [14]. This is confirmed by the fact that the configurations neighboring to such shocks are time-dependent [15, 16]. For such configurations the coplanarity of the boundary states is violated and therefore they are not solutions of the Rankine–Hugoniot problem.

The problem of structural instability of MHD shocks has recently progressed to a new point due to the consideration of a nonplanar shock structure [17–20]. Kennel et al. [19] discussed nonplanar shocks of small amplitude. They have demonstrated that the structure of nonevolutionary, TASWs, is not unique. Namely, the transitions II $\rightarrow$ III can be connected by two integral curves, left-hand and right-hand polarized, and the transitions I $\rightarrow$ III and II $\rightarrow$ IV allow an infinite number of connecting integral curves. These conclusions are in agreement with those of Hau and Sonnerup [18], who analyzed the stationary points of the MHD equations corresponding to the boundary states of the shock transitions in the case where the magnetic diffusivity is the only nonzero transport coefficient. Recall that under the assumption of a planar shock structure the trans-Alfvénic transitions also do not have a unique structure for all values of the dissipative transport coefficients [21-23].

It can be shown [19] that the integral over the shock thickness of the out-of-plane component of the magnetic field is independent of time if the upstream and downstream states of the small amplitude shock satisfy the Rankine–Hugoniot conditions. This means that the flow outside the shock is in one plane. This integral characterizing the nonplanar structure remains constant during the evolution of the initial profile, and it labels uniquely the integral curve that connects the given states. For the evolutionary, I $\rightarrow$ II and III $\rightarrow$ IV, shocks the flow is planar, and the integral is zero. For the shock transition II $\rightarrow$ III the integral takes two values with equal modules and opposite signs, and for the transitions II $\rightarrow$ IV and I $\rightarrow$ III it falls into some interval which describes a one-parameter family of structures.

To remove the ambiguity of the solution for the TASWs, Kennel *et al.* [19] postulated that in addition to the boundary states, the integral characterizing the nonplanar structure should be fixed. The solution of the boundary value problem describing the shock will then be unique. To assure this, however, one must assume that the configuration contains one shock. This is not the only possibility. As discussed above, the trans-Alfvénic shock transitions can be realized also through a set of several shock and rarefaction waves. Thus, the ambiguity of the structure connecting the nonevolutionary boundary states is not lifted [24], and the TASW is structurally unstable. Nevertheless, the conservation of the quantity that fixes the structure of the nonplanar shock is an additional factor that governs the disintegration process.

Indeed, at a TASW the tangential magnetic field changes sign. Consequently, it must change sign at a secondary discontinuity. This may take place either at another TASW or at an Alfvén discontinuity. As is known [25], the Alfvén discontinuity is also nonevolutionary in the presence of arbitrary small, but nonzero dissipation. Since the evolution of a nonplanar shock is related to its structure, it cannot be assumed that the dissipation is absent. Therefore, the Alfvén discontinuity also cannot exist as a stationary configuration. In the present paper we suggest a new scenario for the evolution of the TASW, oscillatory disintegration, i.e., reversible transformation to the Alfvén discontinuity. Such a form of evolution resolves the contradiction inherent in the nonevolutionary configurations.

We consider the small amplitude shocks that propagate almost parallel to the magnetic field. This approximation allows us to use an analytical approach, and, at the same time, to determine some features of the behavior of finite-amplitude discontinuities. In Sec. 2 we obtain the disintegration configuration for the TASW of small amplitude that propagate almost parallel to the magnetic field. In Sec. 3 we discuss the evolutionarity and some other properties of nonplanar shocks. In Sec. 4 we describe the time evolution of the TASW in the case where the transverse magnetic field is not small and in the case of almost parallel propagation. Finally, we present our conclusions in Sec. 5.

2. STRUCTURAL INSTABILITY OF TRANS-ALFVÉNIC SHOCKS

We first consider the disintegration configurations of small-amplitude, almost parallel MHD shocks. We choose the frame of reference in which $\mathbf{B} \parallel \mathbf{v}$ and the x axis is directed along the normal to the discontinuity. We proceed from the following jump conditions at the discontinuity surface:

$$\Delta(\rho v_x) = 0, \tag{1}$$

$$\Delta p + \rho v_x \Delta v_x + \frac{1}{8\pi}\Delta(B_y^2) = 0\,, \tag{2}$$

$$\rho v_x \Delta v_y - \frac{B_x}{4\pi}\Delta B_y = 0\,, \tag{3}$$

$$v_y = v_x B_y / B_x\,, \tag{4}$$

$$\frac{1}{2}\rho v_x \Delta(v^2) + \frac{\gamma}{\gamma - 1}\Delta(p v_x) = 0 . \tag{5}$$

