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SUPERFLUIDITY OF ^{3}He IN AEROGEL AT $T = 0$ IN A MAGNETIC FIELD

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The transition of liquid ^{3}He to the superfluid B phase in aerogel at $T = 0$ is considered. It is shown that in a magnetic field, the quantum phase transition with respect to pressure is split in two. The amount of splitting δP is estimated. The components of the superfluid density tensor are calculated near the critical pressures.

1. The behavior of superfluid ^{3}He in a silica aerogel environment is a subject of recent experimental investigations [1-4]. The scattering of quasiparticles on a random network of SiO_2 strands affects superfluid correlations, thus considerably modifying the phase diagram of liquid ^{3}He in the millikelvin temperature range. Interesting observations [4] were made concerning the behavior of ^{3}He at $T \simeq 0$ where, in contrast to bulk liquid ^{3}He, superfluidity shows up only above some critical density ρ_c (at pressures $P > P_c$).

The situation at $T = 0$ for ^{3}He in aerogel was recently considered theoretically [5]. It is shown that according to a simple model with scattering effects characterized by the quasiparticle mean free path $l = v_F\tau$, the critical pressure P_c is given by the equation

$$T_{c0}(P_c) = \gamma_E/\pi\tau, \qquad \ln\gamma_E = C \simeq 0.577, \tag{1}$$

where $T_{c0}(P)$ is the P-dependent critical temperature of the transition of bulk ^{3}He to the superfluid state. Near P_c the gap function (the order parameter) is

$$\Delta^2(P) = \frac{3}{\tau^2}\theta(P - P_c)\ln\frac{T_{c0}(P)}{T_{c0}(P_c)}. \tag{2}$$

The investigation carried out in Ref. 5 is based on the assumption that the superfluid state at $P > P_c$ is of the B-phase type. It should be remembered that the appearance of a B-phase-like state in aerogel at low pressures is expected when the magnetic contribution to quasiparticle scattering events is suppressed by ^{4}He layers covering the silica strands [3]. In what follows we extend the results of Ref. 5 to the B phase in a magnetic field. Our obvious motivation is to explore an expected magnetic splitting of quantum phase transition at $T = 0$. The behavior of magnetically distorted ^{3}He-B in aerogel at $T \simeq T_c$ has been considered by us in Ref. 6.

2. In what follows we use quasiclassical Green's functions (the ξ-integrated Gorkov functions) in the Matsubara representation:

$$\hat{g}_\omega(\hat{\mathbf{k}}) = \frac{1}{i\pi}\int \hat{G}_\omega(\hat{\mathbf{k}},\xi)d\xi = g_\omega\hat{1} + \mathbf{g}_\omega\hat{\boldsymbol{\sigma}},$$

$$\hat{f}_\omega(\hat{\mathbf{k}}) = \frac{1}{\pi}\int \hat{F}_\omega(\hat{\mathbf{k}},\xi)d\xi = (f_\omega\hat{1} + \mathbf{f}_\omega\hat{\boldsymbol{\sigma}})i\hat{\sigma}_y, \tag{3}$$

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where the unit vector $\hat{\mathbf{k}}$ specifies the location on the normal state Fermi sphere.

In an external magnetic field $\mathbf{H} = H_0\hat{\mathbf{h}}$, the functions $\hat{g}_\omega$ and $\hat{f}_\omega$ satisfy a set of equations ($\boldsymbol{\omega}_0 = \omega_0\hat{\mathbf{h}}$, $\omega_0 = \gamma H_0$):

$$\begin{aligned} g_\omega \boldsymbol{\Delta}_t + \mathbf{g}_\omega \Delta_s + \omega \mathbf{f}_\omega - \frac{i}{2}\boldsymbol{\omega}_0 f_\omega &= \mathbf{0}, \\ g_\omega \Delta_s + \mathbf{g}_\omega \boldsymbol{\Delta}_t + \omega f_\omega - \frac{i}{2}\boldsymbol{\omega}_0 \mathbf{f}_\omega &= 0. \end{aligned} \tag{4}$$

Here spin-singlet and spin-triplet order parameters Δ_s and $\boldsymbol{\Delta}_t$, respectively, are the components of the matrix

$$\hat{\Delta}(\hat{\mathbf{k}}) = (\Delta_s \hat{1} + \boldsymbol{\Delta}_t \hat{\boldsymbol{\sigma}}) i\hat{\sigma}_y, \tag{5}$$

which is found according to the self-consistency equation

$$\hat{\Delta}(\hat{\mathbf{k}}) = \pi T \sum_\omega \left\langle V(\hat{\mathbf{k}}, \hat{\mathbf{k}}') \hat{f}_\omega(\hat{k}') \right\rangle, \tag{6}$$

where $V(\hat{\mathbf{k}}, \hat{\mathbf{k}}')$ is the Cooper pairing interaction and angle brackets denote averaging across the Fermi surface.

