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RARE KAON DECAY $K^+ \to \pi^+\nu\bar{\nu}$ IN $SU(3)_C \otimes SU(3)_L \otimes U(1)_N$ MODELS

Hoang Ngoc Long[a,b*], *Le Phuoc Trung*[c], *Vo Thanh Van*[b]

[a] *Asia Pacific Center for Theoretical Physics Seoul 135-080, Korea*

[b] *Institute of Physics, P. O. Box 429, Bo Ho Hanoi 10000, Vietnam*

[c] *HCM city Branch, Institute of Physics Ho Chi Minh city, Vietnam*

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The rare kaon decay $K^+ \to \pi^+\nu\bar{\nu}$ is considered in the framework of models based on the $SU(3)_C \otimes \otimes SU(3)_L \otimes U(1)_N$ (3–3–1) gauge group. In the 3–3–1 model with right-handed neutrinos, the lower bound of the Z' mass at the value of 3 TeV is derived, and that in the minimal version, at 1.7 TeV.

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1. INTRODUCTION

The kaon is the lightest hadron having a non-zero strangeness quantum number. Due to the weak interactions, the kaon decays into zero-strangeness states containing pions, photons, and/or leptons. The physics of kaons has played a major role in the development of particle physics. The concept of strangeness, with its implications for the quark model, the discovery of the P and CP violation, and the GIM mechanism have all emerged from the study of K mesons. Today, rare kaon decays remain a field of active investigations (see for example [1]). Flavour changing neutral currents (FCNC) are completely suppressed at the tree level by the GIM mechanism in the standard model (SM). In the second or higher order interactions, this suppression is not complete because of different quark masses [2].

The first experimental evidence for atmospheric neutrino oscillations and consequently a non-zero neutrino mass observed at the SuperKamiokande Collaboration calls for an extension of the SM. Among the possible models, those based on the $SU(3)_C \otimes \otimes SU(3)_L \otimes U(1)_N$ (3–3–1) gauge group [3–6] contain a number of intriguing features. First, the models predict three families of quarks and leptons if the anomaly-free condition on $SU(3)_L \otimes U(1)_N$ and the QCD asymptotic freedom are imposed. Second, the Peccei–Quinn symmetry naturally occurs in these models [7]. The third interesting point is that one generation of quarks is treated differently from the other two. This could lead to a natural explanation for the unbalancingly heavy top quark. This family nonuniversality also leads to the FCNC by the Z' currents at the tree level [8, 9]. Finally, the 3–3–1 models predict new physics at a scale only slightly above the SM scale (several TeVs) [8, 9, 10, 11].

In this work, we consider the implications of the main two 3–3–1 models for the rare $K^+ \to \pi^+\nu\bar{\nu}$ decay; our aim is to obtain a bound on the Z' mass.

2. THE RARE KAON DECAY $K^+ \to \pi^+\nu\bar{\nu}$ IN 3–3–1 MODELS

2.1. The decay in the 3–3–1 model with right-handed neutrinos

We first recapitulate the basic elements of the model. The leptons in this model are arranged into triplets, with the third member being a right-handed neutrino [5, 6],

*E-mail: hnlong@iop.ncst.ac.vn

$$f_L^a = \begin{pmatrix} \nu_L^a \\ e_L^a \\ (\nu_L^c)^a \end{pmatrix} \sim (1,3,-1/3), \quad e_R^a \sim (1,1,-1), \quad (1)$$

where $a = 1, 2, 3$ is the family index.

The first two families of quarks are in antitriplets and the third one is in a triplet,

$$Q_{iL} = \begin{pmatrix} d_{iL} \\ -u_{iL} \\ D_{iL} \end{pmatrix} \sim (3, \bar{3}, 0), \quad (2)$$

$$u_{iR} \sim (3, 1, 2/3), \quad d_{iR} \sim (3, 1, -1/3),$$
$$D_{iR} \sim (3, 1, -1/3), \quad i = 1, 2,$$

$$Q_{3L} = \begin{pmatrix} u_{3L} \\ d_{3L} \\ T_L \end{pmatrix} \sim (3, 3, 1/3), \quad (3)$$

$$u_{3R} \sim (3, 1, 2/3), \quad d_{3R} \sim (3, 1, -1/3), \quad T_R \sim (3, 1, 2/3).$$