In solving this set of equations we imply that the variations Δ of all MHD properties, except B_y, are small compared to their values at a reference state, and that $B_y \ll B_x$. To the lowest order in the small parameters B_y/B_x and $\Delta\rho/\rho$ Eqs. (1)–(5) have the following solutions which describe the relationship between the jumps of the MHD properties. For the first four solutions $\Delta B_y \gtrsim B_y$,

$$\Delta B_y = -B_y \left(1 \pm \sqrt{1 + 2\frac{V_{Ax}^2 - V_s^2}{V_{Ay}^2}\frac{\Delta\rho}{\rho}}\right) \equiv B_y A , \tag{6}$$

$$\Delta v_y = -\varepsilon V_{Ay} A, \tag{7}$$

$$\Delta v_x = \varepsilon V \Delta\rho/\rho, \tag{8}$$

$$\Delta p = V_s^2 \Delta\rho, \tag{9}$$

where $V = V_{Ax}$ and $\mathbf{V}_A = \mathbf{B}/\sqrt{4\pi\rho}$ is the Alfvén velocity. Here $\varepsilon = +1$ for the waves moving in the positive x direction and $\varepsilon = -1$ for the waves moving in the opposite direction. We assume for definiteness that $V_{Ax} > V_s$. The plus («+») sign in Eq. (6) will then correspond to a TASW and the minus («−») sign will correspond to a fast shock wave. In zeroth approximation the propagation velocity of both waves is V_{Ax}. The TASW is of the II → III type, i.e.,

$$V_{+1} > v_{x1} > V_{Ax1}, \qquad V_{Ax2} > v_{x2} > V_{-2}$$

if

$$-1 \le \frac{B_{y1}}{B_{y2}} \le -\frac{1}{2}, \tag{10}$$

and of I → III type, i.e,

$$v_{x1} > V_{+1}, \qquad V_{Ax2} > v_{x2} > V_{-2}$$

if

$$-\frac{1}{2} \le \frac{B_{y1}}{B_{y2}} \le 0 . \tag{11}$$

Here

$$V_\pm^2 = \frac{1}{2}\left[V_A^2 + V_s^2 \pm \sqrt{(V_A^2 + V_s^2)^2 - 4V_s^2 V_{Ax}^2}\right]$$

are the phase velocities of the fast (+) and slow (−) small-amplitude waves, and the subscripts «1» and «2» indicate the states upstream and downstream of the shock, respectively.

Two more solutions of the set of equations (1)–(5), for which $\Delta B_y \ll B_y$, correspond to slow shocks. They are given by the formulae

$$\Delta B_y = -\frac{V_s^2 B_y}{V_{Ax}^2 - V_s^2} \frac{\Delta\rho}{\rho}, \tag{12}$$

$$\Delta v_y = \varepsilon \frac{V_s V_{Ay} V_{Ax}}{V_{Ax}^2 - V_s^2} \frac{\Delta\rho}{\rho}, \tag{13}$$

and by Eqs. (8) and (9) with $V = V_s$, where $V_s = \sqrt{\gamma p/\rho}$ is the sound velocity. Expressions (8), (9), (12), and (13) coincide with those for small-amplitude waves. At an Alfvén discontinuity

$$\Delta B_y = B_y(\pm 1 - 1), \qquad \Delta v_y = -\varepsilon V_{Ay}(\pm 1 - 1), \tag{14}$$

where the + sign is used if the discontinuity is absent and the − sign is used if it rotates the magnetic field through 180°. Finally, the only nonzero jump at a contact discontinuity is $\Delta\rho$.

Assume now that the TASW with the amplitude $\Delta_0\rho$ moves in the positive x direction. If the shock transition can be represented as a set of more than one discontinuity, the amplitudes of the secondary discontinuities are determined from the condition that the sums of the jumps of the MHD properties at them are equal to those at the initial shock. It should be mentioned that in so doing the variation of B_y at the secondary waves must be taken into account in Eqs. (6)–(9) and (12)–(14), while the other quantities equal to their values upstream of the initial shock may be substituted. We thus find that in zeroth approximation the trans-Alfvénic shock transition can be realized through a fast shock, with the same amplitude $\Delta_f^{(+1)}\rho = \Delta_0\rho$, and the Alfvén discontinuity moving in the same direction. However, since these secondary waves move with zero velocity with respect to each other, there is no disintegration.

Let us now solve Eqs. (1)–(5), taking into account higher-order terms. For simplicity we assume that $B_y^2/B_x^2 \lesssim \Delta\rho/\rho$. Otherwise (when $B_y^2/B_x^2 \gg \Delta\rho/\rho$), the approximation of almost parallel propagation is violated. The corrections to the quantities (6)–(9) are determined by the expressions that follow from the expansion of Eqs. (1)–(5) in the small parameters B_y/B_x and $\Delta\rho/\rho$,

$$\Delta B_y = B_y(A + a), \tag{15}$$

$$\Delta v_y = -\varepsilon V_{Ay}\left[A + a - \frac{1}{2}(1 + A)\frac{\Delta\rho}{\rho}\right], \tag{16}$$

$$\Delta v_x = \varepsilon V_{Ax}\frac{\Delta\rho}{\rho}\left[1 - \frac{1}{2}\left(1 - \frac{1}{A}\right)\frac{\Delta\rho}{\rho}\right], \tag{17}$$

$$\Delta p = V_s^2\Delta\rho + \frac{V_{Ax}^2(\Delta\rho)^2}{\rho A} - \frac{B_y^2}{4\pi}a(1 + A). \tag{18}$$

