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# ELECTRODYNAMICS AND DISPERSION PROPERTIES OF A MAGNETOPLASMA CONTAINING ELONGATED AND ROTATING DUST GRAINS

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The electrodynamics and dispersion properties of a magnetized dusty plasma containing elongated and rotating charged dust grains are examined. Starting from an appropriate Lagrangian for dust grains, a kinetic equation for the dust grain and the corresponding equations of motion are derived. The expressions for the dust charge and dust current densities are obtained with the finite size (the dipole moment) of elongated and rotating dust grains taken into account. These charge and current densities are combined with the Maxwell–Vlasov system of equations to derive dispersion relations for the electromagnetic and electrostatic waves in a dusty magnetoplasma. The dispersion relations are analyzed to demonstrate that the dust grain rotation introduces new classes of instabilities involving various low-frequency waves in a dusty magnetoplasma. Examples of various unstable low-frequency waves include the electron whistler, dust whistler, dust cyclotron waves, Alfvén waves, electromagnetic ion-cyclotron waves, as well as lower-hybrid, electrostatic ion cyclotron, modified dust ion-acoustic waves, etc. Also found is a new type of unstable waves whose frequency is close to the dust grain rotation frequency. The present results should be useful in understanding the properties of low-frequency waves in cosmic and laboratory plasmas that are embedded in an external magnetic field and contain elongated and rotating charged dust grains.

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## 1. INTRODUCTION

About a decade ago, Shukla and collaborators [1, 2] introduced the idea of considering the dynamics of charged dust grains, which formed the foundation for the dust acoustic waves (DAWs) [1]. In the latter, the restoring force comes from the pressures of the inertialess electrons and ions, while the dust mass provides the inertia to maintain the wave. The phase velocity (the frequency) of DAWs is much smaller than the electron and ion thermal velocities (the dust plasma frequency). On the other hand, when the wave frequency is much larger (smaller) than the dust (ion) plasma frequency, we have the dust ion-acoustic waves (DIAWs) [3] whose phase velocity is much smaller (larger) than the electron (ion) thermal velocity. In DIAWs, the restoring force comes from the pressure of the inertialess electrons, while the ion mass provides the inertia because the massive dust grains remain immobile at the time scale of the DIAWs. Both the dust acoustic and dust ion-acoustic waves are spectacularly verified in several laboratory experiments [4–8]. We note that the previous theories of DAWs and DIAWs and the corresponding laboratory experiments have dealt with spherical dust grains. Comprehensive reviews of waves and instabilities in a weakly coupled unmagnetized dusty plasma with spherical dust grains were given in Refs. [9, 10].

However, elongated charged dust grains are ubiquitous in cosmic and laboratory plasmas [11–14]. The formation of elongated charged dust grains is attributed

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to the coagulation of particulates in partially or fully ionized gases due to some attractive forces. Elongated charged grains can acquire a rotational motion due to their interaction with photons and particles of the surrounding gas or due to the presence of an oscillating electric field in a plasma [11, 15]. In astrophysical objects, the angular frequency of the dust grain rotation can reach a rather large value, viz. between tens of kHz to mega Hertz for thermal dust grains and hundreds and thousands of MHz for super thermal grains [11, 12, 16]. There is an orientation of a different kind involving preferred direction (relative to the galactic disk) of the dust grain angular momentum vector.

In general, elongated charged dust grains have a nonzero dipole moment due to a finite grain size. Accordingly, Mahmoodi et al. [17] investigated the dispersion properties of an unmagnetized dusty plasma in the presence of rotating and elongated dust grains. It was found that the dust rotational energy can be coupled to both the electromagnetic and electrostatic waves. However, cosmic and laboratory plasmas are usually embedded in an external magnetic field that can have substantial effects on the dusty plasma wave spectra when elongated and rotating dust grains are present in a dusty plasma system.

In this paper, we present the electrodynamics and dispersion properties of a dusty magnetoplasma whose constituents are electrons, ions, and finite-size elongated dust grains. In Sec. 2, we find expressions for the charge and current densities of dust grains by including the effect of the dust dipole moment and the dust grain rotation. The forces acting on the dust grains as well as the corresponding dust kinetic equation and the equations of motion are presented in Sec. 3. In Sec. 4, we derive dispersion relations for both the electromagnetic and electrostatic waves. Specific instability results are discussed in Sec. 5. Finally, Sec. 6 contains a brief summary and possible applications of our work to cosmic and laboratory plasmas.

## 2. DERIVATION OF THE CHARGE AND CURRENT DENSITIES FOR DUST GRAINS

We consider a multi-component dusty plasma in the external magnetic field $\hat{\mathbf{z}}B_0$, where $\hat{\mathbf{z}}$ is the unit vector along the $z$ axis and $B_0$ is the strength of the external magnetic field. The dusty plasma constituents are electrons, ions, and negatively charged nonspherical rotating dust grains. The dust sizes are much smaller than the characteristic scale sizes of the inhomogeneities (viz. wavelength of disturbances in our system). To construct the electrodynamics of charged dust grains in a magnetized dusty plasma, we must obtain appropriate expressions for the charge and current densities of dust grains through the dust grain distribution function, taking the size of the dust grain into account. On the other hand, expressions for the charge and current densities of electrons and ions assume the standard form.

For our purposes, we assume that the charged dust grains are a system of discrete parts [18]. The charge microdensity of the grains is represented as

$$\rho_m = \sum_i \left[ \sum_j dq(\mathbf{r}_j)\delta(\mathbf{r}-\mathbf{r}_j) \right], \tag{1}$$

where the summation over $i$ is taken over different grains and the one over $j$ is taken over different parts of the $i$-th grain. Here, $dq_i(\mathbf{r}_j)$ is the charge of the $j$-th part of the $i$-th grain and $\delta(\mathbf{r}-\mathbf{r}_j)$ is the standard Dirac function. If there is a continuous charge distribution onto the grain, the summation over $j$ can be replaced with the integral over the grain volume, and the charge density onto the grain can be introduced. Hence, we have

$$\rho_m = \sum_i \int\limits_{V_i(\mathbf{R_i})} \bar{\rho}_i(\mathbf{r}'-\mathbf{R}_i, \mathbf{R}_i)\delta(\mathbf{r}-\mathbf{r}'), \tag{2}$$

where $\mathbf{R}_i$ is the radius vector of the centre of mass of the grain and the integral is taken over the grain volume $V_i(\mathbf{R}_i)$. In (2), we introduced the density of the charge distribution onto the grain

$$dq_i(\mathbf{r}) = \frac{dq(\mathbf{r})}{d\mathbf{r}}d\mathbf{r} \equiv \bar{\rho}_i(\mathbf{r}-\mathbf{R}_i, \mathbf{R}_i)d\mathbf{r}. \tag{3}$$

For a point grain charge, we have

$$\bar{\rho}_i(\mathbf{r}-\mathbf{R}_i, \mathbf{R}_i) = q_i\delta(\mathbf{r}-\mathbf{R}_i), \tag{4}$$

which leads to the usual expression for the charge microdensity of the grain

$$\rho_m = \sum_i q_i\delta(\mathbf{r}-\mathbf{R}_i), \tag{5}$$

where $q_i$ is the total charge of the $i$-th grain.

