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RADIATION OF A CHARGED PARTICLE IN A RANDOM STACK OF PLATES

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Radiation from a charged particle moving in a system of randomly spaced plates is considered. It is shown that the dominant radiation mechanism is diffusion. The total intensity of radiation is investigated, and its quadratic dependence on particle energy is noted in the optical region. A comparison with Cherenkov radiation is carried out.

1. INTRODUCTION

More than 50 years ago, Ginzburg and Frank [1] showed that radiation is produced when a charged, uniformly moving particle passes through the interface between two media with different dielectric constants. Since then, much research have been done on this problem (for a review, see for example Ginzburg and Tsytovich [2]). It turns out that the dependence of total intensity of radiation at an isolated interface on particle energy is logarithmic in the optical region. To be able to use transition radiation to detect relativistic charged particles, it is desirable to have a stronger energy dependence. In this context, the X-ray region turns out to be more promising, because in this region the energy dependence of the radiation intensity is linear [3]. However, the number of photons emitted at the interface is small. To increase this number, systems of many plates are used. Earlier, when investigating radiation in a stack of plates, mainly the X-ray region was considered (see for example Garibian and Yang [4]). In this region the interaction of the electromagnetic field with each plate is weak, so multiple scattering effects can be neglected.

The objective of our paper is to take these effects into account when charged particles radiate while traversing a random stack of plates. Having considered three-dimensional random media [5], we know that multiple scattering effects in the electromagnetic field play a crucial role in the radiation of a charged particle. Below we show that in the one-dimensional case, these effects play an even more important role, particularly in the optical region.

2. FORMULATION OF THE PROBLEM

The system which we want to study is a stack of plates randomly spaced in a homogeneous medium. Let us assume that the plates fill the regions $z_i - a/2 < z < z_i + a/2$ (where a is the plate thickness and z_i are random coordinates). The permitivity of the system can be represented in the form

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$$\varepsilon(z,\omega) = \varepsilon_0(\omega) + \sum_i [b(\omega) - \varepsilon_0(\omega)] \left[|\theta(z - z_i - a/2) - \theta(z - z_i + a/2)|\right], \tag{1}$$

where $\varepsilon_0(\omega)$ and $b(\omega)$ are the permitivity of the homogeneous medium and the plate, respectively, and $\theta(x)$ is the Heaviside step function. It is convenient to represent the permitivity as a sum of average and varying parts:

$$\varepsilon(z,\omega) = \varepsilon + \varepsilon_r(z,\omega), \qquad \langle\varepsilon_r(z,\omega)\rangle = 0, \tag{2}$$

where $\varepsilon = \langle\varepsilon(z,\omega)\rangle$, and averaging over random coordinates of plates is defined as follows:

$$\langle f(z,\omega)\rangle = \int \prod_i \frac{dz_i}{L_z} f(z, z_i, \omega), \tag{3}$$

where L_z is the system size in the z-direction.

In the Fourier representation, Maxwell's equations for the vector potential **A** of electromagnetic field has the form:

$$\nabla^2 \mathbf{A} + \frac{\omega^2}{c^2}\varepsilon(\mathbf{r},\omega)\mathbf{A}(\mathbf{r},\omega) = \mathbf{j}(\mathbf{r},\omega), \tag{4}$$

where

$$\mathbf{j}(\mathbf{r},\omega) = -\frac{4\pi e}{c}\frac{\mathbf{v}}{v}\delta(x)\delta(y)e^{i\omega z/v}$$

is the current of a charged particle moving uniformly in the z-direction with velocity v. The electric field **E** is related to the potentials by

$$\mathbf{E}(\mathbf{r},\omega) = \frac{i\omega}{c}\mathbf{A}(\mathbf{r},\omega) - \nabla\varphi(\mathbf{r},\omega). \tag{5}$$

Finally we write the condition relating the vector and scalar potentials of the electromagnetic field:

$$\nabla \cdot \mathbf{A} - \frac{i\omega}{c}\varepsilon(\mathbf{r},\omega)\varphi(\mathbf{r},\omega) = 0. \tag{6}$$

One needs the relations (4)–(6) to calculate the intensity of radiation. It follows from the symmetry of the problem that the vector potential **A** points in the z-direction, so $A_i = \delta_{zi}A(\mathbf{r},\omega)$.

