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COLLECTIVE EFFECTS IN DOPED NEMATIC LIQUID CRYSTALS

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We study the collective elastic interaction in a system of many macroparticles embedded in a nematic liquid crystal. A theoretical approach to the interaction of macroparticles via deformation of the director field [1] is developed. It is found that the director field distortion induced by many particles leads to the screening of the elastic pair interaction potential. This screening strongly depends on the shape of the embedded particles: it exists for anisotropic particles and is absent for spherical ones. Our results are valid for the homeotropic and the planar anchoring on the particle surface and for different Frank constants. We apply our results to cylindrical particles in a nematic liquid crystal. In the system of magnetic cylindrical grains suspended in a nematic liquid crystal, the external magnetic field perpendicular to the grain orientation results in inclining the grains to the director and induces an elastic Yukawa-law attraction between the grains. The appearance of this elastic attraction can explain the cellular texture in magnetically doped liquid crystals in the presence of the magnetic field [2].

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1. INTRODUCTION

Suspensions in liquids were recently given an intensive consideration in science and technology. Colloid suspensions of solid particles coated with a surfactant and dispersions of liquid droplets form a medium have attracted great interest in different practical applications including medicine [3, 4]. Small particles suspended in a nematic liquid crystal make a new composite material with unique physical properties that originate from the orientational ordering of the liquid crystal. Mechanical and optical properties of this medium are primarily determined by the collective behavior of these particles. Depending on their size and anchoring energies, the particles form chains [5–7], anisotropic clusters [8–10], or periodic structures [1, 11].

The origin of the structure formation is the overlapping distortions of the director field $\mathbf{n}$ caused by single particles. These distortions interfere and result in a fascinating anisotropic interaction between particles. The director deformations greatly depend on the particle sizes and anchoring energy. For the normal anchoring, the director prefers to be normal to the surface of the particles; for the planar anchoring, the director prefers to be parallel to the surface. For a strong anchoring, the boundary conditions on $\mathbf{n}$ are fixed and impose topological constrains on the director field around the particle. Topological defects arising in this case cannot be removed from the particle. A single spherical particle with a strong normal anchoring induces a point topological defect called the hyperbolic hedgehog. The droplet and the defect form a dipole that was theoretically described with the help of ansatz functions using the electrostatic analogy [5, 12]. Such dipoles play the dominant role in the formation of chains. Terentjev et al. introduced the Saturn-ring configuration with the quadrupole symme-

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try where a $-1/2$ disclination ring surrounds the sphere along the equator. It was investigated by both analytical and numerical methods [13, 14]. For a finite anchoring strength of the molecules at the surface, a ring configuration occurs. In this configuration, the director field is smooth everywhere, and a ring of tangentially oriented molecules is located at the equator of the sphere [14]. Using a Monte-Carlo simulation, Ruhwandl and Terentjev showed that the surface ring is the preferred configuration for a sufficiently small anchoring [15]. Stark showed that the transition from the dipole to the Saturn-ring configuration can be achieved by either decreasing the particle size or applying the magnetic field or decreasing the anchoring [17].

In all the papers cited above, the behavior of spherical droplets in a nematic liquid crystal was studied. However, it was shown that there can be interesting properties and new structures in suspensions of anisotropic particles [1]. In 1970, Brochard and de Gennes for the first time showed that the doping of a nematic liquid crystal with ferromagnetic cylindrical grains leads to a macroscopic collective behavior [19]. This behavior is manifested as a distortion of the uniform molecular orientation of the entire matrix upon application of an external magnetic field. In other words, magnetic grains rule over the orientation of the entire nematic liquid crystal matrix. This was experimentally confirmed by Chen and Amer [2, 20]. They found that the doped nematic exhibits a cellular texture with the cells of the order of tens of micrometers at the critical magnetic field $H \sim 30$ G. The magnetically doped nematic liquid crystal system (DNLC) in the magnetic field was theoretically examined by Burylov and Raikher [21, 22], but because the elastic interaction between the grains was not taken into account, the cellular texture itself has not found a satisfactory theoretical explanation. The first attempt to find the elastic interaction between cylindrical grains in a nematic matrix was made by Lopatnikov and Namiot [24], who found the pair interaction potential for the weak anchoring when the grains lie along the director.

A new approach to finding the pair interaction potential between arbitrarily shaped particles for the weak anchoring was proposed in [1]. This approach allows finding the interaction potential for any orientation of the particles with respect to the director. The pair interaction potential was found as the result of the overlapping of distortions of the director field from two particles. The general potential obtained in [1] reduces to the results of Ramaswamy et al. [23] and Lubensky et al. [12] for spherical particles and to the result of Lopatnikov and Namiot [24] for cylindrical particles. For spherical particles, this potential differs from that obtained by Ruhwandl and Terentjev only by the angular dependence of the interacting macroparticles with respect to the director.

In this paper, we use the approach in [1] to consistently account for the interference of the deformations of the director field from all particles embedded in the liquid crystal. Distortions of the director field from the other particles affect the interaction potential between two chosen particles. We show that the collective screening effect arises when the concentration of particles is high. This screening effect is shown to be essentially depending on the shape and orientation of the particles and on the anchoring strength. For example, the screening is absent for spherical particles and is significant for anisotropic particles, e.g., cylinders. When the cylinders are placed at an angle to the director, the screened Coulomb attraction of the Yukawa form arising between them can lead to nontrivial consequences. In this paper, we show that this potential is responsible for the cellular texture in ferronematics that was observed by Chen and Amer [2]. We show that the effective charge in the screened Coulomb attraction greatly depends on the angle between the grains and the director. It is zero in the equilibrium states when the grains are parallel or perpendicular to the director in the case of the planar or homeotropic anchoring. The external magnetic field that is not parallel to the initial orientation of the magnetic grains takes them out of the equilibrium state; the effective charge then arises because of the angle between the grains and the director. We also show that the screening is not always exponential but can be trigonometrical under some conditions. It can occur only in the presence of the external field when the angle between the grains and the director exceeds the critical threshold.

In Sec. 2, we formulate the problem of finding the elastic energy of a doped nematic liquid crystal following the approach in Ref. [1]. In Subsec. 2.1, we find the director distribution resulting from the interference of the distortions induced by all particles. In Subsec. 2.2, we consider the energy of these director distortions and extract the screened pair interaction potentials from it. In Subsec. 2.3, we find an analytical expression for the pair interaction potential in the diagonal approximation. In Sec. 3, we find the elastic Yukawa attraction of the magnetic grains in the liquid crystal in the presence of the external magnetic field. We consider the system of many particles attracting in accordance with the Yukawa law and find the conditions for the clumping to occur in it. This allows us to explain the cellular texture in ferronematics.