The set of equations (4) should be supplemented by the «boundary» conditions

$$\begin{aligned} g_\omega^2 + \mathbf{g}_\omega^2 + f_\omega^2 + \mathbf{f}_\omega^2 &= 1, \\ g_\omega \mathbf{g}_\omega + f_\omega \mathbf{f}_\omega &= \mathbf{0}. \end{aligned} \tag{7}$$

The structure of Eqs. (4) and (7) implies that

$$f_\omega = a_\omega g_\omega, \qquad \mathbf{f}_\omega = \mathbf{a}_\omega g_\omega, \qquad \mathbf{g}_\omega = -a_\omega \mathbf{a}_\omega g_\omega, \tag{8}$$

and the solution is easily obtained:

$$\begin{aligned} g_\omega &= -\frac{\operatorname{sign}\omega}{\sqrt{1 + a_\omega^2}\sqrt{1 + \mathbf{a}_\omega^2}}, \\ a_\omega &= \frac{i}{2}\frac{2i\Delta_s + \boldsymbol{\omega}_0 \mathbf{a}_\omega}{\omega - \boldsymbol{\Delta}_t \mathbf{a}_\omega}, \quad \mathbf{a}_\omega = \frac{i}{2}\frac{2i\boldsymbol{\Delta}_t + \boldsymbol{\omega}_0 a_\omega}{\omega - \Delta_s a_\omega}. \end{aligned} \tag{9}$$

This is our starting point when considering the properties of the magnetized B phase. In order to take into account the quasiparticles scattering on aerogel spatial irregularities ($1/\tau \neq 0$), the «impurity»-induced renormalization of the frequencies ω and ω_0 and of the order parameter $\hat{\Delta}$ is to be performed according to a standard prescription:

$$\begin{aligned} \omega \to \tilde{\omega} &= \omega + iM_{\tilde{\omega}}, \\ \omega_0 \to \tilde{\omega}_0 &= \omega_0 - 2\hat{\mathbf{h}}\mathbf{M}_{\tilde{\omega}}, \\ \Delta(\hat{\mathbf{k}}) \to \hat{\Delta}_{\tilde{\omega}} &= \hat{\Delta}(\hat{\mathbf{k}}) + \hat{m}_{\tilde{\omega}}, \end{aligned} \tag{10}$$

where the «impurity»-generated self-energies are given by

$$\hat{M}_\omega = M_\omega \hat{1} + \mathbf{M}_\omega \hat{\sigma} = \frac{i}{2\tau}\left\langle \hat{g}_\omega(\hat{\mathbf{k}})\right\rangle,$$
$$\hat{m}_\omega = (m_\omega \hat{1} + \mathbf{m}_\omega \hat{\sigma}) i\hat{\sigma}_y = \frac{1}{2\tau}\left\langle \hat{f}_\omega(\hat{\mathbf{k}})\right\rangle. \tag{11}$$

Before proceeding, an important comment should be made. Since we are going to consider a superfluid state with p-wave Cooper pairing for which $V(\hat{\mathbf{k}}, \hat{\mathbf{k}}') = 3g(\hat{\mathbf{k}}\hat{\mathbf{k}}')$, from Eq. (6) it is certainly clear that $\Delta_s \equiv 0$ and $\mathbf{\Delta}_t$ satisfies the equation

$$\mathbf{\Delta}_t(\hat{\mathbf{k}}) = g\pi T \sum_\omega \left\langle 3\hat{\mathbf{k}}\hat{\mathbf{k}}'\mathbf{f}_{\tilde{\omega}}(\hat{\mathbf{k}}')\right\rangle. \tag{12}$$

This does not mean, however, that in our starting expressions for a_ω and $\mathbf{a}_\omega$ we have to forget about the presence of Δ_s. The point is that when considering the «impurity» renormalization $\hat{m}_\omega$, the contribution stemming from the spin-singlet part m_ω must be taken into account. This involves the calculation of the superfluid density tensor $\rho_{ij}^{(s)}$, which is a response of the system to the superfluid velocity field $\mathbf{v}_s$ and is contained in the expression for the supercurrent

$$\mathbf{j}_s = 2\pi i k_F N_F T \sum_\omega \left\langle \hat{\mathbf{k}} g_{\tilde{\omega}+iq}(\hat{\mathbf{k}})\right\rangle. \tag{13}$$

Here $\mathbf{v}_s$ is absorbed in $q(\hat{\mathbf{k}}) = k_F(\hat{\mathbf{k}}\mathbf{v}_s)$ and N_F denotes the quasiparticle density of states at the Fermi level. As we noted in Ref. 6, to first order in q, the spin-singlet part $m_{\tilde{\omega}+iq}$ is proportional to $(1/\tau)\omega_0 q$ and contributes to $\rho_{ij}^{(s)}$ in the magnetized B phase in the aerogel environment.

