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BOUND FERROMAGNETIC AND PARAMAGNETIC POLARONS AS AN ORIGIN OF THE RESISTIVITY PEAK IN FERROMAGNETIC SEMICONDUCTORS AND MANGANITES

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A theory of resistivity is developed for ferromagnetic semiconductors, including possibly manganites. It is based on the interaction of the free and bound charge carriers with the magnetization of the crystal. The temperature dependence of free energy for nonionized donors and free electrons is calculated for the spin-wave and paramagnetic regions. In addition to the trapping by the ferromagnetic fluctuations (the ferromagnetic polarons), the electron trapping by the random magnetization fluctuations as $T \to \infty$ is taken into account (the paramagnetic polarons). For the nondegenerate semiconductors, the theory makes it possible to explain a nonmonotonic temperature dependence of the activation energy, with $T = 0$ value lower than its value for $T \to \infty$. For degenerate semiconductors, the theory explains a metal–insulator transition that occurs in samples with relatively low charge carrier density with increasing temperature. If the density is larger, a reentrant metal–insulator transition should take place, so that the crystal is highly conductive as $T \to \infty$.

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1. INTRODUCTION

The present paper deals with ferromagnetic semiconductors, both degenerate and nondegenerate. All these semiconductors display a resistivity peak in the vicinity of the Curie point T_C. The heavily doped semiconductors displaying the metallic conductivity at $T = 0$ can remain in the insulating state up to very high temperatures after passing this peak; i.e., the metal–insulator transition takes place with increasing temperature. Still more heavily doped semiconductors return to a highly conductive state after passing the peak, i.e., the metal–insulator transition is reentrant in them. The nondegenerate semiconductors have a temperature-dependent activation energy for the conductivity. This energy passes a maximum in the vicinity of the Curie point. The high-temperature activation energy exceeds its low-temperature value. All these materials display a colossal negative magnetoresistance [1].

In what follows, we talk about the donors and conduction electrons, although all the results obtained below remain in force for the holes and acceptors. A semiqualitative explanation of the properties of nondegenerate ferromagnetic semiconductors was given in Refs. 1–3. The point is that the electron levels decrease with increasing magnetization. But the local magnetization in the vicinity of a nonionized donor is higher than the average magnetization in the crystal, which was first pointed out in Refs. 4 and 5. In fact, since the indirect exchange via free charge carriers is exponentially small in nondegenerate semiconductors, the average ferromagnetic ordering is supported only by the superexchange. In contrast, the magnetization near the donor is supported also by the indirect exchange via the donor electron. Hence, at finite temperatures, the donor magnetization is destroyed to a lesser degree than the average magnetization.

The donor overmagnetization means that with increasing temperature, the donor level depth first increases, since the conduction band bottom rises much more rapidly than the donor level. But, with further increase in temperature, the local ordering begins to disappear. The level depth will then decrease with increasing temperature. As a result, the charge carrier density is minimal and the resistivity is maximal at a certain temperature (to avoid a misunderstanding, the magnetization dependence of the donor level depth and of the free charge carrier density was not investigated

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in Refs. 4 and 5).

Unfortunately, the calculation [1–3] was carried out under the assumption that the electron dwells only at the magnetic atoms nearest to the donor atom, which was also assumed in Refs. 4 and 5. On the other hand, the orbital radius must depend on the magnetization for the same reason as in the antiferromagnetic semiconductors [6]: at finite temperatures, the overmagnetized region close to the donor is a potential well for the donor electron. Hence, the electron is attracted to the donor not only by the Coulomb potential, but also by the magnetic potential well. As a result, the orbital radius must be magnetization-dependent and should be found by a self-consistent calculation similar to that carried out for the antiferromagnetic semiconductors in Ref. 6. In complete analogy with the donors in the antiferromagnetic semiconductors, where magnetized regions arise close to the nonionized impurities, one can use the term «the bound magnetic polaron», or «the bound ferron» for the overmagnetized donors.

Calculations of the bound ferrons in the spin-wave region will be carried out below. In this case the overmagnetized region is determined by enhanced ferromagnetic correlations in the vicinity of the nonionized donor. The bound ferron radius and free energy are determined. This allows us to find the free charge carrier density and its activation energy as functions of temperature. In essence, this part of the paper develops ideas set forth for antiferromagnetic semiconductors in Refs. 1 and 2, although it requires a quite different calculation procedure.

In addition to the already known low-temperature bound ferron, a new type of bound ferron will be considered. It exists in the limit $T \to \infty$ and can be called the paramagnetic bound polaron (ferron). While the ferrons investigated so far are related to a self-consistent enhancement of the ferromagnetic correlations in the region of the electron localization, the correlations are absent here, and only the fluctuating magnetization of the region increases with decreasing size; it is of the order of $1/N_I^{1/2}$, where N_I is the number of magnetic atoms over which the donor electron is spread.

Although the mean local magnetization remains zero, the electron spin adjusts to the fluctuating magnetization, fluctuating jointly with it and thus ensuring the gain in the exchange energy between the electron and the magnetic atoms. This means that the tendency arises for the electron to be concentrated inside a region as small as possible in size. This tendency competes with the Coulomb interaction and kinetic energy in determining the equilibrium orbital radius. The shrinking of the electron orbit caused by the magnetization fluctuations can lead to a considerable lowering of the donor level as compared with its depth at $T = 0$. Hence, the low-temperature activation energy for the resistivity is less than the high-temperature activation energy. The uncorrelated fluctuations possibly also can trap the free charge carriers, but the binding energy of the free fluctuation polarons, if it is nonzero, should be very small.

Calculations carried out for nondegenerate ferromagnetic semiconductors are generalized for the degenerate semiconductors in the following way. The free energy of the impurity metal consisting of the ionized donors and electrons is calculated and compared with the free energy of separate donors. If the former at $T = 0$ is lower than the latter, then an increase in T can cause a crossover, which means that there is a transition to the insulating state. As for the reverse transition at elevated temperatures, it can occur only for those systems in which the donor density markedly exceeds the density of the metal–insulator transition at $T = 0$. If this is not the case, the reverse transition is prohibited due to the paramagnetic ferrons. This result agrees with the experimental data on degenerate ferromagnetic semiconductors presented in Ref. 1.