2. FORMULATION OF THE PROBLEM

The free energy of a nematic liquid crystal is given by

$$F_b = \frac{1}{2}\int d^3r \left\{K_{11}(\operatorname{div}\mathbf{n})^2 + \right.$$
$$\left. + K_{22}(\mathbf{n}\cdot\operatorname{rot}\mathbf{n})^2 + K_{33}(\mathbf{n}\times\operatorname{rot}\mathbf{n})^2\right\}, \quad (1)$$

where K_{ii} are the Frank elastic constants and $\mathbf{n}$ is the director. The macroparticles embedded in a liquid crystal induce deformations of the director field. The surface of these particles can be coated with different surfactants that orient the nematic molecules either normally or tangentially to the surface. The surface energy can be written as

$$F_s = \sum_p \oint ds W(\mathbf{s})(\nu(\mathbf{s})\cdot\mathbf{n}(\mathbf{s}))^2, \quad (2)$$

where $\nu(\mathbf{s})$ is the unit normal to the surface at the point $\mathbf{s}$ on the surface and $W(\mathbf{s})$ is the anchoring coefficient at this point. In the general case, $W(\mathbf{s}) > 0$ corresponds to the planar anchoring and $W(\mathbf{s}) < 0$ corresponds to the normal anchoring. The integral is taken over the entire surface of particle p. We assume that all particles have the same orientation in space (for example, with the help of the external field), but their centers of mass can move freely under the action of the resulting elastic potentials in order to minimize the total free energy of the system. The total free energy is the sum of the bulk and surface energies:

$$F = F_b + F_s. \quad (3)$$

We do not consider the distribution entropy part of the free energy, because it does not affect the director distribution and is irrelevant for finding the elastic interaction potential between particles.

We consider the case of the weak anchoring, where $Wr_0/K \ll 1$ (we imply the absolute value of W), where r_0 is the smallest size of the particle, e.g., the radius of the sphere or the cylinder. For the homeotropic anchoring and spherical droplets, this corresponds to the surface ring configuration [13, 14]. In this case, we assume that the director distortion from the homogeneous state $\mathbf{n}_0$ is small everywhere:

$$\mathbf{n}(\mathbf{r}) = \mathbf{n}_0 + \delta\mathbf{n}(\mathbf{r}), \quad |\delta\mathbf{n}| \ll 1. \quad (4)$$

Under this assumption, the director smoothly varies from point to point and no topological defects arise in the vicinity of particles. This is a consequence of the weak anchoring strength and of the small particle size (we consider particles with the size less than 1 μm, which only slightly distort the director for the real anchoring strength [15]). We can use the Fourier representation for the director in the entire space, thereby considerably simplifying the problem.

In the Fourier representation, we have

$$\delta\mathbf{n}(\mathbf{r}) = \frac{1}{(2\pi)^3}\int d^3q \exp(-i\mathbf{q}\cdot\mathbf{r})\delta\mathbf{n}(\mathbf{q}). \quad (5)$$

Inserting (5) in bulk Frank energy (1), we obtain

$$F_b = \frac{1}{2}\frac{1}{(2\pi)^3}\int d^3q \left\{K_{11}\left|\mathbf{q}\cdot\delta\mathbf{n}(\mathbf{q})\right|^2 + \right.$$
$$\left. + K_{22}\left|[\mathbf{n}\times\mathbf{q}]\cdot\delta\mathbf{n}(\mathbf{q})\right|^2 + K_{33}\left|(\mathbf{n}\cdot\mathbf{q})\delta\mathbf{n}(\mathbf{q})\right|^2\right\}. \quad (6)$$

To simplify this expression, we choose the special basis

$$\mathbf{e}_1 = \frac{(\mathbf{q}_\perp\times\mathbf{n}_0)}{q_\perp}; \quad \mathbf{e}_2 = \frac{\mathbf{q}_\perp}{q_\perp},$$
$$\mathbf{e}_3 = \mathbf{n}_0, \quad \mathbf{q}_\perp = \mathbf{n}_0\times\mathbf{q}. \quad (7)$$

We then have $\mathbf{q} = (q_\perp, 0, q_\parallel)$ and $\delta\mathbf{n} = (\delta n_1, \delta n_2, 0)$, and Eq. (1) therefore reduces to

$$F_b = \frac{1}{2}\frac{1}{(2\pi)^3}\times$$
$$\times\sum_i\int d^3q\left\{K_{ii}q_\perp^2 + K_{33}q_\parallel^2\right\}\left|\delta n_i(\mathbf{q})\right|^2. \quad (8)$$

Because we assume that the director smoothly varies from point to point and relation (4) is true, we can consider the director to have a given value inside the volume of the particle. This assumption is valid if the total volume of the suspended particles is much less than the entire volume of the system, i.e., the volume fraction of the particles is small, $cv \ll 1$, where $c = N/V$ is the concentration and v is the volume of a particle (the «gas» approximation). For the real system [2], $c = 10^{10}$ cm^{-3}, $v \sim 10^{-15}$ cm^3, and $cv \sim 10^{-5}$, and our assumption is therefore true.

The director on the surface can therefore be expressed through the director at the center of mass $\mathbf{R}_p$ of the particle and its derivatives,

$$\delta\mathbf{n}(\mathbf{s}) = \delta\mathbf{n}(\mathbf{R}_p) + (\rho\nabla)\delta\mathbf{n}(\mathbf{R}_p) + \frac{1}{2}(\rho\nabla)^2\delta\mathbf{n}(\mathbf{R}_p),$$

where ρ is the vector drawn from the center of mass to the point $\mathbf{s}$ on the surface. The complete expression for the director on the surface through the director at the center of mass of the particle is therefore given by

$$\mathbf{n}(\mathbf{s}) = \mathbf{n}_0 + \delta\mathbf{n}(\mathbf{R}_p) + (\rho\nabla)\delta\mathbf{n}(\mathbf{R}_p) +$$
$$+ \frac{1}{2}(\rho\nabla)^2\delta\mathbf{n}(\mathbf{R}_p). \quad (9)$$

We now fix a coordinate system (x, y, z) where the z axis is parallel to the undeformed director $\mathbf{n}_0$ and x and y are perpendicular to it. This system is firmly fixed in space. We next substitute director field (9) in the scalar product $(\mathbf{n}(\mathbf{s})\cdot\nu(\mathbf{s}))^2$ and also include the second powers of the perpendicular director deformations δn_x and δn_y. We thus write

$$(\nu(\mathbf{s})\cdot\mathbf{n}(\mathbf{s}))^2 = (\nu\cdot\mathbf{n}_0)^2 + 2(\nu\cdot\mathbf{n}_0)(\nu\cdot\delta\mathbf{n}) + \\ + 2(\nu\cdot\mathbf{n}_0)(\rho\cdot\nabla)(\nu\cdot\delta\mathbf{n}) + \\ + 2(\nu\cdot\mathbf{n}_0)(\rho\cdot\nabla)^2(\nu\cdot\delta\mathbf{n}) + (\nu\cdot\delta\mathbf{n})^2, \quad (10)$$

where

$$\nu = \nu(\mathbf{s}), \quad \delta\mathbf{n} = \delta\mathbf{n}(\mathbf{R}_p).$$

