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NONLINEAR FLUCTUATION PHENOMENA IN THE TRANSPORT PROPERTIES OF SUPERCONDUCTORS

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There exists a wide temperature region ($\mathrm{Gi}T < T - T_c < T\sqrt{\mathrm{Gi}}$), where the influence of fluctuations on the thermodynamic properties of superconductors can be taken into account in the linear (Gaussian) approximation, while their influence on the kinetic properties is strongly nonlinear. The Maki–Thompson cotribution to the conductivity saturates in this region. However, the Aslamazov–Larkin contribution becomes even more singular. This enhancement is related to the fact that nonlinear effects increase the lifetime of fluctuating pairs. Pair-breaking and energy relaxation processes can decrease the nonlinear effects.

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1. INTRODUCTION

The electron scattering off the usual impurities leads to a temperature-independent residual resistance of the normal metal [1]. The conductivity of bulk samples and films can be measured with a very high accuracy. This allows one to study different mechanisms leading to the temperature-dependent conductivity at low temperatures. One of these mechanisms is related to thermal fluctuations above the superconducting transition temperature T_c [2–5]. There are two kinds of fluctuation corrections leading to the temperature-dependent conductivity above T_c. The first one is known as the Maki–Thompson (MT) contribution and the second is the conductivity of fluctuating pairs (the Aslamazov–Larkin (AL) contribution). These corrections depend differently on the spin flip scattering time τ_s. The characteristic temperature range for the contributions of both types is determined by the Ginzburg parameter Gi that depends on dimensionality; for films, to $\mathrm{Gi} = \tau_0 = 1/32\nu Dd = e^2/16\hbar\sigma_\square$, where $\nu = mp^2/2\pi^2$ is the electron density of states per spin, $D = v_F l_{tr}/3$ is the diffusion coefficient, d is the film thickness, l_{tr} is the electron mean free path, p is the Fermi momentum, and $\sigma_\square$ is the conductance of a square film. It has been found in [6] that nonlinear fluctuation phenomena lead to a new temperature scale $T_c\sqrt{\mathrm{Gi}}$ (see also [7–10]). In this paper, we obtain expressions for the conductivity in the temperature region $\mathrm{Gi} < \tau < \sqrt{\mathrm{Gi}}$, where the Gaussian approximation works well and the nonlinear fluctuation effects are important.

In [6], an attempt to find the fluctuating correction to the conductivity was made. The main point was that long-wave fluctuations with $Dk^2 < T\tau$ are essential. These fluctuations can be considered as a Bose condensate. The dynamics of superconductors must be considered in the background of these fluctuations. They lead to a pseudogap in the excitation spectrum. In this paper, we show that short-wave fluctuations with $Dk^2 \gg T\tau$ can be important. It was found in [11] that short-wave fluctuations of the order parameter Δ affect the electron Green's functions as paramagnetic impurities with the depairing factor $\Gamma = \tau_s^{-1} = \langle|\Delta|^2\rangle/\varepsilon$. Essential values of the energy ε are of the order $\varepsilon \sim \Delta \sim T\sqrt{\mathrm{Gi}}$, and therefore, Γ is of the order $T\sqrt{\mathrm{Gi}}$. This large value of the depairing factor leads to the saturation of the MT contribution to conductivity in the temperature region $\tau < \sqrt{\mathrm{Gi}}$.

A more complicated situation occurs for the AL

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contribution. This contribution is proportional to the density of pairs and their lifetime. For sufficiently large values of τ, the time-dependent Ginzburg–Landau (TDGL) equation can be used to obtain this lifetime. It is proportional to $\hbar/(T-T_c)$, and hence, the AL contribution is proportional to τ^{-1}. If the concentration of paramagnetic impurities is large or if the energy relaxation time is short, the TDGL equation can be used for all temperatures T. In this case, the AL contribution is valid in the temperature range $\tau > \mathrm{Gi}$. In the opposite limiting case, the nonlinear fluctuation effects destroy the applicability of the TDGL equation and increase the lifetime of fluctuating pairs. As a result, the AL contribution to the conductivity becomes more singular in the temperature region $\sqrt{\mathrm{Gi}} > \tau > \mathrm{Gi}$.

2. QUALITATIVE PICTURE

In the temperature region $1 \gg \tau \gg \mathrm{Gi}$, thermodynamic fluctuations of the order parameter Δ can be considered to be Gaussian. The corresponding correlator is given by

$$\langle \Delta_k^* \Delta_k \rangle = \frac{T}{\nu d} \frac{1}{\tau + \pi D k^2/8T} = \frac{256}{\pi} \frac{\mathrm{Gi} T^2}{k^2 + 8T\tau/\pi D}. \quad (1)$$

To calculate thermodynamic quantities in the temperature region $\tau > \mathrm{Gi}$, it is sufficient to know this correlator only. However, a more complicated problem must be solved in order to calculate kinetic coefficients. One must find how the Gaussian fluctuations change the one-particle excitation spectrum. The long-wavelength fluctuations with $k^2 < k_{min}^2 = 8T\tau/\pi D$ can be considered as a local condensate. They lead to the formation of a pseudogap in the one-particle spectrum of excitations. It follows from Eq. (1) that the pseudogap is equal to

$$\Delta_{PG} = \frac{8}{\pi} \sqrt{\mathrm{Gi}}\, T. \quad (2)$$

At some distance from the transition (for $\tau > \sqrt{\mathrm{Gi}}$), only the excitations with the energy $\omega > \Delta_{PG}$ are significant. The pseudogap does not play any role in these excitations. It is therefore sufficient to consider fluctuations in the linear approximation only (see [3–5]). It is important, however, that the excitations with the energy $\omega < \Delta_{PG}$ become essential in the temperature region $\tau < \sqrt{\mathrm{Gi}}$. In [6], the fluctuation correction to the conductivity was considered with the pseudogap taken into account in the same way as the gap below the transition temperature. This approximation gives a correct estimate for the width of the temperature region where the non-linear effects are important. However, the model with a constant Δ considered in [6] cannot reproduce the correct temperature dependence of the conductivity in the temperature region $\tau < \sqrt{\mathrm{Gi}}$.

To describe the nonlinear effects, we consider fluctuations of Δ in the statical approximation. This is eligible, because the fluctuation lifetime $(T\tau)^{-1}$ is large compared to the inverse pseudogap. However, the spatial dispersion of the pseudogap changes the physical picture significantly. To take the spatial variations into account, we must calculate the conductivity as a function of the order parameter $\Delta(\mathbf{r})$ that is an arbitrary function of $\mathbf{r}$ and average the result over the Gaussian fluctuations with correlator (1). We accomplish this program up to a numerical coefficient in the limiting case where the energy relaxation rate is large ($\tau_\varepsilon \ll (T\tau)^{-1}$). In the other cases, we obtain a functional form of the temperature dependence of the conductivity with undetermined coefficients.