We separate the electric field into two parts, $\mathbf{E} = \mathbf{E}_0 + \mathbf{E}_r$, to determine the radiation intensity. Here $\mathbf{E}_0$ is the electric field of a charge moving in a homogeneous medium with the dielectric constant ε, and $\mathbf{E}_r$ is the radiation field associated with fluctuations of the dielectric constant. The radiation tensor is

$$I_{ij}(\mathbf{R}) = E_{ri}(\mathbf{R})E^*_{rj}(\mathbf{R}), \tag{7}$$

where **R** is the radius vector of the observation point, which is far from the system ($R \gg L$).

The vector potential can be split in a similar manner, $\mathbf{A} = \mathbf{A}_0 + \mathbf{A}_r$, where $\mathbf{A}_0$ and $\mathbf{A}_r$ satisfy the equations

$$\nabla^2 \mathbf{A}_0 + \frac{\omega^2}{c^2}\varepsilon \mathbf{A}_0 = \mathbf{j}(\mathbf{r},\omega),$$
$$\nabla^2 \mathbf{A}_r + \frac{\omega^2}{c^2}\varepsilon \mathbf{A}_r + \frac{\omega^2}{c^2}\varepsilon_r \mathbf{A}_r = -\frac{\omega^2}{c^2}\varepsilon_r \mathbf{A}_0 . \tag{8}$$

The first equation is easily solved, and for the background field $\mathbf{A}_0$, one has

$$A_0(\mathbf{q}) = -\frac{8\pi^2 e}{c}\frac{\delta(q_z - \omega/v)}{k^2 - q^2}. \tag{9}$$

It is convenient to express the radiation tensor (7) in terms of the radiation potential $\mathbf{A}_r$. Using (5)–(7), we obtain

$$\langle I_{ij}(\mathbf{R})\rangle = \frac{\omega^2}{c^2}\delta_{zi}\delta_{zj}\langle A_r(\mathbf{R},\omega)A_r^*(\mathbf{R},\omega)\rangle + \frac{\delta_{zi}}{\varepsilon}\langle A_r(\mathbf{R},\omega)\frac{\partial^2}{\partial R_j \partial z}A_r^*(\mathbf{R},\omega)\rangle +$$
$$+\frac{\delta_{zj}}{\varepsilon}\left\langle A_r^*(\mathbf{R},\omega)\frac{\partial^2}{\partial R_i \partial z}A_r(\mathbf{R},\omega)\right\rangle + \frac{c^2}{\omega^2\varepsilon^2}\left\langle \frac{\partial^2}{\partial R_i \partial z}A_r(\mathbf{R},\omega)\frac{\partial^2}{\partial R_j \partial z}A_r^*(\mathbf{R},\omega)\right\rangle . \tag{10}$$

We express the radiation potential A_r in terms of the Green's function of the second equation in (8) for averaging in (10):

$$A_r(\mathbf{R}) = -\frac{\omega^2}{c^2}\int \varepsilon_r(\mathbf{r})A_0(\mathbf{r})G(\mathbf{R},\mathbf{r})\,d\mathbf{r}, \tag{11}$$

where the Green's function satisfies

$$\left[\nabla^2 + k^2 + \frac{\omega^2}{c^2}\varepsilon_r(z)\right]G(\mathbf{r},\mathbf{r}') = \delta(\mathbf{r}-\mathbf{r}'), \tag{12}$$

and $k = \omega\sqrt{\varepsilon}/c$.

