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SCALING LAW FOR THE FLUID-SOLID PHASE TRANSITION IN YUKAWA SYSTEMS (DUSTY PLASMAS)

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Yukawa systems serve as models for plasmas and colloidal suspensions of charged particles. The state of these systems is determined by two dimensionless parameters: $k = a/\lambda_D$, which is the ratio of the mean interparticle distance to the Debye length λ_D, and $\Gamma = Z_d^2 e^2/aT_d$, which is the ratio of the Coulomb potential energy to the particle temperature T_d (Z_d is the charge of each particle). We propose an empirical scaling law for the critical coupling parameter Γ_c needed for crystallization in Yukawa systems. The dependence of Γ_c on k is in good agreement with recent molecular dynamics simulations.

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Systems of small solid particles (dust particles) immersed in plasmas have recently attracted much attention. They arise in a wide variety of plasma environments ranging from the interstellar medium to laboratory plasma devices. A dust particle in plasma usually acquires an electric charge and interacts with other particles. The interaction potential between macroscopic dust particles depends on their own physical parameters and those of the ambient plasma. The question of the correct potential of interaction between dust particles is not only fundamental, it is also still open. Such effects as plasma flow anisotropies, dipole effects, and long range attractive interaction due to shadow effects may play a role when considering different plasma conditions. In order to understand the behavior of dusty plasmas in complicated situations, however, the results for simple and basic cases are indispensible. As one of those cases, an isotropic screened Coulomb potential (or Yukawa-type potential) is frequently assumed:

$$\phi(r) = \frac{Z_d^2 e^2}{r} \exp\left(-\frac{r}{\lambda_D}\right), \qquad (1)$$

where Z_d is the particle charge and λ_D is the screening length. For an isotropic and homogeneous plasma,

$$\lambda_D^{-2} = \lambda_{De}^{-2} + \lambda_{Di}^{-2} \approx \lambda_{Di}^{-2}$$

(if as usual $T_e \gg T_i$), where

$$\lambda_{De(i)} = \sqrt{\frac{T_{e(i)}}{4\pi e^2 n_{e(i)}}}$$

is the electron (ion) Debye length.

It was suggested by Ikezi [1] that when the interparticle potential energy exceeds the kinetic energy, particles in a plasma can form crystalline structures. Laboratory experiments under various plasma conditions have recently demonstrated this possibility [2–8]. Such crystal structures have been also observed in colloidal suspensions of charged particles, where the interaction potential (1) can be also adopted.

The conditions of such crystallization of particles interacting via a screened Coulomb potential are under investigation. For example, molecular dynamics simulations were recently used to study phase diagrams of Yukawa systems [9–12]. Although some of the assumptions in the simulations (interaction potential, cubic simulation box with periodic boundary conditions) may be not completely suited to some experiments on Coulomb crystallization in dusty plasmas, the results are revealing. In these simulations the state of a system is determined by only two dimensionless parameters,

$$k = a/\lambda_D \quad \text{and} \quad \Gamma = Z_d^2 e^2/aT_d, \qquad (2)$$

which enter into the equations of motion. Here $a = n_d^{-1/3}$ is the mean interparticle distance and n_d is the particle number density. The coupling parameter Γ is roughly the ratio of the unscreened Coulomb

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potential energy to the kinetic energy per particle (T_d being the particle temperature). Some studies [10,12] have used normalization that differs slightly from (2). Specifically, the Wigner—Seitz radius $\rho = (3/4\pi n_d)^{1/3}$ was used as the length unit instead of a. Note that k and Γ will then be $k' = k(4\pi/3)^{-1/3}$ and $\Gamma' = \Gamma(4\pi/3)^{1/3} \approx 1.612\Gamma$.

In the limit $k \to 0$, the Yukawa potential devolves into the long-range Coulomb potential describing the one-component plasma (OCP) system. In this limit, only the one parameter Γ describes the fluid-solid phase transition. It is well known [13,14] that for the OCP system, Γ must exceed the critical value $\Gamma_c \approx 106$ ($\Gamma_c' \approx 170$) to form a Coulomb lattice. For a system with a Yukawa interaction potential, the transition between fluid and solid phases takes place at some critical value Γ_c that depends on the screening parameter k. By analogy with the OCP system, it was first proposed to introduce a coupling parameter that takes screening into account [1],

$$\Gamma_s = \frac{Z_d^2 e^2}{aT_d} \exp(-a/\lambda_D) \equiv \Gamma \exp(-k), \qquad (3)$$

and use the condition $\Gamma_s > 106$ to describe Coulomb solidification, so that $\Gamma_c \approx 106 \exp k$. However, recent numerical simulations show that Γ_c is a more complicated function of k [12].

We have constructed a function $\Gamma_c = \Gamma_c(k)$ that fits the fluid-solid phase transition data of Hamaguchi, Farouki, and Dubin [12] well over a wide range of k (in Ref. [12], three different polynomial fits (Eqs. (17)–(19)) were used to fit data over different ranges of k). We assume that the critical value of the coupling parameter depends on k as

$$\Gamma_c = \Gamma_c^{OCP}(1 + k + k^2/2)^{-1} \exp k, \qquad (4)$$

where $\Gamma_c^{OCP} \approx 106.6$ as found in Ref. [12].