To consider the spatial dependence of the order parameter, we use the results obtained in [11], where the spatial variations of Δ were shown to act on one-particle excitations in the same way as the magnetic impurities. In this case, the total pair-breaking rate Γ can be written as a sum of the pair-breaking rate due to the magnetic impurities and the fluctuation term. Thus, the self-consistent equation for Γ becomes

$$\Gamma = \int \frac{d^2k}{(2\pi)^2} \frac{\langle \Delta_k^* \Delta_k \rangle}{\omega + Dk^2/2 + \Gamma} + \frac{1}{\tau_s}. \quad (3)$$

It is important to mention that Eq. (3) is exact if either $\omega \gg \Gamma$ or τ_s is very small such that the first term in Eq. (3) is a small correction to the second one. In the other cases, the self-consistent equation (3) can be considered as an approximation and gives the result valid by the order of magnitude only.

In the region where $\omega < \Gamma$ and $\Gamma \gg T\tau$, we obtain from Eqs. (3) and (1) that

$$\Gamma = \frac{8T}{\pi} \left(\mathrm{Gi} \ln \frac{\Gamma}{T\tau} \right)^{1/2}, \quad (4)$$

which coincides with the value obtained in [7, 12] up to the logarithmic term. In what follows, we repeat the derivation from [11] and show that the pseudogap does not change result (4) qualitatively.

We note that the pair-breaking rate Γ is of the order of the pseudogap Δ_{PG}. Thus, a wide maximum of the density of states appears at $\omega \sim \Delta_{PG}$.

As known from [5], the MT correction to the conductivity saturates for $T\tau < \Gamma$ and takes the form

$$\frac{\delta\sigma^{MT}}{\sigma_0} = \frac{8T\mathrm{Gi}}{\pi\Gamma} \ln \frac{\pi\Gamma}{4T\tau}. \quad (5)$$

As can be seen from Eqs. (4) and (5), this saturation occurs for $\tau < \sqrt{\mathrm{Gi}}$. Similar results have been obtained in [7, 8, 10]. However, numerical coefficients are different.

We note that the numerical coefficient in Eq. (5) depends on how the summation of higher-order diagrams is made. However, its exact value is not very important because in the region $T\tau < \Gamma$, the MT contribution is less singular than the AL contribution and can be neglected. The AL contribution does not saturate as T tends to T_c but becomes more and more singular.

To estimate the AL contribution due to the appearance of fluctuating Cooper pairs, we use the simple Drude formula

$$\delta\sigma^{AL} = \frac{ne^2}{m}\tau_{fl}, \tag{6}$$

where n, m, and τ_{fl} are the concentration, the mass, and the lifetime of the fluctuating Cooper pairs. The ratio n/m can be estimated from Eq. (1), while the lifetime follows from the TDGL equations,

$$\left(a\frac{\partial}{\partial t} + Dk^2 + \frac{8}{\pi}T\tau\right)\Delta_k(t) = \zeta(t), \tag{7}$$

where ζ is the Langevin noise. In the two-dimensional case, we have

$$\frac{n}{m} \approx \frac{T}{2\pi d\hbar^2}$$

and

$$\tau_{fl} = \frac{\pi\hbar}{8(T - T_c)a}.$$

At a sufficient distance from the transition ($T\tau > \Delta_{PG}$) or for a very large energy relaxation rate, we can set $a = 1$, because the quasiparticles are at the thermal equilibrium. Thus, we have

$$\frac{\delta\sigma_{AL}}{\sigma} = \frac{\mathrm{Gi}}{\tau}. \tag{8}$$

In the presence of the pseudogap, there is no equilibrium and the coefficient a becomes greater than one. We recall that below the transition temperature, the coefficient a in the TDGL equations for $|\Delta|$ changes similarly (see, e.g., [13–17]). The growth of a and, consequently, the growth of the fluctuation lifetime occur because the quasiparticles require more time to attain the thermal equilibrium (we let τ_e denote the corresponding time). A rough estimate gives $a \sim \Delta_{PG}\tau_e$. For a weak energy relaxation, τ_e must be determined from the diffusion equation with the pseudogap taken into account (see [18–20]). We note that in this complicated case, the coefficient a becomes a nonlocal operator. Rough estimates give the thermal equilibrium transition time $\tau_e \sim (Dk_{min}^2)^{-1} \sim (T\tau)^{-1}$. Taking Eq. (2) into account, we obtain

$$\delta\sigma/\sigma_0 = \mathrm{Gi}^{3/2}/\tau^2. \tag{9}$$

We see that paraconductivity can exceed the normal conductivity σ_0 in the region $\mathrm{Gi}^{3/4} > \tau > \mathrm{Gi}$. We emphasize that corrections to all the thermodynamic coefficients are small in this region and are adequately described by the linear theory.

We now discuss the role of the energy relaxation processes characterized by the quasiparticle lifetime τ_ε. In the two-dimensional case, the nonelastic electron–electron scattering in dirty metals leads to the electron–electron collision time

$$\tau_\varepsilon^{-1} \sim Td\,l\,p^2 \sim \mathrm{Gi}T.$$

Such a large collision time does not change nonlinear effects. However, the nonelastic electron scattering off phonons and other possible collective excitations can decrease τ_ε significantly. These processes together with additional pair-breaking processes (due to magnetic impurities or a magnetic field) decrease the nonlinear effects. The energy relaxation reduces the thermal equilibrium transition time τ_e. If these processes are very strong (for example, if the temperature is relatively high), the transport equation for the distribution function becomes local and in the limit $T\tau \sim Dk^2 \ll \tau_\varepsilon^{-1}$, we can write $\tau_e = \tau_\varepsilon$. Thus, in the temperature region under consideration, we have

$$\frac{\delta\sigma}{\sigma_0} = \frac{\mathrm{Gi}^{3/2}T\tau_\varepsilon}{\tau}. \tag{10}$$

The elastic scattering off magnetic impurities and the magnetic field also tend to diminish the nonlinear fluctuation effects in conductivity, but in a different way. These scattering processes (as well as scatterings off the static fluctuations of the order parameter) do not affect the quasiparticle motion and hence, τ_ε. However, if the pair-breaking rate is sufficiently large ($\Gamma > \Delta_{PG}$), these processes lead to the reduced pseudogap $\Delta_{PG} \sim \langle|\Delta|^2\rangle/\Gamma$ (we recall that $\Delta_{PG} \sim \sqrt{\langle|\Delta|^2\rangle} \sim T\mathrm{Gi}^{1/2}$ without the pair breaking). Thus, the fluctuation correction can be written as

$$\delta\sigma/\sigma_0 = \mathrm{Gi}^2T/\tau^2\Gamma. \tag{11}$$

In the presence of both a strong pair breaking and a large energy relaxation, exact expressions for the coefficient a in the TDGL equation, which is local in this

case, and for paraconductivity can be derived with a logarithmic accuracy. The main contribution to a then comes from the fluctuations with $T\tau < Dk^2 < \tau_\varepsilon^{-1}$. The first inequality allows us to consider only the leading terms in the expansion of a with respect to Δ, and the second one implies a local approximation in the transport equation. The result is

$$a = \frac{\tau_\varepsilon \langle \Delta^2 \rangle}{2\Gamma}, \tag{12}$$

$$\frac{\delta\sigma}{\sigma_0} = \frac{32\mathrm{Gi}^2 T^2 \tau_\varepsilon}{\pi^2 \Gamma \tau} \ln \frac{\pi}{8T\tau_\varepsilon \tau}. \tag{13}$$

