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ELECTRONIC TRANSPORT IN QUASICRYSTALS

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Mobility of electrons in quasicrystals is considered in the framework of the fractional Fermi surface model, i.e., a multiconnected FS with many electron–hole pockets. The Mott law for the variable range hopping conductivity is obtained when intervalley scattering processes with small momentum transfer are taken into account. The transition to the power law temperature dependence is discussed.

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1. INTRODUCTION

Quasicrystals are materials that have a long-range aperiodic atomic order and rotational symmetries that are crystallographically forbidden for periodic structures (e.g. five-, eight-, ten-, and twelwe-fold rotational axes). Quasircystals (QC's) usually are the intermetallic alloys, but their physical properties differ from those of the crystalline and amorphous metallic phases. Like metals, quasicrystals have a nonzero electronic contribution to the specific heat, although it is smaller than the value calculated within the free-electron model. At the same time, the electronic resistivity of quasicrystals at low temperature is anomalously high and increases with increasing the structural order and annealing the defects. The highest resistivity of all the known quasicrystals occurs in icosahedral (i) Al-Pd-Re quasicrystal, where the value of resistivity at 4.2 K exceeds 1 $\Omega \cdot$ cm. Large values of the resistivity ratio $\mathcal{R} = \rho(4.2\ \mathrm{K})/\rho(295\ \mathrm{K})$, up to 200, are also observed for this material (the values of $\mathcal{R}$ are 1.1 for i-Al-Li-Cu, up to 2 for i-Al-Cu-Fe, and about 4 for Al-Cu-Ru), which shows how perfect the sample is [1–5]. In contrast to the Matissen rule, where the resistivities are additive, the conductivity of quasicrystals behaves as $\sigma = \sigma(0) + \Delta\sigma(T)$ over a wide range of temperatures, where $\sigma(0)$ is the residual conductivity at zero temperature, and $\Delta\sigma(T)$ represents the T-dependent part. Usually, $\sigma(0)$ increases with the structural disorder and $\Delta\sigma(T)$ increases with increrasing the temperature as $\Delta\sigma(T) \propto T^{\beta}$ [1–3]. In a series of i-Al-Pd-Re samples with different $\mathcal{R}$, Gignoux et al. [3] obtained $1 < \beta < 1.5$ in the temperature range from 7 K to 700 K. Pierce et al. [4] measured the $\sigma(T)$ dependence of several i-Al-Pd-Re samples (with various $\mathcal{R}$ ratio) and found the power-law dependence with β in the range 0.5 to 1 for temperatures from 0.45 K to 3 K.

Different explanations of the transport properties of quasicrystals have been proposed. Much attention has been given to the power-law temperature dependence of $\sigma(T)$. The role of the pseudogap in the density of states (DOS) at the Fermi level, the role of quantum interference effects (weak localization and electron-electron interactions), proximity to the metal–insulator transition, the spiky structure of the electronic spectrum, and the criticality of wave functions have been discussed in connection with this problem (see the review articles by Poon [1] and Rapp [2]). Fujiwara et al. [6, 7] tried to obtain the $\sigma(T)$ dependence on the basis of the band structure and Fermi surface (FS) calculations for crystalline approximants. Macia [8] gave a phenomenological description of $\sigma(T)$ based on the DOS model that takes the pertinent experimental results into account. The problem was analyzed by Burkov et al. [9], who used the fractional FS model, i.e., a multiconnected FS with many electron–hole pockets. They considered the intravalley and intervalley scattering processes in order to explain the power-law dependence of $\sigma(T)$. They also predicted a zero value of σ at $T = 0$ K for the perfect QC (with no scattering centers) and a small residual conductivity for «dirty» QC.