For the statistical description of a dust grain gas, we must introduce the probability density $D$ for the grain gas state [19, 20]. If all grains are identical, we have

$$D = D(\mathbf{R}_1, \mathbf{v}_1, \mathbf{\Omega}_1, \theta_1, \psi_1, \varphi_1; \\ \mathbf{R}_2, \mathbf{v}_2, \Omega_2, \theta_2, \psi_2, \varphi_2; \ldots, t), \tag{6}$$

where $\mathbf{v}_i$ is the velocity of the centre of mass, $\mathbf{\Omega}_i$ is the angular velocity of the $i$-th grain, and $\theta_i$, $\psi_i$, and $\varphi_i$ (the Euler angles) describe the orientation of elongated grains. For the averaged charge density of the grain, we can then write

$$\rho(\mathbf{r},t) = \int d\Gamma_1, d\Gamma_2, \dots, d\Gamma_N D\rho_m, \tag{7}$$

where $N$ is the total number of grains and

$$d\Gamma_i = d\mathbf{R}_i d\mathbf{v}_i d\mathbf{\Omega}_i d\theta_i d\psi_i d\varphi_i.$$

Introducing the one-particle distribution function for the dust grain

$$f_d(\mathbf{R}_1, \mathbf{v}_1, \mathbf{\Omega}_1, \theta_1, \psi_1, \varphi_1, t) = = N \int d\Gamma_2 d\Gamma_3, \dots, d\Gamma_N D, \tag{8}$$

we can write the charge density of grains as

$$\rho(\mathbf{r},t) = \int d\Gamma_1 \int_{V_1} \bar{\rho}_1(\mathbf{r}'')\delta(\mathbf{r}-\mathbf{R}_1-\mathbf{r}'') \times \times f_d(\mathbf{R}_1, \mathbf{v}_1, \mathbf{\Omega}_1, \theta_1, \psi_1, \varphi_1, t) d\mathbf{r}''. \tag{9}$$

In what follows, we omit the subscript 1 and consider the one-dimensional grain rotation such that the angular velocity is oriented along the external magnetic field direction, $\mathbf{\Omega} = (0,0,\Omega)$. Equation (9) can then be written as

$$\rho(\mathbf{r},t) = \int d\Gamma \hat{\rho}(\mathbf{r}-\mathbf{R},\varphi) f_d(\mathbf{R},\mathbf{v},\Omega,\varphi,t), \tag{10}$$

where the integrand

$$\hat{\rho}(\mathbf{r}-\mathbf{R},\varphi) = \int_V d\mathbf{r}' \bar{\rho}(\mathbf{r}')\delta(\mathbf{r}-\mathbf{R}-\mathbf{r}'), \tag{11}$$

describing the charge distribution onto a single grain, depends on the shape of the grain and the azimuthal orientation of the grain elongation axis. Outside the grain volume, we have $\hat{\rho} = 0$. For identical grains, we can partly determine the dependence of $\hat{\rho}$ on the azimuthal angle $\varphi$. Every given direction of the grain elongation axis, determined by the angle $\varphi$, can be considered as the final position of the axis (and simultaneously the entire grain) rotation from the direction where $\varphi = 0$. This allows us to write

$$\hat{\rho}(\mathbf{r}-\mathbf{R},\varphi) = \hat{\rho}[\overset{\leftrightarrow}{\mathbf{F}}(\varphi)(\mathbf{r}-\mathbf{R}),0] \equiv \equiv \hat{\rho}[\overset{\leftrightarrow}{\mathbf{F}}(\varphi)(\mathbf{r}-\mathbf{R})], \tag{12}$$

where $\overset{\leftrightarrow}{\mathbf{F}}(\varphi)$ is the rotation matrix for the angle $\varphi$,

$$\overset{\leftrightarrow}{\mathbf{F}} = F_{ij}(\varphi) = \begin{pmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{pmatrix}. \tag{13}$$

In the dipole approximation, when the dust grain size $a$ is much smaller than the scale length of the plasma inhomogeneity $\lambda$,

$$a \ll \lambda, \tag{14}$$

we insert (12) in (10) and expand the distribution function $f_d$ around the point $\mathbf{r}$. This gives the grain charge density

$$\rho_d(\mathbf{r},t) = \int (q - \mathbf{d}\cdot\nabla) f_d(\mathbf{r},\mathbf{v},\Omega,\varphi,t) d\Lambda, \tag{15}$$

where $d\Lambda = d\mathbf{v} d\Omega d\varphi$,

$$q = \int d\mathbf{r} \hat{\rho}(\mathbf{r}) \tag{16}$$

is the total charge of the dust grain, and

$$\mathbf{d} = \overset{\leftrightarrow}{\mathbf{F}}{}^{-1}(\varphi) \int d\mathbf{r}\, \mathbf{r} \hat{\rho}(\mathbf{r}) \tag{17}$$

is the dipole moment of the grain. Here, $\overset{\leftrightarrow}{\mathbf{F}}{}^{-1}$ is the inverse matrix of $\overset{\leftrightarrow}{\mathbf{F}}(\varphi)$.

Similar calculations lead to the following expression for the dust current density:

$$\mathbf{J}_d(\mathbf{r},t) = \int d\Lambda \left[\mathbf{v}(q - \mathbf{d}\cdot\nabla) + \mathbf{\Omega}\times\mathbf{d}\right] \times \times f_d(\mathbf{r},\mathbf{v},\Omega,\varphi,t). \tag{18}$$

The first term in the right-hand side of (18) describes the transfer of charge (15) and the second term describes the current arising from the dust grain rotation. In the next section, we show that Eq. (15) and (18) are related to the continuity equation.

## 3. FORCES ACTING ON GRAINS AND THE GRAIN KINETIC EQUATION

To construct the kinetic equation for dust grains, we must completely know the forces that act on dust grains in the presence of electromagnetic fields. Assuming that charged dust grains constitute a discrete system of particles [18], we have the Lagrangian

$$\mathcal{L} = \sum_i \frac{\Delta m_i u_i^2}{2} + \frac{1}{c}\sum_i \Delta q_i[\mathbf{v}_i\cdot\mathbf{A}(\mathbf{r},t)] - - \sum_i \Delta q_i \phi(\mathbf{r},t), \tag{19}$$

where $\Delta m_i$ and $\Delta q_i$ are the mass and the charge of the $i$-th part of the grain, respectively, $\mathbf{r}_i$ and $\mathbf{u}_i$ are its coordinate and velocity, $\mathbf{A}$ and $\phi$ are the vector and scalar potentials, respectively, and $c$ is the speed of light in vacuum. Separating the center-of-mass motion and the rotation around the center of mass, we can write

$$\mathbf{u}_i = \mathbf{v} + \mathbf{\Omega} \times \Delta\mathbf{r}_i \quad \text{and} \quad \mathbf{r}_i = \mathbf{r} + \Delta\mathbf{r}_i,$$

where $\mathbf{v}$ and $\mathbf{r}$ are the velocity and the position of the center of mass, $\Delta\mathbf{r}_i$ is the coordinate of the $i$-th part of the grain relative to the center of mass, and $\mathbf{\Omega}$ is the angular velocity of the dust grain. Assuming that the inhomogeneity scale $\lambda$ of the electromagnetic field is much larger than the grain size $a$, we can use dipole approximation (14) up to the third order in the small parameter $a/\lambda$ and expand the potentials as

$$\mathbf{A}(\mathbf{r}_i, t) = \mathbf{A}(\mathbf{r}, t) + (\Delta\mathbf{r}_i \cdot \nabla)\mathbf{A}(\mathbf{r}, t) + \frac{1}{2}(\Delta\mathbf{r}_i \cdot \nabla)^2\mathbf{A}(\mathbf{r}, t) + \ldots, \quad (20)$$

and

$$\phi(\mathbf{r}_i, t) = \phi(\mathbf{r}, t) + (\Delta\mathbf{r}_i \cdot \nabla)\phi(\mathbf{r}, t) + \frac{1}{2}(\Delta\mathbf{r}_i \cdot \nabla)^2\phi(\mathbf{r}, t) + \ldots \quad (21)$$

Accordingly, Lagrangian (19) becomes

$$\mathcal{L} = \frac{m_d\mathbf{v}^2}{2} + \frac{1}{2}I_{\alpha\beta}\Omega_\alpha\Omega_\beta + \frac{q}{c}\mathbf{v} \cdot \mathbf{A}(\mathbf{r}, t) - q\phi(\mathbf{r}, t) + \mathbf{m} \cdot \mathbf{B} + \left[\mathbf{d} + \frac{1}{2}\sum_i \Delta q_i \Delta\mathbf{r}_i(\Delta\mathbf{r}_i \cdot \nabla)\right] \times \left(\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B}\right), \quad (22)$$

where

$$m_d = \sum_i \Delta m_i, \quad q = \sum_i \Delta q_i$$

are the total mass and charge of the grain,

$$I_{\alpha,\beta} = \sum_i \Delta m_i[(\Delta\mathbf{r}_i)^2\delta_{\alpha\beta} - (\Delta\mathbf{r}_i)_\alpha \cdot (\Delta\mathbf{r}_i)_\beta]$$

is the inertia moment tensor,

$$\mathbf{d} = \sum_i \Delta q_i \Delta\mathbf{r}_i$$

is the dipole moment of the elongated grain, and

$$\mathbf{m} = (1/2c)\sum_i \Delta q_i(\Delta\mathbf{r} \times \mathbf{U}_i)$$

(with $\mathbf{U}_i = \mathbf{\Omega} \times \Delta\mathbf{r}_i$ being the rotation velocity) is the magnetic moment of the grain. The electric and magnetic fields are

$$\mathbf{E} = -\nabla\phi - c^{-1}\partial_t\mathbf{A}(\mathbf{r}, t), \quad \mathbf{B} = \nabla \times \mathbf{A}(\mathbf{r}, t),$$

respectively. In deriving (22), we used the relation

$$d\mathbf{d}/dt = \mathbf{\Omega} \times \mathbf{d}.$$

In the presence of the gravity field $\mathbf{g}$, we must add the term $m_d\mathbf{g} \cdot \mathbf{r}$ to the right-hand side of (22). In what follows, we neglect the second term in the square bracket in the right-hand side of (22), which is associated with the multidipole effect.