The gauge bosons in this model are the photon (A), $Z, Z', W^{\pm}, Y^{\pm}$, and complex neutral bosons X^0 and X^{*0},

$$\sqrt{2}\, W_\mu^+ = W_\mu^1 - iW_\mu^2, \quad \sqrt{2}\, Y_\mu^- = W_\mu^6 - iW_\mu^7,$$
$$\sqrt{2}\, X_\mu^0 = W_\mu^4 - iW_\mu^5,$$

$$\begin{aligned} A_\mu &= s_W W_\mu^3 + c_W \left(-\frac{t_W}{\sqrt{3}} W_\mu^8 + \sqrt{1 - \frac{t_W^2}{3}} B_\mu \right), \\ Z_\mu &= c_W W_\mu^3 - s_W \left(-\frac{t_W}{\sqrt{3}} W_\mu^8 + \sqrt{1 - \frac{t_W^2}{3}} B_\mu \right), \\ Z'_\mu &= \sqrt{1 - \frac{t_W^2}{3}}\, W_\mu^8 + \frac{t_W}{\sqrt{3}}\, B_\mu, \end{aligned} \quad (4)$$

where we use the notation $s_W \equiv \sin\theta_W$ and $t_W \equiv \equiv \operatorname{tg}\theta_W$. The physical states are a mixture of Z and Z',

$$Z_1 = Z \cos\phi - Z' \sin\phi,$$

$$Z_2 = Z \sin\phi + Z' \cos\phi,$$

where ϕ is the mixing angle.

The interactions between fermions and Z_1, Z_2 are given by

$$\begin{aligned} \mathcal{L}^{NC} &= \frac{g}{2c_W} \times \\ &\times \left\{ \bar{f}\gamma^\mu [a_{1L}(f)(1-\gamma_5) + a_{1R}(f)(1+\gamma_5)] f Z_\mu^1 + \right. \\ &+ \left. \bar{f}\gamma^\mu [a_{2L}(f)(1-\gamma_5) + a_{2R}(f)(1+\gamma_5)] f Z_\mu^2 \right\}, \end{aligned} \quad (5)$$

where

$$\begin{aligned} a_{1L,R}(f) &= [T^3(f_{L,R}) - s_W^2 Q(f)] \cos\phi - \\ &- c_W^2 \left[\frac{3N(f_{L,R})}{(3 - 4s_W^2)^{1/2}} - \right. \\ &- \left. \frac{(3 - 4s_W^2)^{1/2}}{2c_W^2} Y(f_{L,R}) \right] \sin\phi, \\ a_{2L,R}(f) &= c_W^2 \left[\frac{3N(f_{L,R})}{(3 - 4s_W^2)^{1/2}} - \right. \\ &- \left. \frac{(3 - 4s_W^2)^{1/2}}{2c_W^2} Y(f_{L,R}) \right] \times \\ &\times \cos\phi + [T^3(f_{L,R}) - s_W^2 Q(f)] \sin\phi, \end{aligned} \quad (6)$$

where $T^3(f)$ and $Q(f)$ are, respectively, the third component of the weak isospin and the charge of the fermion f. The mixing angle ϕ is constrained to be very small [6], $-2.8 \cdot 10^{-3} \le \phi \le 1.8 \cdot 10^{-4}$ and can therefore be neglected.

Because one family of left-handed quarks is treated differently from the other two, the N charges for left-handed quarks are also different (see Eq. (3)). Therefore, the flavour-changing neutral current Z' occurs through a mismatch between weak and mass eigenstates. We diagolize the mass matrices by three biunitary transformations

$$\begin{aligned} U'_L &= V_L^U U_L, \quad U'_R = V_R^U U_R, \\ D'_L &= V_L^D D_L, \quad D'_R = V_R^D D_R, \end{aligned} \quad (7)$$

where $U \equiv (u, c, t)^T$ and $D \equiv (d, s, b)^T$. The usual Cabibbo–Kobayashi–Maskawa matrix is given by

$$V_{CKM} = V_L^{U+} V_L^D. \quad (8)$$

Using unitarity of the V^D and V^U matrices, we obtain the flavour-changing neutral interactions [9]

$$\mathcal{L}_{ds}^{NC} = \frac{g c_W}{2\sqrt{3 - 4s_W^2}} \left[V_{Lid}^{D*} V_{Lis}^D \right] \bar{d}_L \gamma^\mu s_L Z'_\mu, \quad (9)$$

where i denotes the number of «different» quark families, i.e., the $SU(3)_L$ quark triplet. It was shown in Ref. [9] that i must be equal to 3, i.e., the third family of quarks must be different from the first two.