We note that this expression involves two smallness parameters. The first is the perpendicular component of the director,

$$|\delta n_x|, |\delta n_y| \sim \varepsilon, \quad \delta n_3 \sim \varepsilon^2,$$

and the second is the ratio $\varrho = \rho/l_n$ of the particle size to the average deformation length l_n of the director. In [1], the respective terms proportional to ε, $\varrho\varepsilon$, and $\varrho^2\varepsilon$ were taken into account. The expansion in ϱ is equivalent to the multipole expansion in [12]. In this paper, we also take the last term proportional to ε^2 into account. This term is not essential at the distances comparable to the average distance between particles, as we see below, and it can therefore be omitted for the systems considered in [6, 7, 12], where the concentration of dispersed particles is small. It becomes essential for dense colloids, where there are too many particles and where the interference of the distortions from all particles is considerable. In [12], $|\delta n_\mu|$ (with $\mu = x, y$) was shown to fall off as R^{-2} and R^{-3}, depending of the dipole or quadrupole symmetry. We thus conclude that $\varepsilon \sim \varrho^2$ for the third term, which has the dipole symmetry (and $\varepsilon^2 \ll \varrho\varepsilon$ in this case), and $\varepsilon \sim \varrho^3$ ($\varepsilon^2 \ll \varrho^2\varepsilon$) for the fourth term with the quadrupole symmetry. In any case, taking the last term into account gives only small corrections at the average distances and for a small number of particles. As we see below, this is essential in the collective effect of the screening at large distances, where the concentration of particles is high. We specifically clarify this problem in what follows.

For this, we write the scalar products $(\nu(\mathbf{s})\cdot\mathbf{n}(\mathbf{s}))$ in the local basis $(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$ associated with each particle. For example,

$$\nu(\mathbf{s})\cdot\mathbf{n}(\mathbf{s}) = \sum_{l=1,2,3} (\nu\cdot\mathbf{k}_l)(\delta\mathbf{n}\cdot\mathbf{k}_l) = \nu_l\delta n_\mu k_{l_\mu} + \nu_l\delta n_3 k_{l_3},$$

where

$$\nu_l = (\nu\cdot\mathbf{k}_l),, \quad k_{l_3} = \mathbf{k}_l\cdot\mathbf{n}_0,$$

$$\delta n_3 = -\frac{1}{2}(\delta n_x^2 + \delta n_y^2).$$

The surface energy is then written as

$$F_s = F_s^{(0)} + F_s^{(1)} + F_s^{(2)}, \quad (11)$$

$$F_s^{(0)} = N\oint ds\, W(\mathbf{s})(\nu(\mathbf{s})\cdot\mathbf{n}_0)^2, \quad (12)$$

$$F_s^{(1)} = \sum_p \oint ds W(s)\nu_l\nu_m k_{l_3} \times \\ \times \left\{2\delta n_\mu k_{l_\mu} + 2(\rho\nabla)\delta n_\mu k_{l_\mu} + (\rho\nabla)^2\delta n_\mu k_{l_\mu}\right\}, \quad (13)$$

$$F_s^{(2)} = \sum_p \oint ds W(s)\nu_l\nu_m \times \\ \times \left[\delta n_\mu\delta n_\nu k_{l_\mu}k_{m_\nu} - (\delta n_x^2 + \delta n_y^2)k_{l_3}k_{m_3}\right], \quad (14)$$

where N is the total number of particles in the entire volume V of the liquid crystal matrix and the respective surface terms $F_s^{(1)}$ and $F_s^{(2)}$ are linear and quadratic in δn_μ. Following [1], we now define several tensors in the basis $(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$ that characterize the anchoring energy,

$$\alpha_{kl} = 2\oint ds W(s)\nu_k(s)\nu_l(s), \\ \beta_{klm} = 2\oint ds W(s)\nu_k(s)\nu_l(s)\rho_m(s), \quad (15) \\ \gamma_{klmn} = \oint ds W(s)\nu_k(s)\nu_l(s)\rho_m(s)\rho_n(s).$$

The elastic energy F_{el}, i.e., the energy of deformations of the director field, is then given by

$$F_{el} = F_b + F_s^{(1)} + F_s^{(2)}, \quad (16)$$

$$F_s^{(1)} = \sum_p \{\alpha_{lm} + \beta_{lms}(\mathbf{k}_s\cdot\nabla) + \\ + \gamma_{lmst}(\mathbf{k}_s\cdot\nabla)(\mathbf{k}_t\cdot\nabla)\}\delta n_\mu k_{l_\mu}k_{m_3}, \quad (17)$$

$$F_s^{(2)} = \frac{1}{2}\times \\ \times\sum_p \alpha_{lm}\left[\delta n_\mu\delta n_\nu k_{l_\mu}k_{m_\nu} - (\delta n_x^2 + \delta n_y^2)k_{l_3}k_{m_3}\right]. \quad (18)$$

The main difference between this paper and [1] is in taking term (18) into account. It is quadratic in the director deformations and can be regarded as the contribution of all particles to the interference of distortions. Precisely this term leads to the screening effects.

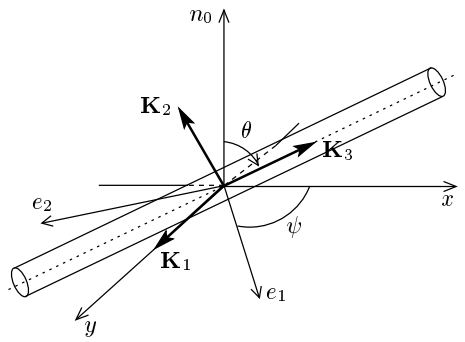

Fig. 1. Orientation of a cylindrical particle. The particle lies in the xn_0 plane at the angle θ with the director

Some of its features can be considered without finding the director. For example, it is clearly seen that it vanishes for spherical particles. Indeed, $\alpha_{lm} = \alpha\delta_{lm}$ and $k_{l_\mu}k_{l_\nu} = \delta_{\mu\nu}$ for the sphere, and therefore, $F_s^{(2)} \equiv 0$. For any other shape, Eq. (18) does not vanish. To describe its effect analytically, we go to the continuum limit in this expression and replace the summation with the integration over the entire space,

$$\sum_p \to c\int dV,$$

where as before, $c = N/V$ is the concentration of particles:

$$\begin{aligned} F_s^{(2)} &= \frac{c}{2}\int dV\widetilde{a}_{\mu\nu}\delta n_\mu(\mathbf{x})\delta n_\nu(\mathbf{x}),\\ \widetilde{a}_{\mu\nu} &= \alpha_{lm}\left[k_{l_\mu}k_{m_\nu} - k_{l_3}k_{m_3}\delta_{\mu\nu}\right]. \end{aligned} \tag{19}$$

We thus consider the interference of only longwavelengh distortions of the director field. In the Fourier representation, we have

$$F_s^{(2)} = \frac{c}{2\,(2\pi)^3}\int d^3q\widetilde{a}_{\mu\nu}\delta n_\mu(\mathbf{q})\delta n_\nu^*(\mathbf{q}). \tag{20}$$