It should be pointed out that earlier explanation of the temperature-induced metal–insulator transition in degenerate ferromagnetic semiconductors was given in terms of a modified Mott theory, in which only the stability of the metallic state was investigated [1, 7]. But this approach seems to be less fruitful than that used here. In particular, it does not lead to the insulating state as $T \to \infty$, i.e., it does not allow to explain some essential features of the degenerate ferromagnetic semiconductors. In addition, it can be used if only the donor density is very close to the density at which the Mott transition takes place at $T = 0$. The approach used here is more general, allowing to overcome these limitations and to prove the similarity of the physical mechanisms leading to the resistivity peak in the nondegenerate ferromagnetic semiconductors and to the metal–insulator transitions in degenerate ferromagnetic semiconductors. But some problems treated in Refs. 1 and 7 remain beyond the scope of our paper (e.g., the charge carrier mobility). Thus, the present approach and that adopted in Refs. 1 and 7 are complementary.

2. INDIRECT EXCHANGE HAMILTONIAN FOR A NONIONIZED DONOR

To analyze the magnetic properties of the nonionized donors, it is advisible to begin with construction of the magnetic Hamiltonian describing the indirect exchange via the donor electron. It must differ strongly from the RKKY indirect Hamiltonian since the latter assumes that the conduction electrons are completely spin-depolarized to the zero approximation. Certainly, the situation with a sole donor electron is quite opposite.

As usual, the *s*-*d* model is used. The Hamiltonian of the system in the coordinate representation is given by

$$\begin{aligned} &H = H_s(\mathbf{r}) + H_{sd}(\mathbf{r}) + H_{dd}, \\ &H_s = -\frac{\Delta}{2m} - \frac{e^2}{\varepsilon r}, \\ &H_{sd} = -A\sum_{\mathbf{g}}(\mathbf{S_g s})D(\mathbf{r}-\mathbf{g}), \\ &H_{dd} = -\frac{I}{2}\sum_{\mathbf{g},\Delta}(\mathbf{S_g S_{g+\Delta}}), \end{aligned} \tag{1}$$

where $\mathbf{S_g}$ is the *d*-spin of the atom $\mathbf{g}$, $\mathbf{s}$ is the conduction electron spin, $D(\mathbf{r}-\mathbf{g})$ is equal to unity inside the unit cell $\mathbf{g}$ and zero outside it, m is the electron effective mass, ε is the dielectric constant, Δ is the vector connecting the nearest neighbors, $\hbar = 1$. The *s*-*d* exchange integral A is assumed to be positive.

In what follows, the inequality $AS \ll W$ is assumed to be met where S is the *d*-spin magnitude, W is the conduction band width. This inequality is certainly met in rare-earth semiconductors (EuO, EuS, and others) [1] and can also be satisfied in transition metal compounds. In particular, it can possibly be met in colossal magnetoresistance manganites, although the experimental situation is not clear about them yet. Many investigators believe that the holes in them move not over the Mn ions but over the oxygen ions [8–10]. In this case the *s*-*d* exchange is relatively weak and the band width is relatively large, in contrast with to the double-exchange case where holes move over the Mn ions.

As usual, in the theory of the indirect exchange, the adiabatic approximation is used when, in dealing with the *s*-electron, the *d*-spins are considered as the classical vectors. In the first approximation in AS/W, the wave function of the system can be separated into the orbital part and the spin part:

$$\Psi(\mathbf{r}, \{S^z\}, \sigma) = \psi(\mathbf{r})\eta(\{S^z\}, \sigma), \tag{2}$$

where the normalized magnetic wave function η of the set of the *d*-spin variables $\{S^z\}$ and *s*-electron spin variable σ will be specified below as a functional of the donor ground-state orbital wave function ψ. After constructing the wave equation with the Hamiltonian (1) and wave function (2), multiplying it by $\psi(\mathbf{r})$ from the left side, and integrating over $\mathbf{r}$, we obtain the wave equation for the magnetic subsystem

$$\begin{aligned} &H_{av}\eta = (E - E_I)\eta, \qquad E_I = \int d^3r\psi H_s\psi, \\ &H_{av} = -A\sum w(\mathbf{g})(\mathbf{S_g s}), \quad w(\mathbf{g}) = \psi^2(\mathbf{g})a^3, \end{aligned} \tag{3}$$

where E_I is the energy of the *s*-electron bound to the impurity, and a is the lattice constant.

The eigenfunction of H_{av} is represented in the form

$$\eta(\{S^z\}, \sigma) = \phi(\{S^z\})\delta(\sigma, 1/2) + \chi(\{S^z\})\delta(\sigma, -1/2), \tag{4}$$

where $\delta(\sigma, \pm 1/2)$ is the *s*-electron spin wave function with $\delta(x, y) = 1$ for $x = y$ and $\delta(x, y) = 0$ for $x \neq y$, (ϕ, χ) is the two-component wave function of the *d*-system.

Using Eqs. (3) and (4), we can represent the wave equation in the form (E_I is omitted)

$$\begin{aligned} &\frac{AL^+}{2}\phi + \left(E - \frac{AL^z}{2}\right)\chi = 0, \\ &\frac{AL^-}{2}\chi + \left(E + \frac{AL^z}{2}\right)\phi = 0, \\ &\mathbf{L} = \sum w(\mathbf{g})\mathbf{S_g}, \qquad L^{\pm} = L^x \pm iL^y. \end{aligned} \tag{5}$$

In the particular case of $w(\mathbf{g}) = 1/N_I$, the system of equations (5) can be solved exactly. Accordingly, we use the following relations, which are valid for any function $f(S^z)$ of S^z:

$$S^- f(S^z) = f(S^z + 1)S^-, \qquad L^-L^+ = L^2 - L^z(L^z + 1).$$

They follow from the definition of the S^- operator and from the commutation rules for the spin operators. The exact result for $w(\mathbf{g}) = \text{const} \equiv 1/N_I$ is

$$\left(E + \frac{A}{2N_I}\right)^2 \phi = \left(\frac{A}{2N_I}\right)^2 \left(\mathbf{L}^2 + \frac{1}{4}\right)\phi,$$

which corresponds to the effective magnetic Hamiltonian

$$H_{mI} = \frac{A}{2N_I}\left(\frac{1}{2} \pm \sqrt{\mathbf{L}^2 + \frac{1}{4}}\right). \tag{6}$$

The double sign in Eq. (6) corresponds to two possible spin projections of the conduction electron onto the total moment $\mathbf{L}$ of the *d*-spins. As should be the case, the exact eigenvalues of the Hamiltonian (6) are $(-AL/2)$ and $A(L + 1)/2$.