3. GREEN'S FUNCTION

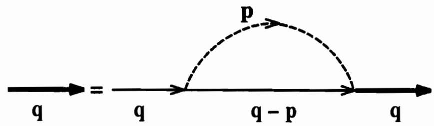
The bare ($\varepsilon_r = 0$) Green's function can easily be obtained from (12):

$$G_0(\mathbf{q}) = \frac{1}{k^2 - q^2 + i\delta}. \tag{13}$$

In the coordinate representation one has from (13)

$$G_0(r) = -\frac{1}{4\pi r}e^{ikr}. \tag{14}$$

To perform the averaging, we use the impurity-diagram method [6]. Summing the diagrams in the independent-scatterer approximation, we obtain the Dyson equation for the average Green's function:



(15)

The dashed line denotes the Fourier component $B(\mathbf{p}) = (2\pi)^2\delta(\mathbf{p}_\rho)B(|p_z|)$, of the correlation function of the one-dimensional random field:

$$B(|z-z'|) = \frac{\omega^4}{c^4}\langle \varepsilon_r(z)\varepsilon_r(z')\rangle, \tag{16}$$

where $\mathbf{p}_\rho$ is the transverse component of $\mathbf{p}$. The solution of Eq. (15) can be represented in the form

$$G(\mathbf{q}) = \frac{1}{k^2 - q^2 + i\mathrm{Im}\Sigma(\mathbf{q})}, \tag{17}$$

in which the imaginary part of the self-energy is given by Ward's identity:

$$\mathrm{Im}\,\Sigma(\mathbf{q}) = \int \frac{d\mathbf{p}}{(2\pi)^3}B(\mathbf{p})\,\mathrm{Im}\,G_0(\mathbf{q}-\mathbf{p}) = \frac{1}{4\sqrt{k^2-q_\rho^2}} \times$$
$$\times\left[B\left(\left|q_z - \sqrt{k^2-q_\rho^2}\right|\right) + B\left(\left|q_z + \sqrt{k^2-q_\rho^2}\right|\right)\right], \quad |\mathbf{q}_\rho| < k. \tag{18}$$

The dephasing length of a pseudophoton in the z-direction is determined by the imaginary part of the self-energy:

$$l(\mathbf{q}) = \frac{\sqrt{k^2-q_\rho^2}}{\mathrm{Im}\,\Sigma(\mathbf{q})}. \tag{19}$$

As expected, the dephasing length depends on the direction of the pseudophoton momentum. When the momentum is directed along z, one obtains from (18) and (19)

$$l(\vartheta = 0) = \frac{4k^2}{B(0) + B(2k)}. \tag{20}$$

From this point on, we shall call this quantity the pseudophoton mean free path.

Using (1)–(3) and (16), one can find for correlation function

$$B(q_z) = \frac{4(b-\varepsilon)^2 n \sin^2(q_z a/2)}{q_z^2}\,\frac{\omega^4}{c^4}. \tag{21}$$

Here $n = N/L_z$ is the density of plates in the system. Using (21), it is easy to see that $B(2k)/B(0) \sim 1/(ka)^2 \ll 1$ when $ka \gg 1$. Therefore, the photon mean free path is

$$l \equiv l(\vartheta = 0) \approx \begin{cases} 4k^2/B(0), & ka \gg 1 \\ 2k^2/B(0), & ka \ll 1\,. \end{cases} \tag{22}$$

The foregoing only holds in the weak scattering regime, for which $\mathrm{Im}\,\Sigma(\mathbf{q})/(k^2 - q_\rho^2) \ll 1$. Substituting (18) into this condition, we obtain

$$\frac{B(0)+B(2k|\cos\vartheta|)}{4k^3|\cos\vartheta|^3} \ll 1. \tag{23}$$

It follows from (23) that at $\vartheta \approx \pi/2$, the weak-scattering condition is not satisfied. This is natural, because in this case the pseudophoton moves parallel to the plates. Taking $\vartheta = \pi/2-\delta$ and using (22) and (23), one has $\delta \gg (1/kl)^{1/3}$.