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Currently, the physical origin of the high resistivity of intermetallic quasicrystalline alloys compared to other systems composed only of metals is not well understood, and this has challenged the experimentalists to examine the possibility of a metal–insulator transition (MIT) in quasicrystalline systems and to investigate whether these materials are metallic. Recent experiments on the perfect icosahedral $Al_{70.5}Pd_{21}Re_{3.5}$ quasicrystals have shown that at low temperatures, their conductivity follows the Mott law for the variable range hopping (VRH) conductivity,

$$\sigma = \sigma_0 \exp\left[-(T_0/T)^p\right], \qquad (1)$$

where p = 1/4 [10–13]. The temperature range where the Mott law is fulfilled was found to be 0.45–10 K, 0.02–0.6 K, and 0.5–7 K according to Guo and Poon [10], Delahaye et al. [12], and Wang et al. [13], respectively. In i-$Al_{70.5}Pd_{21}Re_{8.5-x}Mn_x$, it was found that the VRH including the Coloumb interaction ($p = 1/2$) can describe the experimental data for $2 < x < 4$ [10]. The experimental data on bulk i-Al-Pd-Re samples are rather contradictory. Different authors quote different values of T_0. According to Refs. [10, 11, 13], T_0 reaches 100 K and is higher for samples with higher $\mathcal{R}$. On the other hand, in Ref. [12], the very low value of $T_0 \sim 1$ mK was given. However, the lowest temperature reached in this experiment was 20 mK and the value of T_0 was determined by extrapolation. Moreover, in Ref. [10], a small but finite value of $\sigma(0)$ was obtained, although it decreased with increasing the perfectness of the sample [11]. At the same time, in Refs. [12, 13], the conductivity was fitted to Eq. (1) without including any extra $\sigma(0)$ term, while it was used by Guo and Poon [10] in order to analyse their experimental data on conductivity. We note that the presence of a residual conductivity is quite possible, because the different conductivity channels in QC are parallel, and a nearly vanishing conductivity $\sigma(0)$ cannot hide the VRH conductivity.

The occurrence of the Mott law (Eq. (1)) shows that electronic states in QC are localized and the sample is on the insulating side of the MIT. Qualitatively, the possible role of the hopping conduction between localized states in QC at low temperatures have been previously discussed by several authors on experimental grounds [1, 3, 5, 14]. Poon [1], Pierce et al. [14], and Mayou et al. [5] discussed the possibility of explaining the power-law dependence of σ by a hopping mechanism taking the criticality of the wave functions into account. Janot [15] considered this problem in the framework of the hierarchical cluster model and predicted the conductivity that scales roughly as $T^{3/2}$. But the localization lengths ξ obtained experimentally (we recall that $T_0 \propto \xi^{-3}$ in accordance with the Mott theory) are much larger than the separation, ~ 20 Å, of the ideal clusters that are assumed to be structure units between which the electrons hop in the Janot model. In addition, in the subsequent paper [16], Janot proposed that « ... the power law T^β may be lost experimentally because of extrinsic effects due to structural defects, boundaries, and periodic approximant distorsion which may restore the $T^{-1/4}$ law of the Mott model». That is, the existence of the Mott law was related to the presence of disorder in QC, which contradicts the experimental data. A rather interesting but unrealistic idea was put forward by Rivier and Durand [17] based on the results obtained for the one-dimensional model. In order to obtain the VRH conductivity, they suggested that the electronic structure of a quasicrystal looks somewhat similar to highly doped, p-type semiconductors, but no reliable explanation of the Mott law, Eq. (1), for quasicrystals was given.

In this paper, we proceed on the ground of the band structure theory to explain the VRH conductivity in QC. Although the Bloch theorem does not apply to a quasicrystal the ideas of the band structure theory can be used to describe the transport properties of QC, with the quasicrystalline state considered as a structural limit of a sequence of rational periodic approximants with increasing periods. Therefore, we take the band structure effects into account that are specific for the quasicrystalline symmetry and use the fractional Fermi surface model [9] to explain the origin of the VRH conductivity in QC. However, the hopping mechanism of conductivity involves hops between localized states, and we therefore begin with the discussion of the nature of localization in the regular and perfect QC without phasons and other distortions.

The paper is organized as follows. In Sec. 2, the localization of electrons in quasicrystals is discussed. The VRH conductivity is discussed in Sec. 3. In Sec. 4, the crossover to the power-law temperature dependence of conductivity is considered.

2. LOCALIZATION OF ELECTRONS IN QUASICRYSTALS

For amorphous alloys, granular metal films, and doped semiconductors the electronic localization plays an important role in the low-temperature electron transport. For the above systems, the localization of electrons is known to arise from disorder. But the object of our discussion is the origin of localization in

QC. The Al-Pd-Re QC is a highly ordered material with very sharp X-ray diffraction spots, and as mentioned above, improving the perfection of the quasilattice order has been found to lead to increasing of the resistivity.