For an arbitrary $w(\mathbf{g})$, the system of equations (5) can be solved with accuracy of $1/2SN_I$, where N_I is the number of magnetic atoms over which the localized electron is spread. The terms of this order omitted below arise as a result of commuting L^- and

$(E - AL^z/2)^{-1}$ after exclusion of χ from the second equation in (5). In this case, one obtains the following relation with the accuracy pointed out above:

$$H_{mI} = \pm\frac{A}{2}\sqrt{\sum_{\mathbf{g},\mathbf{f}} w(\mathbf{g})w(\mathbf{f})(\mathbf{S_g S_f})}. \qquad (7)$$

In contrast with the RKKY Hamiltonian, the Hamiltonians (6) and (7) are linear and not quadratic in A. More importantly, they describe not only the bilinear exchange but also the multispin exchange in which up to $N_I(N_I - 1)$ spins take part simultaneously. The Hamiltonian (7) contains also the biquadratic terms and terms of higher orders in the scalar product of the spins, as well as terms of still more complicated structure. This is seen from Eq. (7), if one separates the diagonal terms with $\mathbf{g} = \mathbf{f}$ and then expands Eq. (7) in terms of the nondiagonal terms with $\mathbf{g} \neq \mathbf{f}$.

The strength of the indirect exchange between the spins does not depend on the distance between them, but depends on the distance of each d-spin from the donor atom. Obviously, the Hamiltonians (6) and (7) are isotropic, and there is no gap in the spectrum for the uniform spin rotation, as should be the case. At $T = 0$, the Hamiltonians (6) and (7) correctly reproduce the s-d exchange energy for the complete ferromagnetic ordering (the latter with accuracy of $1/2SN_I$).

3. DONOR STATES AND THE RESISTIVITY PEAK IN NONDEGENERATE FERROMAGNETIC SEMICONDUCTORS (SPIN-WAVE REGION)

In this section our first task is to calculate the free energy for a ferromagnetic system containing nonionized donors. This calculation is inapplicable to the manganites since the undoped manganites are antiferromagnetic, and only the heavily doped manganites are ferromagnetic. The problem can be solved by using a variational procedure for the free energy under condition that the donor electron is in the ground state with the wave function

$$\psi(r) = \sqrt{\frac{x^3}{\pi a_B^3}}\exp\left(-\frac{xr}{a_B}\right), \qquad a_B = \frac{\varepsilon}{me^2}, \qquad (8)$$

where x is the variational parameter.

In addition to the electronic energy, the total free energy includes contribution from the magnons, whose frequencies are renormalized as a result of their interaction with the donor electron, realizing the indirect exchange between the d-spins in the vicinity of the donor atom. The state of the magnon subsystem is determined from the spin-wave Hamiltonian, including the direct d-d exchange from Eq. (1) and indirect exchange (7). It is obtained from these equations after the Holstein—Primakoff transformation

$$S_\mathbf{g}^z = S - b_\mathbf{g}^* b_{-\mathbf{g}}, \quad S_\mathbf{g}^+ = \sqrt{2S}b_\mathbf{g}, \quad S_\mathbf{g}^- = \sqrt{2S}b_\mathbf{g}^*, \quad (9)$$

where the electron distribution $w(\mathbf{g})$ corresponds to Eq. (8):

$$H_{mg} = IS\sum_{\mathbf{g},\Delta}(b_\mathbf{g}^* b_\mathbf{g} - b_\mathbf{g}^* b_{\mathbf{g}+\Delta}) + \\ + \frac{A}{2}\sum_{\mathbf{g},\mathbf{f}} w(\mathbf{g})w(\mathbf{f})(b_\mathbf{g}^* b_\mathbf{g} - b_\mathbf{g}^* b_\mathbf{f}). \quad (10)$$

The last term $\sim b_\mathbf{g}^* b_\mathbf{f}$ in Eq. (10) is basically important to ensure the absence of the gap in the magnon spectrum. But it does not influence the bulk of the magnon frequencies. For example, if $w(\mathbf{g}) = 1/N_I$, only the $q = 0$ magnon has the zero frequency. In the absence of the d-d exchange, $N_I - 1$ magnons with other wave vectors have the same frequency $A/2N_I$. Hence, in calculating the free energy we can use the following Hamiltonian for the magnon frequencies:

$$H_{mg} = IS\sum_{\mathbf{g},\Delta}(b_\mathbf{g}^* b_\mathbf{g} - b_\mathbf{g}^* b_{\mathbf{g}+\Delta}) + H(\mathbf{g})b_\mathbf{g}^* b_\mathbf{g}, \\ H(\mathbf{g}) = \frac{A\psi^2(\mathbf{g})a^3}{2}. \qquad (11)$$

But the Hamiltonian (11), written with allowance for Eq. (8), is still too complicated to be diagonalized exactly. The perturbation theory also cannot be used here. To carry out an approximate calculation, we must replace the magnon potential hump $H(\mathbf{g})$ of a compicated shape in Eq. (11) by a rectangular potential hump with the height h and radius ρ equal to the mean height of the hump (11) and the mean radius of the electron wave function:

$$h = \sum H(g)\psi^2(g) = \frac{Ax^3}{16\pi b^3}, \\ \rho = \sum g\psi^2(g) = \frac{3a_B}{2x}, \qquad b = \frac{a_B}{a}. \qquad (12)$$

This means that the magnon frequency in the region close to a nonionized donor is given by

$$\omega_\mathbf{q}^I(x) = \Omega_\mathbf{q} + h, \qquad (13)$$

$$\Omega_\mathbf{q} = J(1 - \gamma_\mathbf{q}), \quad J = zIS, \quad \gamma_\mathbf{q} = \frac{1}{z}\sum_\Delta \exp(i\mathbf{q}\Delta),$$

where z is the coordination number.

To calculate the density of the conduction electrons in a nondegenerate semiconductor, it is necessary to write the spin-wave Hamiltonian with allowance for the conduction electrons. The relative number of the donors ζ is assumed to be small.