The tensor $\widetilde{a}_{\mu\nu}$ is here taken in the (x, y, z) coordinate system, which is not convenient. It is much more suitable to write the surface energy and the bulk energy in Eq. (8) in the same basis $(\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$. This basis is rotated by the angle $\psi(\mathbf{q})$ with respect to (x, y, z) around the z axis (Fig. 1). In the new basis, the director has the components $\delta\mathbf{n} = (\delta n_1, \delta n_2, 0)$ and $\delta n_\mu = \varpi_{\mu i}\delta n_i$ (with $\mu = x, y$ and $i = 1, 2$). The rotation matrix is given by

$$\varpi_{\mu i} = \begin{bmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{bmatrix}.$$

In the basis $(\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$, the surface energy becomes

$$\begin{aligned} F_s^{(1)} = \sum_p \frac{1}{2\,(2\pi)^3}\int d^3q \times\\ \times\{\exp(-i\mathbf{q}\cdot\mathbf{r}_p)a_m^*(\delta\mathbf{n}(\mathbf{q})\cdot\mathbf{k}_m) +\\ + \exp(i\mathbf{q}\cdot\mathbf{r}_p)a_m(\delta\mathbf{n}^*(\mathbf{q})\cdot\mathbf{k}_m)\}, \end{aligned} \tag{21}$$

$$a_m = (\mathbf{k}_l\cdot\mathbf{n}_0)\left[\alpha_{lm} + i\beta_{lms}(\mathbf{q}\cdot\mathbf{k}_s) - \gamma_{lmst}(\mathbf{q}\cdot\mathbf{k}_s)(\mathbf{q}\cdot\mathbf{k}_t)\right],$$

$$F_s^{(2)} = \frac{c}{2\,(2\pi)^3}\int d^3q a_{ij}\delta n_i(\mathbf{q})\delta n_j^*(\mathbf{q}), \tag{22}$$

$$a_{ij} = \varpi_{i\mu}^T\widetilde{a}_{\mu\nu}\varpi_{\nu j}. \tag{23}$$

We now add the bulk energy F_b and the surface energy F_s and find the total energy of the system

$$F_{total} = F_s^{(0)} + F_{el}$$

with the elastic energy

$$\begin{aligned} F_{el} = \frac{1}{2\,(2\pi)^3}\int d^3qV_{ij}(\mathbf{q})\delta n_i(\mathbf{q})\delta n_j^*(\mathbf{q}) +\\ + b_i^*(\mathbf{q})\delta n_i(\mathbf{q}) + b_i(\mathbf{q})\delta n_i^*(\mathbf{q}), \end{aligned} \tag{24}$$

$$V_{ij}(\mathbf{q}) = (K_{ii}q_\perp^2 + K_{33}q_\parallel^2)\delta_{ij} + ca_{ij}, \tag{25}$$

$$b_i(\mathbf{q}) = \sum_p \exp(i\mathbf{q}\cdot\mathbf{r}_p)a_m k_{m_i}. \tag{26}$$

Here, $m = 1, 2, 3$, $i, j = 1, 2$, and $\delta n_i(\mathbf{q})$ and k_{m_i} are the projections of these vectors on the basis $(\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$.

2.1. Director distribution in the doped nematic liquid crystal

Having found the complete expression for the elastic energy of a liquid crystal with particles, we can find the director at any point of the system from the extremum condition

$$\begin{aligned} &\frac{\delta}{\delta n_j^*(\mathbf{q})}F_{el} = V_{ij}(\mathbf{q})\delta n_i(\mathbf{q}) + b_j(\mathbf{q}) = 0,\\ &\delta n_i(\mathbf{q}) = -V_{ij}^{-1}(\mathbf{q})b_j(\mathbf{q}). \end{aligned} \tag{27}$$

In the matrix form, the last equation becomes

$$\begin{aligned} \begin{pmatrix} \delta n_1(\mathbf{q}) \\ \delta n_2(\mathbf{q}) \end{pmatrix} =\\ = -\frac{1}{D}\begin{bmatrix} V_{22} & -V_{12} \\ -V_{12} & V_{11} \end{bmatrix}\cdot\begin{pmatrix} b_1(\mathbf{q}) \\ b_2(\mathbf{q}) \end{pmatrix}, \end{aligned} \tag{28}$$

where

$$D = V_{11}V_{22} - V_{12}^2.$$

We can make some general conclusions from this expression concerning the behavior of the director. Distortions decrease far from the particle. However, the denominator D can vanish for some wave vectors $\mathbf{q}$, which corresponds to the appearance of oscillations of the director field. The origin of the director oscillations is purely collective, because the effect depends on the concentration $V_{12} \sim ca_{12}$. It also depends on the shape of the particles and their orientation with respect to the undeformed director $\mathbf{n}_0$. We assume that these oscillations can be observed experimentally, because they must lead to the scattering of electromagnetic waves with wave lengths commensurate to the oscillations period. Equation (28) seems to imply that

$$\lambda \sim \sqrt{K/c\alpha},$$

where α is the average value of the tensor α_{lm}; from Eq. (15), we have

$$\alpha_{lm} \sim WS,$$

where W is the anchoring energy and S is the area of the particle. The resonance wave length is therefore roughly estimated as

$$\lambda_{res} \sim \sqrt{K/WcS}. \tag{29}$$

For example, in the experiment with cylindrical grains [2], the parameters are given by $c \approx 10^{10}$ cm^{-3}, $S \approx 2\pi RL$, the radius of the grain is $R \approx 0.05\,\mu$m, the length $L \approx 0.5\,\mu$m, the elastic constant $K \sim 10^{-7}$ dyn and the anchoring energy $W \sim 10^{-3}$ dyn/cm. We then find $\lambda_{res} \approx 30\,\mu$m. Because it is always possible to vary the concentration and the anchoring energy, the resonance wave length can be in the range $\lambda \approx 10$–$100\,\mu$m. In any case, this length must be larger than the average distance between the particles, $\lambda \gg \langle l \rangle$. For the experiment in [2], $\langle l \rangle \approx 4\,\mu$m and all the assumptions are therefore valid. In the domain with the size about $30\,\mu$m, there are approximately 500 particles and their collective interaction can induce long-wavelength oscillations of the director field.