We note that Eqs. (9)–(13) are valid only if the parameters Γ and τ_ε are such that the contribution to the conductivity $\delta\sigma$ is larger than the usual Aslamazov–Larkin contribution in Eq. (8). If $\Gamma > T$ and $T\tau_\varepsilon < \sqrt{\mathrm{Gi}}$ or if $T^2\tau_\varepsilon/\Gamma < \mathrm{Gi}$, the nonlinear effects are negligible and the usual result (8) is valid for all $\tau > \mathrm{Gi}$. We note that the MT contribution saturates at the temperatures such that $T\tau \sim \max\left[\Gamma,\, 1/\tau_\varepsilon,\, T\sqrt{\mathrm{Gi}}\right]$.

3. DEPAIRING FACTOR INDUCED BY FLUCTUATIONS

A nonzero fluctuating order parameter Δ and the Gor'kov Green's function β [6] exist above the transition temperature. In the temperature region $\tau > \mathrm{Gi}$, the main contribution to the order parameter Δ arises from zero «frequency». The momentum space can be split into two parts, $\pi Dk^2/8T < \tau$ and $\pi Dk^2/8T > \tau$. The fluctuations with $\pi Dk^2/8T > \tau$ can be considered as «fast» variables created in the background of slow fluctuations with $\pi Dk^2/8T < \tau$. The «fast» fluctuations induce the intrinsic depairing factor Γ even if the external depairing factor related to paramagnetic impurities is missing ($\tau_s \to \infty$). A similar phenomenon was studied in [11]. Using the method developed in that paper, we obtain expressions for the statical Green's functions α and β and the depairing factor Γ. We start from the Usadel equation for the Green's functions α and β in the dirty limit (see [6, 21]),

$$\Delta\alpha - \omega\beta + \frac{D}{2}\left(\alpha\nabla^2\beta - \beta\nabla^2\alpha\right) = \alpha\beta\Gamma. \tag{14}$$

Following [11], we present the Green's functions α and β in the field of «fast» fluctuations of the order parameter $\Delta(k)$ as

$$\alpha = \langle\alpha\rangle + \alpha_1, \quad \beta = \langle\beta\rangle + \beta_1. \tag{15}$$

The deviations of the Green's functions from their mean values can be found in the perturbation theory. We have [11]

$$\alpha_1(k) = -\frac{\Delta(k)\langle\alpha\rangle\langle\beta\rangle}{\omega\langle\alpha\rangle + \langle\Delta\rangle\langle\beta\rangle + Dk^2/2}. \tag{16}$$

The «mean» Green's functions $\langle\alpha\rangle$ and $\langle\beta\rangle$ are solutions of the system of equations

$$\langle\alpha\rangle^2 + \langle\beta\rangle^2 = 1, \quad \langle\alpha\rangle\langle\Delta\rangle - \omega\langle\beta\rangle = \langle\alpha\rangle\langle\beta\rangle\Gamma. \tag{17}$$

The value of the parameter Γ is determined by Eq. (16) and is equal to

$$\Gamma = \int \frac{d^2k}{(2\pi)^2} \frac{\langle\Delta_k^*\Delta_k\rangle}{\langle\alpha\rangle\omega + \langle\Delta\rangle\langle\beta\rangle + Dk^2/2}, \tag{18}$$

where $\langle\Delta\rangle = \langle|\Delta|^2\rangle^{1/2}$. The quantity $\langle\Delta\rangle$ in Eqs. (16) and (17) must be understood as the integral over k of expression (18) over the range $\pi Dk^2/8T \le \tau$; it then becomes

$$\langle\Delta\rangle = \left[\frac{T}{\nu d}\int \frac{d^2k}{(2\pi)^2}\frac{1}{\tau + \pi Dk^2/8T}\right]^{1/2} \approx \approx T\left[\frac{64\mathrm{Gi}}{\pi^2}\right]^{1/2}. \tag{19}$$

From Eqs. (1) and (18), we obtain

$$\Gamma = \frac{16T\mathrm{Gi}}{\pi\tau}\frac{1}{(\pi/4T\tau)(\omega\langle\alpha\rangle + \langle\Delta\rangle\langle\beta\rangle) - 1} \times \times \ln\left(\frac{\pi(\omega\langle\alpha\rangle + \langle\Delta\rangle\langle\beta\rangle)}{4T\tau}\right). \tag{20}$$

As can be seen from Eq. (20), $\Gamma(\omega)$ is a function of the energy ω. In the range $\tau \le \sqrt{\mathrm{Gi}}$, essential values of ω are of the order Γ. Thus, Γ itself is of the order $\langle\Delta\rangle$ (see (19)). This order of Γ is related to fluctuations of the order parameter modulus. This value is much larger than the one due to the phase fluctuations of the order parameter (see [6]).

4. EQUATIONS FOR THE TIME-DEPENDENT ORDER PARAMETER

The static Ginzburg–Landau equations are valid in the wide temperature region

$$\mathrm{Gi} \ll |1 - T/T_c| \ll 1. \tag{21}$$

The TDGL equations are valid if the energy relaxation time τ_ε or the pair-breaking time $\tau_s = \Gamma^{-1}$ is sufficiently short [13–16]. For large τ_ε, the dynamics

of normal excitations becomes essential. As a result, the dynamical term in the equation for the order parameter becomes more complicated. We now derive the corresponding equation.

The order parameters $\Delta_{1,2}(t)$ can be written as

$$\Delta_{1,2}(t) = \frac{\pi\lambda_{eff}}{2} F^K_{1,2}(t,t), \tag{22}$$

with the Green's function $\hat{G}$ presented in the form [18]

$$\hat{G} = \begin{pmatrix} G^R, & G^K \\ 0, & G^A \end{pmatrix}, \tag{23}$$

where $G^{R,A,K}$ are the retarded, advanced, and Keldysh Green's functions. Each of these is a Gor'kov–Nambu matrix

$$\begin{aligned} G^{R,A,K} &= \begin{pmatrix} g_1 & F_1 \\ -F_2 & g_2 \end{pmatrix}^{R,A,K}, \\ \tilde{\Delta} &= \begin{pmatrix} 0 & \Delta_1 \\ -\Delta_2 & 0 \end{pmatrix}, \end{aligned} \tag{24}$$

where $\Delta_2(\omega) = \Delta_1^*(-\omega)$.