This makes it possible to disregard the interaction between s-electrons. We can divide all regular magnetic atoms into those which enter spheres of radius ρ surrounding donors and those which are outside these spheres (the number of the latter greatly exceeds the total number of the former). Using the expression for the conduction-electron-magnon Hamiltonian H_{mg} (11), (13), we can represent the total electron-magnon Hamiltonian in the form

$$\begin{aligned} H = \sum n_{I,i}\left(E_I + \sum \omega^I_{\mathbf{q}} m_{\mathbf{q},i}\right) + \\ + \sum (1-n_{I,i})\,\Omega_{\mathbf{q}} m_{\mathbf{q},i} + \sum E_{\mathbf{k}} n_{\mathbf{k}} + \\ + \sum B_{\mathbf{q}} n_{\mathbf{k}} m_{\mathbf{q}} + \sum \Omega_{\mathbf{q}} m_{\mathbf{q}} - \mu \sum (n_{I,i} + n_{\mathbf{k}}), \end{aligned} \tag{14}$$

where $m_{\mathbf{q},i}$ and $m_{\mathbf{q}}$ are the magnon operators for the i-th sphere and outside the spheres that surrounds donors, respectively. Since the magnon number operators for different donor regions and outside them are constructed of magnon operators $b^*_{\mathbf{g}}$ and $b_{\mathbf{g}}$ with different $\mathbf{g}$, all the operators $m_{\mathbf{q},i}$ and $m_{\mathbf{q}}$ are independent.

Further, $n_{I,i}$ and $n_{\mathbf{k}}$ are the operators for an electron in the localized state at the donor i and for the delocalized electrons with the quasimomentum $\mathbf{k}$, respectively, The spin index is absent from the electron operators since the electrons are completely spin-polarized in the spin-wave region. For the same reason, the s-d exchange energy $(-AS/2)$ is the same for all the electronic states considered and therefore can be omitted as an additive constant. The quantity μ is the chemical potential.

The energy E_I of an electron at the donor is given by Eqs. (3) and (8). At low temperatures we can set x =1 in Eq. (8), so that $E_I = -E_B = -e^2/2\varepsilon a_B$. The quantity $B_{\mathbf{q}}$, which describes the s-d interaction of the delocalized electrons with magnons when the electron quasimomenta are small compared to the magnon quasimomenta, has the form [1, 11]

$$B_{\mathbf{q}} = \frac{Aq^2}{2N(p^2+q^2)}, \qquad p^2 = 2mAS, \tag{15}$$

where m is the s-electron effective mass, and N is the total number of atoms.

With allowance for mutual independence of $m_{\mathbf{q},i}$ and $m_{\mathbf{q}}$, the mean number of electrons at a donor is given by the expression (the index of the donor is omitted)

$$\begin{aligned} \langle n_I\rangle = \sum_m \exp[-(E_I-\mu)/T - \sum_{\mathbf{q}} \omega^I_{\mathbf{q}} m_{\mathbf{q}}/T] \times \\ \times \left(\sum_m \left\{ \exp[-(E_I-\mu)/T \; - \right.\right. \\ \left.\left. - \sum_{\mathbf{q}} \omega^I_{\mathbf{q}} m_{\mathbf{q}}/T] + \exp[-\sum_{\mathbf{q}} \Omega_{\mathbf{q}} m_{\mathbf{q}}/T] \right\}\right)^{-1}. \end{aligned} \tag{16}$$

In Eq. (16) the summation over m denotes summation over $m_{\mathbf{q}}$. Carrying out the summation, we find

$$\begin{aligned} \langle n_I\rangle = \\ = \left\{1+\exp\left(\frac{E_I-\mu}{T}\right)\frac{\prod_{\mathbf{q}}[1-\exp(-\omega^I_{\mathbf{q}}/T)]}{\prod_{\mathbf{q}}[1-\exp(-\Omega_{\mathbf{q}}/T)]}\right\}^{-1} = \\ = \left[1+\exp\left(\frac{E_I+\delta F_{mI}-\mu}{T}\right)\right]^{-1}, \end{aligned} \tag{17}$$

$$\delta F_{mI} = F_{mI} - F^0_{mI} = N_I(f_I - f_0),$$

where F_{mI} and F^0_{mI} are the magnon free energies for a region of radius ρ containing the nonionized and ionized donor, respectively,

$$\begin{aligned} f_I &= T\frac{a^3}{(2\pi)^3}\int d^3q \ln\left[1-\exp\left(-\frac{\omega^I_{\mathbf{q}}}{T}\right)\right], \\ f_0 &= T\frac{a^3}{(2\pi)^3}\int d^3q \ln\left[1-\exp\left(-\frac{\Omega_{\mathbf{q}}}{T}\right)\right], \\ N_I &= \frac{4\pi\rho^3}{3a^3}. \end{aligned} \tag{18}$$

A similar calculation is carried out for the mean number $\langle n_{\mathbf{k}}\rangle$ of electrons with the quasimomentum $\mathbf{k}$:

$$\begin{aligned} \langle n_{\mathbf{k}}\rangle = \left\{1+\exp\left(\frac{E_{\mathbf{k}}-\mu}{T}\right)\times\right. \\ \left.\times\frac{\prod_{\mathbf{q}}[1-\exp(-\Omega_{\mathbf{q}}/T - B_{\mathbf{q}}/T)]}{\prod_{\mathbf{q}}[1-\exp(-\Omega_{\mathbf{q}}/T)]}\right\}^{-1} = \\ = \left[1+\exp\left(\frac{E_{\mathbf{k}}+\delta F_{mC}-\mu}{T}\right)\right]^{-1}, \end{aligned} \tag{19}$$

$$\delta F_{mC} = F_{mC} - F^0_{mC} = N(f_C - f_0),$$

$$f_C = T\frac{a^3}{(2\pi)^3}\int d^3q \ln\left[1-\exp\left(-\frac{\Omega_{\mathbf{q}}}{T}-\frac{B_{\mathbf{q}}}{T}\right)\right]. \tag{20}$$

Keeping in mind the fact that $B_{\mathbf{q}} \sim 1/N$, we can write

$$\delta F_{mC} = \frac{Aa^3}{2(2\pi)^3}\int d^3q \frac{q^2}{p^2+q^2}\,\frac{1}{[\exp(\Omega_{\mathbf{q}}/T)-1]}. \tag{21}$$

Equating the number of ionized donors with the total number of the conduction electrons, we find an expression for the charge carrier density n_{cc} for $E_{\mathbf{k}} = k^2/2m$:

$$n_{cc} = \sqrt{nn_{eff}}\exp(-E_B + \delta F_m/2T), \qquad n_{eff} = \frac{(mT)^{3/2}}{2\sqrt{2\pi}} \qquad \delta F_m = \delta F_{mI} - \delta F_{mC}, \tag{22}$$

where n_{eff} is the effective density of states in the conduction band, and $n = \zeta/a^3$ is the donor density.