The equations of motion for the charged dust grains can be readily deduced from (22) as

$$\frac{d\mathbf{p}}{dt} = (q + \mathbf{d} \cdot \nabla)\left(\mathbf{E} + \frac{1}{c}\mathbf{v} \times \mathbf{B}\right) + \frac{1}{c}(\mathbf{\Omega} \times \mathbf{d}) \times \mathbf{B} + (\mathbf{m} \times \nabla) \times \mathbf{B} \quad (23)$$

and

$$\frac{dM_\alpha}{dt} = -\frac{1}{2}S_{\alpha\beta}\left[\frac{\partial B_\beta}{\partial t} + (\mathbf{v} \cdot \nabla) B_\beta\right] + \left[\mathbf{d} \times \left(\mathbf{E} + \frac{1}{c}\mathbf{v} \times \mathbf{B}\right)\right]_\alpha + (\mathbf{m} \times \mathbf{B})_\alpha, \quad (24)$$

where $\mathbf{p} = m_d\mathbf{v}$ is the momentum,

$$S_{\alpha\beta} = c^{-1}\sum_i \Delta q_i\left[(\Delta\mathbf{r}_i)^2\delta_{\alpha\beta} - (\Delta\mathbf{r}_i)_\alpha(\Delta\mathbf{r}_i)_\beta\right],$$

and $M_\alpha = I_{\alpha\beta}\Omega_\beta$ is the angular momentum of the grain. If we choose the principal axis of the moment of inertia, then

$$M_x = I_x\Omega_x, M_y = I_y\Omega_y, \quad M_z = I_z\Omega_z.$$

The kinetic equation for the dust grains can now be written as

$$\frac{\partial f_d}{\partial t} + \mathbf{v} \cdot \frac{\partial f_d}{\partial \mathbf{r}} + \mathbf{\Omega} \cdot \frac{\partial f_d}{\partial \varphi} + \frac{d\mathbf{p}}{dt} \cdot \frac{\partial f_d}{\partial \mathbf{p}} + \frac{d\mathbf{M}}{dt} \cdot \frac{\partial f_d}{\partial \mathbf{M}} = 0, \quad (25)$$

where the respective forces $d\mathbf{p}/dt$ and $d\mathbf{M}/dt$ are defined by Eqs. (23) and (24). Kinetic equation (25) and definitions (15) and (18) imply that the dust grain charge and the current densities satisfy the continuity equation

$$\frac{\partial \rho_d}{\partial t} + \frac{\partial \mathbf{J}_d}{\partial \mathbf{r}} = 0. \quad (26)$$

Using the expressions for $\rho_d$ and $\mathbf{J}_d$, we can construct the kinetics and electrodynamics of a dusty plasma with elongated and rotating dust grains. In what follows, we consider the wave dynamics of such a magnetized dusty plasma.

## 4. DIELECTRIC PERMITTIVITY

We assume that the dust grain size is much smaller than the grain gyroradius and that the dust grain thermal velocity is smaller than the characteristic velocity of our problem. Under these conditions taken together with (14), equations of motion (23) and (24) can be simplified. For simplicity, we furthermore consider the one-dimensional case of the dust grain rotation; we then have $\mathbf{M} = (0, 0, M)$, where $M = I\Omega$ and $I$ is the $z$ component of the principal moment of inertia. The kinetic equation for the dust grain (25) then assumes the form

$$\frac{\partial f_d}{\partial t} + \mathbf{v}\cdot\frac{\partial f_d}{\partial \mathbf{r}} + \Omega\frac{\partial f_d}{\partial \varphi} + (\mathbf{d}\times\mathbf{E})_z\frac{\partial f_d}{\partial M} + q\left(\mathbf{E} + \frac{1}{c}\mathbf{v}\times\mathbf{B}_0\right)\cdot\frac{\partial f_d}{\partial \mathbf{p}} = 0. \quad (27)$$

For electrons and ions, we have the well-known kinetic equation

$$\frac{\partial f_\alpha}{\partial t} + \mathbf{v}\cdot\frac{\partial f_\alpha}{\partial \mathbf{r}} + e_\alpha\left[\mathbf{E} + \frac{1}{c}\mathbf{v}\times(\mathbf{B}_0 + \mathbf{B})\right]\cdot\frac{\partial f_\alpha}{\partial \mathbf{p}} = 0, \quad (28)$$

where $\alpha$ equals $e$ for electrons and $i$ for ions, and $e_\alpha$ is the charge of the species $\alpha$.

Assuming that the wave electric and magnetic field perturbations are small, we can express the perturbed distribution function as

$$\delta f_d = f_d - f_{d0} \ll f_{d0} \quad \text{and} \quad \delta f_\alpha = f_\alpha - f_{\alpha 0} \ll f_\alpha.$$

The equilibrium distribution functions are [21]

$$f_{d0} = \frac{n_{d0}}{2\pi(2\pi m_d T_d)^{3/2}}\frac{1}{(2\pi I T_d)^{1/2}} \times \exp\left[-\frac{p^2}{2m_d T_d} - \frac{(M - M_0)^2}{2IT_d}\right], \quad (29)$$

and

$$f_{\alpha 0} = \frac{n_{\alpha 0}}{(2\pi m_\alpha T_\alpha)^{3/2}}\exp-\left(\frac{p^2}{2m_\alpha T_\alpha}\right), \quad (30)$$

where $n_{\beta 0}$ and $T_\beta$ ($\beta = e, i, d$) are the unperturbed number density and the temperature of the species $\beta$. We assumed that the dust grains rotate with a preferred angular velocity $\Omega_0$, and therefore, $M_0 = I\Omega_0$.

The components of the dust dipole moment are

$$d_x = d\cos\varphi, \quad d_y = d\sin\varphi. \quad (31)$$

Thus, the perturbed dust grain distribution function is represented as

$$\delta f_d = \sum_{n=-\infty}^{\infty}\delta f_n \exp(in\varphi), \quad (32)$$

and therefore, Eqs. (27) and (28) give [22]

$$\frac{\partial \delta f_n}{\partial t} + \mathbf{v}\cdot\frac{\partial \delta f_n}{\partial \mathbf{r}} + in\Omega\delta f_n - \omega_{cd}\frac{\partial \delta f_n}{\partial \psi} = -q\mathbf{E}\cdot\frac{\partial f_{d0}}{\partial \mathbf{p}}\Delta(n) - \frac{i}{2}\frac{\partial f_{d0}}{\partial M} \times d\left[(E_x - iE_y)\Delta(n-1) - (E_x + iE_y)\Delta(n+1)\right] \quad (33)$$

and

$$\frac{\partial \delta f_\alpha}{\partial t} + \mathbf{v}\cdot\frac{\partial \delta f_\alpha}{\partial \mathbf{r}} - \omega_{c\alpha}\frac{\partial \delta f_\alpha}{\partial \psi} = -e_\alpha\mathbf{E}\cdot\frac{\partial f_{\alpha 0}}{\partial \mathbf{p}}, \quad (34)$$

where

$$\omega_{cd} = qB_0/m_d c, \quad \omega_{c\alpha} = e_\alpha B_0/m_\alpha c$$

are the cyclotron frequencies of the dust grain and the species $\alpha$, respectively. Furthermore, $\Delta(n)$ equals 1 for $n = 0$ and 0 for $n \neq 0$. The symbol $\psi$ is the azimuthal angle in the momentum space [22],

$$p_x = p_\perp\cos\psi, \quad p_y = p_\perp\sin\psi.$$

In accordance with (31), only $n = 0, \pm 1$ give a contribution to the summation in (32).