3. Now we turn to the calculation of equilibrium properties of magnetized ^{3}He-B in a quasiparticle scattering medium. Noticing that in the absence of superflow ($q = 0$) $\hat{\Delta}(\hat{\mathbf{k}})$ is not renormalized in nonmagnetic scattering, and addressing Eqs. (9), it can be shown that to lowest order in the magnetic field strength,

$$\mathbf{f}_{\tilde{\omega}}(\hat{\mathbf{k}}) = \frac{1}{\sqrt{\tilde{\omega}^2 + \Delta^2(\hat{\mathbf{k}})}}\left[\mathbf{\Delta} - \frac{1}{4}\left(\mathbf{\Delta}_\| - \frac{3}{2}\frac{\Delta_\|^2}{\tilde{\omega}^2 + \Delta^2}\mathbf{\Delta}\right)\frac{\tilde{\omega}_0^2}{\tilde{\omega}^2 + \Delta^2}\right]. \tag{14}$$

In this expression the longitudinal component of $\mathbf{\Delta} = \mathbf{\Delta}_\| + \mathbf{\Delta}_\perp$ is given by

$$\mathbf{\Delta}_\|(\hat{\mathbf{k}}) = (\hat{\mathbf{h}}\mathbf{\Delta}(\hat{\mathbf{k}}))\hat{\mathbf{h}} = \Delta_\|(P, T)(\hat{\mathbf{l}}\hat{\mathbf{k}})\hat{\mathbf{h}}, \tag{15}$$

where the magnetic-field induced orbital anisotropy axis $\hat{\mathbf{l}}$ is defined as $\hat{l}_i = \hat{h}_\mu R_{\mu i}$ with $R_{\mu i}$ being the components of an orthogonal matrix of $3D$ relative spin-orbit rotations. We note also that

$$\Delta^2(\hat{\mathbf{k}}) = \Delta_\|^2(P, T)(\hat{\mathbf{l}}\hat{\mathbf{k}})^2 + \Delta_\perp^2(P, T)(\hat{\mathbf{l}} \times \hat{\mathbf{k}})^2, \tag{16}$$

and in zero magnetic field $\Delta_\| = \Delta_\perp = \Delta$.

Using Eqs. (12) and (14), and taking into account that

$$\tilde{\omega} = \omega - \frac{1}{2\tau}\langle g_{\tilde{\omega}}(\hat{\mathbf{k}})\rangle \tag{17}$$

with

$$g_{\tilde{\omega}} \simeq -\frac{\tilde{\omega}}{\sqrt{\tilde{\omega}^2+\Delta^2}}\left[1+\frac{3}{8}\frac{\tilde{\omega}_0^2}{\tilde{\omega}^2+\Delta^2}\frac{\Delta_{\parallel}^2}{\tilde{\omega}^2+\Delta^2}\right], \tag{18}$$

equations for amplitudes $\Delta_{\parallel}(P,T)$ and $\Delta_{\perp}(P,T)$ can be readily derived.

Let us consider first the planar phase ($\Delta_{\parallel}=0$). Near the transition to the normal state for $\Delta_{\perp}(\hat{\mathbf{k}})$ we obtain

$$\left[\ln\frac{T}{T_{c0}}+\psi\left(\frac{1}{2}+w\right)-\psi\left(\frac{1}{2}\right)\right]\Delta_{\perp}(\hat{\mathbf{k}})=\frac{3}{4}\frac{\psi^{(2)}(1/2+w)}{(2\pi T)^2}\langle\hat{\mathbf{k}}\hat{\mathbf{k}}'|\Delta_{\perp}(\hat{\mathbf{k}}')|^2\Delta_{\perp}(\hat{\mathbf{k}}')\rangle+$$
$$+\frac{w}{12}\frac{\psi^{(3)}(1/2+w)}{(2\pi T)^2}\langle\Delta_{\perp}^2\rangle\Delta_{\perp}(\hat{\mathbf{k}}), \tag{19}$$

where $w(T)=1/4\pi T\tau$. After simple averaging we obtain an equation for $\Delta_{\perp}(P,T)$:

$$\Delta_{\perp}\left\{\ln\frac{T_{c0}}{T}+\psi\left(\frac{1}{2}\right)-\psi\left(\frac{1}{2}+w\right)+\left[\frac{1}{5}\psi^{(2)}\left(\frac{1}{2}+w\right)+\frac{w}{18}\psi^{(3)}\left(\frac{1}{2}+w\right)\right]\left(\frac{\Delta_{\perp}}{2\pi T}\right)^2\right\}=0. \tag{20}$$