It can be ascertained that the activation energy in Eq. (22) increases with temperature in the spin-wave region. It is sufficient to consider the case of $J \gg h, p^2 \ll 1$. Using Eqs. (17)–(22), we find

$$\delta F_m = -\frac{7Aa^3}{32(2\pi)^3}\int d^3q \frac{1}{[\exp(\Omega_{\mathbf{q}}/T) - 1]}. \tag{23}$$

In other words, δF_m is negative, and its absolute value increases with temperature. This conclusion is confirmed by numerical calculations. For example, for $A = 2$, $I = 0.02$ (in the E_B units), $\delta F_m = -0.022$ at $T = 0.01$, but $\delta F_m = -0.214$ at $T = 0.03$. The fact that the activation energy increases with temperature in the spin-wave region suggests that the resistivity peak at elevated temperatures is caused by a minimum in the charge carrier density.

4. TEMPERATURE-INDUCED METAL–INSULATOR TRANSITION (SPIN-WAVE REGION)

In this section we investigate the transition of a degenerate ferromagnetic semiconductor into the insulating state, which occurs with increasing temperature. We will compare the free energy of the highly conductive state with that of the insulating state. First, using Eqs. (1), (8), (12), and (13), we will write the total free energy of a separated nonionized donor in the E_B units:

$$F_I(x) = (x^2 - 2x) + \delta F_{mI}(x). \tag{24}$$

If one considers the term $\delta F_{mI}(x)$ in Eq. (24) as a small perturbation, the optimal value of x is

$$x = 1 - \frac{1}{2}\frac{d\delta F_{mI}}{dx}(1) \tag{25}$$

and, to a first approximation, the optimal free energy is

$$F_I^{opt} = -1 + \delta F_{mI}(1). \tag{26}$$

Since $\delta F_{mI}(x)$ [Eqs. (17) and (18)] decreases with decreasing x and, hence, the last term in Eq. (25) is negative, the parameter x increases with temperature, and, accordingly, the electronic radius decreases. This is a manifestation of the ferronic effect: the electron is dragged in by the region of the enhanced magnetization and simultaneously supports it, realizing the indirect exchange inside it. Temperature-induced decrease in the electron radius points to the tendency of the temperature-induced transition from the metallic to the insulating state if at $T = 0$ the system is metallic. In fact, if at $T = 0$ the orbit overlapping of neighboring atoms is sufficient for metallization, at finite temperatures this overlapping is insufficient, and the transition to the insulating state should take place. To prove the possibility of such a transition, one should compare the free energy of separated nonionized donors and that of the impurity metal which consists of ionized donors and delocalized electrons.

Under typical conditions for degenerate ferromagnetic semiconductors, due to a relatively small electron density in them, the condition $\mu < AS$ is met (here μ is the Fermi energy [1]). In other words, the electron gas is completely spin-polarized in the spin-wave region. Using expressions for the energy of the electron gas from Refs. 12 and 13, we find the following expression for the donor metal energy per donor atom (unlike the «magnetic» index m, the index M denotes metal):

$$E_M^F = E(k=0) + \frac{3(6\pi^2 n)^{2/3}}{10m} + E_{ex}(n), \tag{27}$$

where $E(k = 0)$ is the electron energy at the conduction band bottom, E_{ex} is the exchange energy between conduction electrons, and n is the electron (or donor) density. Under condition of the complete spin polarization, $E_{ex}(n)$ can be easily obtained by generalization of the corresponding Bloch expression for the completely spin-depolarized electron gas, e. g., in Ref. 12:

$$E_{ex}(n) = -\frac{3}{4}\left(\frac{6n}{\pi}\right)^{1/3}\frac{e^2}{\varepsilon}. \tag{28}$$

To calculate the energy $E(k = 0)$, we will use the Wigner—Seitz procedure (see, e. g., Ref. 13). Each ionized donor is surrounded by a sphere of radius $L = (3/4\pi n)^{1/3}$. Inside of each Wigner—Seitz shell, the electron wave function Φ corresponding to $k = 0$, satisfies the wave equation

$$\left(-\frac{\Delta}{2m} - \frac{e^2}{\varepsilon r} - E(k=0)\right)\Phi(r) = 0 \tag{29}$$

with the boundary condition

$$\frac{d\Phi}{dr}(L) = 0. \tag{30}$$

As is well known from the theory of metal adhesion, the wave function Φ should be almost constant with the boundary condition (30). A special analysis shows that for relative densities ζ between 0.001 and 0.1 the

Φ = const approximation ensures an accuracy in energy higher than 1%. With sufficient accuracy, we can therefore set

$$E(k=0) = -3\left(\frac{4\pi n}{3}\right)^{1/3} E_B. \qquad (31)$$

With allowance for Eqs. (28) and (31), the energy (27) in the E_B units takes the form

$$E_M^F = \frac{3}{5}(6\pi^2\zeta)^{2/3}b^2 - (36\pi\zeta)^{1/3} - \frac{3}{2}\left(\frac{6\zeta}{\pi}\right)^{1/3} b, \qquad (32)$$

where $\zeta = na^3$, and $b = a_B/a$.

At finite temperatures, the free energy of the donor metal with the volume V is given by the expression

$$\begin{gathered} G^F(n) = nVE_M^F(n) + Nf_M, \\ f_M = T\frac{a^3}{(2\pi)^3}\int d^3q \ln\left[1 - \exp\left(-\frac{\omega_{\mathbf{q}}^M}{T}\right)\right], \end{gathered} \qquad (33)$$

where, with allowance for the non-RKKY indirect exchange in our case (since $\mu < AS$), the magnon frequencies are given by the expression [1, 11], see also Eq. (15)

$$\omega_{\mathbf{q}}^M = \Omega_{\mathbf{q}} + \frac{Aq^2\zeta}{p^2 + q^2}, \qquad p^2 = 2mAS. \qquad (34)$$

Equating the energy $E_M^F(n)$ (32) with the donor energy $E_I = -E_B$, we find that the density n_0, at which the electron delocalization takes place at $T = 0$, obeys the relation $n_0^{1/3}a_B = C$, where $C = 0.208$, which is slightly lower than the value of 0.25 found by Mott.