Here

$$a = \frac{2A^3 + (\gamma + 4)A^2 + b_1 A + b_2}{2A(1 + A)} \frac{\Delta\rho}{\rho}, \tag{19}$$

$$b_1 = 2 + 2b_2 - (\gamma + 3)\frac{V_{Ax}^2}{V_{Ay}^2}\frac{\Delta\rho}{\rho}, \qquad b_2 = 2\frac{V_s^2}{V_{Ay}^2}\frac{\Delta\rho}{\rho}. \tag{20}$$

The velocity in front of such shocks is

$$v_x = -\varepsilon V_{Ax}\left[1 + \frac{1}{2}\left(1 + \frac{1}{A}\right)\frac{\Delta\rho}{\rho}\right]. \tag{21}$$

Next, we set equal again the sums of the jumps at the secondary waves to those at the initial TASW. In substituting the relationship between the MHD properties for the waves absent in zeroth approximation it is sufficient to use Eqs. (6)–(9), (12), and (13), which are valid in the lowest order, because the amplitudes of these waves are small compared to the amplitude of the initial shock.

As a result, we find that the fast wave moving in the direction opposite to the initial shock has the amplitude

$$\Delta_f^{(-1)}\rho = \frac{1}{2}\frac{V_{Ay1}^2 A_-(1 + A_-)\Delta_0\rho}{V_{Ax}^2 - V_s^2}, \tag{22}$$

the amplitude of the contact discontinuity is

$$\Delta_c\rho = -(\gamma - 1)\Delta_0\rho\frac{V_{Ay1}^2}{V_s^2}\sqrt{1 + 2\frac{V_{Ax}^2 - V_s^2}{V_{Ay1}^2}\frac{\Delta_0\rho}{\rho}}, \tag{23}$$

and the amplitudes of the slow waves moving in the positive and negative x directions are

$$\Delta_s^{(\varepsilon)}\rho = -\frac{1}{2}\Delta_c\rho + \varepsilon\frac{V_{Ax}}{V_s}\Delta_f^{(-1)}\rho + \frac{\rho(1+A_-)V_{Ay1}^2}{2(V_{Ax}^2 - V_s^2)}\left[a_+ + a_- + \varepsilon\frac{V_{Ax}}{V_s}\left(a_+ + a_- - \frac{1}{2}\frac{\Delta_0\rho}{\rho}\right)\right]. \tag{24}$$

Here A and a are given by Eqs. (6), (19), and (20), in which $B_y = B_{y1}$ and $\Delta\rho = \Delta_0\rho$, and the subscripts «+» and «−» correspond to the sign in Eq. (6). It can be shown, with the help of Eqs. (17) and (21), that the absolute value of the normal flow velocity behind the fast shock moving in the positive x direction is larger than the normal Alfvén velocity in front of the Alfvén discontinuity. Hence, these waves become separated as time goes on (Fig. 1).

Note that if V_s is comparable with V_{Ax}, then the flow in the fast and trans-Alfvénic shocks, which is determined by Eqs. (6)–(9), is isentropic, but only to the lowest order. The jump of the entropy at such shocks is

$$\Delta s = \frac{\gamma}{4}\frac{V_{Ay}^2}{V_s^2}\frac{\Delta\rho}{\rho}A^2. \tag{25}$$

In case where $\Delta\rho/\rho \ll V_{Ay}^2/V_{Ax}^2$ this expression coincides with that obtained by Bazer and Ericson [26]. After the disintegration, the difference between the entropy jumps at the trans-Alfvénic and the fast shock is taken by the contact discontinuity at which the entropy jump is of the same order of magnitude as the density jump

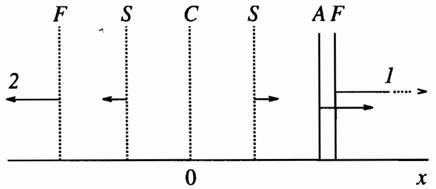

Fig. 1. Disintegration configuration for the trans-Alfvénic shock in the case $V_s \gg V_{Ay}$. In zeroth approximation the dotted lines are absent

$$\Delta s = \left(\frac{\partial s}{\partial \rho}\right)_p \Delta\rho = -\frac{\gamma}{\gamma - 1}\frac{\Delta\rho}{\rho}. \tag{26}$$

Thus, the consideration of the corrections to zeroth approximation reveals two properties of the disintegration configuration. First, the fast shock and the Alfvén discontinuity moving in the same direction as the initial shock acquire a small relative velocity. Second, the amplitudes of the discontinuities, which have finite velocity with respect to each other, become nonzero.