In Table, the values of Γ_c found via numerical simulation [12] and normalized by the rhs of Eq. (4) for various values of k are summarized. It can be seen that for $k \leq 1.61$ ($k' \leq 1$), the deviation between simulations and Eq. (4) is less than 1%. This range of k is very often applied to dust crystals in laboratory experiments. For all values of k except the last point $k = 8.06$, Eq. (4) fit simulation results to within 10%.

In Figure, the values of $\Gamma_c \exp(-k)/\Gamma_c^{OCP}$ calculated from the solid-fluid phase transition data of Ref. [12] are plotted versus k. In addition, the function $(1 + k + k^2/2)^{-1}$ is plotted. The error bar at $k = 4.84$ ($k' = 3.0$) represents the simulation uncertainties estimated in Ref. [12]. It seems that for all values of k (except $k = 8.06$), Eq. (4) holds to within the simulation errors. Figure also shows that Γ_s introduced by Eq. (3) is not an appropriate measure to describe the fluid-solid phase transition.

k	$\frac{\Gamma_c}{\Gamma_c^{OCP}} \exp(-k)(1 + k + k^2/2)$
0	1.00
0.32	1.01
0.65	1.01
0.97	1.01
1.29	1.00
1.61	0.99
1.93	0.98
2.26	0.95
3.22	0.96
4.19	0.93
4.84	0.96
5.80	0.99
6.45	1.00
7.42	1.08
8.06	1.15

The form of the melting curve (4) can be obtained using a very simple approach. We consider a one-dimensional lattice of dust particles interacting via a screened Coulomb potential. Moreover, we assume that it is sufficient to include only interactions between nearest neighbor particles. Then the characteristic oscillation frequency of a given particle about its equilibrium position with all other particles held fixed is [15]

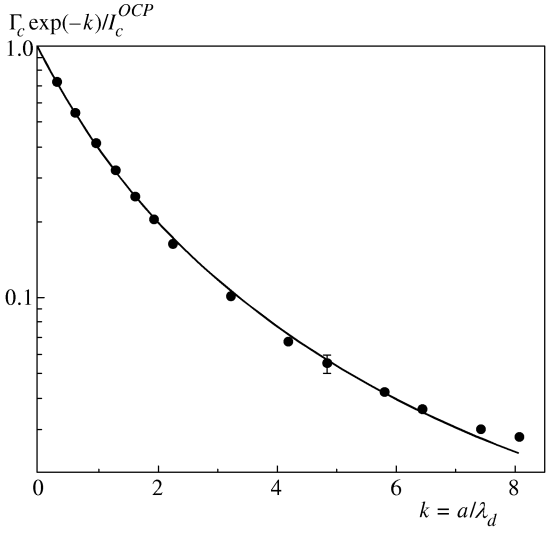
$$\omega_0^2 = \frac{4Z_d^2 e^2}{m_d a^3} \left(1 + k + \frac{k^2}{2}\right) \exp(-k),$$

where m_d is the dust particle mass. The mean squared displacement of particles around their equilibrium positions is $\langle \delta u^2 \rangle \sim T_d/m_d\omega_0^2$. According to Lindenmann's rule for the melting transition $\langle \delta u^2 \rangle / a^2 = \text{const}$, we have

$$\frac{Z_d^2 e^2}{aT_d} \left(1 + k + \frac{k^2}{2}\right) \exp(-k) = \text{const} \qquad (5)$$

at the melting curve. Extrapolating (5) to the limit $k \to 0$ we finally arrive at Eq. (4).

Surely this crude model cannot serve as a physical basis for Eq. (4). It considers a one-dimensional lattice (although simulations were performed in $3D$) and includes only interactions with nearest neighbors (this assumption is valid only when $k \gg 1$). At the present time, Eq. (4) must therefore be considered as an empirical relation.



Filled circles are the values of $\Gamma_c \exp(-k)/\Gamma_c^{OCP}$ for various values of k, calculated from the solid-fluid phase transition data of Hamagushi et al. [12]. The error bar at $k = 4.84$ ($k' = 3.0$) represents the simulation uncertainties. The solid curve is the function $(1 + k + k^2/2)^{-1}$

To conclude, we propose a scaling law for the critical coupling parameter Γ_c needed for crystallization in Yukawa systems. The dependence of Γ_c on k is determined by Eq. (4). This dependence is consistent with the recent fluid-solid phase transition simulation data obtained by Hamaguchi et al. [12] to within the simulation errors over a wide range of k. The empirical condition for crystallization,

$$\frac{Z_d^2 e^2}{aT_d}\left(1 + k + \frac{k^2}{2}\right)\exp(-k) \geq 106,$$

can be very useful in a variety of experimental contexts ranging from dusty plasmas to colloidal suspensions.

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