4. RADIATION INTENSITY

We now turn to a close examination of radiation intensity. First we consider the single-scattering approximation. In this approximation, the Green's function in (11) is simply replaced by bare one. Substituting (11) into (10) and using the relations

$$G_0(\mathbf{R},\mathbf{r}) \approx -\frac{1}{4\pi R}e^{ik(R-\mathbf{nr})}, \quad \frac{\partial^2 G_0(\mathbf{R},\mathbf{r})}{\partial R_i \partial z} \approx \frac{k^2 n_i n_z}{4\pi R}e^{ik(R-\mathbf{nr})}, \; R \gg r \tag{24}$$

and (14), after simple transformations we obtain the following expression for the single-scattering contribution to radiation intensity $I(\mathbf{n}) = (c/2)R^2 I_{ii}(\mathbf{R})$:

$$I^0(\mathbf{n}) = \frac{\pi e^2}{c}\delta(0)\frac{B(|k_0-kn_z|)n_\rho^2}{\left[k^2n_z^2-k_0^2\right]^2}\frac{\omega^2}{c^2}. \tag{25}$$

Here $\mathbf{n} = \mathbf{R}/R$ is the unit vector in the direction of the observation point $\mathbf{R}$ and $k_0 = \omega/v$; the δ-type singularity of (25) results from the infinite path of the charged particle in the medium. If one takes into account the finite size of the system, $\delta(0)$ must be replaced by $L_z/2\pi$. To analyze the angular dependence of (25), it is convenient to represent it in the form

$$I^0(\theta) = \frac{e^2}{2c}\frac{L_z B(|k_0-k\cos\vartheta|)\sin^2\vartheta}{\left[\gamma^{-2}+(k^2/k_0^2)\sin^2\vartheta\right]^2}\frac{\omega^2}{k_0^4c^2}, \tag{26}$$

where $\gamma = (1-\varepsilon v^2/c^2)^{-1/2}$ is the Lorentz factor of the particle in the medium, $n_z = \cos\vartheta$, and $n_\rho = \sin\vartheta$.

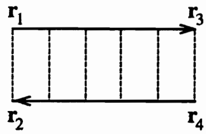
Note the key features of the single-scattering contribution I^0. It follows from (26) and the form (21) of the correlation function B that at relativistic energies ($\gamma \gg 1$, $k_0 \to k$), the maximum of radiation lies in the range of angles $\vartheta \sim \gamma^{-1}$ in the forward direction. Integrating (21) over the angles, it is easy to see that the dependence of total intensity on particle energy is logarithmic, $I^0 \propto \ln\gamma$. As $B \propto n$, the dependence of radiation intensity on the number of plates is linear. All of these results are consistent with previous results [2, 4].

We now consider the diffusion contribution to the radiation intensity. Using (10), (11) and (22), one can represent the diffusion contribution to the radiation tensor in the form

$$\begin{aligned} I_{ij}^D(\mathbf{R}) = \frac{k^2}{16\pi^2R^2\varepsilon}\int d\mathbf{r}\,d\mathbf{r}'B(\mathbf{r}-\mathbf{r}')A_0(\mathbf{r})A_0^*(\mathbf{r}')\int d\mathbf{r}_1 d\mathbf{r}_2 d\mathbf{r}_3 d\mathbf{r}_4 \times \\ \times \exp\left[-ik\mathbf{n}(\mathbf{r}_1-\mathbf{r}_2)\right]P(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4)G(\mathbf{r}_3,\mathbf{r})G^*(\mathbf{r},\mathbf{r}_4)\times \\ \times\left[\delta_{zi}\delta_{\hat{z}j}+n_in_jn_z^2-\delta_{\hat{z}i}n_jn_z-\delta_{\hat{z}j}n_in_z\right], \end{aligned} \tag{27}$$

where $P(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4)$ is the diffusion propagator:

$$P(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4) = \Sigma \tag{28}$$



We find the diffusion propagator as in the three-dimensional case [5]. As follows from (28), P can be represented in the form

$$P(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4) = B(\mathbf{r}_1 - \mathbf{r}_2)B(\mathbf{r}_3 - \mathbf{r}_4)P(\mathbf{R}', \mathbf{r}_1 - \mathbf{r}_2, \mathbf{r}_3 - \mathbf{r}_4), \tag{29}$$