In the dirty limit, we have the system of equations for $G^{R,A}$ (see [19])

$$\begin{aligned} -D\partial_\mp \left(g^{R,A}\partial_\mp F^{R,A}_{1,2} - F^{R,A}_{1,2}\frac{\partial g^{R,A}}{\partial r}\right) + 2i\Delta_{1,2} g^{R,A} - \\ - 2i\varepsilon F^{R,A}_{1,2} + \frac{2}{\tau_s} g^{R,A} F^{R,A}_{1,2} = -I^{Ph(R,A)}_{1,2}, \end{aligned} \tag{25}$$

where $I^{Ph(R,A)}_{1,2}$ is the electron–phonon collision integral; in the vicinity of the transition temperature T_c for small energy values $|\varepsilon| \ll T$, it is equal to

$$I^{Ph(R,A)}_{1,2} = \pm\frac{1}{\tau_\varepsilon} F^{R,A}_{1,2}. \tag{26}$$

The Keldysh Green's function G^K can be written as [20]

$$G = \int dt_1 (G^R \hat{f} - \hat{f} G^A), \tag{27}$$

where the distribution function $\hat{f}$ is given by [20]

$$\hat{f} = f + \tau_z f_1. \tag{28}$$

Equations for the distribution functions $f_{1,2}$ have been derived in [20] and are given by

$$\begin{aligned} &- D\frac{\partial}{\partial r}\left\{ \frac{\partial f}{\partial r}(1 - G^R G^A)\right\} - D\frac{\partial}{\partial r}(f_1 j_\varepsilon) + \\ &+ 2\frac{\partial f}{\partial t}\operatorname{Sp}\alpha + \frac{\partial f}{\partial \varepsilon} \times \\ &\times \left\{ eD\frac{\partial A}{\partial t} j_\varepsilon - 2\operatorname{Sp}\frac{\partial \hat{\Delta}}{\partial t}\delta \right\} + 4I^{Ph}_1(f) = 0, \\ &- D\frac{\partial}{\partial r}\operatorname{Sp}\left\{ \frac{\partial f_1}{\partial r}(1 - \tau_z G^R \tau_z G^A)\right\} - D\frac{\partial f}{\partial r} j_\varepsilon + \\ &+ 2\frac{\partial}{\partial t}(f_1 \operatorname{Sp}\alpha) - 4i f_1 \operatorname{Sp}(\gamma\hat{\Delta}) + \\ &+ 2\frac{\partial f}{\partial \varepsilon}\operatorname{Sp}\left\{ e\frac{\partial\varphi}{\partial t}\alpha - \frac{\partial\hat{\Delta}}{\partial t}\tau_z\gamma + \frac{i}{2}\frac{\partial^2\hat{\Delta}}{\partial t^2}\frac{\partial\delta}{\partial\varepsilon}\right\} + \\ &+ 4I^{Ph}_2(f_1) = 0, \end{aligned} \tag{29}$$

where

$$\begin{aligned} j_\varepsilon &= \operatorname{Sp}\tau_z (G^R \partial G^R - G^A \partial G^A), \\ \partial &= \frac{\partial}{\partial r} - ieA\tau_z, \end{aligned} \tag{30}$$

$$2\alpha = G^R\tau_z - \tau_z G^A, \quad 2\delta = G^R - G^A, \quad 2\gamma = G^R + G^A.$$

In the important limiting case where $\varepsilon \sim \Gamma \gg \Delta$, Eqs. (25) and (29) can be simplified and we obtain

$$\begin{aligned} F^{R,A}_1 &= -i\left(\Gamma \mp i\varepsilon - \frac{D}{2}\frac{\partial^2}{\partial r^2}\right)^{-1}\Delta, \\ F^{R,A}_2 &= -i\left(\Gamma \mp i\varepsilon - \frac{D}{2}\frac{\partial^2}{\partial r^2}\right)^{-1}\Delta^*, \end{aligned} \tag{31}$$

$$\begin{aligned} &- D\frac{\partial^2 f}{\partial r^2} - \frac{D}{4}\frac{\partial}{\partial r}(j_\varepsilon f_1) + \frac{\partial f}{\partial t} + \frac{1}{4}\frac{\partial f}{\partial\varepsilon} \times \\ &\times \left\{ \frac{\partial\Delta_1}{\partial t}(F^R_2 - F^A_2) + \frac{\partial\Delta_2}{\partial t}(F^R_1 - F^A_1)\right\} + I^{Ph}_1(f) = 0, \\ &- D\frac{\partial^2 f_1}{\partial r^2} - \frac{D}{4} j_\varepsilon \frac{\partial f_1}{\partial r} + \frac{\partial f_1}{\partial t} + \\ &+ \frac{i}{2} f_1 \left(\Delta(F^R_2 + F^A_2) + \Delta^*(F^R_1 + F^A_1)\right) + \frac{\partial f}{\partial\varepsilon} \times \\ &\times \left\{ e\frac{\partial\varphi}{\partial t} + \frac{1}{4}\left(-\frac{\partial\Delta_1}{\partial t}(F^R_2 + F^A_2) + \frac{\partial\Delta_2}{\partial t}(F^R_1 + F^A_1)\right)\right\} + \\ &+ I^{Ph}_2(f_1) = 0, \end{aligned}$$

where

$$\begin{aligned} j_\varepsilon = -F^R_1\frac{\partial F^R_2}{\partial r} + F^R_2\frac{\partial F^R_1}{\partial r} + \\ + F^A_1\frac{\partial F^A_2}{\partial r} - F^A_2\frac{\partial F^A_1}{\partial r}. \end{aligned} \tag{32}$$

The general expression for the collision integrals $I^{Ph}_{1,2}$ is given in [20, 22]. For small energy values $|\varepsilon| \ll T$, these integrals can be taken in the simple form

$$I^{Ph}_1(f) = \frac{1}{\tau_\varepsilon}\left(-\mathrm{th}\left(\frac{\varepsilon}{2T}\right) + f\right), \qquad (33)$$

$$I^{Ph}_2(f_1) = \frac{1}{\tau_\varepsilon} f_1,$$

$$\tau_\varepsilon^{-1} = 7\zeta(3)\pi\nu g^2 T^3/2(sp)^2,$$

where s is the sound velocity in the metal and g is the electron–phonon coupling constant.

In the limiting case of strong energy relaxation with $\tau_\varepsilon\Delta \ll 1$, the distribution function $\hat{f}$ can be taken as the equilibrium one,

$$f = \mathrm{th}(\varepsilon/2T), \quad f_1 = 0. \qquad (34)$$

In this case, Eqs. (22), (31), and (34) allow us to obtain the time-dependent Ginzburg–Landau equation in the standard form

$$\left(1 - T/T_c - \frac{7\zeta(3)}{8\pi^2T^2}|\Delta|^2\right)\Delta + \frac{\pi D}{8T}\partial_-^2\Delta - \\ - \frac{\pi}{8T}\left(\frac{\partial}{\partial t} + 2ie\varphi\right)\Delta = 0. \qquad (35)$$

If the condition $\tau_\varepsilon\Delta \ll 1$ is not satisfied, the deviation of the distribution function $\hat{f}$ from its equilibrium value can change the last term in Eq. (35).