Assuming that the perturbed quantities are proportional to $\exp(-i\omega t + i\mathbf{k}\cdot\mathbf{r})$, where $\omega$ and $\mathbf{k}$ are the frequency and the wave vector, respectively, we obtain [22] the following solutions of Eqs. (33) and (34):

$$\delta f_0 = \frac{q\mathbf{E}}{\omega_{cd}}\int_{\pm\infty}^{\psi} d\psi'\frac{\partial f_{d0}}{\partial \mathbf{p}} \times \exp\left\{-i\int_{\psi'}^{\psi}\left[\frac{\omega - \mathbf{k}\cdot\mathbf{v}(\psi'')}{\omega_{cd}}\right]d\psi''\right\}, \quad (35)$$

$$\delta f_{\pm 1} = \pm\frac{i}{2}\frac{d(E_x \mp iE_y)}{\omega_{cd}}\int_{\pm\infty}^{\psi} d\psi'\frac{\partial f_{d0}}{\partial M} \times \exp\left\{-i\int_{\psi'}^{\psi}\left[\frac{\omega \mp \Omega - \mathbf{k}\cdot\mathbf{v}(\psi'')}{\omega_{cd}}\right]d\psi''\right\}, \quad (36)$$

$$\delta f_\alpha = \frac{e_\alpha \mathbf{E}}{\omega_{c\alpha}} \cdot \int\limits_{\pm\infty}^{\psi} d\psi' \frac{\partial f_{\alpha 0}}{\partial \mathbf{p}} \times \\ \times \exp\left\{ -i \int\limits_{\psi'}^{\psi} \left[ \frac{\omega - \mathbf{k}\cdot\mathbf{v}(\psi'')}{\omega_{c\alpha}} \right] d\psi'' \right\}. \quad (37)$$

Inserting Eqs. (35) and (36) in (18) and also inserting (37) in the expression for the electron and ion current densities

$$\mathbf{J}_\alpha = e_\alpha \int d\mathbf{p}\mathbf{v} f_\alpha, \quad (38)$$

we obtain the total current density

$$J_i = \left[ \sigma^r_{ij}(\omega, \mathbf{k}) + \sum_{\beta=e,i,d} \sigma^\beta_{ij}(\omega, \mathbf{k}) \right] E_j, \quad (39)$$

where the first term in the right-hand side is related to the rotational motion of the dust grain and the second term represents the contributions of the electrons and ions including the center-of-mass motion of the grains. The various components [25] of $\sigma^r_{ij}$ and the dielectric permittivity are given in the Appendix. For $k_\perp^2 V_{td}^2 \ll \omega_{cd}^2$ and $|\omega \pm \Omega_0| \gg K V_{td}$, the dust grains are assumed to be cold and the rotational part of the dielectric tensor (cf. Eq. (A.15) in the Appendix) is given by

$$\epsilon^r_{ij} = \begin{pmatrix} \epsilon^r_\perp & ig^r & 0 \\ -ig^r & \epsilon^r_\perp & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad (40)$$

where

$$\epsilon^r_\perp = -\frac{\Omega_r^2}{(\omega - \Omega_0)^2} - \frac{\Omega_r^2}{(\omega + \Omega_0)^2} \quad (41)$$

and

$$g^r = \frac{\Omega_r^2}{(\omega - \Omega_0)^2} - \frac{\Omega_r^2}{(\omega + \Omega_0)^2}. \quad (42)$$

We note that this involves a new characteristic frequency

$$\Omega_r = (4\pi n_{d0} d^2/4I)^{1/2}$$

for dust grains that have a non-zero dipole moment. This frequency is of the same order as the dust plasma frequency $\omega_{pd}$.

## 5. DISPERSION PROPERTIES

The general analysis of the dispersion relation

$$\left| k^2 \delta_{ij} - k_i k_j - \frac{\omega^2}{c^2} \epsilon_{ij}(\omega, \mathbf{k}) \right| = 0 \quad (43)$$

for waves in a magnetized dusty plasma is rather complicated, because the number of wave branches is large. Here, we present the dispersion properties of some most interesting modes and describe the underlying approximations required for the existence of these modes. We first consider waves that are propagating along $\hat{\mathbf{z}} B_0$. For waves in a cold dusty plasma with

$$k_\perp V_{t\alpha} \ll \omega_{c\alpha}, \; |k_z| V_{t\alpha} \ll \omega \\ \text{and } |\omega \pm n\omega_{c\alpha}| \gg |k_z| V_{t\alpha}, \quad (44)$$

we have

$$\epsilon_{xx} = \epsilon_{yy} = \epsilon_\perp = \\ = 1 - \sum_\beta \frac{\omega_{p\beta}^2}{\omega^2 - \omega_{c\beta}^2} - \frac{\Omega_r^2}{(\omega - \Omega_0)^2} - \frac{\Omega_r^2}{(\omega + \Omega_0)^2}, \quad (45)$$

$$\epsilon_{xy} = -\epsilon_{yx} = ig = -i \sum_\beta \frac{\omega_{p\beta}^2 \omega_{c\beta}}{\omega(\omega^2 - \omega_{c\beta}^2)} + \\ + i\frac{\Omega_r^2}{(\omega - \Omega_0)^2} - i\frac{\Omega_r^2}{(\omega + \Omega_0)^2}, \quad (46)$$

$$\epsilon_{zz} = \epsilon_\parallel = 1 - \sum_\beta \frac{\omega_{p\beta}^2}{\omega^2}, \quad (47)$$

and

$$\epsilon_{xz} = \epsilon_{zx} = \epsilon_{yz} = \epsilon_{zy} = 0. \quad (48)$$

The electric field components are determined by the set of equations

$$\left( k_z^2 - \frac{\omega^2}{c^2}\epsilon_\perp \right) E_x - i\frac{\omega^2}{c^2} g E_y - k_\perp k_z E_z = 0, \quad (49)$$

$$\frac{\omega^2}{c^2} g E_x + \left( k^2 - \frac{\omega^2}{c^2}\epsilon_\perp \right) E_y = 0, \quad (50)$$

$$-k_\perp k_z E_x + \left( k_\perp^2 - \frac{\omega^2}{c^2}\epsilon_\parallel \right) E_z = 0. \quad (51)$$

We note that for $k_\perp = 0$ (i.e., for $\mathbf{k} = \hat{\mathbf{z}} k_z$), we have $\epsilon_\parallel = 0$ if $E_z \neq 0$, which shows that the dust grain rotation does not affect the longitudinal waves. Obviously, the dust grain rotation can act on the waves when the electric field is in the rotation plane. The energy exchange between the dust grain rotation and such a wave is most efficient when the rotation frequency is close to the wave frequency.

For the circularly polarized electromagnetic waves, we have

$$\frac{k^2c^2}{\omega^2} = 1 - \sum_\beta \frac{\omega_{p\beta}^2}{\omega(\omega \mp \omega_{c\beta})} - \frac{2\Omega_r^2}{(\omega \pm \Omega_0)^2}, \tag{52}$$

where $\pm$ in the denominators corresponds to the left/right-hand circularly polarized waves. By replacing $\Omega_0$ with $-\Omega_0$, we can make the dust grain rotation direction coincide with the wave polarization direction.