In the limit $T\to 0$ ($w\to\infty$), it is found that for the planar phase

$$\Delta_{\perp}^2(P)=\frac{15}{16}\Delta^2(P), \tag{21}$$

where $\Delta^2(P)$ is given by the Mineev solution (at $\omega_0=0$), Eq. (2). The coefficient 15/16 in Eq. (21) is due to the averaging at $\Delta_{\parallel}=0$ (which gives an answer analogous to the A phase).

As will be seen below, the solution (21) extends up to the pressure $P=P_{\parallel}$ where $\Delta_{\parallel}$ first appears. The new critical pressure $P_{\parallel}$ is given by

$$\ln\frac{T_{c0}(P_{\parallel})}{T_{c0}(P_c)}=\frac{4}{9}(\omega_0\tau)^2. \tag{22}$$

To show this we turn back to Eqs. (12) and (14) and, after simple calculations, a set of equations for $\Delta_{\perp}$ and $\Delta_{\parallel}$ is obtained (again in the limit $T=0$):

$$\begin{aligned}\Delta_{\perp}\left[\Delta_{\perp}^2+\frac{1}{15}(\Delta_{\perp}^2-\Delta_{\parallel}^2)-\Delta^2\right]&=0,\\ \Delta_{\parallel}\left[\Delta_{\parallel}^2+\frac{2}{15}(\Delta_{\parallel}^2-\Delta_{\perp}^2)-\Delta^2+\frac{3}{2}\omega_0^2\right]&=0.\end{aligned} \tag{23}$$

The solution of Eqs. (23) for $\Delta_{\parallel}\neq 0$, $\Delta_{\perp}\neq 0$ is ($P\geq P_{\parallel}$)

$$\begin{aligned}\Delta_{\parallel}^2(P)&=\Delta^2-\frac{4}{3}\omega_0^2=\frac{3}{\tau^2}\ln\frac{T_{c0}(P)}{T_{c0}(P_{\parallel})},\\ \Delta_{\perp}^2(P)&=\Delta_{\parallel}^2(P)+\frac{5}{4}\omega_0^2.\end{aligned} \tag{24}$$

It can be verified that $\Delta_{\perp}^2(P_{\parallel})$ matches the solution (21).

Finally we conclude that at $P < P_c$ the normal state is realized; in the pressure range $P_c < P < P_{\parallel}$ the planar phase is stabilized; and at $P > P_{\parallel}$ a magnetically distorted B phase appears.

At $P_{\parallel}$ a discontinuity of the magnetic susceptibility takes place (similar to a discontinuity of the compressibility at P_c). In order to demonstrate this property the superfluid contribution Φ_S to the thermodynamic potential density is to be constructed. In the Ginzburg—Landau region (which we consider)

$$\Phi_S = \frac{2}{3}N_F\left\{-\ln\frac{T_{c0}(P)}{T_{c0}(P_c)}\Delta_{\perp}^2 - \frac{1}{2}\left[\ln\frac{T_{c0}(P)}{T_{c0}(P_c)} - \frac{1}{2}(\omega_0\tau)^2\right]\Delta_{\parallel}^2 + \right.$$
$$\left. + \frac{\tau^2}{6}\left[\Delta_{\perp}^4 + \frac{1}{2}\Delta_{\parallel}^4 + \frac{1}{15}(\Delta_{\perp}^2 - \Delta_{\parallel}^2)^2\right]\right\}. \tag{25}$$

It is easely verified that the solutions of Eqs. (23) realize the minima of Φ_S.

At zero magnetic field ($\omega_0 = 0$) $\Delta_{\perp} = \Delta_{\parallel} = \Delta$, and in this case (see Ref. 5)

$$\Phi_S = \Phi_{S0} = N_F\left[-\ln\frac{T_{c0}(P)}{T_{c0}(P_c)}\Delta^2 + \frac{\tau^2}{6}\Delta^4\right]. \tag{26}$$

It can be shown that in equilibrium the magnetic field contribution $\Phi_{SM} = \Phi_S - \Phi_{S0}$ is

$$\Phi_{SM}^{(equ)} = \frac{1}{\tau^2}N_F\begin{cases}\dfrac{9}{16}\ln^2\dfrac{T_{c0}(P)}{T_{c0}(P_c)}, & P_c \le P \le P_{\parallel},\\[2ex] \dfrac{1}{2}\left[\ln\dfrac{T_{c0}(P)}{T_{c0}(P_c)} - \dfrac{2}{9}(\omega_0\tau)^2\right](\omega_0\tau)^2, & P \ge P_{\parallel}.\end{cases} \tag{27}$$