2.2. Elastic energy and the pair interaction potential between particles

Having found the director field, we insert Eq. (28) in (24) and obtain the elastic energy of the director deformations in the DNLC:

$$F_{el} = -\frac{1}{2\,(2\pi)^3}\int d^3q V_{ij}^{-1}(\mathbf{q}) b_i^*(\mathbf{q}) b_j(\mathbf{q}) < 0. \tag{30}$$

The negative sign implies that the total free energy

$$F = F_s^{(0)} + F_{el}$$

evaluated for solution (28) is less than the energy $F = F_s^{(0)}$ for the undeformed director field $\mathbf{n}_0$. The total energy F_{el} can be represented as the sum of the pair potentials between two particles. Indeed, we introduce the operator $\widehat{A}_m$ such that

$$\widehat{A}_m e^{i\mathbf{q}\cdot\mathbf{r}} = a_m e^{i\mathbf{q}\cdot\mathbf{r}}, \tag{31}$$

$$\widehat{A}_m = (\mathbf{k}_l \cdot \mathbf{n}_0)\,[\alpha_{lm} + \beta_{lms}(\mathbf{k}_s \cdot \nabla) + \\ + \gamma_{lmst}(\mathbf{k}_s \cdot \nabla)(\mathbf{k}_t \cdot \nabla)]. \tag{32}$$

The elastic energy F_{el} then takes the form

$$F_{el} = \frac{1}{2}\sum_{p,p'} U_{pp'}, \tag{33}$$

$$U_{pp'} = -\frac{1}{(2\pi)^3}\widehat{A}_m^p \widehat{A}_{m'}^{p'} \times \\ \times \int d^3q \exp\left[i\mathbf{q}(\mathbf{r}_p - \mathbf{r}_{p'})\right] V_{ij}^{-1}(\mathbf{q}) k_{m_i} k_{m'_j}. \tag{34}$$

The expression $U_{pp'}$ has the meaning of the pair interaction potential between particles p and p' that is caused by long-range deformations of the director field. The subscript p indicates that we must substitute

$$\nabla = \frac{\partial}{\partial \mathbf{r}_p}$$

in the operator $\widehat{A}_m^p$. This expression is valid for particles of the ordinary shape and orientation. It accounts for screening effects that arise from the interference of the director field distortions by all particles.

2.3. Pair potential in the diagonal approximation. Analytical results

Although expression (34) is exact, it is too difficult to find the results analytically. In the most general case, the pair potential $U(\mathbf{R}, \mathbf{\Omega})$ depends on all the three components of the radius vector $\mathbf{R} = \mathbf{r}_p - \mathbf{r}_{p'}$ and on three Euler angles $\mathbf{\Omega}$ that determine the orientation of particles in space (we assume that all particles are oriented in the same way, and all of them therefore have the same Euler angles). To take the screening effects into account analytically, we consider particles with the rotational symmetry around one axis. For such particles, the pair potential $U(\mathbf{R}, \theta)$ depends on the angle θ

between this symmetry axis and the director. If $\theta = 0$, all particles are parallel to the director. In this case, the entire DNLC has the rotational symmetry around the director $\mathbf{n}_0$ and the pair potential $U(R_\perp, R_\parallel)$ depends on the perpendicular and parallel projections of $\mathbf{R}$ with respect to $\mathbf{n}_0$. But in the case where $\theta \neq 0$, the second preferential direction arises in the system, the direction along which all the particles lie. We project this direction on the plane perpendicular to the director and let $\mathbf{s}$ denote the projection,

$$\mathbf{s} \cdot \mathbf{n}_0 = 0.$$

We then have the potential $U = U(R_\perp, R_\parallel, \varphi, \theta)$, where φ is the azimuthal angle between $\mathbf{R}$ and $\mathbf{s}$.

To obtain analytical results, we average over the angle φ. For this purpose, we average the tensor a_{ij} in Eq. (23) over the angle ψ and drop the off-diagonal terms; we call this the diagonal approximation:

$$a_{ij}(\psi,\theta) \to \langle a_{ij}\rangle_\psi = \\ = \frac{1}{2}\begin{bmatrix} \widetilde{a}_{11}+\widetilde{a}_{22} & 0 \\ 0 & \widetilde{a}_{11}+\widetilde{a}_{22} \end{bmatrix} = a(\theta)\delta_{ij}, \quad (35)$$

where

$$a(\theta) = \frac{1}{2}(\widetilde{a}_{11}+\widetilde{a}_{22}).$$

This approximation makes the propagator $V_{ij}^{-1}(\mathbf{q})$ diagonal and allows us to take all the integrals analytically. The diagonal approximation is exact only for $\theta = 0$, when the entire system has the rotational symmetry in the xy plane. The coefficient $a(\theta)$ depends on the shape of the particles. For example, for cylinders with $R \ll L$, we have

$$a(\theta) = \pi RLW(2 - 3\sin^2\theta),$$

and for flat (pancake) particles with $R \gg h$,

$$a(\theta) = 2\pi R^2 W(1 - 3\cos^2\theta)$$

(where θ is the angle between the normal to the pancake plane and the director). In the diagonal approximation, the propagator therefore becomes

$$V_{ij}^{-1}(\mathbf{q}) = (K_{ii}q_\perp^2 + K_{33}q_\parallel^2 + ca(\theta))^{-1}\delta_{ij} \quad (36)$$

and the pair potential is given by

$$U_{pp'} = -\frac{1}{(2\pi)^3}\widehat{A}_l^p \widehat{A}_{l'}^{p'}\left[I_{ll'}(\mathbf{R})\right], \quad (37)$$

$$I_{ll'}(\mathbf{R}) = I_{1ll'}(\mathbf{R}) + I_{2ll'}(\mathbf{R}), \quad (38)$$

$$I_{1ll'}(\mathbf{R}) = \\ = \int d^3q e^{i\mathbf{q}\cdot\mathbf{R}} \frac{(\mathbf{k}_l\cdot[\mathbf{q}_\perp\times\mathbf{n}_0])\,(\mathbf{k}_{l'}\cdot[\mathbf{q}_\perp\times\mathbf{n}_0])}{q_\perp^2\left(K_{11}q_\perp^2 + K_{33}q_\parallel^2 + ca(\theta)\right)}, \quad (39)$$

$$I_{2ll'}(\mathbf{R}) = \\ = \int d^3q e^{i\mathbf{q}\cdot\mathbf{R}} \frac{(\mathbf{k}_l\cdot\mathbf{q}_\perp)\,(\mathbf{k}_{l'}\cdot\mathbf{q}_\perp)}{q_\perp^2\left(K_{22}q_\perp^2 + K_{33}q_\parallel^2 + ca(\theta)\right)}. \quad (40)$$

It is easy to integrate over $\mathbf{q}$ in Eqs. (39) and (40) using the coordinate system with the basis

$$\mathbf{r}_1 = \frac{\mathbf{R}_\perp\times\mathbf{n}_0}{R_\perp}, \quad \mathbf{r}_2 = \frac{\mathbf{R}_\perp}{R_\perp}, \\ \mathbf{r}_3 = \mathbf{n}_0, \quad \mathbf{R}_\perp = \mathbf{n}_0\times\mathbf{R}. \quad (41)$$

This basis is rotated with respect to the one in (7) by some angle φ about the axis $\mathbf{n}_0$. The quantities $q_\perp$ and $q_\parallel$ are similar in both bases. We therefore have

$$\exp(-i\mathbf{q}\cdot\mathbf{R}) = \exp\left\{-i\left[q_\perp R_\perp\cos\varphi + q_\parallel R_\parallel\right]\right\}$$

and the denominators of the fractions involved in (39) and (40) do not depend on the angle φ. Integrating over φ, we obtain

$$I_{\mu ll'}(\mathbf{R}) = \pi\int_0^\infty dq_\perp q_\perp \times \\ \times\left\{Q_{l,l'}^{+}J_0(q_\perp R_\perp) + (-1)^\mu Q_{l,l'}^{-}J_2(q_\perp R_\perp)\right\}\times \\ \times\int_{-\infty}^{\infty} dq_\parallel \frac{\exp(-iq_\parallel R_\parallel)}{(K_{\mu\mu}q_\perp^2 + K_{33}q_\parallel^2 + ca(\theta))}, \quad (42)$$

where $\mu = 1, 2$ and

$$Q_{l,l'}^{(\pm)} = (\mathbf{r}_1\cdot\mathbf{k}_l)(\mathbf{r}_1\cdot\mathbf{k}_{l'}) \pm (\mathbf{r}_2\cdot\mathbf{k}_l)(\mathbf{r}_2\cdot\mathbf{k}_{l'})$$

and J_0 and J_2 are the Bessel functions.