We emphasize that the absence of disintegration in zeroth approximation is essentially a consequence of the assumption that the tangential magnetic field is small. This result is consistent with the fact that the exactly parallel TASW can be represented as a switch-on and a switch-off shock, but they do not become separated [10, 11]. However, the parallel shock disintegrates when it collides with small-amplitude shocks incident on both sides of the discontinuity surface [11]. It should be mentioned that Eqs. (6) and (7) are not valid for the exactly parallel shock, because it necessarily has a finite amplitude if V_s is not close to V_{Ax}.

We can also consider the case $V_s \ll V_{Ax}$, which was discussed by Kennel et al. [19]. Note that, on the other hand, V_s^2 must be much larger than $V_{Ay}^2\Delta\rho/\rho$ (or V_{Ay}^4/V_{Ax}^2) for a small-amplitude shock. The character of the disintegration changes significantly when $V_s \lesssim V_{Ay}$. Under this condition the flow is not isentropic in the lowest order (see Eqs. (23), (25), and (26)). To this order the jumps $\Delta_c s$, $\Delta_c\rho$, and $\Delta_s^{(\varepsilon)}\rho$, are not equal to zero, in contrast with the case $V_s \gg V_{Ay}$. At the same time, the velocity of the slow shocks V_s is small. Therefore, in zeroth approximation they are not separated from the contact discontinuity which is at rest with respect to the medium.

As a result, the initial TASW disintegrates into two structures. The first structure, denoted by $A + F$, is formed by the Alfvén discontinuity and the fast shock, which are at rest with respect to each other. The second, denoted by $S + C + S$, is formed by two slow shocks and the contact discontinuity. These structures have the finite relative velocity V_{Ax}. The peculiarity of such a configuration is that the only nonzero total jump at the structure $S + C + S$ is $\Delta_c s$, because, as can be seen from Eqs. (22)–(24), the total density jump equals zero when $V_s \lesssim V_{Ay}$. Consequently, the disintegration in this case takes place in the lowest order, although its only manifestation is the time-dependent entropy profile along the x axis (Fig. 2). In higher orders the profiles of the other MHD properties also become nonsteady.

3. EVOLUTIONARITY OF NONPLANAR SHOCKS

Next, we consider the influence of the nonplanar structure on the shock evolution. If the boundary states of the shock are coplanar, the evolution of the configuration is characterized by additional conservation laws. This can be understood, for example, from the z-component of the induction equation,

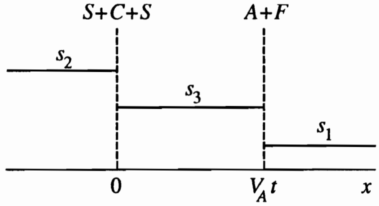

Fig. 2. Time-dependent entropy profile after the disintegration of the trans-Alfvénic shock in case $V_s \lesssim V_{Ay}$

$$\frac{\partial B_z}{\partial t} = -\frac{\partial}{\partial x}\left(B_z v_x - B_x v_z - \nu_m \frac{\partial B_z}{\partial x}\right), \tag{27}$$

where ν_m is the magnetic diffusivity. Since the flow outside the discontinuity is planar and homogeneous, the integration over the thickness of the transition layer $\Delta x = x_2 - x_1$ yields

$$\frac{\partial}{\partial t}\int_{x_1}^{x_2} B_z dx = 0. \tag{28}$$

Similarly, it follows from the z-component of the momentum equation,

$$\frac{\partial \rho v_z}{\partial t} = -\frac{\partial}{\partial x}\left(\rho v_x v_z - \frac{1}{4\pi}B_x B_z - \eta\frac{\partial v_z}{\partial x}\right), \tag{29}$$

that the integral of the quantity ρv_z is also conserved in the process of time evolution of the configuration,

$$\frac{\partial}{\partial t}\int_{x_1}^{x_2} \rho v_z dx = 0. \tag{30}$$

Here η is the shear viscosity.

As shown in Ref. [19], if the integral of B_z over the profile of the small amplitude almost parallel II $\to$ III shock is prescribed, then the ratio B_{y1}/B_{y2} of the upstream to the downstream values of the transverse magnetic field is fixed. Thus, the state behind the shock is unambiguously determined by the state in front of it. This gives us an additional equation for the amplitudes of the waves that occur after the disintegration. Assuming that this equation is valid to the second order in the amplitude, it can be shown with the help of Eqs. (12), (13), and (15)–(18) that the amplitudes of the initial and secondary TASWs are equal, while all other secondary waves are absent. The same is true for the I $\to$ III shock. The difference is that in the latter case there is no additional equation, because the given integral over the profile allows infinite number of boundary states [19]. However, the fast wave moving in the direction of the initial shock is absent. Therefore, the number of equations is again equal to the number of secondary waves. These equations have only a trivial solution. Consequently, in the approximation under consideration a TASW cannot appear after the disintegration as a secondary wave.