In the range $\Gamma \gg \Delta$, the crossing term in Eq. (31) has the smallness $(\Delta/\Gamma)^2$. In the leading approximation, system (31) is therefore diagonal.

With the aid of Eqs. (27), (28), and (31), we can rewrite Eq. (22) as

$$\left[\tau + \frac{\pi}{8T}\left(-i\omega_1 - D\frac{\partial^2}{\partial r^2}\right)\right]\Delta_1 - \frac{\pi}{2}\int\limits_{-\infty}^{\infty}\frac{d\varepsilon}{2\pi}\times \\ \times\left[\delta f(F_1^R - F_1^A) - f_1(F_1^R + F_1^A)\right] = 0, \\ \left[\tau + \frac{\pi}{8T}\left(-i\omega_1 - D\frac{\partial^2}{\partial r^2}\right)\right]\Delta_2 - \frac{\pi}{2}\int\limits_{-\infty}^{\infty}\frac{d\varepsilon}{2\pi}\times \\ \times\left[\delta f(F_2^R - F_2^A) - f_1(F_2^R + F_2^A)\right] = 0, \qquad (36)$$

where we set

$$f = \mathrm{th}(\varepsilon/2T) + \delta f. \qquad (37)$$

In (36), the contributions of the second terms are of the order $(\Delta/\Gamma)^2$. This result is due to the cancellation of the terms coming from δf and f_1. But in the next orders of the perturbation theory, the quantity f_1 becomes small and the main contribution arises from the distribution function δf beyond perturbation theory.

5. THE CONDUCTIVITY OF FLUCTUATING PAIRS (THE ASLAMAZOV–LARKIN CONTRIBUTION)

The conductivity of fluctuating pairs is given by the diagrams in Fig. *a*. In what follows, we assume that the order parameters $\Delta_{1,2}$ can be written as the sums of two terms. One of them is related to the statical thermodynamic fluctuations Δ and Δ^*. In the range $\tau >$ Gi, these fluctuations are Gaussian with the correlator given by Eq. (1). The wavy line in Fig. *a* gives the dynamical fluctuations $\tilde{\Delta}_{1,2}$ of the order parameter. The correlators of these fluctuations $\hat{K}_{ij}$ must be found in the background of thermodynamic fluctuations,

$$\hat{K}_{ij}(\omega_1) = \nu\langle\Delta_i^*\Delta_j\rangle_{\omega_1}. \qquad (38)$$

The contribution to the conductivity can be expressed through the correlators $\hat{K}$ in the same way as for weak fluctuations [3].

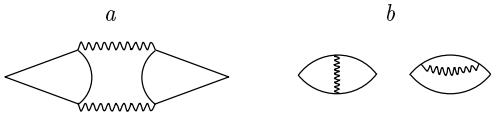
We must first find the conductivity as a function of the Matzubara frequency ω_0 and then perform the analytical continuation in ω_0. The correction to the current was found in [6] with the aid of the equations for the Green's function in the dirty limit in high-frequency fields,

$$j^\alpha_{\omega_0} = \frac{1}{2d}\int d^2r_1 T\sum_{\omega_1}\mathrm{Sp}\,\hat{L}^\alpha_r\hat{K}(\omega_1+\omega_0, r, r_1)\times \\ \times\hat{L}^\beta_{r_1}\hat{K}(\omega_1, r_1, r)A^\beta_{\omega_0}, \qquad (39)$$

where A_{ω_0} is the vector potential of the external field and the matrix $\hat{L}$ is given by

$$\hat{L}^\alpha_{12} = L^\alpha_{21} = 0, \quad \hat{L}^\alpha_{11}(r) = -\frac{\pi eD}{2T}\frac{\partial}{\partial r^\alpha}, \\ \hat{L}^\alpha_{22} = -\hat{L}^\alpha_{11}. \qquad (40)$$

After the analytical continuation with respect to ω_0



The Aslamov–Larkin contribution to the conductivity (*a*); the Maki–Thompson contributions to the conductivity (*b*)

in Eq. (39), we obtain

$$j_\omega^\alpha = -\frac{1}{2d}\int d^2r_1 \frac{iT}{2\pi}\int\limits_{-i\infty}^{i\infty} d\omega_1 \left[\frac{1}{\omega_1 - i\omega - \delta} - \frac{1}{\omega_1+\delta}\right]\times$$
$$\times \mathrm{Sp}\left(\hat{L}_r^\alpha \hat{K}(\omega_1 - i\omega + \delta, r, r_1)\times\right.$$
$$\left.\times \hat{L}_{r_1}^\beta \hat{K}(\omega_1 - \delta, r_1, r)\right) A_\omega^\beta. \quad (41)$$

It was found in [6] that the fluctuations are weak in the range $\tau > \mathrm{Gi}^{1/2}$. In this region, we have

$$K_{11}(\omega_1+\delta) = K_{22}(\omega_1+\delta) = \frac{1}{\tau + (\pi/8T)(\omega_1 + Dk^2)}. \quad (42)$$

From Eqs. (40)–(42), we obtain the well-known result for the paraconductivity [3],

$$\sigma^{(a)}/\sigma_0 = \mathrm{Gi}/\tau. \quad (43)$$

To obtain the conductivity in the temperature region $\tau < \mathrm{Gi}^{1/2}$, we must find the correlation functions $\hat{K}$ in the field of thermodynamic fluctuations Δ. We must then average the expression for conductivity over Δ. The correlation functions $\hat{K}$ can be found from Eq. (36),

$$\hat{K}^{-1} = \begin{pmatrix} \tau + \dfrac{\pi}{8T}\left(\omega_1 - D\dfrac{\partial^2}{\partial r^2}\right) - C_{11} & -C_{12} \\ -C_{21} & \tau + \dfrac{\pi}{8T}\left(\omega_1 - D\dfrac{\partial^2}{\partial r^2}\right) - C_{22} \end{pmatrix}, \quad (44)$$

where the operators C_{ij} are given by

$$C_{11} = \frac{\pi}{2}\int\limits_{-\infty}^{\infty}\frac{d\varepsilon}{2\pi}\left[\left(F_1^R - F_1^A\right)\delta f^{(1)} - f_1^{(1)}\left(F_1^R + F_1^A\right)\right],$$
$$C_{12} = \frac{\pi}{2}\int\limits_{-\infty}^{\infty}\frac{d\varepsilon}{2\pi}\left[\left(F_1^R - F_1^A\right)\delta f^{(2)} - f_1^{(2)}\left(F_1^R + F_1^A\right)\right],$$
$$C_{21} = \frac{\pi}{2}\int\limits_{-\infty}^{\infty}\frac{d\varepsilon}{2\pi}\left[\left(F_2^R - F_2^A\right)\delta f^{(1)} + \left(F_2^R + F_2^A\right)\delta f_1^{(1)}\right], \quad (45)$$
$$C_{22} = \frac{\pi}{2}\int\limits_{-\infty}^{\infty}\frac{d\varepsilon}{2\pi}\left[\left(F_2^R - F_2^A\right)\delta f^{(2)} + \left(F_2^R + F_2^A\right)\delta f_1^{(2)}\right].$$