This expression is certainly continuous at $P = P_{\parallel}$. On the other hand,

$$\frac{\partial^2\Phi_{SM}^{(equ)}}{\partial H_0^2} = \begin{cases}0, & P = P_{\parallel}^-,\\[1ex] -\dfrac{4}{9}\gamma^2 N_F(\omega_0\tau)^2, & P = P_{\parallel}^+,\end{cases} \tag{28}$$

which signals the discontinuity of magnetic susceptibility at $P_{\parallel}$.

Now we shall estimate the value of $\delta P = P_{\parallel} - P_c$, which characterizes the magnetic splitting of the superfluid phase transition of ^{3}He in aerogel at $T = 0$. According to Eq. (22),

$$\frac{1}{T_{c0}(P_c)}\left(\frac{\partial T_{c0}}{\partial P}\right)_{P_c}\delta P \simeq \frac{4}{9}(\omega_0\tau)^2, \tag{29}$$

so that $\delta P = \alpha H_0^2$, where

$$\alpha = \left(\frac{2\gamma_E}{3\pi}\right)^2\frac{(\hbar\gamma/k_B)^2}{T_{c0}(P_c)(\partial T_{c0}/\partial P)_{P_c}}. \tag{30}$$

Using experimental data on $T_{c0}(P)$, we find that at $P_c = 7$ bar the coefficient $\alpha \simeq 2\cdot 10^{-2}$ bar/(kG)^2 and increases gradually to $\alpha \simeq 2.8\cdot 10^{-2}$ bar/(kG)^2 at $P_c = 10$ bar.

4. Now we turn to the calculation of the superfluid density tensor

$$\rho_{ij}^{(s)} = \rho_{\parallel}^{(s)}\hat{l}_i\hat{l}_j + \rho_{\perp}^{(s)}(\delta_{ij} - \hat{l}_i\hat{l}_j). \tag{31}$$

For this purpose Eq. (13) is to be used and $g_{\tilde{\omega}+iq}$ should be constructed. Returning to Eqs. (9) and performing «impurity» renormalizations according to Eqs. (10), it can be shown that

$$g_{\tilde{\omega}+iq}^{(1)} = -i\left\{\frac{\Delta^2 q + i\tilde{\omega}(\Delta \mathbf{m}_{\tilde{\omega}}^{(1)})}{(\tilde{\omega}^2+\Delta^2)^{3/2}} - \frac{3}{2}\frac{\tilde{\omega}_0^2}{(\tilde{\omega}^2+\Delta^2)^{5/2}}\left(1-\frac{5}{4}\frac{\Delta^2}{\tilde{\omega}^2+\Delta^2}\right)\Delta_{\parallel}^2 q - \right.$$
$$\left. -\frac{3}{4}\frac{i\tilde{\omega}\tilde{\omega}_0^2}{(\tilde{\omega}^2+\Delta^2)^{5/2}}\left[\Delta_{\parallel}\mathbf{m}_{\tilde{\omega}}^{(1)} - \frac{5}{2}\frac{\Delta_{\parallel}^2}{\tilde{\omega}^2+\Delta^2}(\Delta \mathbf{m}_{\tilde{\omega}}^{(1)})\right] - \frac{\tilde{\omega}_0}{(\tilde{\omega}^2+\Delta^2)^{3/2}}\left(1-\frac{3}{2}\frac{\Delta^2}{\tilde{\omega}^2+\Delta^2}\right)(\hat{\mathbf{h}}\Delta)m_{\tilde{\omega}}^{(1)}\right\}. \tag{32}$$

As will be seen shortly, $m_{\tilde{\omega}}^{(1)}$ and $\mathbf{m}_{\tilde{\omega}}^{(1)}$ are proportional to v_s.