Dispersion relation (52) can be written as

$$\frac{k^2c^2}{\omega^2} = \epsilon(\omega) - \frac{2\Omega_r^2}{(\omega - \Omega_0)^2}, \tag{53}$$

where

$$\epsilon(\omega) = 1 - \sum_\beta \frac{\omega_{p\beta}^2}{\omega(\omega + \omega_{c\beta})}. \tag{54}$$

Introducing a small frequency shift $\Delta$ around $\Omega_0$, we set $\omega = \Omega_0 + \Delta$, where $\Delta \ll \Omega_0$, and express (53) as

$$\frac{k^2c^2}{\Omega_0^2} - \epsilon(\Omega_0) + \Delta\frac{\partial}{\partial\Omega_0}\left[\frac{k^2c^2}{\Omega_0^2} - \epsilon(\Omega_0)\right] = -\frac{2\Omega_r^2}{\Delta^2}. \tag{55}$$

We now assume that $\Omega_0$ is far from the characteristic frequency $\omega_0$ of the magnetized dusty plasma, which satisfies

$$H(\omega_0) = \frac{k^2c^2}{\omega^2} - \epsilon(\omega_0) = 0. \tag{56}$$

The condition

$$\left|\frac{H(\Omega_0)}{\Omega_0\,(dH(\Omega_0)/d\Omega_0)}\right| \gg \frac{\Delta}{\Omega_0}, \tag{57}$$

is then satisfied (this case is referred to as the nonresonance case) and we obtain

$$\Delta = \pm i\sqrt{2}\frac{\Omega_r}{kc}\Omega_0\left[1 + \frac{\Omega_0^2}{k^2c^2}\epsilon(\Omega_0)\right], \tag{58}$$

where we also assumed that $\Omega_0^2 \ll k^2c^2$. Equation (58) describes a new type of unstable transversal waves whose frequency is close to the rotation frequency $\Omega_0$. In the resonance case, when inequality (57) is reversed, $\Omega_0$ is close to some characteristic frequency of the magnetized dusty plasma,

$$H(\Omega_0) = 0, \tag{59}$$

and we obtain the frequency shift

$$\Delta = \left[-\frac{2\Omega_r^2}{\Omega_0^3\partial H(\Omega_0)/\partial\Omega_0}\right]^{1/3} \times \Omega_0\begin{pmatrix} 1 \\ -\dfrac{1 \pm i\sqrt{3}}{2}\end{pmatrix}. \tag{60}$$

Equation (60) exhibits an unstable root with a substantial growth rate that is proportional to $\Omega_r^{2/3}$. This was expected because dispersion relation (53) is formally similar to the dispersion relation for a two-stream instability discussed in [23].

We now present several examples of the magnetized dusty plasma wave spectra for the resonance case. Because $\Omega_0$ is small in most of the astrophysical and terrestrial environments, we consider low-frequency regimes of the plasma oscillations.

For $|\omega_{cd}|, \omega_{ci} \ll \omega \ll |\omega_{ce}|$, we have

$$H(\omega) = \frac{k^2c^2}{\omega^2} - \frac{\omega_{pe}^2}{\omega|\omega_{ce}|}. \tag{61}$$

Setting

$$\omega = \Omega_0 + i\gamma,$$

where

$$\Omega_0 \approx \omega_0 = k^2c^2|\omega_{ce}|/\omega_{pe}^2$$

(the electron whistler waves), we obtain the growth rate

$$\gamma \approx \Omega_0\left[2\frac{\Omega_r^2}{k^2c^2}\right]^{1/3}. \tag{62}$$

In the frequency regime where $|\omega_{cd}| \ll \omega \ll \omega_{ci}$, we have

$$H(\omega) = \frac{k^2c^2}{\omega^2} - \frac{\omega_{pd}^2}{\omega|\omega_{cd}|}. \tag{63}$$

In deriving (63), we used the dusty plasma quasi-neutrality condition at equilibrium

$$|e|n_{e0} + |q|n_{d0} = e_i n_{i0}. \tag{64}$$

Setting

$$\omega = \Omega_0 + i\gamma,$$

where

$$\Omega_0 \approx \omega_0 = k^2c^2|\omega_{cd}|/\omega_{pd}^2$$

(the dust whistler wave [2, 24–26]), we obtain the growth rate

$$\gamma \approx \left(2\frac{\Omega_r^2}{k^2c^2}\right)^{1/3}\Omega_0. \tag{65}$$

On the other hand, for $\omega \sim \omega_{cd}$, we have

$$H(\omega) = \frac{k^2c^2}{\omega^2} - \frac{\omega_{pd}^2}{\omega(|\omega_{cd}| - \omega)}. \tag{66}$$

In this case, setting

$$\omega = \Omega_0 + i\gamma,$$

where

$$\Omega_0 = \omega_0 = |\omega_{cd}| \left(1 - \omega_{pd}^2/k^2c^2\right)$$

(the electromagnetic dust cyclotron wave), we obtain the growth rate

$$\gamma \approx \left(2\frac{\Omega_r^2}{k^2c^2}\frac{\omega_{pd}^2}{k^2c^2}\right)^{1/3} |\omega_{cd}|. \qquad (67)$$

For the frequency range $\omega \approx \omega_{ci}$ (ion cyclotron waves), the growth rate is given by

$$\gamma \approx \left(2\frac{\Omega_r^2}{k^2c^2}\frac{\omega_{pi}^2}{k^2c^2}\right)^{1/3} \omega_{ci}. \qquad (68)$$

We now take the thermal motion of the electrons into account assuming that

$$V_{td}, V_{ti} \ll \frac{\omega}{|k_z|} \ll V_{te}. \qquad (69)$$

We restrict ourself to the wavelengths longer than the Larmor radii

$$k_\perp^2 V_{t\alpha}^2, \quad |k_z|^2 V_{t\alpha}^2 \ll \omega_{c\alpha}^2.$$

From (A.9)–(A.11), we then obtain the dielectric permittivity tensor components

$$\epsilon_{xx} = \epsilon_{yy} = 1 + \frac{c^2}{V_A^2} - \frac{\omega_{pd}^2}{\omega^2 - \omega_{cd}^2} - \frac{\Omega_r^2}{(\omega - \Omega_0)^2} - \frac{\Omega_r^2}{(\omega + \Omega_0)^2}, \qquad (70)$$

$$\epsilon_{xy} = -\epsilon_{yx} = -i\frac{\omega_{pd}^2}{\omega^2 - \omega_{cd}^2}\frac{\omega}{\omega_{cd}} + i\frac{\Omega_r^2}{(\omega - \Omega_0)^2} - i\frac{\Omega_r^2}{(\omega + \Omega_0)^2}, \qquad (71)$$

$$\epsilon_{zz} = 1 - \frac{\omega_{pi}^2}{\omega^2} - \frac{\omega_{pd}^2}{\omega^2} + \frac{\omega_{pe}^2}{k_z^2 V_{te}^2}, \qquad (72)$$

$$\epsilon_{xz} = \epsilon_{zx} = \epsilon_{yz} = \epsilon_{zy} = 0, \qquad (73)$$

where we used (64) and set

$$V_A = \frac{B_0}{\sqrt{4\pi m_i n_{i0}}}.$$

We also ignored the Landau damping on electrons. Dispersion relation (43) separates into two equations:

$$\epsilon_{zz}(\omega, k_z) = 0, \qquad (74)$$

which is not influenced by the rotation of the grain, and

$$H(\omega) = \frac{k^2c^2}{\omega^2} - \frac{c^2}{V_A^2} + \frac{\omega_{pd}^2}{\omega_{cd}(\omega \pm \omega_{cd})} = -\frac{2\Omega_r^2}{(\omega \mp \Omega_0)^2}. \qquad (75)$$

We now assume that $\omega \ll |\omega_{cd}|$. Setting

$$\omega = \Omega_0 + i\gamma,$$

where

$$\Omega_0 = kV_A$$

(Alfvén waves), and using (58), we obtain the growth rate

$$\gamma \approx \left(\frac{\Omega_r^2}{k^2c^2}\right)^{1/3} kv_A. \qquad (76)$$

We next consider the longitudinal waves for which the dispersion relation assumes the form

$$\frac{k_\perp^2}{k^2}\epsilon_{xx}(\omega, \mathbf{k}) + \frac{k_z^2}{k^2}\epsilon_{zz}(\omega, \mathbf{k}) = 0 \qquad (77)$$

where the components $\epsilon_{xx}$ and $\epsilon_{zz}$ for the cold plasma are defined by Eqs. (70) and (72). Inserting the latter equation in (77), we obtain

$$1 - \frac{k_\perp^2}{k^2}\sum_\beta \frac{\omega_{p\beta}^2}{\omega^2 - \omega_{c\beta}^2} - \frac{k_z^2}{k^2}\sum_\beta \frac{\omega_{p\beta}^2}{\omega^2} = \frac{k_\perp^2}{k^2}\left[\frac{\Omega_r^2}{(\omega - \Omega_0)^2} + \frac{\Omega_r^2}{(\omega + \Omega_0)^2}\right]. \qquad (78)$$

It follows from (78) that the dust grain rotation contributes only for waves with $k_\perp \neq 0$, because the electric field of the longitudinal waves then has a component that lies in the dust grain rotation plane.