To find the transition temperature from the the highly conductive state to the insulating state for a material with n exceeding n_0, one should equate the metal free energy G^F (33) with the free energy of the localized state found with the use of Eqs. (26) and (17):

$$F^I = \zeta N(E_I + N_I f_I) + Nf_0(1 - \zeta N_I).$$

For n sufficiently close to n_0, we then obtain the following implicit expression for the transition temperature:

$$\begin{gathered} (\zeta - \zeta_0)\frac{d(nE_M^F)}{dn} = \zeta N_I(f_I - f_0) + (f_0 - f_M) \\ \zeta_0 = n_0 a^3. \end{gathered} \qquad (35)$$

Numerical calculations based on Eq. (32) show that the quantity $d(nE_M^F)/dn$ is negative for $\zeta < 0.2$. This does not mean that the system is unstable since this derivative is not the electron Fermi energy. It does mean that the energy of the donor metal changes as a result of the change in the number of the donor atoms by unity. The expression on the right-hand side of Eq. (35) is also negative for x close to unity, which is seen from numerical calculations. The proof of this statement is especially simple in the case $S \gg 1$ if one considers the region $T_C/S \ll T \ll T_C$ and uses Eqs. (12), (13), (15), (18), and (33) (T_C is the Curie point). This means that the equality (35) can be met for ζ that exceeds ζ_0 only moderately, and the transition from the metallic state to the insulating state should take place with increasing temperature. But for large densities, $\zeta > 0.2$, this transition is prohibited at least in the spin-wave region which agrees with the experimental data cited in the Introduction.

5. FLUCTUATION TRAPPING IN THE PARAMAGNETIC REGION AND RESISTIVITY OF NONDEGENERATE SEMICONDUCTORS

Calculations carried out in this section and in the next possibly are also applicable to the manganites. First, the expression (7) will be analyzed in the limit $T \to \infty$. Although the correlations between the d-spins are absent, the s-d exchange energy remains nonzero in the first order in AS/W. We see from Eq. (6) that in this case

$$E_{mI} = \pm\frac{A}{2}\sqrt{\frac{S(S+1)}{N_I}}. \qquad (36)$$

The physical meaning of result (36) is clear if we recall that, according to the mathematical statistics, a system of N noninteracting spins should possess the total moment on the order of $N^{-1/2}$ of their maximal moment NS. The direction of this moment is not fixed but fluctuates freely, so that its mean value should vanish. But the spin of the s-electron adjusts to the direction of the fluctuating moment and fluctuates jointly with it, providing the maximum gain in the s-d exchange energy for the energetically favored direction of the s-electron spin relative to the total spin of its localization region. This gain should be on the order of the total moment per atom, i.e., $\sim AS/N^{1/2}$, as is the case for Eq. (36). The term of order AS/W is essential only for orbital radii that are sufficiently small. For larger radii, the terms of the next order in AS/W should be taken into account.

Let us now consider the bound ferron at $T \gg T_C$, taking into account the fluctuation lowering of the energy discussed above. When the correlations between the d-spins are weak, the donor magnetic Hamiltonian (7), jointly with the direct d-d exchange Hamiltonian

(1), can be represented in the Heisenbergian form

$$\begin{gathered}H_{mP} = -\frac{A}{2}\sqrt{P} + H_H, \\ H_H = -\frac{1}{2}\sum_{\mathbf{g}\neq\mathbf{f}} I_{tI}(\mathbf{g},\mathbf{f})(\mathbf{S}_\mathbf{g}\mathbf{S}_\mathbf{f}), \\ I_{tI}(\mathbf{g},\mathbf{f}) = \frac{A}{2\sqrt{P}} w(\mathbf{g})w(\mathbf{f}) + I(\mathbf{g}-\mathbf{f}), \\ P = S(S+1)\sum w^2(\mathbf{g}).\end{gathered} \tag{37}$$

The free energy of the system is obtained by the high-temperature expansion to the first order in $1/T$:

$$\begin{aligned}F^{PI} &= E_I - \frac{A}{2}\sqrt{P} + F_{mP}, \\ F_{mP} &= -T\ln\operatorname{Tr}\exp\left(-\frac{H_H}{T}\right) = \\ &= -NT\ln(2S+1) - \frac{S^2(S+1)^2}{12T}\sum I_{tI}^2(\mathbf{g},\mathbf{f}).\end{aligned} \tag{38}$$

Calculating the electron energy E_I using the Hamiltonian H_s (1) and the trial wave function (8), and keeping in mind that the direct exchange integral $I(\mathbf{g}-\mathbf{f})$ is nonzero only in the nearest-neighbor approximation, we can write the x-dependent portion of the free energy (38) in the form (for $z=6$)

$$F^{PI}(x) = (x^2-2x)E_B - Lx^{3/2} - \frac{Q(x)}{T}, \tag{39}$$

where

$$L = \frac{A}{2}\sqrt{\frac{S(S+1)}{8\pi}}\, b^{-3/2}, \qquad b = \frac{a_B}{a},$$

$$\begin{aligned}Q(x) = {}& \frac{L^2x^3}{12} + \\ &+ AIS^{3/2}(S+1)^{3/2}\sqrt{\frac{x^3b^3}{32\pi}}\left(1+2xb+\frac{4x^2b^2}{3}\right)e^{-2xb}.\end{aligned}$$

In writing Eq. (39), we calculate the integral

$$\sum w(\mathbf{g})w(\mathbf{g}+\Delta)$$

in $Q(x)$ in elliptic coordinates. The entropy term $-NT\ln(2S+1)$ is omitted from the free energy here and below.

Minimizing the free energy (39) with respect to x, we obtain its optimal value and inverse orbital radius in the limit $T\to\infty$ (in the E_B and $1/a_B$ units, respectively)

$$F_\infty = -\frac{8}{3}l^3[l+\sqrt{1+l^2}] - \frac{8}{3}l\sqrt{1+l^2} - 4l^2 - 1, \tag{40}$$

$$x_\infty = [l+\sqrt{1+l^2}]^2; \qquad l = \frac{3L}{8E_B} \sim \frac{AS}{(We^2/\varepsilon a)^{1/2}}. \tag{41}$$

If one sets $a_B = a$, then for AS/E_B varying from 1 to 5, the energy F_∞ varies from -1.104 to -1.659, and the radius x_∞ from 1.077 to 1.1445. Hence, the electron interaction with random (uncorrelated) magnetization fluctuations leads to a marked decrease in the donor ionization energy and in the orbital radius; this applies to any type of magnetic ordering at $T=0$. The corresponding electron state can be called the bound paramagnetic fluctuation polaron (ferron).