In order to calculate these integrals, we must thoroughly scrutinize the function $a(\theta)$. As mentioned above,

$$a(\theta) = \pi RLW(2 - 3\sin^2\theta)$$

for cylindrical grains. The case where $W > 0$ corresponds to the planar anchoring, and $W < 0$ corresponds to the normal anchoring. For the planar anchoring, the equilibrium state of the grains is $\theta = 0$ and

$$a_{planar}(0) = 2\pi RLW > 0,$$

and for the normal anchoring, the equilibrium state is $\theta = \pi/2$ and

$$a_{normal}(\pi/2) = -\pi RLW > 0.$$

We thus see that $a(\theta) > 0$ in the equilibrium state independently of the anchoring type. But if the grains have a magnetic moment, the external magnetic field can lead the grains into the states where $a(\theta) < 0$. This occurs if

$$\arcsin\sqrt{2/3} < \theta < \pi/2$$

for the planar anchoring and if

$$0 < \theta < \arcsin\sqrt{2/3}$$

for the homeotropic anchoring. These states exist only because of the magnetic field. Both cases must therefore be considered. We first consider the case where $a(\theta) > 0$, which corresponds to the equilibrium states or the case of weak external fields. We write $I^{exp}_{\mu ll'}(\mathbf{R})$ for $I_{\mu ll'}(\mathbf{R})$ in this case. We introduce

$$p_\mu = \sqrt{\frac{K_{\mu\mu}}{K_{33}}}R_\parallel, \quad s = R_\perp, \quad z_\mu = \sqrt{\frac{ca(\theta)}{K_{\mu\mu}}}.$$

After the integration over the $q_\parallel$, $I^{exp}_{\mu ll'}(\mathbf{R})$ becomes

$$I^{exp}_{\mu ll'}(\mathbf{R}) = \frac{\pi^2}{\sqrt{K_{\mu\mu}K_{33}}}\int\limits_0^\infty dq_\perp q_\perp \times \\ \times \frac{\exp\left(-p_\mu\sqrt{q_\perp^2+z_\mu^2}\right)}{\sqrt{q_\perp^2+z_\mu^2}} \times \\ \times \left\{Q^+_{l,l'}J_0(sq_\perp) + (-1)^\mu Q^-_{l,l'}J_2(sq_\perp)\right\}. \quad (43)$$

For the Bessel functions, we have the relation

$$2\nu J_\nu(x) = xJ_{\nu+1}(x) + xJ_{\nu-1}(x),$$

which for $\nu = 1$ gives

$$J_2(x) = \frac{2}{x}J_1(x) - J_0(x).$$

The corresponding integrals involving $J_1(x)$ and $J_0(x)$ are given by

$$\int\limits_0^\infty dq_\perp q_\perp \frac{e^{-p_\mu\sqrt{q_\perp^2+z_\mu^2}}}{\sqrt{q_\perp^2+z_\mu^2}}J_0(sq_\perp) = \frac{e^{-z_\mu\sqrt{p_\mu^2+s^2}}}{\sqrt{p_\mu^2+s^2}}, \quad (44)$$

$$\int\limits_0^\infty dq_\perp \frac{e^{-p_\mu\sqrt{q_\perp^2+z_\mu^2}}}{\sqrt{q_\perp^2+z_\mu^2}}J_1(sq_\perp) = \\ = \frac{1}{sz_\mu}\left[e^{-p_\mu z_\mu} - e^{-z_\mu\sqrt{p_\mu^2+s^2}}\right]. \quad (45)$$

Using these relations, we find

$$I^{exp}_{\mu ll'}(\mathbf{R}) = \frac{\pi^2}{\sqrt{K_{\mu\mu}K_{33}}}\left\{\left[Q^+_{l,l'} + (-1)^{\mu+1}Q^-_{l,l'}\right]\times \right. \\ \times \frac{\exp\left(-z_\mu\sqrt{p_\mu^2+s^2}\right)}{\sqrt{p_\mu^2+s^2}} + (-1)^\mu Q^-_{l,l'}\frac{2}{s^2z_\mu}\times \\ \left.\times\left[\exp\left(-p_\mu z_\mu\right) - \exp\left(-z_\mu\sqrt{p_\mu^2+s^2}\right)\right]\right\}. \quad (46)$$

The pair interaction potential is then given by

$$U_{pp'} = -\frac{1}{(2\pi)^3}\times \\ \times \widehat{A}^p_l\widehat{A}^{p'}_{l'}\left[I^{exp}_{1ll'}(\mathbf{r}_p - \mathbf{r}_{p'}) + I^{exp}_{2ll'}(\mathbf{r}_p - \mathbf{r}_{p'})\right]. \quad (47)$$

This is the potential of the elastic interaction between any particles in the diagonal approximation. It depends on the three components of the radius-vector $\mathbf{R} = \mathbf{r}_p - \mathbf{r}_{p'}$ between particles.

In the one-constant approximation where $K_{\mu\mu} = K_{33} = K$, the potential depends only on the scalar of the vector $\mathbf{R}$,

$$U_{pp'} = -\frac{Q^+_{l,l'}}{4\pi K}\widehat{A}^p_l\widehat{A}^{p'}_{l'}\left[\frac{\exp(-\xi\,|\mathbf{r}_p - \mathbf{r}_{p'}|)}{|\mathbf{r}_p - \mathbf{r}_{p'}|}\right], \quad (48)$$

$$\xi^{-1}(\theta) = \sqrt{\frac{K}{ca(\theta)}}. \quad (49)$$

It is clearly seen that collective distortions of the director lead to the screening of the pair interaction potential with the screening length

$$\xi^{-1} \approx \sqrt{K/WcS}$$

(where W is the absolute value of the anchoring energy and S is the area of the particle). This screening occurs both for the homeotropic and for the planar anchoring. Because the concentration is only involved in the inverse screening length ξ, the limit as $c \longrightarrow 0$ gives $\xi = 0$ and brings us back to the unscreened result of Lev and Tomchuk [1], which is equivalent to the result of Lopatnikov and Namiot [24] for asymmetric cylinders. All this is true only if $\xi^{-1} \gg \langle l\rangle$, where $\langle l\rangle = 1/\sqrt[3]{c}$ is the average distance between particles. We thus write the condition on the anchoring strength under which our approach is applicable:

$$W \ll \frac{K}{\sqrt[3]{c}S}. \quad (50)$$

2.3.1. Field-induced trigonometric screening

If the grains have the magnetic or electric moment, the external magnetic or electric field can change the angle θ between them and the director, which can result in $a(\theta) < 0$. To find the potential in this case, we must replace

$$z_\mu \to \pm i z_\mu$$

in (46) and take half the sum of the two expressions,

$$I^{trig}_{\mu ll'}(\mathbf{R}) = \frac{1}{2}\left[I^{exp}_{\mu ll'}(iz_\mu, \mathbf{R}) + I^{exp}_{\mu ll'}(-iz_\mu, \mathbf{R})\right], \quad (51)$$

which gives

$$I^{trig}_{\mu ll'}(\mathbf{R}) = \frac{\pi^2}{\sqrt{K_{\mu\mu}K_{33}}}\left\{\left[Q^+_{l,l'} + (-1)^{\mu+1}Q^-_{l,l'}\right]\times\right.$$
$$\times \frac{\cos\left(z_\mu\sqrt{p_\mu^2+s^2}\right)}{\sqrt{p_\mu^2+s^2}} + (-1)^{\mu+1}Q^-_{l,l'}\times$$
$$\left.\times\frac{2}{s^2 z_\mu}\left[\sin(p_\mu z_\mu) - \sin(z_\mu\sqrt{p_\mu^2+s^2})\right]\right\}. \quad (52)$$

The pair interaction potential then takes the form

$$U_{pp'} = -\frac{1}{(2\pi)^3}\times$$
$$\times \widehat{A}^p_l \widehat{A}^{p'}_{l'}\left[I^{trig}_{1ll'}(\mathbf{r}_p - \mathbf{r}_{p'}) + I^{trig}_{2ll'}(\mathbf{r}_p - \mathbf{r}_{p'})\right]. \quad (53)$$

In the one-constant approximation, this becomes

$$U_{pp'} = -\frac{Q^+_{l,l'}}{4\pi K}\widehat{A}^p_l\,\widehat{A}^{p'}_{l'}\left[\frac{\cos(\xi\,|\mathbf{r}_p - \mathbf{r}_{p'}|)}{|\mathbf{r}_p - \mathbf{r}_{p'}|}\right], \quad (54)$$

where

$$\xi^{-1}(\theta) = \sqrt{\frac{K}{c\,|a(\theta)|}}.$$

The screening becomes trigonometrical. We have obtained this result in the diagonal approximation after averaging over the azimuthal angle φ. Beyond the diagonal approximation, the screening length $\xi^{-1}(\varphi,\theta)$ actually depends on the azimuth, and the exponential screening $\xi^{-1}(\varphi,\theta_c)$ is therefore different in different directions. Changing the external field changes the angle θ, and at a certain critical angle θ_c, the screening length $\xi^{-1}(\varphi,\theta_c)$ can become infinite in some directions determined by φ; the screening thus vanishes along these directions. Subsequently increasing the field makes the screening trigonometrical along these directions. The screening is therefore exponential along certain directions and is trigonometrical along others, but it is absent on the intersections.

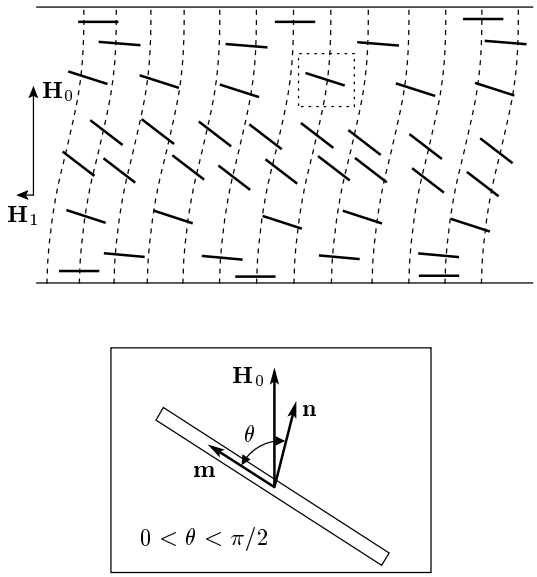

Fig. 2. Aggregation of magnetic grains in a ferronematic upon application of the magnetic field

3. EXPLANATION OF THE CELLULAR TEXTURE IN FERRONEMATICS

In 1970, Brochard and de Gennes proposed doping the liquid crystal matrix with ferromagnetic grains to allow the coupling of the liquid crystal molecular orientation to weak external fields [19]. The authors treated such a system theoretically and predicted the Freedericks effect in weak magnetic fields $H \sim 10$ G. The doped matrix therefore exhibits a collective orientational distortion in weak magnetic fields. They also predicted segregation effects, i.e., a smooth change of the grain concentration $c(\mathbf{R})$ from point to point in the magnetic field. In [2], the authors experimentally observed the collective behavior in the MBBA doped with magnetic grains, which is exhibited as a long-range uniform distortion of the molecular orientation of the entire sample upon application of a weak magnetic field $H < 1$ G. In that experiment, the grains were coated with DMOAP, which provides homeotropic anchoring on their surfaces, thereby making the magnetic grains lie perpendicular to the nematic director in the absence of the magnetic field.

This system was theoretically studied by Burylov and Raikher [21, 22]. It was shown that under applying the magnetic field H, there is an angle between the grain dipole moment direction $\mathbf{m}$ (which is the unit vector along the grain) and the director $\mathbf{n}_0$; the angle is different from $\pi/2$ or 0 for a finite anchoring, as shown in Fig. 2.

To describe the experimental results on the dependence of the field-induced birefringence on the strength of the applied field, on the concentration of the magnetic dopant, and on the thickness of the nematic cell, Burylov and Raikher proposed the free energy density functional

$$F = \frac{1}{2}\left[K_{11}(\operatorname{div}\mathbf{n})^2 + K_{22}(\mathbf{n}\cdot\operatorname{rot}\mathbf{n})^2 + K_{33}[\mathbf{n}\times\operatorname{rot}\mathbf{n}]^2\right] - M_s f(\mathbf{m}\cdot\mathbf{H}) + \frac{f k_b T \ln f}{v} - \frac{fWA(\mathbf{n}\cdot\mathbf{m})^2}{d}, \quad (55)$$

where $f = cv$ is the volume fraction occupied by the particles, v is the particle volume, M_s is the magnetization inside the grains, d is the diameter, and $A \sim 1$ is a constant. This functional differs from the one proposed by Brochard and de Gennes only by the last term. The last term accounts for the weak anchoring under which $0 < \theta < \pi/2$. Minimization with respect to f (keeping the number of particles fixed) leads to