To obtain the growth rates for longitudinal waves, we use the same procedure as was used to deduce Eqs. (58) and (60).

We now consider the lower hybrid waves with $|k_z|V_{te}$, $|k_z|V_{ti}$, $\omega_{ci} \ll \omega \ll |\omega_{ce}|$. Setting

$$\omega = \Omega_0 + i\gamma,$$

where

$$\Omega_0 = \omega_0 = \frac{\omega_{pi}\omega_{ce}}{\sqrt{\omega_{pe}^2 + \omega_{ce}^2}}, \qquad (79)$$

we find the growth rate

$$\gamma = \left(\frac{1}{2}\frac{\Omega_r^2}{\omega_{pi}^2}\right)^{1/3} \Omega_0. \qquad (80)$$

Next, we consider the frequency regime where

$$|k_z|V_{td}, \quad |k_z|V_{ti} \ll \omega \ll |k_z|V_{te}.$$

The dielectric permittivity components in Eq. (77) are now defined by Eqs. (72) and (78). Using these expressions, we obtain the dispersion relation

$$1+\frac{1}{k^2r_{De}^2}-\frac{k_\perp^2}{k^2}\left(\frac{\omega_{pi}^2}{\omega^2-\omega_{ci}^2}+\frac{\omega_{pd}^2}{\omega^2-\omega_{cd}^2}\right)-\frac{k_z^2}{k^2}\frac{\omega_{pi}^2}{\omega^2} = \frac{k_\perp^2}{k^2}\left[\frac{\Omega_r^2}{(\omega-\Omega_0)^2}+\frac{\Omega_r^2}{(\omega+\Omega_0)^2}\right]. \quad (81)$$

This equation can be analyzed in two limiting cases. First, we consider the ion-cyclotron waves with $\omega_{cd} \ll \omega \approx \Omega_{ci}$ and $k_z \ll k_\perp$. Setting

$$\omega \approx \omega_0 + i\gamma,$$

we then obtain the growth rate

$$\gamma \approx \left(\frac{\Omega_r^2}{2\omega_{pi}^2}\frac{k_\perp^4 c_s^4}{\Omega_0^4}\right)^{1/3}\Omega_0, \quad (82)$$

where

$$\Omega_0 = \omega_0 = (\omega_{ci}^2 + k_\perp^2 c_s^2)^{1/2}, \quad c_s = \left(\frac{n_{i0}T_e}{n_{e0}m_i}\right)^{1/2}$$

is the ion acoustic speed.

Second, we consider the modified dust ion-acoustic waves (MDIAWs) characterized by $\omega_{pd}, \omega_{cd} \ll \omega \ll \omega_{ci}$. In this case, Eq. (81) gives

$$1 + k^2r_{De}^2 + k_\perp^2\rho_s^2 - \frac{k_z^2c_{ss}^2}{\omega^2} = k_\perp^2 r_{De}^2\left[\frac{\Omega_r^2}{(\omega-\Omega_0)^2}+\frac{\Omega_r^2}{(\omega+\Omega_0)^2}\right], \quad (83)$$

where

$$\rho_s = \frac{r_{De}\omega_{pi}}{\omega_{ci}} \equiv \frac{c_s}{\omega_{ci}}.$$

Equation (83) admits an instability of the MDIAWs with the frequency

$$\Omega_0 = \omega_0 = \frac{k_z c_s}{(1 + k^2r_{De}^2 + k_\perp^2\rho_s^2)^{1/2}},$$

and the growth rate is

$$\gamma \approx \left(\frac{k_\perp^2}{2k^2}\frac{\Omega_r^2}{\omega_{pi}^2}\right)^{1/3}\Omega_0. \quad (84)$$

Finally, we consider coupled dust acoustic-dust cyclotron waves in a dust-electron plasma (without ions) with positive dust grains [27]. For $kV_{td} \ll \omega \ll k_zV_{te}$, we then have

$$1 + \frac{1}{k^2r_{De}^2} - \frac{k_\perp^2}{k^2}\frac{\omega_{pd}^2}{\omega^2-\omega_{cd}^2} - \frac{k_z^2}{k^2}\frac{\omega_{pd}^2}{\omega^2} = \frac{k_\perp^2}{k^2}\left[\frac{\Omega_r^2}{(\omega-\Omega_0)^2}+\frac{\Omega_r^2}{(\omega+\Omega_0)^2}\right]. \quad (85)$$

For $\omega \ll |\omega_{cd}|$, Eq. (85) admits an instability of short wavelength DIWs when

$$\Omega_0 = \omega_0 = \frac{kr_{De}\omega_{pd}}{(1 + k^2r_{De}^2 + k_\perp^2\rho_{sd}^2)^{1/2}},$$

where

$$\rho_{sd} = \lambda_{De}\omega_{pd}/\omega_{cd}.$$

The growth rate of this instability is

$$\gamma \approx \left(\frac{1}{2}\frac{k_\perp^2}{k_z^2}\frac{\Omega_r^2}{\omega_{pd}^2}\right)^{1/3}\Omega_0. \quad (86)$$

On the other hand, for $\omega \sim |\omega_{cd}|$, $k_\perp \gg k_z$, and $k^2r_{De}^2 \ll 1$, an instability of the dust cyclotron waves occurs when

$$\Omega_0 = \omega_0 = (\omega_{cd}^2 + k^2r_{De}^2\omega_{pd}^2)^{1/2}.$$

The growth rate of the instability is

$$\gamma \approx \left(\frac{1}{2}k_\perp^4 r_{De}^4\frac{\Omega_r^2}{\Omega_0^2}\frac{\omega_{pd}^2}{\Omega_0^2}\right)^{1/3}\Omega_0. \quad (87)$$

It is interesting to note that a dust-electron plasma with positively charged grains can occur in the Earth's polar mesosphere [28, 29], where the grains are irradiated by the sun light, in which case the grains act as a source of electrons and collect ions from the ambient plasma to become positively charged. There also is the prediction [30] that positively charged dust grains in retrogate orbits are most likely to be observed by the Cosmic Dust Analyzer aboard the Cassini Orbiter mission to Saturn. Furthermore, the dust electron plasma can also be created in a laboratory discharge when the dust grains are irradiated by ultraviolet (UV) radiation [31, 32, 33, 34].