The experimental results for i-Al-Pd-Re show that at low temperatures, the regular and perfect quasicrystal behaves as a material in the Fermi-glass state, that is, the DOS is finite at the Fermi level, but the electrons are localized. This localization in a QC is a consequense of the coherent interference of the electronic states caused by the specific symmetry and the structure of the material, and the more perfect the material is, the more localized the electrons are. Whereas in a disordered metal or a heavily doped semiconductor, the origin of the localization is the destruction of the phase coherency of the wave functions due to disorder (the Anderson localization), in the QC, the phase coherency of the wave functions is the main source of localization. The following simple observations can justify this conclusion.

First, within the six-dimensional periodic description of the icosahedral structure, it is obvious that each scattering wave vector in the quasicrystal corresponds to a reciprocal wave vector in the periodic structure of a higher dimension. Thus, the set of the reciprocal lattice vectors densely fills the reciprocal space of the quasicrystal, and all the electron states at the Fermi level have zero group velocity (the standing waves) due to the Bragg reflections (evidently, with different intensities), i.e., due to the constructive interference of the electron states at the Fermi level.

Second, it is convenient to elucidate this picture by considering the quasicrystal as a structural limit of a sequence of rational approximants (crystal analogs) with an increasing lattice period. The Brillouin zone (BZ) volume is diminished with increasing the order of the approximant because the lattice period increases, and the BZ volume becomes infinitely small ($\sim h^3$) in the quasicrystalline limit. Therefore, employing the usual approach to the construction of the FS [18], one can see that in the hierarchy of higher-order approximants, the energy bands are folded down and the FS becomes fractional in the quasicrystalline limit, namely, it is multiconnected with a large number of electron and hole «pockets», and for atomically ordered perfect QC as was pointed out by Poon [1], the electron states must be localized at zero temperature, because the strong localization condition $k_F^i\, l \sim 1$ (where l is the mean free path and k_F^i is Fermi momentum) is satisfied for the electrons in each valley i. Therefore, each valley plays a role similar to that of a localization center in a disordered object. Hence, there is a formal analogy between a well-ordered quasiperiodic object with a fractional FS and a disordered metal or a heavily doped semiconductor. This analogy helps one to explain the occurence of the Mott law in perfect quasicrystals at very low temperatures, because the electrons in localized states can participate in conductivity only via hopping between localization sites.

3. VRH CONDUCTIVITY IN QC

Mott was the first to point out that at low temperatures, the most frequent hopping process would not be the hopping to a nearest neighbour [19]. To explain the conduction with an activation energy monotonically decreasing with decreasing the temperature, Mott proposed a model where in strongly localized systems with a sufficiently high density of states $N(E_F)$ near the Fermi level, the states that are optimal for conduction accumulate closer and closer to the Fermi level as T decreases. Thus, the activation energy decreases, while the hop length grows with decreasing the temperature. The simplest arguments were as follows. Within a radius R around a given site, the total number of electron states near the Fermi energy is

$$\frac{4\pi}{3} R^3 N(E_F), \tag{2}$$

and the lowest activation energy ΔE for a hopping process at the distance R is reciprocal to Eq. (2),

$$\Delta E = \frac{3}{4\pi}\, \frac{1}{R^3 N(E_F)}. \tag{3}$$

Therefore, ΔE decreases with increasing the hopping range. But hopping by a large distance involves tunneling with the probability proportional to $\exp(-2\alpha R)$, where $1/\alpha = \xi$ is the decay length of the localized wave function. Therefore, there exists an optimum hopping distance R_{opt}, for which the expression

$$\exp(-2\alpha R)\exp(-\Delta E/k_B T) \tag{4}$$

has a maximum. This maximum occurs at the minimum value of the exponent

$$2\alpha R + \frac{3}{4\pi R^3}\, \frac{1}{N(E_F) k_B T}, \tag{5}$$

which gives

$$R_{opt} = \left(\frac{9}{8\pi N(E_F)\alpha k_B T} \right)^{1/4}. \tag{6}$$

Inserting R_{opt} in (4), we obtain that the hopping probability, and hence the conductivity, is given by Eq. (1) with $p = 1/4$ and