In Eqs. (45), the operators $\delta f^{(1,2)}$ and $f_1^{(1,2)}$ are such that

$$\delta f = \delta f^{(1)}\tilde{\Delta}_1 + \delta f^{(2)}\tilde{\Delta}_2, \quad (46)$$

$$f_1 = f_1^{(1)}\tilde{\Delta}_1 + f_1^{(2)}\tilde{\Delta}_2,$$

with δf and f_1 being the respective solutions of system (31) in the field of $\tilde{\Delta}_1$ and $\tilde{\Delta}_2$. System (31) cannot be solved analytically for an arbitrary function $\Delta(r)$. Nevertheless, in the range $\tau < \mathrm{Gi}^{1/2}$, the expression for the correlation functions $\hat{K}$ can be found with the logarithmic accuracy if the value of the external depairing factor Γ is larger than Δ. In this case, simple expressions for the Green's functions $F_{1,2}^{R,A}$ can be used,

$$F_1^{R,A} = \frac{-i\Delta}{\Gamma \mp i\varepsilon}, \quad F_2^{R,A} = \frac{-i\Delta^*}{\Gamma \mp i\varepsilon}. \quad (47)$$

If $Dk^2 \gg |\Delta|^2/\Gamma$, the contribution of $\delta f^{(1,2)}$ is cancelled out in the expressions for C_{11} and C_{22}. We note that $Dk^2 \ll |\Delta|^2/\Gamma$ implies $f_1^{(1,2)} \ll \delta f^{(1,2)}$. Thus, this region gives the dominant contribution to C_{ij}. Equations (44) and (45) can then be reduced to

$$\left[\tau+\frac{\pi}{8T}\Big(\omega_1-D\frac{\partial^2}{\partial r^2}\Big)\right]K_{11}+\frac{\pi\omega_1\Delta}{16T\Gamma}\Big(\omega_1+\tau_\varepsilon^{-1}-D\frac{\partial^2}{\partial r^2}\Big)^{-1}\Big(\Delta^*K_{11}\Big)+$$
$$+\frac{\pi\omega_1\Delta}{16T\Gamma}\Big(\omega_1+\tau_\varepsilon^{-1}-D\frac{\partial^2}{\partial r^2}\Big)^{-1}\Big(\Delta K_{21}\Big)=\delta(r-r_1),$$
$$\left[\tau+\frac{\pi}{8T}\Big(\omega_1-D\frac{\partial^2}{\partial r^2}\Big)\right]K_{12}+\frac{\pi\omega_1\Delta}{16T\Gamma}\Big(\omega_1+\tau_\varepsilon^{-1}-D\frac{\partial^2}{\partial r^2}\Big)^{-1}\Big(\Delta^*K_{12}\Big)+$$
$$+\frac{\pi\omega_1\Delta}{16T\Gamma}\Big(\omega_1+\tau_\varepsilon^{-1}-D\frac{\partial^2}{\partial r^2}\Big)^{-1}\Big(\Delta K_{22}\Big)=0,$$
$$\left[\tau+\frac{\pi}{8T}\Big(\omega_1-D\frac{\partial^2}{\partial r^2}\Big)\right]K_{21}+\frac{\pi\omega_1\Delta^*}{16T\Gamma}\Big(\omega_1+\tau_\varepsilon^{-1}-D\frac{\partial^2}{\partial r^2}\Big)^{-1}\Big(\Delta K_{21}\Big)+ \tag{48}$$
$$+\frac{\pi\omega_1\Delta^*}{16T\Gamma}\Big(\omega_1+\tau_\varepsilon^{-1}-D\frac{\partial^2}{\partial r^2}\Big)^{-1}\Big(\Delta^*K_{11}\Big)=0,$$
$$\left[\tau+\frac{\pi}{8T}\Big(\omega_1-D\frac{\partial^2}{\partial r^2}\Big)\right]K_{22}+\frac{\pi\omega_1\Delta^*}{16T\Gamma}\Big(\omega_1+\tau_\varepsilon^{-1}-D\frac{\partial^2}{\partial r^2}\Big)^{-1}\Big(\Delta K_{22}\Big)+$$
$$+\frac{\pi\omega_1\Delta^*}{16T\Gamma}\Big(\omega_1+\tau_\varepsilon^{-1}-D\frac{\partial^2}{\partial r^2}\Big)^{-1}\Big(\Delta^*K_{12}\Big)=\delta(r-r_1).$$

This system can be solved with the logarithmic accuracy for strong energy relaxation $\tau_\varepsilon^{-1}>T\tau$. In this region, it follows from Eq. (1) that

$$\left\langle\Delta^*\Big(\omega_1+\tau_\varepsilon^{-1}-D\frac{\partial^2}{\partial r^2}\Big)^{-1}\Delta\right\rangle=$$
$$=\frac{64\mathrm{Gi}}{\pi^2}T^2\tau_\varepsilon\ln\left(\frac{\pi}{8T\tau\tau_\varepsilon}\right). \tag{49}$$

Equations (1), (48), and (49) now imply the relations for the correlators $\hat K$,

$$\left\{\tau+\frac{\pi D}{8T}k^2+\frac{4\mathrm{Gi}T\tau_\varepsilon\omega_1}{\pi\Gamma}\ln\left(\frac{\pi}{8T\tau\tau_\varepsilon}\right)-\right.$$
$$\left.-\frac{2}{\tau}\left(\frac{4\mathrm{Gi}T\tau_\varepsilon\omega_1}{\pi\Gamma}\right)^2 I\right\}K_{11}=1,\quad K_{22}=K_{11}, \tag{50}$$

where

$$I=\int\limits_0^\infty\frac{dx\,dy}{(x+1)(y+1)\sqrt{(x-y)^2+2(x+y)a+a^2}},$$
$$a=1+\frac{4\omega_1T\tau_\varepsilon\mathrm{Gi}}{\pi\Gamma\tau}\ln\left(\frac{\pi}{8T\tau\tau_\varepsilon}\right). \tag{51}$$

The nondiagonal elements in $\hat K$ give a logarithmically small contribution to the conductivity. As a result, we obtain

$$\frac{\sigma^a}{\sigma_0}=\frac{32\mathrm{Gi}^2T^2\tau_\varepsilon}{\pi^2\Gamma\tau}\ln\left(\frac{\pi}{8T\tau\tau_\varepsilon}\right). \tag{52}$$