## 6. SUMMARY AND CONCLUSION

In this paper, we have developed the electrodynamics of a magnetized dusty plasma taking the finite size of elongated and rotating charged dust grains into account. Starting from an appropriate

Lagrangian for charged dust grains, we have derived the dust charge and dust current densities, as well as a kinetic equation for charged dust grains and the corresponding equations of motion in the external magnetic field. The effects of the dipole moment and the principal moment of inertia of the elongated and rotating dust grains are self-consistently incorporated. The newly derived dust charge and dust current densities, together with the corresponding quantities for electrons and ions, are combined with the Maxwell–Vlasov system of equations to obtain dielectric response functions for a magnetized dusty plasma. For a cold dust gas, we have obtained explicit expressions for the permittivities associated with the dust grain rotation and for those of the ambient plasma species. The dispersion relations for transverse and longitudinal waves were then derived. Our analytical results exhibit instabilities of the electron whistler, the dust whistler, the Alfvén waves, electromagnetic ion and dust cyclotron waves, as well as lower-hybrid, electrostatic ion-cyclotron, and coupled dust acoustic and dust cyclotron waves. The instability arises due to the resonance interaction between waves and elongated rotating dust grains. The free energy stored in the dust grain rotational motion is basically coupled to both the electromagnetic and electrostatic waves, driving them at nonthermal levels. The presence of nonthermal fluctuations can be used for diagnostic purposes. For example, coherent or incoherent scatterings of star light and/or electromagnetic waves off nonthermal fluctuations in cosmic plasmas may yield valuable information regarding the light polarization, the dust number density and the dust charge in situ, and other plasma parameters including the external magnetic field strength. We stress that the oscillating electric fields of electromagnetic waves may produce dust grain rotation, the energy of which is required for driving waves at nonthermal levels. In conclusion, we emphasize that the present investigation should be useful for understanding waves and instabilities in astrophysical and laboratory plasmas that contain elongated and rotating charged dust grains. Finally, we suggest that new laboratory experiments in a weakly coupled dusty magnetoplasma must be designed to test the ideas described in this paper. A recent experimental work by Molotkov et al. [14] has conclusively demonstrated the Coulomb crystallization of 300-$\mu m$ highly charged elongated cylindrical grains (with $|q| \sim 7.7 \cdot 10^5$ and with the length-to-diameter ratio 20–40) of the mass density 1.1 g/cm$^3$ and the diameters 15 and 7.5$\mu m$ in a low pressure gas discharge plasma, where the electron energy ranges between 1–10 eV. Thus, a sheath electric field of the order 30 V/cm can levitate the grain. Molotkov et al. [14] have discussed the role of the induced dipole moment that can influence the grain orientation. At small pressures (0.1 Torr), they also observed oscillations with the wavelength $\sim$ 1 mm and the frequency 20–50 Hz. The latter can be associated with the dust acoustic waves that are deduced from our Eq. (83). Furthermore, by applying the external magnetic field 1–6 kGs and with the plasma ($n_{i0} \sim 10^9$ cm$^{-3}$ and $n_{d0} \sim 10^3$ cm$^{-3}$) and dust parameters similar to those in Molotkov et al. [14], one should be able to observe the magnetization of ions and the electrostatic ion-cyclotron wave instability described by (81). Finally, we mention that several authors [35–38] have experimentally observed rotation of spherical dust grains by magnetic fields. The rotation is attributed to the azimuthal $\mathbf{E} \times \mathbf{B}_0$ ion drift, which also drags the dust grain along due to the space charge electric field that is set up between ions and grains.

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## APPENDIX: VARIOUS COMPONENTS OF $\sigma_{ij}^r$

The components of $\sigma_{ij}^r$ are given by

$$\sigma_{ij}^r(\omega, \mathbf{k}) = \begin{pmatrix} \sigma_{xx}^r & \sigma_{xy}^r & 0 \\ -\sigma_{xy}^r & \sigma_{yy}^r & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad (A.1)$$

where

$$\sigma_{xx}^r = \sigma_{yy}^r = i\frac{n_{d0}d^2}{4I}\frac{1}{KV_{td}} \times \\ \times \sum_{n=-\infty}^{\infty} \exp(-z_d) I_n(z_d) \left(\Phi_-^n + \Phi_+^n\right), \qquad (A.2)$$

$$\sigma^r_{xy} = \frac{n_{d0}d^2}{4I}\frac{1}{KV_{td}} \times \\ \times \sum_{n=-\infty}^{\infty} \exp(-z_d) I_n(z_d) \left(\Phi^n_- - \Phi^n_+\right), \quad \text{(A.3)}$$

with

$$\Phi^n_\pm = \frac{k_z^2}{K^2}\frac{KV_{td}}{\omega \pm \Omega_0 - n\omega_{cd}} J_+\left(\frac{\omega \pm \Omega_0 - n\omega_{cd}}{KV_{td}}\right) - \\ - \left(\frac{\kappa^2}{K^2}\frac{\omega - n\omega_{cd}}{KV_{td}} \mp \frac{k_z^2}{K^2}\frac{\Omega_0}{KV_{td}}\right) \times \\ \times \left[1 + J_+\left(\frac{\omega \pm \Omega_0 - n\omega_{cd}}{KV_{td}}\right)\right]. \quad \text{(A.4)}$$

Here,

$$V_{td} = (T_d/m_d)^{1/2}$$

is the grain thermal velocity,

$$z_\beta = \frac{k_\perp^2 V_{t\beta}^2}{\omega_{c\beta}^2}, \quad K = \sqrt{k^2 + \kappa^2}, \quad \kappa = \sqrt{\frac{m_d}{I}},$$

$I_n(z)$ is the Bessel function of an imaginary argument, and the function

$$J_+(x) = \frac{x}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dt \exp\left(-\frac{t^2}{2}\right)(x-t)^{-1}$$

has the asymptotic behavior

$$J_+(x) = 1 + \frac{1}{x^2} + \frac{3}{x^2} + \ldots - i\sqrt{\frac{\pi}{2}} x \exp(-x^2/2) \quad \text{(A.5)}$$

for $|x| \gg 1$, $|\operatorname{Re} x| \gg |\operatorname{Im} x|$, and $\operatorname{Im} x < 0$ and

$$J_+(x) \approx -i\sqrt{\frac{\pi}{2}} x \quad \text{(A.6)}$$

for $|x| \ll 1$.

For the tensor $\sigma^\beta_{ij}(\omega, \mathbf{k})$, we have

$$\sigma^\beta_{ij}(\omega, \mathbf{k}) = \frac{e_\beta^2}{\omega_{c\beta}} \int d\mathbf{p} v_i(\psi) \int_{\pm\infty}^{\psi} d\psi' \left(\frac{\partial f_{\beta 0}}{\partial p_j}\right)_{\psi'} \times \\ \times \exp\left\{ i \int_{\psi'}^{\psi} \left[\left(\omega - \mathbf{k}\cdot\mathbf{v}(\psi'')\right)/\omega_{c\beta}\right] d\psi'' \right\}, \quad \text{(A.7)}$$

where $e_\beta$ is $q$ for $\beta = d$.

Straightforward calculations lead to the following expressions for the dielectric permittivity tensor [22]:

$$\epsilon_{ij}(\omega, \mathbf{k}) = \hat{\epsilon}_{ij}(\omega, \mathbf{k}) + \epsilon^r_{ij}(\omega, \mathbf{k}), \quad \text{(A.8)}$$

where

$$\hat{\epsilon}_{xx} = 1 - \sum_\beta \sum_n \frac{n^2\omega_{p\beta}^2}{\omega(\omega - n\omega_{c\beta})} \frac{A_n(z_\beta)}{z_\beta} J_+(\xi_n), \quad \text{(A.9)}$$

$$\hat{\epsilon}_{yy} = \hat{\epsilon}_{xx} + \\ + 2\sum_\beta \sum_n \frac{\omega_{p\beta}^2 z_\beta}{\omega(\omega - n\omega_{c\beta})} A'_n(z_\beta) J_+(\xi_n), \quad \text{(A.10)}$$

$$\hat{\epsilon}_{xy} = -\hat{\epsilon}_{yx} = \\ = -i\sum_\beta \sum_n \frac{n^2\omega_{p\beta}^2}{\omega(\omega - n\omega_{c\beta})} A'_n(z_\beta) J_+(\xi_n), \quad \text{(A.11)}$$

$$\hat{\epsilon}_{xz} = \hat{\epsilon}_{zx} = \\ = \sum_\beta \sum_n \frac{n\omega_{p\beta}^2 k_\perp}{\omega\omega_{c\beta} k_z} \frac{A_n(z_\beta)}{z_\beta} \left[1 - J_+(\xi_n)\right], \quad \text{(A.12)}$$

$$\hat{\epsilon}_{yz} = -\hat{\epsilon}_{zy} = \\ = -i\sum_\beta \sum_n \frac{\omega_{p\beta}^2 k_\perp}{\omega\omega_{c\beta} k_z} A'_n(z_\beta) \left[1 - J_+(\xi_n)\right], \quad \text{(A.13)}$$

$$\hat{\epsilon}_{zz} = 1 + \sum_\beta \sum_n \frac{\omega_{p\beta}^2(\omega - n\omega_{c\beta})}{\omega k_z^2 V_{t\beta}^2} \times \\ \times A_n(z_\beta) \left[1 - J_+(\xi_n)\right], \quad \text{(A.14)}$$

and

$$\epsilon^r_{ij}(\omega, \mathbf{k}) = \frac{4\pi i}{\omega} \sigma^r_{ij}(\omega, \mathbf{k}). \quad \text{(A.15)}$$

Here,

$$A_n(z) = \exp(-z) I_n(z), \quad \xi_n = \frac{\omega - n\omega_{c\beta})}{|k_z| V_{t\beta}},$$

$$\omega_{p\beta} = \sqrt{\frac{4\pi e_\beta^2 n_{\beta 0}}{m_\beta}}$$

is the plasma frequency of the species $\beta$, and $\mathbf{k} \equiv (k_\perp, 0, k_z)$.