It should be mentioned that the conclusion about the relationship between the integral quantity, which characterizes the structure, and the amplitude of the wave is made in Ref. [19] only for isentropic flows. As shown in Sec. 2, this is not always the case for the almost parallel propagation. Nevertheless, this conclusion may be also made on the basis of the following reasoning. The consideration of the stationary points of the MHD equations, corresponding to the boundary states of the shock transitions, shows that the transition II $\to$ III can be realized through two integral curves, right-hand and left-hand polarized, in contrast with the transitions I $\to$ III (II $\to$ IV) and I $\to$ IV which are described by one- and two-parameter families of curves, respectively (see, e.g., Ref. [18]). Since the structure of the II $\to$ III shock does not contain free parameters other than the amplitude (although it is nonunique), for the given quantity characterizing the structure and the given state in front of the shock its amplitude is fixed.

Below we restrict the discussion to the shocks of the II $\to$ III type. The evolution of these shocks is the most important feature for the following reason. In contrast with the other types, the nonevolutionarity of the II $\to$ III shocks is essentially based on the fact that for normally propagating waves the equations for the Alfvén perturbations are separated from those for the magnetosonic and entropy perturbations [10]. Therefore, the evolutionarity criterion must be satisfied separately for both groups of waves. This makes the shock nonevolutionary, although the total number of perturbations is compatible with the total number of boundary conditions. Since under the assumption of a nonplanar shock structure the separation does not take place [20], this argument formally does not hold. Nevertheless, the coupling of Alfvén modes to magnetosonic and entropy modes does not alter the conclusions made on the basis of the evolutionarity principle. Let us clarify this point.

In the linear approximation Eqs. (27) and (29) for the perturbations proportional to $\exp(i\omega t)$ take the following form after the integration over x

$$i\omega \int\limits_{x_1}^{x_2} \delta B_z dx = -\Delta \left(v_x \delta B_z - B_x \delta v_z - \nu_m \frac{\partial \delta B_z}{\partial x} \right), \tag{31}$$

$$i\omega \int\limits_{x_1}^{x_2} v_z \delta\rho dx + i\omega \int\limits_{x_1}^{x_2} \rho \delta v_z dx = -\Delta \left(\rho v_x \delta v_z - \frac{1}{4\pi} B_x \delta B_z - \eta \frac{\partial \delta v_z}{\partial x} \right), \tag{32}$$

where δ is the small perturbation, and the unperturbed quantities correspond to the stationary shock. The term responsible for the coupling is the first integral on the left-hand side of Eq. (32).

Let us assume now that the Alfvén wave, which transfers only the perturbations δv_z and δB_z, is incident on the II $\to$ III shock. In this case there is one outgoing Alfvén wave, whose amplitude is the unknown parameter that should be determined from Eqs. (31) and (32). Because the perturbation $\delta\rho$ enters into Eq. (32), when $v_z \neq 0$, the latter becomes an additional equation for the amplitudes of outgoing magnetosonic and entropy waves which may be generated by the incident Alfvén wave. The perturbation $\delta\rho$ inside the transition layer depends on the amplitudes of the waves outside it and on the stationary shock structure. Since $\delta\rho$ should be determined from Eq. (32), the term with $\delta\rho$ cannot be much smaller than all other terms. Estimating the first terms on the left- and right-hand sides of Eq. (32), we obtain in the order of magnitude

$$\omega v_{z0} \delta\rho_0 \Delta x \sim \rho_0 v_{x0} \Delta \delta v_z, \tag{33}$$

where the subscript 0 indicates some characteristic values.

In general, $\Delta\delta v_z \sim \delta v_{z0}$. In addition, the unperturbed quantities and the discontinuity thickness do not depend on the frequency of the perturbation. Consequently, for the perturbation with small enough ω the quantity $\delta\rho_0/\rho_0$ is arbitrarily large compared to $\delta v_{z0}/v_{z0}$. However, this result is valid for any nonevolutionary shock; namely, as discussed in Sec. 1, infinitesimal incident perturbation (δv_z in the present case) causes a finite variation of the flow ($\delta\rho$). Therefore, since the coupling of the modes is weak, the contradiction inherent in the II $\to$ III shock is not resolved even in the absence of the separation of the boundary conditions.

In connection with the nonunique shock structure, Wu [20] argued that the shocks of the remaining types become evolutionary if the free parameters characterizing their structure are added to the total number of perturbations of the shock. However, the additional free parameters on their own also do not resolve the contradiction. In the presence of dissipation such parameters are provided, in particular, by the amplitudes of purely dissipative waves, which are absent in the ideal medium [25]. The problem is that the boundary conditions, that follow from the conservation laws at the shock, are incompatible. Therefore, a free parameter contributes to the evolutionarity only if it enters into the conservation laws. For example, purely dissipative waves damp within the length of the order of the shock thickness. Consequently, their amplitudes do not enter into the boundary conditions, and they should be disregarded when solving the problem of evolutionarity of a shock wave, unless it is of switch-off or switch-on type [25]. At the same time, it can be shown that in the case of dissipative discontinuities inside inviscid shock waves the additional dissipative waves affect the evolutionarity condition. Apparently, the structure variations are also confined within the transition layer.