We consider the decay

$$K^+(p1) \to \pi^+(p2) \nu(k1) \bar{\nu}(k2), \quad (10)$$

where the symbols in parentheses stand for the momenta of the particles. The one-loop effective SM Lagrangian for this process was calculated by Inami and Lim and other authors [2]. Due to family nonuniversality in the 3–3–1 models, the decay can be mediated by

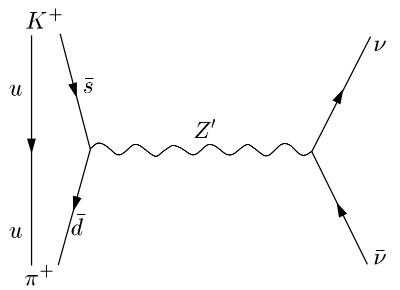

Fig. 1. Feynman diagram for $K^+ \to \pi^+\nu\bar{\nu}$ in the 3–3–1 models

Z' at the tree level. The Feynman diagram contributing to the above decay is depicted in Fig. 1.

The decay amplitude is given by

$$\mathcal{M}(K^+ \to \pi^+\nu\bar{\nu}) = \frac{G_F}{\sqrt{2}}\frac{m_W^2}{M_{Z'}^2}V_{Lbd}^{D*}V_{Lbs}^D \times \\ \times \langle\pi^+(p2)|\bar{s}_L\gamma_\mu d_L|K^+(p1)\rangle\bar{\nu}_L(k1)\gamma^\mu\nu_L(k2), \quad (11)$$

where m_W and $M_{Z'}$ stand for the W and Z' boson masses respectively.

For our initial purpose, we present the well measured semileptonic decay $K^+(p1) \to \pi^0(p2) \times \times e^+(k1)\nu(k2)$. The tree-level amplitude for this process can be written as

$$\mathcal{M}(K^+ \to \pi^0 e^+\nu) = \frac{G_F}{\sqrt{2}}V_{us}^* \times \\ \times \langle\pi^0(p2)|\bar{s}_L\gamma_\mu u_L|K^+(p1)\rangle\bar{\nu}_{eL}(k1)\gamma^\mu e_L(k2). \quad (12)$$

The isospin symmetry relates hadronic matrix elements in (11) to (12) to a very good precision [12],

$$\langle\pi^+(p2)|\bar{s}_L\gamma_\mu d_L|K^+(p1)\rangle = \\ = \sqrt{2}\langle\pi^0(p2)|\bar{s}_L\gamma_\mu u_L|K^+(p1)\rangle. \quad (13)$$

Neglecting differences in the phase space of two considered decays occurring because $m_{\pi^+} \neq m_{\pi^0}$ and $m_e \neq 0$, we sum over the three neutrino flavours and obtain

$$\frac{Br^{rhn}(K^+ \to \pi^+\nu\bar{\nu})}{Br(K^+ \to \pi^0 e^+\nu)} = \\ = 6\left(\frac{m_W^2}{M_{Z'}^2}\right)^2 \frac{|V_{Lbd}^{*D}V_{Lbs}^D|^2}{|V_{us}^*|^2}, \quad (14)$$

where the symbol rhn added to the branching ratio indicates the case under consideration. We now apply the simple Fritzsch [13] scheme as

$$V_{ij}^D \approx \left(\frac{m_i}{m_j}\right)^{1/2}, \quad i < j. \quad (15)$$

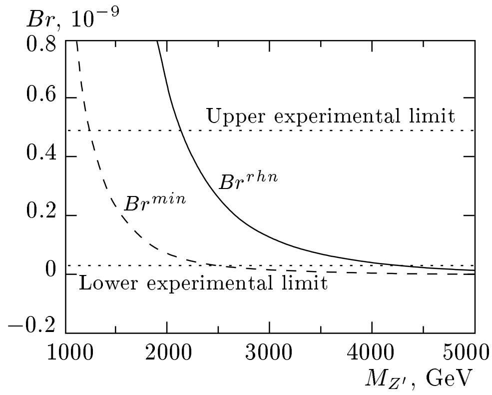

Fig. 2. Branching ratio (Br) as a function of $M_{Z'}$

Inserting (15) in (14), we obtain

$$Br^{rhn}(K^+ \to \pi^+\nu\bar{\nu}) = 6\left(\frac{m_W^2}{M_{Z'}^2}\right)^2 \times \\ \times \frac{m_d m_s}{m_b^2 V_{us}^2}Br(K^+ \to \pi^0\ e^+\ \nu). \quad (16)$$

In Fig. 2, we plot Br^{rhn} as a function of $M_{Z'}$, using the data [14]

$$\begin{aligned} &m_W = 80.41\ \text{GeV}, \quad |V_{us}| = 0.2196, \\ &m_d = 7\ \text{MeV}, \\ &m_s = 115\ \text{MeV}, \quad m_b = 4.3\ \text{GeV}, \\ &Br(K^+ \to \pi^0 e^+\nu) = 4.42\cdot 10^{-2}. \end{aligned} \quad (17)$$

The horizontal lines are the upper $(4.9\cdot 10^{-10})$ and the lower $(0.3\cdot 10^{-10})$ experimental data [15].