where $\mathbf{R}' = (1/2)(\mathbf{r}_3 + \mathbf{r}_4 - \mathbf{r}_1 - \mathbf{r}_2)$ and P satisfies the equation

$$\int \frac{d\mathbf{p}}{(2\pi)^3}\left[1 - \int \frac{d\mathbf{q}}{(2\pi)^3} f(\mathbf{q}, \mathbf{K})B(\mathbf{p} - \mathbf{q})\right] P(\mathbf{K}, \mathbf{p}, \mathbf{q}') = f(\mathbf{q}', \mathbf{K}), \tag{30}$$

where

$$f(\mathbf{q}, \mathbf{K}) = G(\mathbf{q} + \mathbf{K}/2)G^*(\mathbf{q} - \mathbf{K}/2). \tag{31}$$

As will be seen, one has to know P when $\mathbf{K} \to 0$. In this limit, the diffusion propagator has the form [5]

$$P(\mathbf{K} \to 0, \mathbf{p}, \mathbf{q}) = \frac{\operatorname{Im} G(\mathbf{p}) \operatorname{Im} G(\mathbf{q})}{\operatorname{Im}\Sigma(\mathbf{q})} A(\mathbf{K}), \tag{32}$$

where

$$A(\mathbf{K}) = \left[\int \frac{(\mathbf{qK})^2 \operatorname{Im} G(\mathbf{q})}{\operatorname{Im}^2\Sigma(\mathbf{q})} \frac{d\mathbf{q}}{(2\pi)^3}\right]^{-1}. \tag{33}$$

Substituting (17) and (18) into (33), choosing $\mathbf{K} \parallel \hat{z}$, and calculating the integral, we obtain

$$A(K) = \frac{1}{k} \frac{20\pi}{K^2 l^2}. \tag{34}$$

When we know the form of the diffusion propagator, we can calculate the diffusion contribution to the radiation intensity. Transforming variables in (27) and going to the Fourier representation, we obtain

$$I^D(\mathbf{n}) = \frac{k^2 c}{32\pi^2\varepsilon}(1 - n_z^2) \int \frac{d\mathbf{q}_1 d\mathbf{q}_2 d\mathbf{q}_3 d\mathbf{q}_4}{(2\pi)^{12}} |A_0(\mathbf{q}_1)|^2 B(\mathbf{q}_2)B(\mathbf{q}_3)B(\mathbf{q}_4) \times \\ \times P(\mathbf{K} \to 0, -\mathbf{q}_3 - k\mathbf{n}, \mathbf{q}_1 + \mathbf{q}_2 + \mathbf{q}_4)|G(\mathbf{q}_1 + \mathbf{q}_2)|^2. \tag{35}$$

Substituting (9) into (35) and integrating using Ward's identity (18), we have

$$I^D(\mathbf{n}) = \frac{e^2}{2c\varepsilon}(1 - n_z^2)A(\mathbf{K})k^2 \operatorname{Im}\Sigma(k\mathbf{n})L_z \times \\ \times \int \frac{d\mathbf{q}_\rho}{(2\pi)^2} \frac{1}{(q_\rho^2 + k_0^2 - k^2)^2} \frac{B\left(|k_0 + \sqrt{k^2 - q_\rho^2}|\right) + B\left(|k_0 - \sqrt{k^2 - q_\rho^2}|\right)}{B(0) + B\left(2\sqrt{k^2 - q_\rho^2}\right)}. \tag{36}$$