## REFERENCES


1. N. N. Rao, P. K. Shukla, and M. Y. Yu, Planet Space Sci. **38**, 345 (1990).
2. P. K. Shukla, Physica Scripta **45**, 504 (1992).
3. P. K. Shukla and V. P. Silin, Physica Scripta **45**, 508 (1992).
4. A. Barkan, R. L. Merlino, and N. D'Angelo, Phys. Plasmas **2**, 3563 (1995); C. Thomson, A. Barkan, N. D'Angelo, and R. L. Merlino, ibid. **4**, 2331 (1997); R. L. Merlino, A. Barkan, C. Thompson, and N. D'Angelo, ibid. **5**, 1607 (1998).
5. H. R. Prabhakara and V. L. Tanna, Phys. Plasmas **3**, 1212 (1996).
6. E. Thomas Jr. and M. Watson, Phys. Plasmas **6**, 4111 (1999); V. E. Fortov, A. G. Kharpak, S. A. Kharpak, V. I. Molotkov, A. P. Nefedov, O. F. Petrov, and V. M. Torchinsky, ibid. **7**, 1374 (2000); V. I. Moltokov, A. P. Nefedov, V. M. Torchinskii, V. E. Fortov, and A. G. Kharpak, Zh. Eksp. Teor. Fiz. **116**, 902 (1999).
7. A. Barkan, N. D'Angelo, and R. L. Merlino, Planet Space Sci. **44**, 239 (1996).
8. Y. Nakamura, H. Bailung, and P. K. Shukla, Phys. Rev. Lett. **83**, 1602 (1999).
9. P. K. Shukla, in *Frontiers in Dusty Plasmas*, ed. by Y. Nakamura, T. Yokota, and P. K. Shukla, Elsevier, Amsterdam (2000), p. 3.
10. F. Verheest, *Waves in Dusty Plasmas*, Kluwer Academic Publishers, Dordrecht, The Netherlands (2000).
11. M. Harwit, *Astrophysical Concepts*, Springer-Verlag, New York (1988), p. 405.
12. L. Spitzer, Jr., *Physical Processes in the Interstellar Medium*, John Wiley and Sons, New York (1977), p. 182.
13. U. Mohideen, H. U. Rahman, M. A. Smith, M. Rosenberg, and D. A. Mendis, Phys. Rev. Lett. **81**, 349 (1998); H. U. Rahman, U. Mohideen, M. A. Smith, M. Rosenberg, and D. A. Mendis, Physica Scripta T **89**, 186 (2000).
14. V. I. Molotkov, A. P. Nefedov, M. Yu. Putsyl'ink, V. M. Torchinsky, V. E. Fortov, A. G. Kharpak, and K. Yoshino, JETP Lett. **71**, 102 (2000).
15. D. Tskhakaya and P. K. Shukla, Submitted to Phys. Lett. A **274**, 243 (2001).
16. N. V. Voshchinnikov and D. A. Semenov, Astronmy Lett. **26**, 697 (2000).
17. J. Mahmoodi, P. K. Shukla, N. L. Tsintsadze, and D. D. Tskhakaya, Phys. Rev. Lett. **84**, 2626 (2000). We have noted an error in Eq. (27), which must contain an addition term $(k_\perp/k^2)\Omega_r^2/(\omega+\Omega_0)^2$, respectively, on its right-hand side. However, this term does not affect our final finding [viz. our Eq (31) remains intact], as we have dealt with the resonance case $\omega \simeq \Omega_0$.
18. L. D. Landau and E. M. Lifshitz, *The Classical Theory of Fields*, Pergamon Press, Oxford (1971).
19. N. N. Bogolyubov, *Dinamicheskie Problemy Statisticheskoj Fiziki* [Dynamic Problems of Statistical Physics, in Russian] Gostekhizdat, Moscow (1946).
20. Yu. L. Klimontovich, *Kinetic Theory of Nonideal Gases and Nonideal Plasmas*, Pergamon Press, New York (1982).
21. L. D. Landau and E. M. Lifshitz, *Statistical Physics*, Pergamon Press, Oxford (1989).
22. A. F. Alexandrov, L. S. Bogdankevich, and A. A. Rukhadze, *Principles of Plasma Electrodynamics*, Springer-Verlag, Berlin (1984).
23. A. B. Mikhailovskii, *Theory of Plasma Instabilities: Instabilities of a Homogenous Plasma*, Consultants Bureau, New York (1974), Vol. 1.
24. M. Rosenberg and D. A. Mendis, IEEE Trans. Plasma Sci. **20**, 929 (1992); D. A. Mendis and M. Rosenberg, Ann. Rev. Astron. Astrophys. **32**, 410 (1994).
25. P. K. Shukla and H. U. Rahman, Phys. Plasmas **3**, 430 (1996).
26. P. K. Shukla, Phys. Lett. A **252**, 340 (1999).
27. P. K. Shukla, Phys. Rev. E **61**, 7249 (2000).
28. O. Havnes, J. Troim, T. Blix et al., J. Geophys. Res. **101**, 10839 (1996).
29. M. Horányi, B. Walch, S. Robertson et al., J. Geophys. Res. **103**, 8575 (1998).
30. J. E. Howard, H. R. Dullin, and M. Horányi, Phys. Rev. Lett. **84**, 3244 (2000).
31. V. W. Chow, D. A. Mendis, and M. Rosenberg, IEEE Trans. Plasma Sci. **22**, 179 (1994); M. Rosenberg and D. A. Mendis, ibid. **23**, 177 (1995); M. Rosenberg, D. A. Mendis, and D. P. Sheehan, ibid. **27**, 239 (1999).
32. V. E. Fortov, A. P. Nefedov, O. F. Petrov et al., Phys. Lett. A **219**, 89 (1996).

**33**. V. E. Fortov, A. P. Nefedov, and O. S. Vaulina, Zh. Eksp. Teor. Fiz. **114**, 2004 (1998); V. Fortov, V. I. Molotkov, A. P. Nefedov, and O. F. Petrov, Phys. Plasmas **6**, 1759 (1999).

**34**. A. A. Samaryan, O. S. Vaulina, A. P. Nefedov, O. F. Petrov, and V. E. Fortov, J. Eksp. Teor. Fiz. **118**, 119 (2000); A. A. Sickafoose, J. W. Colwell, M. Horányi, and S. Robertson, Phys. Rev. Lett. **84**, 6034 (2000).

**35**. H. Fujiyama, H. Kawasaki, S. C. Yang, and Y. Matsuda, Jap. J. Appl. Phys. **33**, 4216 (1994); S. Nunomura, N. Ohno, and S. Takamura, ibid. **36**, 877 (1997).

**36**. G. Uchida, R. Ozaki, S. Iizuka, and N. Sato, in *Proc. of Int. Congress on Plasma Physics*, ed. by P. Pavlo, European Physical Society, Prague (1998), p. 2557; N. Sato, G. Uchida, R. Ozaki, S. Iizuka, and T. Kamimura, in *Frontiers in Dusty Plasmas*, ed. by Y. Nakamura, T. Yokota, and P. K. Shukla, Elsevier, Amsterdam (2000), p. 329.

**37**. D. W. Law, H. Steel, B. M. Annaratone, and J. E. Allen, Phys. Rev. Lett. **80**, 4189 (1998).

**38**. U. Konopka, D. Samsonov, A. V. Ivlev, J. Goree, V. Steinberg, and G. E. Morfill, Phys. Rev. E **61**, 1890 (2000).