The renormalized frequencies $\tilde{\omega}$ and $\tilde{\omega}_0$ obey

$$\tilde{\omega} = \omega + \frac{\tilde{\omega}}{2\tau}\left\langle \frac{1}{(\tilde{\omega}^2+\Delta^2)^{1/2}} + \frac{3}{8}\frac{\tilde{\omega}_0^2\Delta_{\parallel}^2}{(\tilde{\omega}^2+\Delta^2)^{5/2}}\right\rangle, \tag{33}$$

$$\tilde{\omega}_0 = \omega_0 + \frac{\tilde{\omega}_0}{2\tau}\left\langle \frac{\Delta_{\parallel}^2}{(\tilde{\omega}^2+\Delta^2)^{3/2}}\right\rangle. \tag{34}$$

In order to construct equations for $m_{\tilde{\omega}}^{(1)}$ and $\mathbf{m}_{\tilde{\omega}}^{(1)}$ we have to address the expressions for $f_{\tilde{\omega}+iq}$ and $\mathbf{f}_{\tilde{\omega}+iq}$. In the lowest order in ω_0 and Δ_s

$$f_{\omega} = \frac{i}{2}\frac{\omega\omega_0(\hat{\mathbf{h}}\Delta) - 2i\omega^2\Delta_s}{(\omega^2+\Delta^2)^{3/2}}, \tag{35}$$

$$f_{\tilde{\omega}}^{(1)} \simeq \frac{\tilde{\omega}^2}{(\tilde{\omega}^2+\Delta^2)^{3/2}}\left\{m_{\tilde{\omega}}^{(1)} + \frac{\tilde{\omega}_0}{\tilde{\omega}}\left[\left(1-\frac{3}{2}\frac{\Delta^2}{\tilde{\omega}^2+\Delta^2}\right)(\hat{\mathbf{h}}\Delta)\frac{q}{\tilde{\omega}} + \right.\right.$$
$$\left.\left. + \frac{i}{2}\left(\hat{\mathbf{h}}\mathbf{m}_{\tilde{\omega}}^{(1)} - 3\frac{(\hat{\mathbf{h}}\Delta)(\Delta\mathbf{m}_{\tilde{\omega}}^{(1)})}{\tilde{\omega}^2+\Delta^2}\right)\right]\right\}. \tag{36}$$

Now, according to Eq. (11) and using Eq. (33) for the renormalized frequency $\tilde{\omega}$, we easily obtain the following equations for $m_{\tilde{\omega}}^{(1)}$:

$$\left(\omega + \frac{\tilde{\omega}}{2\tau}\frac{\Delta^2}{(\tilde{\omega}^2+\Delta^2)^{3/2}}\right)m_{\tilde{\omega}}^{(1)} =$$
$$= \frac{1}{2\tau}\frac{\tilde{\omega}_0\tilde{\omega}^2}{(\tilde{\omega}^2+\Delta^2)^{3/2}}\left[\frac{i}{2}\frac{\tilde{\omega}^2}{\tilde{\omega}^2+\Delta^2}\hat{\mathbf{h}}\mathbf{m}_{\tilde{\omega}}^{(1)} + \left(1-\frac{3}{2}\frac{\Delta^2}{\tilde{\omega}^2+\Delta^2}\right)\frac{Q}{\tilde{\omega}}\right], \tag{37}$$

where $Q = \left\langle(\hat{\mathbf{k}}\Delta)q\right\rangle = \frac{1}{3}k_F\Delta_{\parallel}\hat{\mathbf{l}}\mathbf{v}_s$.

A little more algebra is needed to construct an equation for $\mathbf{m}_{\tilde{\omega}}^{(1)}$:

$$\left(\omega\mathbf{m}_{\tilde{\omega}}^{(1)} + \frac{\tilde{\omega}}{2\tau}\left\langle\frac{(\Delta\mathbf{m}_{\tilde{\omega}}^{(1)})\Delta}{(\tilde{\omega}^2+\Delta^2)^{3/2}}\right\rangle\right) =$$
$$= \frac{\tilde{\omega}}{2\tau}\left\{-i\left\langle\frac{\tilde{\omega}\Delta q}{(\tilde{\omega}^2+\Delta^2)^{3/2}}\right\rangle + \frac{3}{4}\frac{\tilde{\omega}_0^2}{(\tilde{\omega}^2+\Delta^2)^{3/2}}\left[\left(\frac{i\tilde{\omega}Q}{\tilde{\omega}^2+\Delta^2} - \frac{1}{3}\hat{\mathbf{h}}\mathbf{m}_{\tilde{\omega}}^{(1)}\right)\hat{\mathbf{h}} + \right.\right.$$
$$\left. + \left\langle\hat{\mathbf{h}}\Delta\frac{(\hat{\mathbf{h}}\mathbf{m}_{\tilde{\omega}}^{(1)})\Delta + (\Delta\mathbf{m}_{\tilde{\omega}}^{(1)})\hat{\mathbf{h}}}{\tilde{\omega}^2+\Delta^2}\right\rangle - \frac{5}{2}\left\langle\frac{(\hat{\mathbf{h}}\Delta)^2(i\tilde{\omega}q + \Delta\mathbf{m}_{\tilde{\omega}}^{(1)})}{(\tilde{\omega}^2+\Delta^2)^2}\Delta\right\rangle\right] +$$
$$\left. + \frac{i}{2}\frac{\tilde{\omega}_0\tilde{\omega}^3}{(\tilde{\omega}^2+\Delta^2)^{5/2}}m_{\tilde{\omega}}^{(1)}\hat{\mathbf{h}}\right\}. \tag{38}$$