4. TIME EVOLUTION OF NONEVOLUTIONARY DISCONTINUITIES

Let us now turn to the time evolution of the TASWs. For an illustration we first consider the case in which the ratio B_y/B_x is not small. To the first order in $\Delta\rho/\rho$ we can then reduce the expressions for the jumps of the MHD properties at the TASW as follows [12]:

$$\Delta_0 Q_j = \Delta_A Q_j + A_{0j}\Delta_0\rho. \tag{34}$$

Here $\mathbf{Q} = (\rho, p, v_x, v_y, B_y)$ is the vector of state, i.e., the set of MHD properties, A_{0j} are the known coefficients, and Δ_A are the jumps at the Alfvén discontinuity given by Eq. (14). In the present case the jump $\Delta|B_y|$ is small compared to $|B_y|$, hence, the inequality (10) is satisfied, and the TASW is of the II $\to$ III type.

The jumps at the fast and slow shock (or rarefaction) waves coincide with those at the corresponding small-amplitude waves. As a result, the equations that determine the disintegration configuration take the form

$$\sum_i A_{ij}\Delta_i\rho + \Delta_A Q_j = A_{0j}\Delta_0\rho + \Delta_A Q_j, \tag{35}$$

where the subscript i indicates the type of the discontinuity, and A_{ij} are known quantities. The solution of these algebraic equations

$$\Delta_i\rho = a_i\Delta_0\rho \tag{36}$$

expresses the amplitudes of secondary waves in terms of the amplitude of the initial TASW. The explicit expressions for the quantities a_i are given in Ref. [12].

Thus, the initial nonevolutionary shock may disintegrate into a contact discontinuity, magnetosonic waves, and an Alfvén discontinuity. At the same time, as shown by Roikhvarger and Syrovatskii [25], the Alfvén and the contact discontinuities are also nonevolutionary in the presence of an arbitrarily small but nonzero dissipation and heat conduction. This stems from the fact that the flow velocity in this case is equal to the phase velocity of the Alfvén and the entropy wave, respectively. As a result, the wavelength of the small perturbation with a fixed frequency tends to zero. This makes it necessary to account for the dissipation and, as a consequence, leads to the occurrence of additional outgoing (dissipative) waves that damp within the length much larger than the discontinuity thickness.

However, the nonevolutionarity does not lead to the disintegration of the discontinuity if it cannot be represented as a set of several discontinuities. This is the case for a contact discontinuity. Indeed, under the condition that B_y is not small the jumps Δv_y and ΔB_y at a TASW or at an Alfvén discontinuity are much larger than all other jumps. To provide no field reversal in sum, the disintegration configuration must contain two discontinuities at which B_y and v_y change sign or it must not contain them at all. In the latter case the equations for the jumps have only the trivial solution. It can be readily verified, with the help of Eqs. (14) and (35), that in the former case Δv_y and ΔB_y cannot be compensated simultaneously, since Δv_y depends on the direction of propagation, while ΔB_y does not. Hence, the contact discontinuity is structurally stable. In contrast, the Alfvén discontinuity is unstable, and it may disintegrate.

On this basis we suggest a new scenario for evolution of the configurations with the magnetic field reversal, oscillatory disintegration. Under the action of an infinitesimal perturbation the TASW disintegrate into a system of waves including the Alfvén discontinuity. In this process the integrals (28) and (30) over the nonplanar profile of the initial wave are conserved and are equal to the integrals over the profile of the Alfvén discontinuity, while the flow in the remaining secondary waves is plane. In contrast with a shock wave, the out-of-plane structure of an Alfvén discontinuity is not related to the boundary states due to their degeneration. Therefore, such a disintegration configuration can always be adjusted to the initial discontinuity. The Alfvén discontinuity, in turn, also disintegrates, producing the TASW with an amplitude equal to that of the initial wave, which is unambiguously determined by the quantity fixing the structure.

The amplitudes of the remaining waves satisfy Eq. (35), in which $\Delta_0\rho$ is replaced on the right-hand side. Therefore, they are given by Eq. (36), in which a_i is replaced by $-a_i$; i.e., shock waves instead of rarefaction waves occur and vise versa, compared to the case of disintegration of the TASW. After that the process is repeated. The waves of different types may catch up with and outrun each other during their propagation. Since the waves are structurally stable, their types are not changed in this process. If the characteristic time between the disintegrations is not small, the waves of the same types catch up with each other at infinite time. In the approximation of small amplitudes the secondary waves do not interact with each other. This means that the feedback effect of the disintegration on the nonevolutionary discontinuity manifests itself in higher orders.

It should be mentioned that the contradiction associated with the nonevolutionarity of the Alfvén discontinuity is also resolved if it has a time-dependent thickness. To some extent, the situation is similar to that for the corrugationally unstable shocks in the nonmagnetic hydrodynamics [27, 28]. As is known [29, 30], such shock transitions can always be represented as a combination of several discontinuities. This makes it possible to assume that the unstable shocks may disintegrate rather than undergo the growing undulation [30]. However, as well as in the present case, the physical mechanism that distinguishes whether the discontinuity maintains itself during the evolution or transforms to another configuration remains unclear.