Formally, random fluctuations could cause the trapping of a charge carrier in the absense of the impurity potential (the free paramagnetic ferron). In contrast with the ferron self-trapping, which occurs in the region of the enhanced magnetization, no ferromagnetic correlations between d-spins appear in the region of the electron localization. Mathematically, jointly with the solution $x=0$ corresponding to a free electron, solution of Eq. (39) with $x=4l^2$ exists. The corresponding free energy of the trapped electron is

$$F_t = -\frac{16l^4}{3} \sim AS\left(\frac{AS}{W}\right)^3. \tag{42}$$

According to Eq. (42), the depth of the levels corresponding to these trapped states is very small: it is beyond the accuracy in AS/W adopted here. For this reason, the free fluctuation ferrons will not be considered in what follows. In the limit $T\to\infty$, we then obtain the following equation for the charge carrier density similar to Eq. (22):

$$n_{cc} = \sqrt{nn_{eff}}\exp\left(\frac{F_\infty}{2T}\right). \tag{43}$$

We see from comparison of Eqs. (43) and (22) that the high-temperature activation energy of the conductivity $(-F_\infty/2)$ exceeds the low-temperature activation energy.

6. TEMPERATURE-INDUCED METAL–INSULATOR TRANSITION IN THE PARAMAGNETIC REGION

At finite temperatures, from Eqs. (38) and (39) we obtain the total free energy of a system of N magnetic atoms and nV donors,

$$\begin{aligned}F(T) = nF_\infty - \frac{nVQ(x_\infty) + NS^2(S+1)^2I^2/2}{T} - \\ - NT\ln(2S+1),\end{aligned} \tag{44}$$

and for the donor orbital radius we have

$$x(T) = x_\infty + \frac{1}{2T(1-lx_\infty^{-1/2})}\frac{dQ}{dx}(x_\infty), \tag{45}$$

where x_∞ is given by Eq (41). We can prove that the second term on the right-hand side of Eq. (45) is positive if the parameter

$$c = \sqrt{32\pi S(S+1)b^3}\frac{I}{A}$$

is in the range between -1 and 40. With $I > 0$, for any actual parameter values, the inequality $c < 40$ is guaranteed. On the other hand, it can be satisfied even if $I < 0$ but dominates the indirect exchange, ensuring the total ferromagnetic ordering at $T = 0$. In fact, the intensity of the indirect exchange is proportional to $A^2S^2\zeta^{1/3}/W$, which can exceed the intensity of the d-d exchange, zIS^2, if the latter quantity is small compared with A^2S^2/W. The fact that the second term in Eq. (45) is positive means that the radius of the donor orbital state decreases with decreasing temperature. This points to the tendency for the electron localization at lower temperatures if the electrons are delocalized in the limit $T \to \infty$.

Let us now investigate in more detail the temperature-induced transition from the metallic to the insulating state, which can occur with decreasing temperature. In the high-temperature limit, the total free energy of the donor metal is given by

$$G^{PM} = nVE_M^{PM} + \delta G^{PM}. \tag{46}$$

The energy of a nonmagnetized crystal per donor atom, instead of Eq. (32), is given by the following expression, which includes the correlation contribution [12]:

$$\begin{aligned} E_M^{PM} = {} & \frac{3}{5}(3\pi^2\zeta)^{2/3}b^2 - (36\pi\zeta)^{1/3}b - \\ & - \frac{3}{2}\left(\frac{3\zeta}{\pi}\right)^{1/3} b - \frac{0.113\zeta^{1/3}b}{0.1216 + \zeta^{1/3}b}, \end{aligned} \tag{47}$$

where $\zeta = na^3$ and $b = a_B/a$.

In complete analogy with Eq. (38), the magnetic free energy is given by

$$\delta G^{PM} = -NT\ln(2S+1) - \frac{S^2(S+1)^2\sum I_{tM}^2(\mathbf{q})}{12T}, \tag{48}$$

$$I_{tM}(\mathbf{q}) = I(\mathbf{q}) + I_{in}(\mathbf{q}), \qquad I(\mathbf{q}) = I\gamma_{\mathbf{q}}.$$

The structure of the indirect exchange integral $I_{in}(\mathbf{q})$ corresponds to the RKKY theory which can be used because electron gas is fully spin-depolarized in the paramagnetic region:

$$I_{in}(\mathbf{q}) = \frac{3nA^2a^3}{8\mu}\left(1 + \frac{4k_F^2 - q^2}{4k_Fq}\ln\frac{2k_F + q}{|2k_F - q|}\right), \tag{49}$$

$$\mu = \frac{(3\pi^2 n)^{2/3}}{2m}, \qquad k_F = \sqrt{2m\mu}.$$

First, it will be proved that a sample which was in the highly conductive state at $T = 0$ can become insulating at an elevated temperature and remain nonmetallic up to arbitrarily high temperatures. This stems from the fact that the fluctuations lower the donor level strongly, and, for delocalized electrons, such lowering is absent. As a result, according to Eq. (32), at $T = 0$ the delocalization of the donor electrons occurs at the density n_0 which corresponds to the Mott-like equality $n_0^{1/3}a_B = 0.208$. But, equating the energy (47) to the energy F_∞ (40), we find that the delocalization density n_∞ as $T \to \infty$ exceeds the $T = 0$ value n_0 if the ratio AS/E_B exceeds 1.27. Normally, this ratio is essentially larger, and for $AS/E_B = 5$ the Mott-like relation takes the form $n_\infty^{1/3}a_B = 0.378$. Hence, normally, n_∞ exceeds n_0 considerably.

This fact results in a nontrivial temperature dependence of the electric properties of a degenerate ferromagnetic semiconductor. For the donor density n in the range between n_0 and n_∞ at low temperatures the system behaves like a metal, but remains insulating up to arbitrarily high temperatures after its transition from the metallic state to the insulating state. If the density n exceeds n_∞, then the reentrant metal–insulator transition takes place with increasing temperature. This suggests a high resistivity peak at elevated temperatures of the order of the Curie point. Using Eqs. (44), (46), and (48), we find the following expression for the temperature at which the temperature-induced metal–insulator transition occurs when the donor density n exceeds n_∞:

$$\begin{aligned} \frac{1}{T} &= -\frac{(\zeta - \zeta_\infty)}{\zeta Q - R}\,\frac{d(nE_M^{PM})}{dn}, \\ R &= \frac{S^2(S+1)^2}{12(2\pi)^3}\int d^3q I_{in}^2(\mathbf{q}). \end{aligned} \tag{50}$$

In writing Eq. (50) we took into account that $d(nE_M^{PM})/dn$ is negative. This fact was established by numerical calculations, which show that at least to $\zeta = 0.2$ this derivative is about -2 in the E_B units.