From the figure, we see that the lower bound on the Z' mass is in the range from 2.3 TeV to 4.35 TeV. This bound is approximately twice bigger than that derived from the mass difference of the kaon mixing system Δm_K [9]. We thus arrive at the previous conclusion again: for the Z' mass to be relatively low, the third family of quarks must be different from the other two.

2.2. The decay in the minimal 3–3–1 model

This model treats the leptons as $SU(3)_L$ antitriplets [4, 10], with the third element being the antilepton (the name of this version comes from the fact that no new leptons are introduced)

$$f_L^a = \begin{pmatrix} e_L^a \\ -\nu_L^a \\ (e^c)^a \end{pmatrix} \sim (1, \bar{3}, 0). \quad (18)$$

Of the nine gauge bosons $W^a (a = 1, 2, \ldots, 8)$ and B of $SU(3)_L$ and $U(1)_N$, four are light: the photon (A), Z, and $W^{\pm}$. The remaining five correspond to new heavy gauge bosons Z' and $Y^{\pm}$ and the doubly charged bileptons $X^{\pm\pm}$. They are expressed in terms of W^a and B as [10]

$$\sqrt{2}\, W_\mu^+ = W_\mu^1 - iW_\mu^2, \quad \sqrt{2}\, Y_\mu^+ = W_\mu^6 - iW_\mu^7,$$
$$\sqrt{2}\, X_\mu^{++} = W_\mu^4 - iW_\mu^5;$$

$$\begin{aligned} A_\mu &= s_W W_\mu^3 + c_W \left(\sqrt{3} t_W W_\mu^8 + \sqrt{1-3\, t_W^2} B_\mu\right), \\ Z_\mu &= c_W W_\mu^3 - s_W \left(\sqrt{3} t_W W_\mu^8 + \sqrt{1-3 t_W^2} B_\mu\right), \\ Z'_\mu &= -\sqrt{1-3\, t_W^2}\;\; W_\mu^8 + \sqrt{3}\, t_W\; B_\mu. \end{aligned} \tag{19}$$

As before, the physical states are a mixture of Z and Z',

$$Z_1 = Z\cos\phi - Z'\sin\phi,$$
$$Z_2 = Z\sin\phi + Z'\cos\phi,$$

and the mixing angle ϕ is also bounded to be very small. We can therefore assume $\phi \approx 0$. Applying Eq. (4.4) in [10], we obtain the interactions among Z' and neutrinos,

$$a'_V(\nu) = -a'_A(\nu) = \frac{1}{2\sqrt{3}}\sqrt{1-4s_W^2}. \tag{20}$$

One necessary vertex, namely the FCNC, is given in [8],

$$\mathcal{L}_{ds}^{NC} = \frac{g c_W}{2\sqrt{3(1-4s_W^2)}} \left[V_{Lid}^{D*} V_{Lis}^D\right] \bar{d}_L \gamma^\mu s_L Z'_\mu. \tag{21}$$

Combining (20) and (21), we obtain the decay amplitude

$$\begin{aligned} \mathcal{M}^{min}(K^+ \to \pi^+ \nu\bar{\nu}) &= \frac{1}{3}\frac{G_F}{\sqrt{2}}\frac{m_W^2}{M_{Z'}^2} V_{Lbd}^{D*} V_{Lbs}^D \times \\ &\times \langle \pi^+(p2)|\bar{s}_L \gamma_\mu d_L|K^+(p1)\rangle \bar{\nu}_L(k1)\gamma^\mu \nu_L(k2). \end{aligned} \tag{22}$$

From Eq. (22), it is straightforward to obtain

$$\begin{aligned} &\frac{Br^{min}(K^+ \to \pi^+ \nu\bar{\nu})}{Br(K^+ \to \pi^0 e^+ \nu)} = \\ &\qquad = \frac{2}{3}\left(\frac{m_W^2}{M_{Z'}^2}\right)^2 \frac{|V_{Lbd}^{*D} V_{Lbs}^D|^2}{|V_{us}^*|^2}. \end{aligned} \tag{23}$$

As in the previous section, we plot Br^{min} as a function of $M_{Z'}$ in Fig. 2. As a consequence, the lower bound on the Z' mass is in the range from 1.25 TeV to 2.45 TeV. This bound is bigger than the one derived from the mass difference of the kaon mixing system Δm_K (see Dumm et al. in [8]). For the Z' mass to be relatively low, the third family of quarks must be different from the other two. It is worth mentioning that the branching ratio is not sensitive to the value for $\sin^2\theta_W$, while the expression for Δm_K in the minimal version is very sensitive due to the factor $(1-4s_W^2)^{-1}$.