$$T_0 \sim \frac{1}{\xi^3\ N(E_F)}. \qquad (7)$$

We now consider the quasicrystalline state. In the fractional FS model with a practically infinite number of valleys, all the electrons in the atomically ordered quasicrystal are localized at zero temperature. At a finite temperature, the scattering of an electron from a particular valley to the state in a neighboring valley can occur not only due to thermal exitation but also due to tunneling via the gap formed by Bragg reflections. At very low temperatures, the process with a higher probability is the scattering with a small momentum transfer; in the real space, that corresponds to a hopping by a large distance. But the tunneling and correspondingly, the large-distance hopping is a necessary process for the VRH mechanism. Thus, following the Mott procedure [19] and using expression (5), we immediately obtain Eq. (1). The Mott formalism usually fails when $R < \xi$ or $T_0 < T$. But in the case of a quasicrystal, it is always possible to find a state for which $T_0 < T$, even though $R > \xi$. As a matter of fact, the material with a fractional FS has a hierarchy of localization lengths. Therefore, the characteristic temperature T_0 can change from sample to sample in an arbitrary way.

This mesoscopical situation is typical of QC and conventional disordered systems in the vicinity of MIT [20]. It is known that in amorphous alloys and heavily doped semiconductors, the electron wave functions show a characteristic change from localized to extended behavior because of this transition. This corresponds to a change from the states that do not enable transport in the limit of vanishing temperature to the states that do, thereby distinguishing the insulating and metallic character. In the localized regime, the spatial behavior of the wave functions is usually described by an exponential decay length reflecting the spatial extent of the wave function, whereas on the metallic side, the wave functions are extended. As the MIT is approached, the localization length diverges. Close to the MIT, the localization length is already much greater than the numerically accessible system size, and therefore, there can be no direct reflection of the localization in the calculated eigenstates. Exactly at the MIT, where the characteristic length scale is absent, the eigenstates show fractal characteristics and the wave functions are critical. However, characterizing the eigenstates and wave functions at the transition point requires a more general concept of multifractality. This implies that different parts of the same eigenstate must scale with different exponents, thus extending the simple fractal picture that comprised only one scaling exponent. Schriber and Grussbach found that at the crytical point of the $3D$ Anderson model, strong fluctuations of the wave function amplitudes display the multifractal character on all length scales and the singularity spectrum of the critical wave function does not depend on the system size [21]. The same state is typical of QC [11]. As shown in Ref. [22], most of the wave functions in a three-dimensional icosahedral QC are critical and their electronic spectrum contains a singular part. The situation is mesoscopic, and it is possible to experimentally obtain different values of T_0 for different samples (see Refs. [10–13]). We also note that the fractional FS model implies that the VRH mechanism ceases to work beginning with some ξ_{min}, when tunneling probability becomes negligibly small.

The VRH mechanism in the fractional FS model depends on the structure of the FS; for a perfect material, the VRH conductivity must always exist in bulk samples and in films. Recently, it was shown by Rosenbaum et al. [23] that some thin (2200 Å) icosahedral films of $Al_{72}Pd_{20}Re_8$ prepared by magnetron sputtering exhibit insulating transport properties down to 0.07K where their resistivity follows an activated Mott VRH law. Although we did not consider the role of electron–electron interactions, it is possible to assume that with decreasing the temperature, the Mott law ($T^{-1/4}$) is followed by the Efros–Shklovskii law ($T^{-1/2}$). Although it is difficult to distinguish between $T^{-1/4}$ and $T^{-1/2}$ dependences experimentally, one could observe this crossover on the high-quality i-Al-Pd-Re samples with a high resistance ratio $\mathcal{R}$.

In contrast to the Anderson localization, the localization of electrons in quasicrystals is due to constructive interference (phase coherence) of the wave functions and is unstable with respect to small perturbations [22]. The system can therefore be driven to the metallic side of MIT by increasing the temperature or the number of imperfections. Moreover, in a «dirty» object at the temperatures larger than approximately 10 K, the electronic states are smeared by the inelastic scattering processes that wash out the fine details of the FS with a large number of pockets and lead to the FS with an effectively finite number of pockets, depending on the perturbations [13]. When the number of FS pockets is finite, the VRH mechanism does not work, and the crossover to another temperature dependence must occur.