Numerical calculations show also that at $I = 0$ and $\zeta = \zeta_\infty$ the denominator in Eq. (50) for $1/T$ is positive, which accounts for the positive transition temperature T_{tr}. It decreases with increasing density ζ and depends on the direct exchange integral I. For example, for $AS/E_B = 5$, the difference $\zeta Q - R$ is equal to 0.008 for $I = 0.02$, to 0.005 for $I = 0$, and to 0.001 for $I = -0.02$ (a negative I value corresponds to initially antiferromagnetic systems such as the manganites which means that in them the transition from the metallic state to the insulating state is also possible).

7. DISCUSSION OF THE RESULTS

The main results of the present treatment can be formulated as follows. For the nondegenerate semiconductors it is established that the activation energy of the conductivity in the spin-wave region is determined not only by the depth of the donor level, but also by the difference in the magnon free energies for a delocalized electron and for a localized electron. As this difference increases with temperature, the activation energy E_A also increases. In the paramagnetic region the activation energy decreases with temperature. Qualitatively, the activation energy behaves like the difference between the local magnetization in the vicinity of a nonionized donor and the mean magnetization over the crystal: with increasing temperature, it first increases and then decreases, passing through a maximum at a temperature comparable with the Curie point. The resistivity peak for the nondegenerate semiconductors is located at the temperature at which $dE_A(T)/dT = 0$.

A very important result is the fact that the high-temperature activation energy exceeds its low-temperature value. This is a consequence of the fluctuation lowering of the donor level which is caused by the fact that the moment of a region in which the localized electron dwells remains finite even when $T \to \infty$. The direction of this moment fluctuates in space so that its mean value vanishes. But the s-electron spin adjusts to the direction of the moment of the region and fluctuates jointly with the moment. The gain in the s-d exchange energy therefore remains finite for the localized electron, although it diminishes with increasing size of the region. For a delocalized electron such a fluctuation lowering is absent. Obviously, the trapping by random fluctuations is possible not only in ferromagnetic semiconductors but also in all magnetic semiconductors independently of their ground-state magnetic ordering.

Let us now discuss in greater detail the more heavily doped ferromagnetic semiconductors which are in the metallic state at $T = 0$. The same reason as for nondegenerate semiconductors—increase in the stability of the localized states as compared with the delocalized states—leads to their transition from the metallic state to the insulating state with increasing temperature. The high-temperature fluctuation lowering of the donor levels again plays an important part. Because of this circumstance, the low-temperature electron delocalization density n_0 turns out to be less than than the high-temperature delocalization density n_∞.

There are two possible scenarios of the temperature-induced metal–insulator transition. The first corresponds to the case where the donor density exceeds n_0 but is less than n_∞. Then, with increase in temperature, the system undergoes a transition from the metallic state to the insulating state and remains in the latter state as the temperature is raised arbitrarily high. The second scenario corresponds to the case where the donor density exceeds both n_0 and n_∞. Then, with increase in temperature, first, the transition from the highly conductive state to the insulating state takes place and then the reverse transition occurs. Obviously, the temperature range of the insulating state should decrease with increasing density. Then the reentrant metal–insulator transition should manifest itself as a resistivity peak, whose height decreases with increasing density [14].

The following remark is likely to be appropriate here. Many investigators use the terms «insulating» or «semiconducting» to denote the high-temperature state of heavily doped ferromagnetic semiconductors, since the resistivity ρ decreases with increasing temperature. In doing so, they ignore the fact that the resistivity exceeds the typical values for nondegenerate semiconductors by many orders of magnitude; it is on the of order the resistivity typical of degenerate semiconductors. If one accepts this terminology, the state of a nondegenerate semiconductor in the portion of the resistivity peak where $d\rho/dT > 0$ should be called metallic, despite its giant value. Hence, this terminology is misleading.

Strictly speaking, these results correspond to the materials which are ferromagnetic when undoped. A situation is more complicated in the case where the undoped crystal is antiferromagnetic and becomes ferromagnetic as a result of doping (e.g., the manganites). The difficulty in finding n_0 is attributed to the formation of bound magnetic polarons (ferrons) in the vicinity of the donors when electrons are localized, and to the ferromagnetic–antiferromagnetic phase separation in the delocalized state of the electrons which occurs at $T = 0$. It is still more difficult to investigate the temperature dependence of the critical density at low temperatures.

A direct analysis of such materials is therefore carried out here only in the high-temperature limit. It is established on the basis of Eq. (50) that as the temperature is lowered, such materials can undergo a transition from the metallic state to the insulating state. The fact that the electron delocalizes at $T = 0$, is sufficient to conclude that the resistivity peak should exist at intermediate temperatures in these materials. Moreover, one can state that in the materials with the initial antiferromagnetic ordering the resistivity peak should be very close to T_C, in contrast with the materials with the initial ferromagnetic ordering, where they can be well separated. In fact, in the former the localization of charge carriers leads to disappearance of the indirect exchange producing the ferromagnetic long-range order. Thus, after the charge carrier localization in the initially ferromagnetic materials the ferromagnetic order is supported by the *d*-*d* exchange, and it is destroyed only due to the thermal fluctuations of the magnetization. In the initially antiferromagnetic materials the ferromagnetic exchange disappears simultaneously.

These theoretical results which disregard the polaronic effects and which are based only on the *s*-*d* model make it possible to explain electric properties of degenerate ferromagnetic semiconductors presented in Ref. 1, including doped manganites. Many investigators believe that one should take into account the Jahn—Teller (JT) and lattice polarization effects to describe properties of the manganites adequately. As for the former, it should pointed out that the resistivity peak in the vicinity of T_C and collossal magnetoresistance are observed in several tens of the non-JT systems [1], so that the JT effect cannot be the origin of the specific features of ferromagnetic semiconductors. The same mechanisms as in other ferromagnetic semiconductors function in the manganites, leading to the same specific features.

Search for additional mechanisms in the manganites would be justified, if the the resistivity peaks in the vicinity of T_C in them had been considerably higher than in other ferromagnetic semiconductors. The resistivity peak height in the manganites, however, is many orders of magnitude lower than in such ferromagnetic semiconductors as EuO, EuS, and others. Formally, therefore, one should conclude that the JT and polaronic effects rather hinder the manifestation of the particular properties of these materials. I do not insist on this conclusion but am certain that the specific features of the ferromagnetic semiconductors are not related to the lattice effects. The role of the polaronic effects in manganites is discussed in more detail in Ref. 15.

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