The singularity of the radiation intensity results from the diffusion pole. When one takes into account the finite size of the system, the diffusion paths of the photon are cut off at the system size, and therefore $1/K^2$ can be replaced by L_z^2 as $K \to 0$ (we assume that $L_z < L_x, L_y$). It follows from (36) that for particle energies $k_0 \to k, \gamma \gg 1$, the main contribution to the integral over q_ρ comes from $q_\rho \to 0$. The correlation function B in (36) varies slowly when $q_\rho \to 0$ provided that $\gamma^2 \gg ak_0$ (we discuss this condition in more detail in the next section); therefore, taking $q_\rho \approx 0$ (under the condition $\gamma \gg ak$) and substituting (34) and (16) into (36), we obtain

$$I^D(\omega, \vartheta) = \frac{5}{2} \frac{e^2\gamma^2}{\varepsilon c} \left(\frac{L_z}{l(\omega)}\right)^3 \frac{\sin^2\vartheta}{|\cos\vartheta|}. \tag{37}$$

Note the main features of the diffusion contribution (37). Comparing (37) with the single-scattering contribution (26), we see that $I^D/I^0 \sim L_z^2/l^2 \gg 1$. This means that in the wavelength range $\lambda \ll l(\lambda) \ll L_z$, the dominant radiation mechanism is diffusion. Note the strong dependence of spectral intensity on the particle energy, which also holds for the total intensity (integrated over frequencies and angles). Recall that this dependence in conventional transition radiation is logarithmic in the optical region. When $L_z \sim N$, then from (37) the radiation intensity has a strong dependence on the number of plates, $I \propto N^3$.

We now discuss a reasons for the strong dependence of radiation intensity on particle energy. It is convenient to represent the background field in the form

$$A_0(\mathbf{q}) \propto \frac{\delta(q_z - k_0)}{q_\rho^2 + k_0^2\gamma^{-2}} . \tag{38}$$

It follows from (38), that at relativistic energies $\gamma \gg 1$, most pseudophotons have momentum with transverse component $q_\rho \to 0$. It is easy to see from (38) that the total number of pseudophotons $N_{ps} \propto \int A_0^2(\mathbf{q})d\mathbf{q}$ is proportional to γ^2. Each pseudophoton must be scattered to be converted into a real photon. The probability of large-angle scattering of pseudophotons is low in single scattering. Therefore, only a small contigent of pseudophotons is converted into photons.

This picture changes dramatically in multiple scattering, for which almost all pseudophotons are converted into photons via multiple scattering by the plates. As the total number of pseudophotons is proportional to γ^2, the radiation intensity (total number of photons) is also proportional to γ^2.

5. COHERENCE LENGTH

It is known (see for example [1]) that the coherence length (or radiation formation zone) is the distance at which the intrinsic field of charged particle separates from the radiation field. In other words, it is the length at which the interference term becomes small. The interference term consists of expressions like $I^i \sim A_0^*(\mathbf{R}, \omega) < A_r(\mathbf{R}, \omega)$. Using (9) and going to the coordinate representation, one obtains the following expression for the background field:

$$A_0^*(\mathbf{R}) = \frac{2e}{\pi^2 c} e^{-ik_0 z} K_0\left(\frac{k_0 \rho}{\gamma}\right), \tag{39}$$

where K_0 is the modified Bessel function (see for example [7]).

Using (11), we obtain the averaged radiation potential

$$\langle A_r(\mathbf{R}) \rangle = -\frac{\omega^2}{c^2} \int d\mathbf{r} A_0(\mathbf{r}) \langle \varepsilon_r(\mathbf{r}) G(\mathbf{R}, \mathbf{r}) \rangle. \tag{40}$$

Using the impurity diagrams (15), one can represent the average in (40) in the form

$$\langle \varepsilon_r(\mathbf{r}) G(\mathbf{R}, \mathbf{r}) \rangle = \int d\mathbf{r}_1 G_0(\mathbf{R}, \mathbf{r}_1) B(\mathbf{r}_1 - \mathbf{r}) G(\mathbf{r}_1 - \mathbf{r}). \tag{41}$$

Using the approximations (24), for an observation point R far from the system, $R \gg r_1$, we finally obtain

$$I^i(\mathbf{R}) \sim \frac{\exp(ikR - ik_0 z)}{4\pi R} K_0 \left(\frac{k_0 \rho}{\gamma}\right) \int d\mathbf{r}\, d\mathbf{r}_1 A_0(\mathbf{r}) B(\mathbf{r}_1 - \mathbf{r}) G(\mathbf{r}_1 - \mathbf{r}) \exp(-ik\mathbf{n}\mathbf{r}_1). \tag{42}$$