It is evident that $\hat{\mathbf{h}}\mathbf{m}_{\tilde{\omega}}^{(1)}$, which appears in Eq. (37), must be calculated at $\omega_0 = 0$. Addressing Eq. (38), we readily find that in this case $\hat{\mathbf{h}}\mathbf{m}_{\tilde{\omega}}^{(1)}$ is given by

$$\left(\omega + \frac{\tilde{\omega}}{2\tau}\frac{\Delta^2}{(\tilde{\omega}^2+\Delta^2)^{3/2}}\right)\hat{\mathbf{h}}\mathbf{m}_{\tilde{\omega}}^{(1)} = -\frac{i}{2\tau}\frac{\tilde{\omega}^2}{(\tilde{\omega}^2+\Delta^2)^{3/2}}Q. \tag{39}$$

Since in what follows we consider the Ginzburg—Landau region (near T_c or near P_c at $T = 0$), a set of equations is to be used:

$$m_{\tilde{\omega}}^{(1)} \simeq \frac{\tilde{\omega}_0}{\tilde{\omega}}\frac{(i/2)\hat{\mathbf{h}}\mathbf{m}_{\tilde{\omega}}^{(1)} + Q/\tilde{\omega}}{2\tau|\omega| + \Delta^2/\tilde{\omega}^2},$$

$$\hat{\mathbf{h}}\mathbf{m}_{\tilde{\omega}}^{(1)} \simeq -\frac{3iQ/\tilde{\omega}}{6\tau|\omega| + \Delta^2/\tilde{\omega}^2}. \tag{40}$$

In the denominators of these expressions the term $\Delta^2/\tilde{\omega}^2$ is retained. When considering the vicinity of T_c, it is to be dropped as a higher-order correction. On the other hand, at $T = 0$ this term must be preserved, as we shall see shortly, in order to avoid unphysical divergences stemming from the vicinity of $\omega = 0$.

The appearance of $m_{\tilde{\omega}}^{(1)} \neq 0$ is due to a mixing of states with $S_z = 0$ of spin-singlet ($S = 0$) and spin-triplet ($S = 1$) configurations in the presence of three factors: quasiparticle scattering ($1/\tau \neq 0$), magnetic field ($\omega_0 \neq 0$), and supercurrent ($Q \neq 0$).

According to the definition of Q, the spin-singlet part $m_{\tilde{\omega}}^{(1)}$ is absent from the planar phase ($\Delta_{\parallel} = 0$). We begin our consideration of $\rho_{ij}^{(s)}$ with just this simple case, for which in the Ginzburg—Landau regime,

$$g_{\tilde{\omega}}^{(1)} \simeq -\frac{i}{|\tilde{\omega}|^3}\left[\Delta_{\perp}^2 q + i\tilde{\omega}\Delta_{\perp}\mathbf{m}_{\tilde{\omega}}^{(1)}\right], \tag{41}$$

$$\Delta_{\perp}\mathbf{m}_{\tilde{\omega}}^{(1)} \simeq -\frac{i}{\tilde{\omega}}\frac{3\langle\Delta_{\perp}q\rangle\Delta_{\perp}}{6\tau|\omega| + \Delta_{\perp}^2/\tilde{\omega}^2}. \tag{42}$$

Now, using Eq. (13) for the supercurrent, it can be easily shown that for the planar phase

$$\frac{\rho_{\parallel}^{(s)}}{\rho} = \pi T\sum_{\omega}\frac{2}{5}\frac{\Delta_{\perp}^2}{|\tilde{\omega}|^3},$$
$$\frac{\rho_{\perp}^{(s)}}{\rho} = \pi T\sum_{\omega}\frac{\Delta_{\perp}^2}{|\tilde{\omega}|^3}\left[\frac{4}{5} + \frac{1}{6\tau|\omega| + \Delta_{\perp}^2/\tilde{\omega}^2}\right] \tag{43}$$

with $\tilde{\omega} = \omega + (1/2\tau)\operatorname{sign}\omega$.