3. CONCLUSIONS

We have considered the rare kaon decay $K^+ \to \pi^+ \nu\bar{\nu}$ in the 3–3–1 models at the tree level. It was shown that in the model involving right-handed neutrinos, the decay width is by about one order bigger than in the minimal version. As a result, we obtained bounds on the Z' mass in the range from 2.3 TeV to 4.3 TeV in the model with right-handed neutrinos and from 1.2 TeV to 2.4 TeV in the minimal version. There is a point worth noting: these mass limits are in agreement with the recent analysis [16] showing that there are indications of Z' in electroweak precision data. We do hope that the new experimental data from the Collaborations at BNL and Fermilab will bring new indications of the Extra neutral gauge boson Z' — one of the best motivated extensions of the SM.

In this work, we considered only the CP conservating kaon decay $K^+ \to \pi^+ \nu\bar{\nu}$. Implications for the CP violating K and B decays are subjects of future studies.

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REFERENCES

1. J. Bijnens, G. Ecker, and J. Gasser, in *The DAPHNE physics handbook*, Vol. I, ed. by L. Maiani, G. Pancheri, and N. Paver, Frascati (1994), p. 115. J. Bijnens, G. Colangelo, G. Ecker, J. Gasser, and M. E. Sainio, Nucl. Phys. B **508**, 263 (1997). A. Buras, in *Probing of the Standard Model of Particles Interactions*, ed. by F. David and R. Gupta, Elsevier, Amsterdam (1998), (E-print archive, hep-ph/9806471). G. Buchalla, CERN-TH/2000-057 (E-print archive, hep-ph/0002207). G. D'Ambrosio, INFNNA-IV-2000/5 (E-print archive, hep-ph/0002254). A. Buras et al., TUM-HEP-382/00 (E-print archive, hep-ph/0007313).

2. T. Inami and C. S. Lim, Prog. Theor. Phys. **65**, 297 (1981). A. J. Buras, Phys. Rev. Lett. **46**, 1354 (1981).

3. F. Pisano and V. Pleitez, Phys. Rev. D **46**, 410 (1992). R. Foot, O. F. Hernandez, F. Pisano, and V. Pleitez, Phys. Rev. D **47**, 4158 (1993).

4. P. H. Frampton, Phys. Rev. Lett. **69**, 2889 (1992).

5. R. Foot, H. N. Long, and Tuan A. Tran, Phys. Rev. D **50**, R34 (1994).

6. H. N. Long, Phys. Rev. D **54**, 4691 (1996).

7. P. B. Pal, Phys. Rev. D **52**, 1659 (1995).

8. D. G. Dumm, F. Pisano, and V. Pleitez, Mod. Phys. Lett. A **9**, 1609 (1994). J. T. Liu and D. Ng, Phys. Rev. D **50**, 548 (1994).

9. H. N. Long and V. T. Van, J. Phys. G **25**, 2319 (1999).

10. D. Ng, Phys. Rev. D **49**, 4805 (1994).

11. P. Jain and S. D. Joglekar, Phys. Lett. B **407**, 151 (1997). H. Frampton and M. Harada, Phys. Rev. D **58**, 095013 (1998). H. N. Long and T. Inami, Phys. Rev. D **61**, 075002 (2000). M. B. Tully and G. C. Joshi, Phys. Lett. B **466**, 333 (1999). N. A. Ky, H. N. Long, and D. V. Soa, Phys. Lett. B **486**, 140 (2000).

12. W. Marciano and Z. Parsa, Phys. Rev. D **53**, R1 (1996).

13. H. Fritzsch, Phys. Lett. B **73**, 317 (1978); Nucl. Phys. B **155**, 189 (1979).

14. Caso et al., Particle Data Group: Eur. Phys. J. C **3**, 1 (1998).

15. S. Adler, M. S. Atiya, I.-H. Chiang et al., Phys. Rev. Lett. **84**, 3768 (2000).

16. J. Erler and P. Langacker, Phys. Rev. Lett. **84**, 212 (2000).