For our purposes, it suffices to consider only the oscillating part of (42),

$$I^i(\mathbf{R}) \propto \exp(ikR - ik_0 R \cos\vartheta). \tag{43}$$

The interference term will be small when the oscillations are strong, $R(k - k_0 \cos\theta) \gg 2\pi$ [1]. In this case any integration will make the interference contribution negligible. Consequently, the coherence length in our case has the form

$$l_c(\vartheta) = \frac{2\pi}{|k - k_0 \cos\vartheta|}. \tag{44}$$

Now consider some special cases. For relativistic energies $k_0 \to k$ and small angles $\vartheta \approx 0$, taking into account the definition of k and k_0, one finds from (44)

$$l_c \equiv l_c(0) \approx \frac{4\pi\gamma^2}{k_0}. \tag{45}$$

For angles $\theta \approx \pi$, the coherence length has the form

$$l_c(\pi) \approx \frac{2\pi}{k + k_0}. \tag{46}$$

As expected, the coherence length in the direction of particle motion ($\vartheta \approx 0$) is much greater than in the backward direction ($\vartheta \approx \pi$), where it is of the order of the wavelength.

Now the meaning of the condition $\gamma^2 \gg ak_0$, which we used in the previous section, becomes clearer. It means that many plates must be placed at the coherence length, $l_c \gg a$, in order for multiple scattering effects of the pseudophoton to play an important role.

6. CONCLUSIONS

We have considered the diffusion contribution to the radiation intensity of a relativistic particle traversing a stack of randomly spaced plates. It was shown that for a large number of plates ($N \gg 1$), in the wavelength range $\lambda \ll l$, for angles $|\cos\vartheta| \gg (1/kl)^{1/3}$, and coherence length much greater than the plate thickness ($l_c \gg a$), the diffusion contribution is dominant. Note that the backward and forward intensities of relativistic charged particle radiation intensity are equal, whereas in a regular stack, a relativistic particle radiates mainly in the forward direction.

Note that we did not take photon absorption into account. This is correct provided that $l \ll l_{in}$, where l_{in} is the photon inelastic mean free path in the medium. In the theory of diffusive propagation of waves, weak absorption ($l \ll l_{in}$) is taken into account in the following way [8]. If the absorption is so weak that $L_z < \sqrt{ll_{in}}$, then the expression (37) remains unchanged. When $L_z > \sqrt{ll_{in}}$, one must replace L_z^2 with ll_{in} in (37):

$$I^D(\omega, \vartheta) = \frac{5}{2} \frac{e^2\gamma^2}{\varepsilon c} \frac{L_z l_{in}(\omega)}{l^2(\omega)} \frac{\sin^2\vartheta}{|\cos\vartheta|}. \tag{47}$$

It follows from (47) that in this case the dependence of radiation intensity on the number of plates is weaker, $I \propto N$.

Note that absorption changes the frequency dependence of the spectral intensity.

Now compare the radiation considered above with Cherenkov radiation for the corresponding values of particle energy, which, however, are on opposite sides of the critical value $c/\sqrt{\varepsilon}$. The intensity of Cherenkov radiation has the form [8]

$$I^{Ch}(\omega) = \frac{e^2\omega d}{c^2}\left(1 - \frac{c^2}{v^2\varepsilon}\right), \tag{48}$$

where d is the path of the charged particle, which traverses a medium with dielectric constant ε. Comparing (48) with (37), we have

$$\frac{I^D(\omega)}{I^{Ch}(\omega)} \sim \frac{\gamma^2}{kl}\left(\frac{L_z}{l}\right)^2. \tag{49}$$

Note that the Cherenkov intensity is greater than the single-scattering contribution, $I^0/I^{Ch} \sim \sim 1/kl \ll 1$ [9]. From (49), the diffusion radiation, in contrast, can be stronger than the Cherenkov radiation.

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