Considering the limit $T = 0$, we convert the ω-summation to integration and, using the frequency renormalization equation

$$1 - \frac{1}{2\tau}\left\langle\frac{1}{[\tilde{\omega}^2 + \Delta_{\perp}^2(\hat{\mathbf{k}})]^{1/2}}\right\rangle = \frac{\omega}{\tilde{\omega}}, \tag{44}$$

pass to a new variable $z(\omega) = 2\tau\tilde{\omega}(\omega)$. For $\epsilon_{\perp} = 2\tau\Delta_{\perp} \ll 1$,

$$2\tau\omega \simeq z - 1 + \frac{1}{3}\frac{\epsilon_\perp^2}{z^2} + \ldots, \tag{45}$$

so that at $T = 0$

$$\pi T \sum_\omega (\ldots) \simeq \frac{1}{2\tau} \int\limits_{1-\epsilon_\perp^2/3}^{\infty} dz \left(1 - \frac{2}{3}\frac{\epsilon_\perp^2}{z^3}\right)(\ldots). \tag{46}$$

Now, from Eqs. (43) it is readily obtained that at $T = 0$ and $P_c < P < P_\parallel$

$$\frac{\rho_\parallel^{(s)}}{\rho} \simeq \frac{1}{5}\epsilon_\perp^2, \qquad \frac{\rho_\perp^{(s)}}{\rho} \simeq \frac{2}{5}\epsilon_\perp^2 \left(1 + \frac{5}{6}\ln\frac{3}{\epsilon_\perp^2}\right). \tag{47}$$

The case with $\Delta_\perp \neq 0$ and $\Delta_\parallel \neq 0$ needs much more effort. Here both $\mathbf{m}_{\tilde\omega}^{(1)}$ and $m_{\tilde\omega}^{(1)}$ contribute to the supercurrent. Starting from the general expression (13), linearizing with respect to q and using Eq. (32), after quite lengthy calculations the following answers for $\rho_\perp^{(s)}$ and $\rho_\parallel^{(s)}$ are obtained at $T = 0$ near the critical pressure $P_\parallel$ ($\epsilon_\parallel = 2\tau\Delta_\parallel$):

$$\frac{\rho_\perp^{(s)}}{\rho} = \frac{1}{2}\left(\frac{4}{5}\epsilon_\perp^2 + \frac{1}{5}\epsilon_\parallel^2\right) + \frac{1}{3}\epsilon_\perp^2 \ln\frac{3}{\epsilon_\perp^2}, \tag{48}$$

$$\frac{\rho_\parallel^{(s)}}{\rho} = \frac{1}{2}\left(\frac{2}{5}\epsilon_\perp^2 + \frac{3}{5}\epsilon_\parallel^2\right) + \frac{1}{3}\epsilon_\parallel^2 \ln\frac{3}{\epsilon_\perp^2} + \frac{1}{6}\epsilon_\parallel^2\frac{\omega_0^2}{\Delta^2} - \frac{3}{16}(2 - \ln 3)\frac{\Delta_\parallel^2}{\Delta^2}\frac{\omega_0^2}{\Delta^2}. \tag{49}$$

The last term in $\rho_\parallel^{(s)}$ is a contribution of the spin-singlet correlations described by $m_{\tilde\omega}^{(1)}$.

In zero magnetic field ($\omega_0 = 0$), from Eqs. (48) and (49) we immediately obtain that

$$\frac{\rho_\parallel^{(s)}}{\rho} = \frac{\rho_\perp^{(s)}}{\rho} = \frac{1}{2}\epsilon^2\left(1 + \frac{2}{3}\ln\frac{3}{\epsilon^2}\right). \tag{50}$$

As a final remark we point out that the results found in Ref. 5 can be readily transcribed to the case of $T = 0$ properties of nonmagnetic impurity-containing HTSC. In this situation, the transition temperature to the superconducting state T_{c0} for a pure sample depends on the level of the hole doping x so that the critical concentration of holes x_c at which a quantum phase transition should occur at $T = 0$ is given by Eq. (1) with the pressure P being substituted by the hole concentration x. The superconducting order parameter $\Delta(x)$ near x_c is given by

$$\Delta^2(x) \simeq \frac{a}{\tau^2}\ln\frac{T_{c0}(x)}{T_{c0}(x_c)} \begin{cases} \Theta(x - x_c), & (\partial T_{c0}/\partial x)_{x_c} > 0, \\ \Theta(x_c - x), & (\partial T_{c0}/\partial x)_{x_c} < 0. \end{cases} \tag{51}$$

Here we have used a simple d-wave pairing model, where in the weak coupling approximation the coefficient $a = 6/5$. The two possibilities in Eq. (51) take into account that in general $T_{c0}(x)$ is a nonmonotonic function of x. For instance, in $La_{2-x}Sr_xCuO_4$, $T_{c0}(x)$ is bell-shaped with a maximum at an optimal doping x_{opt}.

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