Let us return to the almost parallel shocks. In this case the equations that determine the disintegration configuration are not linear equations, in contrast with Eq. (35). Nevertheless, to the lowest order, the amplitudes of waves occurring after the disintegration of the Alfvén discontinuity and the TASW are also equal in absolute value and have opposite signs. This can be shown by indicating that the expressions for the variations of the MHD properties Δ at the rarefaction waves coincide with those for shock waves. In the approximation of small B_y the equations describing the rarefaction waves (see, e.g., Ref. [31]) take the form

$$\rho\frac{dB_y^2}{d\rho} = B^2 - 4\pi\rho V_s^2 \pm \left(|B_x^2 - 4\pi\rho V_s^2| + \frac{(B_x^2 + 4\pi\rho V_s^2)B_y^2}{|B_x^2 - 4\pi\rho V_s^2|}\right), \tag{37}$$

$$\frac{dv_y}{dB_y} = -\frac{1}{\sqrt{4\pi\rho}}, \tag{38}$$

$$\frac{dv_x}{d\rho} = \frac{V_A}{\rho}, \tag{39}$$

$$\frac{dp}{d\rho} = V_s^2. \tag{40}$$

We thus obtain to the lowest order Eqs. (6)–(9), (12), and (13) for the differences Δ of the downstream to the upstream values.

In the case of small B_y the difference between the velocity of the fast shock and rarefaction waves is small. Therefore, if the disintegration takes place at finite intervals, these waves may catch up with each other at a finite time. Moreover, under the condition $V_s \lesssim V_{Ay}$, the feedback effect of the disintegration on the initial wave is of the same order of magnitude as its amplitude. A disintegration scheme in this case is presented in Fig. 3. In this figure the lowest-order waves only are shown, $U = S + C + S$, and the TASW are denoted by TA. As can be seen from Fig. 3, after the fast rarefaction R_3 catches up with the fast shock F_2 the configuration U_4 that moves toward TA_3 remains there. Since the TASW are nonevolutionary and structurally unstable, their interaction results in disintegration.

After the complete cycle of the oscillatory disintegration the system comes to the state shown in Fig. 4. The quantities s_3 and s_4 are

$$s_3 = s_1 + \frac{\gamma}{4}\frac{V_{Ay1}^2}{V_s^2}\frac{\Delta_0\rho}{\rho}\left(1 - \sqrt{1 - \frac{V_{Ax}^2}{V_{Ay1}^2}}\right)^2, \tag{41}$$

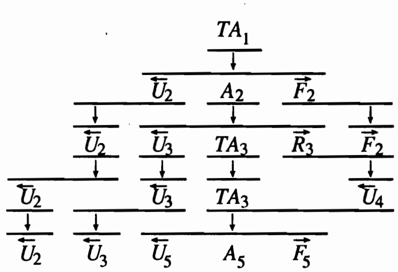

Рис. 3

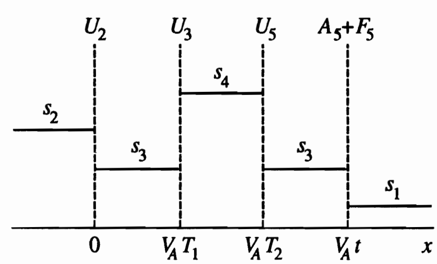

Рис. 4

Fig. 3. Scheme of oscillatory disintegration of the initial trans-Alfvénic shock (denoted by TA) into the Alfvén discontinuity (A), the fast shock (F) and rarefaction (R) waves, and the structure $U = S + C + S$. The vertical arrows show the time evolution and the horizontal arrows indicate the velocity with respect to the nonevolutionary discontinuity

Fig. 4. Entropy profile in the state of the system after the complete cycle of the disintegration. T_1 and T_2 are the moments of time at which the disintegration takes place

$$s_4 = s_1 + \gamma \frac{V_{Ay1}^2}{V_s^2} \frac{\Delta_0 \rho}{\rho} \left(1 + \frac{V_{Ax}^2}{V_{Ay1}^2} \right) . \tag{42}$$

This state resembles the state shown in Fig. 2, except for the configurations U_3 and U_5 that compensate each other. Thus, the nonevolutionary shock emits the discontinuities $S + C + S$ in the process of its evolution.

5. CONCLUSIONS

We have examined the disintegration of small-amplitude nonevolutionary shock waves. We have shown that in case of almost parallel propagation to the magnetic field the shock is structurally unstable in the second order in its amplitude. Such a shock transition can be represented as a set of several discontinuities moving with respect to each other. As a result, the shock structure is ambiguous not only because of the boundary states are connected by a nonunique integral curve, but also because of the shock transition can be realized through the single shock and through the configuration that consists of more than one discontinuity.

However, the disintegration configuration necessarily includes an Alfvén discontinuity that is also nonevolutionary. The contradiction can be resolved if the further time evolution has the form of oscillatory disintegration, i.e., reversible transformation to the Alfvén discontinuity. In this process shock and rarefaction waves, as well as contact discontinuities, which move with respect to each other, are emitted.