The situation becomes more complicated if the energy relaxation time τ_ε is large. From (48), we then obtain the equation for the correlator K_{11}

$$\left[\tau+\frac{\pi}{8T}\Big(\omega_1-D\frac{\partial^2}{\partial r^2}\Big)\right]K_{11}+$$
$$+\frac{\pi\omega_1\Delta}{16T\Gamma}\Big(\omega_1+\tau_\varepsilon^{-1}-D\frac{\partial^2}{\partial r^2}\Big)^{-1}\Big(\Delta^*K_{11}\Big)-$$
$$-\left(\frac{\pi\omega_1}{16T\Gamma}\right)^2\Delta\Big(\omega_1+\tau_\varepsilon^{-1}-D\frac{\partial^2}{\partial r^2}\Big)^{-1}\times$$
$$\times\Delta\left[\tau+\frac{\pi}{8T}\Big(\omega_1-D\frac{\partial^2}{\partial r^2}\Big)+\right.$$
$$\left.+\frac{\pi\omega_1}{16T\Gamma}\Delta^*\Big(\omega_1+\tau_\varepsilon^{-1}-D\frac{\partial^2}{\partial r^2}\Big)^{-1}\Delta\right]^{-1}\times$$
$$\times\Delta^*\Big(\omega_1+\tau_\varepsilon^{-1}-D\frac{\partial^2}{\partial r^2}\Big)^{-1}\Big(\Delta^*K_{11}\Big)=$$
$$=\delta(r-r_1). \tag{53}$$

We next find the mean value

$$\left\langle\Delta^*\Big(\omega_1-D\frac{\partial^2}{\partial r^2}\Big)^{-1}\Big(\Delta\exp(ikr)\Big)\right\rangle=$$
$$=\frac{64T^2\mathrm{Gi}}{\pi^2}\,\frac{1}{Dk^2+8T\tau/\pi}\ln\left(\frac{\pi(Dk^2+8T\tau/\pi)^2}{8T\tau\omega_1}\right). \tag{54}$$

This implies that the coefficient at ω_1 in the equation for K_{11} is logarithmically large. Contrary to the previ-

ous case ($\tau_\varepsilon^{-1} \gg T\tau$), the last term in the right-hand side of Eq. (53) is essential; together with off-diagonal elements in $\hat{K}$, it leads to the cancellation of large terms in the conductivity. To verify this, we must find the mean value of the product of four Δ in the last term in Eq. (53). We have

$$I_1 = \left(\frac{\pi}{16T\Gamma}\right)^2 \left\langle \Delta \left(\omega_1 - D\frac{\partial^2}{\partial r^2}\right)^{-1} \Delta \left[\tau + \frac{\pi}{8T}\left(\omega_1 - D\frac{\partial^2}{\partial r^2}\right) + \frac{\pi\omega_1}{16T\Gamma} \times \right.\right.$$
$$\left.\left. \times\ \Delta^* \left(\omega_1 - D\frac{\partial^2}{\partial r^2}\right)^{-1} \Delta\right]^{-1} \Delta^* \left(\omega_1 - D\frac{\partial^2}{\partial r^2}\right)^{-1} \Delta^* \exp(ikr)\right\rangle = \exp(ikr)\left(\frac{\pi}{16T\Gamma\nu d}\right)^2 \int \frac{d^2k_1}{(2\pi)^2} \int \frac{d^2k_2}{(2\pi)^2} \times$$
$$\times \left[\left(\tau + \frac{\pi D}{8T}k_1^2\right)\left(\tau + \frac{\pi D}{8T}k_2^2\right)\right]^{-1} \left[(\omega_1 + D(k-k_2)^2)(\omega_1 + D(k-k_1)^2)\ \left(\tau + \frac{\pi}{8T}Dk_3^2 + \omega_1\alpha_{k_3}\right)\right]^{-1}, \quad (55)$$

where

$$k_3 = k - k_1 - k_2,$$
$$\alpha_k = \frac{4T\mathrm{Gi}}{\pi\Gamma}\frac{1}{Dk^2 + 8T\tau/\pi}\ln\left(\frac{\pi\left(Dk^2 + 8T\tau/\pi\right)^2}{8T\tau\omega_1}\right). \quad (56)$$

The $\ln^2$ term can be easily separated in expression (55). As the result, we obtain

$$I_1 = \frac{1}{\tau + (\pi D/8T)k^2 + \omega_1\alpha_k} \times$$
$$\times \left\{\alpha_k^2 - \frac{4\pi\alpha_k\mathrm{Gi}}{\Gamma}\int \frac{d^2k_1}{(2\pi)^2}\frac{(\pi D/8T)(k_1^2 - k^2) + \omega_1(\alpha_{k_1} - \alpha_k)}{(\tau + (\pi D/8T)k_1^2)\,(k_1 - k)^2\,(\tau + (\pi D/8T)(k_1 - k)^2 + \omega_1\alpha_{k_1 - k})}\right\}. \quad (57)$$

In Eq. (55), we omitted the «diagonal» term with the denominator of the type $[\omega_1 + D(k + k_1)^2]^2$. This term leads to a small correction to the coefficient at ω_1 in (53).

With the same accuracy, we now present the expression for the nondiagonal elements K_{12} and K_{21} as

$$K_{21} = -\frac{\pi\omega_1}{16T\Gamma}\int \frac{d^2k_1 d^2k_2}{(2\pi)^4}\frac{\Delta_{k_1}^*\Delta_{k_2}^* K_{11}(k)}{(\omega_1 + D(k-k_1)^2)\left[\tau + (\pi D/8T)(k - k_1 - k_2)^2 + \omega_1\alpha_{k-k_1-k_2}\right]},$$
$$K_{12} = -\frac{\pi\omega_1}{16T\Gamma}\int \frac{d^2k_3 d^2k_4}{(2\pi)^4}\frac{\Delta_{k_3}\Delta_{k_4} K_{22}(k)}{(\omega_1 + D(k+k_3)^2)\left[\tau + (\pi D/8T)(k + k_3 + k_4)^2 + \omega_1\alpha_{k+k_3+k_4}\right]}. \quad (58)$$

Using Eqs. (57) and (58), we obtain the correction to the conductivity as

$$\frac{\sigma^a}{\sigma_0} \approx \frac{4T\mathrm{Gi}^2}{\pi\Gamma\tau^2}. \quad (59)$$

This expression is valid up to a numerical factor of the order unity.

If the external depairing factor Γ is zero (a superconductor without paramagnetic impurities), the quantity Γ in Eqs. (51) and (59) must be replaced by its intrinsic value

$$\Gamma \approx T\mathrm{Gi}^{1/2} \quad (60)$$

(see Eq. (18)). In the temperature region $\mathrm{Gi} < \tau < \mathrm{Gi}^{1/2}$, we then obtain

$$\sigma^a/\sigma_0 \approx 4\mathrm{Gi}^{3/2}/\pi\tau^2. \quad (61)$$

Equation (61) implies that the AL contribution to the conductivity is strongly enhanced in the temperature region $\mathrm{Gi} < \tau < \mathrm{Gi}^{1/2}$.