$$f = f_0 \exp\left[\frac{\mu(\mathbf{m}\cdot\mathbf{H})}{k_b T} + \frac{WAv(\mathbf{n}\cdot\mathbf{m})^2}{dk_b T}\right], \quad (56)$$

where f_0 is found from the total number of grains

$$\overline{f} = Nv = \int f(r)dV.$$

It was found that the particles accumulate in the center of the cell under applying the magnetic field (Fig. 2). For weak fields $H < 10$ G, the dependence $f(z)$ (where z is the axis perpendicular to the cell, with $z = 0$ in the center) is given by [21]

$$f(z) = \overline{f}\left[1 + \frac{\rho^2 D^2(1 - 12z^2/D^2)}{48\lambda^2}\right] \quad (57)$$

where

$$\lambda = \left(\frac{K_{33}v}{2\overline{f}k_b T}\right)^{1/2},$$

D is the thickness of the cell ($D \geq 100\,\mu$m),

$$\rho = M_s v H/k_b T, \quad M_s \sim 340 \text{ G},$$

and $\nu \approx 2\cdot 10^{-15}$ cm^3. At higher fields, the concentration in the center is increased faster, which was proved by computer simulations. But on reaching the field $H \sim 30$ G, experiment shows [2] that the uniform orientational distortion is replaced by a new field-induced cellular texture with the cells having dimensions on the order of tens micrometers. At the critical concentration in the center, magnetic particles clump into aggregates. This clumping had no explanation, because the magnetic dipole–dipole interaction is much smaller than the interaction with the external magnetic field. Indeed, the magnetic moment $\mu = M_s v$ induces the interaction

$$E_{dd} = \mu^2/R^3,$$

where R is the average distance between particles, $R^{-3} \sim c \sim 10^{10}$ cm^3, and therefore, $E_{dd} \sim 4\cdot 10^{-15}$ erg. The energy of the interaction with the external magnetic field $H \sim 10$ G is $E_H = \mu H \sim 3\cdot 10^{-12}$ erg, and hence, $E_H \gg E_{dd}$.

We explain this field-induced cellular texture by the clumping of the grains caused by elastic deformations of the director, i.e., by the elastic interaction between particles. In the one-constant approximation, this potential is given by Eq. (48). In the operators $\widehat{A}_l^p$ in (32), we keep only the first term

$$\widehat{A}_l = \alpha_{lm}\,(\mathbf{k}_l\cdot\mathbf{n}_0)\,,$$

because the other terms give higher powers in $1/R$. For the cylinder, the tensor

$$\alpha_{lm} = 2\oint dsW(s)\nu_l(s)\nu_m(s)$$

has the components

$$\alpha_{11} = \alpha_{22} = dL\pi W, \quad \alpha_{33} = d^2\pi W,$$

and $\alpha_{lm} = 0$ for the others. Hence,

$$\alpha_{33}/\alpha_{11} = d/L \approx 0.1,$$

and we can neglect α_{33}. We thus obtain

$$U_{cyl}(R) = -\frac{\alpha_{11}^2 \sin^2\theta\cos^2\theta}{4\pi K}\,\frac{\exp(-\xi(\theta)\,R)}{R}. \quad (58)$$

Cylindrical grains therefore attract each other in accordance with the Yukawa law if $\theta \neq 0, \pi/2$, which is possible in the inclined external field. In the absence of the field, the equilibrium orientations are $\theta = 0, \pi/2$ (dependening on the planar or normal anchoring [25]) and the potential becomes that obtained by Lopatnikov and Namiot [24], which is proportional to $1/R^3$. We set

$$e = \frac{\alpha_{11}^2 \sin^2\theta\cos^2\theta}{4\pi K}.$$

We next consider the system of particles with the concentration c and the interaction law

$$U(r) = -e\frac{\exp(-\xi r)}{r}.$$

The free energy density of this system is written as

$$F = \frac{kT}{v}\int f(\mathbf{R})\ln f(\mathbf{R})dV + \\ + \frac{1}{2v^2}\int f(\mathbf{R})f(\mathbf{R}+\mathbf{r})U(\mathbf{r})d\mathbf{R}d\mathbf{r}. \quad (59)$$

We must find the condition for the loss of stability in this system of attracting particles. We write the concentration as

$$f(\mathbf{R}) = f_0 + \delta f(\mathbf{R}),$$

where f_0 is the ground volume fraction. Expanding

$$f(\mathbf{R}+\mathbf{r}) \approx f(\mathbf{R}) + (\mathbf{r}\cdot\nabla)f(\mathbf{R}) + \frac{1}{2}(\mathbf{r}\cdot\nabla)^2 f(\mathbf{R}),$$

we obtain

$$F - F_0 = \frac{1}{2}\int N\delta f^2(\mathbf{R}) + M\left(\nabla\delta f\right)^2, \quad (60)$$

$$N = \frac{2kT}{v} + \frac{1}{v^2}\int\limits_{r_0}^{\infty} U(r)d\mathbf{r},$$

$$M = -\frac{1}{2v^2}\int\limits_{r_0}^{\infty} U(r)r^2 d\mathbf{r},$$

where r_0 is the size of the particle. Inasmuch as $U < 0$, a phase transition occurs for $N < 0$. In our case, $\xi r_0 \ll 1$ and we can therefore write

$$N \approx \frac{2kT}{f_0 v} - \frac{4\pi e}{\xi^2 v^2},$$

$$M \approx \frac{12\pi e}{\xi^4 v^2}.$$

Below the critical point,

$$N \approx \frac{4\pi e}{\xi^2 v^2}.$$

The length of the first instability is

$$l_{inst} = \sqrt{\frac{2M}{N}} \approx \frac{1}{\xi}. \quad (61)$$

As discussed above, $l_{inst} \approx 30\,\mu$m, which is in a good agreement with the experimental size of the cells [2].

4. CONCLUSIONS

We have derived the potential interaction for particles of the ordinary shape doped in the nematic liquid crystal. We have taken the collective screening effects into account, which is essential for the real colloid systems. It is found that the shape of the particles essentially influences the screening effects, which exist for both the homeotropic and the planar anchoring. Screening is absent for spherical particles. Anisotropic particles (e.g., cylinders) with the magnetic or electric moment in the presence of the inclined external magnetic or electric field induce oscillations in the director distribution with the period about $\lambda \sim 10$–$100\,\mu$m depending on the anchoring, the concentration, and the magnitude of the external field. In this case, selective scattering of the electromagnetic waves on these oscillations may be observed for electromagnetic waves in this range.

It is found that cylindrical grains inclined to the director attract via the Yukawa law. This explains the cellular texture in ferronematics. Application of the external magnetic field changes the orientation of the magnetic grains with respect to the director, which can lead to essentially changing the screening effects. In particular, this can lead to the trigonometric screening of the pair interaction.

Collective effects in doped nematic liquid crystals strongly depend on the anchoring strength, on the particle shape and concentration, and on external fields and make DNLC a marvellous medium for a further experimental and theoretical exploration of the different structures originating from deformations of the director field.

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