Such a process is similar to spontaneous emission of small-amplitude waves by a shock wave without a magnetic field. This phenomenon was observed in the laboratory experiments [32, 33]. It appears in the special case of corrugational instability of the shock when its small perturbation does not grow with time, but propagates away from the discontinuity surface in the form of nondamping waves, whose energy is supplied from the whole moving medium. The similarity is natural, because in this case the reflection and refraction coefficients at the discontinuity surface tend to infinity in the presence of incident waves (see, e.g., Ref. [34]). Consequently, such a shock is nonevolutionary, since the problem of small perturbation does not have a solution [24].

At the same time, the oscillatory disintegration has two significant distinctions. First, the amplitudes of the emitted waves, i.e, those occurring after the disintegration, are comparable with the amplitude of the initial wave. Second, the emission is associated with the transition from one type of the discontinuity to the other, rather than with the oscillation of the discontinuity surface.

Thus, the scenario of time evolution of a trans-Alfvénic shock wave suggested by us is in agreement with the viewpoint according to which the shock cannot exist as a stationary configuration.

References

1. N. E. Kotchine, Rendiconti del Circolo Matematico di Palermo **50**, 305 (1926).
2. H. A. Bethe, Office of Scientific Research and Development, Rep. № 445 (1942).
3. G. Ya. Lyubarskii and R. V. Polovin, Zh. Eksp. Teor. Fiz. **35**, 1291 (1958).
4. V. V. Gogosov, Prikl. Mat. Mekh. **25**, 108 (1961).

5. L. D. Landau, Zh. Eksp. Teor. Fiz. **14**, 240 (1944).
6. R. Courant and K. O. Fridrichs, *Supersonic Flows and Shock Waves*, Interscience Publ., New York (1948).
7. P. Lax, Comm. Pure Appl. Math. **10**, 537 (1957).
8. A. I. Akhiezer, G. Ya. Lyubarskii, and R. V. Polovin, Zh. Eksp. Teor. Fiz. **35**, 731 (1958).
9. V. M. Kontorovich, Zh. Eksp. Teor. Fiz. **35**, 1216 (1958).
10. S. I. Syrovatskii, Zh. Eksp. Teor. Fiz. **35**, 1466 (1958).
11. G. Ya. Lyubarskii and R. V. Polovin, Zh. Eksp. Teor. Fiz. **36**, 1272 (1959).
12. R. V. Polovin and K. P. Cherkasova, Zh. Eksp. Teor. Fiz. **41**, 263 (1961).
13. K. P. Cherkasova, Zh. Prikl. Mekh. Tekh. Fiz. №6, 169 (1961).
14. A. Kantrovitz and H. Petschek, in *Plasma Physics in Theory and Application*, ed. by W. B. Kunkel, McGraw-Hill, New York (1966), p. 148.
15. C. C. Wu and C. F. Kennel, Phys. Rev. Lett. **68**, 56 (1992).
16. C. C. Wu and C. F. Kennel, Phys. Fluids B **5**, 2877 (1993).
17. C. C. Wu, J. Geophys. Res. **93**, 987 (1988).
18. L.-N. Hau and B. U. Ö. Sonnerup, J. Geophys. Res. **94**, 6539 (1989).
19. C. F. Kennel, R. D. Blandford, and C. C. Wu, Phys. Fluids B **2**, 987 (1990).
20. C. C. Wu, J. Geophys. Res. **95**, 8149 (1990).
21. P. Germain, Rev. Mod. Phys. **32**, 951 (1960).
22. A. G. Kulikovskii and G. A. Liubimov, Prikl. Mat. Mekh. **25**, 125 (1961).
23. J. E. Anderson, *Magnetohydrodynamic Shock Waves*, M.I.T. Press, Cambridge, Massachusetts (1963).
24. S. A. Markovskii and B. V. Somov, Space Sci. Rev. **78**, 443 (1996).
25. Z. B. Roikhvarger and S. I. Syrovatskii, Zh. Eksp. Teor. Fiz. **66**, 1338 (1974).
26. J. Bazer and W. B. Ericson, Astrophys. J. **129**, 758 (1959).
27. S. P. D'yakov, Zh. Eksp. Teor. Fiz. **27**, 288, (1954).
28. V. M. Kontorovich, Zh. Eksp. Teor. Fiz. **33**, 1525 (1957).
29. C. S. Gardner, Phys. Fluids **6**, 1366 (1963).
30. N. M. Kuznetsov, Zh. Eksp. Teor. Fiz. **88**, 470, (1985).
31. A. G. Kulikovskii and G. A. Lyubimov, *Magnetohydrodynamics*, Fizmatgiz, Moscow (1962).
32. R. W. Griffits, R. J. Sandeman, and H. G. Hornung, J. Phys. D **8**, 1681 (1975).
33. T. I. Mishin, A. P. Bedin, N. I. Yushenkova, G. E. Skvortsov, and A. P. Ryasin, Zh. Tekh. Fiz. **51**, 2315 (1981).
34. L. D. Landau and E. M. Lifshitz, *Fluid Dynamics*, Nauka, Moscow (1986), p. 476.