6. THE MAKI–THOMPSON CONTRIBUTION TO CONDUCTIVITY IN THE NONLINEAR FLUCTUATION REGION

The general expression for the MT contribution to the conductivity (σ^b) was given in [6]. Equation (28) in [6] can be considered as the interpolation of the MT contribution that is valid in the entire temperature re-

gion $\tau > \mathrm{Gi}$. The depairing factor Γ in Eq. (28) in [6] must be changed to a sum of two terms: the external depairing factor τ_s^{-1} related to the spin flip scattering on magnetic impurities and the intrinsic depairing factor given by Eq. (20). As a result, we obtain

$$\frac{\sigma^b}{\sigma_0} = \frac{\pi}{8d\nu}\int \frac{d^2k}{(2\pi)^2}\,\frac{1}{\Gamma + Dk^2/2}\,\frac{1}{\tau + (\pi D/8T)k^2} = \\ = \frac{2\mathrm{Gi}}{\tau}\,\frac{1}{\pi\Gamma/4T\tau - 1}\ln\left(\frac{\pi\Gamma}{4T\tau}\right). \quad (62)$$

In the range $\mathrm{Gi} < \tau < \mathrm{Gi}^{1/2}$, the MT contribution reaches its saturation value and effectively becomes temperature independent,

$$\frac{\sigma^b}{\sigma_0} = \mathrm{Gi}^{1/2}\ln\left(\frac{\mathrm{Gi}^{1/2}}{\tau}\right). \quad (63)$$

The correction remains small in the entire region $\mathrm{Gi} < \tau < \mathrm{Gi}^{1/2}$ where nonlinear effects are important.

We note that real superconductors are always inhomogeneous. The finite value of the transition width leads to the appearance of an effective depairing factor [11]. The value of this depairing factor can be sufficiently large in the units of $T\mathrm{Gi}$. In this case, the MT contribution to the conductivity is small compared to the AL contribution in the entire temperature region.

7. CONCLUSIONS

We have seen that nonlinear fluctuation effects are much stronger in kinetics phenomena than in thermodynamics. If the external depairing factor is absent, the nonlinear effects lead to a saturation of the MT contribution to the conductivity in the temperature region $\tau \leq \mathrm{Gi}^{1/2}$. In this temperature region, the AL contribution becomes even stronger and grows as $\sigma^a/\sigma_0 \approx \mathrm{Gi}^{3/2}/\tau^2$. In a superconductor with a sufficiently large external depairing factor $\Gamma = \tau_s^{-1} > T\mathrm{Gi}^{1/2}$ or a short energy relaxation time $\tau_\varepsilon^{-1} > T\mathrm{Gi}^{1/2}$, the MT contribution saturates in the temperature region $T\tau \leq \Gamma$ or $T\tau \leq \tau_\varepsilon^{-1}$. It is not very sensitive to nonlinear effects. Magnetic impurities and the energy relaxation act on the AL contribution in different ways. Energy relaxation leads to the appearance of a collision integral in the kinetic equation for the distribution functions of normal excitations. This collision integral diminishes the nonequilibrium contributions to the distribution functions. Magnetic impurities and the magnetic field act only on the superconductivity and do not lead to the relaxation of the distribution functions. However, the TDGL equation essentially depends on the electron distribution function. If $\tau_\varepsilon^{-1} > T\mathrm{Gi}^{1/2}$, the nonlinear fluctuation effects are not essential and the AL contribution remains the same, $\sigma^a/\sigma_0 = \mathrm{Gi}/\tau$, in the entire temperature region $\tau > \mathrm{Gi}$. If the inequality $\tau_\varepsilon^{-1} < T\mathrm{Gi}^{1/2}$ is satisfied, the law $\sigma^a/\sigma_0 \approx \mathrm{Gi}^{3/2}/\tau^2$ applies in the temperature region $T\tau > \tau_\varepsilon^{-1}$. In the region $(T\tau_\varepsilon)^{-1} > \tau > \mathrm{Gi}$, the correction to the conductivity is given by $\sigma^a/\sigma_0 \sim \mathrm{Gi}^{3/2}T\tau_\varepsilon/\tau$ (see Eq. (52)). Magnetic impurities (or a current) suppress nonlinear fluctuation effects in σ^a, but the effect is not as strong as for the energy relaxation. In the range $T\mathrm{Gi}/\Gamma > \tau > \mathrm{Gi}$, the correction to the conductivity σ^a is given by Eq. (59), $\sigma^a/\sigma_0 \sim T\mathrm{Gi}^2/(\Gamma\tau^2)$. In the temperature region $\tau > T\mathrm{Gi}/\Gamma$, the correction σ^a is given by (43) in the linear approximation.

It is essential that the conductivity of fluctuating pairs can be larger than the conductivity of normal electrons in the temperature region where the correction to the thermodynamic quantities is still small (see Eq. (61)).

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REFERENCES

1. A. A. Abrikosov, L. P. Gor'kov, and I. E. Dzyaloshinskii, *Methods of Quantum Field Theory in Statistical Physics*, Prentice-Hall, Englewood Cliffs, N. Y. (1963).
2. R. E. Glover, Phys. Lett. A **25**, 542 (1967).
3. L. G. Aslamazov and A. I. Larkin, Fiz. Tverd. Tela **10**, 1106 (1968); Phys. Lett. A **26**, 238 (1968).
4. K. Maki, Progr. Theor. Phys. (Kyoto) **39**, 897 (1968).
5. R. S. Thompson, Phys. Rev. B **1**, 327 (1970).
6. A. I. Larkin and Yu. N. Ovchinnikov, J. Low Temp. Phys. **10**, 407 (1973).
7. B. R. Patton, Phys. Rev. Lett. **27**, 1273 (1971).
8. J. Keller and V. Korenman, Phys. Rev. B **5**, 4367 (1972).
9. A. A. Varlamov and V. V. Dorin, Zh. Eksp. Teor. Fiz. **84**, 1868 (1983).

10. M. Reiser, Phys. Rev. B **45**, 12949 (1992).

11. A. I. Larkin and Yu. N. Ovchinnikov, JETP **34**, 1144 (1972).

12. W. Breing, M. C. Chang, E. Abrahams, and P. Wölfle, Phys. Rev. B **31**, 7001 (1985).

13. E. Abrachams and T. Zsuneto, Phys. Rev. **152**, 416 (1966).

14. A. Schmid, Phys. Conden. Mater. **5**, 302 (1966).

15. L. P. Gor'kov and G. M. Eliashberg, Zh. Eksp. Teor. Fiz. **54**, 612 (1968).

16. A. Schmid and G. Schön, J. Low Temp. Phys. **20**, 207 (1975).

17. G. Schön and V. Ambegaokar, Phys. Rev. B **19**, 3515 (1979).

18. A. I. Larkin and Yu. N. Ovchinnikov, JETP **41**, 960 (1976).

19. A. I. Larkin and Yu. N. Ovchinnikov, in *Nonequilibrium superconductivity*, ed. by D. N. Langenberg and A. I. Larkin, Elsevier Science Publishers B. V. (1986).

20. A. I. Larkin and Yu. N. Ovchinnikov, JETP **46**, 155 (1978).

21. K. S. Usadel, Phys. Rev. Lett. **25**, 507 (1970).

22. Yu. N. Ovchinnikov, J. Low Temp. Phys. **31**, 785 (1977).