4. HIGH-TEMPERATURE CONDUCTIVITY

In a real quasicrystal one must take the smearing of electron states in the momentum space into account. Because the energy of a quasiparticle is defined with the uncertainity $\delta\varepsilon \sim \max(T, \hbar/\tau)$, where T is the temperature and τ is the electron relaxation time, the splitting of the FS within the procedure described above makes sense as long as the characteristic size of the pockets is larger than $\delta\varepsilon$. This leads to the FS with a finite number of electron–hole pockets with the size of the pocket defined by the uncertainity of the electron energy. This also leads to a nonmetallic regime of the conductivity caused by an intravalley scattering. At these conditions, the VRH mechanism does not work, because the intervalley scattering with the small momentum transfer is ineffective. On the contrary, the momentum transfer that is now required for the intervalley scattering is large (of the order $1/a$, where a is the quasilattice constant). In the FS model with a finite number of valleys, the temperature-dependent conductivity is governed by the intravalley and intervalley processes. As shown in Ref. [9], σ is inversely proportional to the scattering relaxation time and should increase with increasing the temperature according to a power law dependence. However, the high-temperature region ($T > u/a \sim \Theta_D$, where u is the sound velocity, and Θ_D is the Debye temperature) has not been considered previously, and we now appropriately analyze this regime. In this region, the temperature dependence of σ is governed by the electron–phonon intervalley scattering processes and a sharp decrease of the electron–phonon scattering time should be observed, accompanied by the corresponding change in the character of conductivity.

We consider the probability for an electron with the momentum $\mathbf{k}$ in a tiny pocket of the Fermi surface to be scattered by a phonon to the free electron state with the momentum $\mathbf{k}'$. This probability is given by

$$w(\mathbf{k},\mathbf{k}') = g^2(\mathbf{K})\left[\delta(\epsilon_{\mathbf{k}'} - \epsilon_{\mathbf{k}} - \hbar\omega_{\mathbf{q}})n^0_{\mathbf{q}} f^0_{\mathbf{k}}(1 - f^0_{\mathbf{k}'}) + \right.$$
$$\left. + \delta(\epsilon_{\mathbf{k}'} - \epsilon_{\mathbf{k}} + \hbar\omega_{\mathbf{q}})(n^0_{\mathbf{q}} + 1) f^0_{\mathbf{k}'}(1 - f^0_{\mathbf{k}})\right]. \quad (8)$$

Here, $\mathbf{K} = \mathbf{k}' - \mathbf{k}$, $\mathbf{q} + \mathbf{g} = \mathbf{k}' - \mathbf{k}$ (where $\mathbf{g}$ is a reciprocal lattice vector), $\omega_{\mathbf{q}}$ is the phonon energy, $n_{\mathbf{q}}$ is the Bose–Einstein distribution, $f_{\mathbf{k}}$ is the Fermi–Dirac distribution, and $g(\mathbf{K}) = \pi(n_e/E_F)(\mathbf{Ke})/\sqrt{N_i M_i \omega_{\mathbf{q}}}$ is the matrix element of the electron–phonon interaction, where n_e is the electron density at the Fermi level, $\mathbf{e}$ is the phonon polarization vector, N_i and M_i are the ion density and mass, respectively (in the case where the Fermi surface pocket is considered as a sphere of the diameter μ_0, we have $n_e/E_F = 2\mu/3$). To obtain the electron–phonon relaxation time τ_0, we must integrate Eq. (8) over $\mathbf{k}$ and $\mathbf{k}'$. This integration procedure is similar to those for the electron–phonon relaxation time calculation in usual metals: the integration over the modulus of $\mathbf{k}'$ eliminates the delta function; the intergration of the Fermi distribution function over the modulus of $\mathbf{k}$ gives the so-called structural factor. The remaining angular integrations can then be easily reduced to an integral over the angle Θ between the vectors $\mathbf{k}$ and $\mathbf{k}'$. After the final integration over Θ, we find the equation for τ_0 for the hypothetical metal with one tiny valley,

$$\tau_0^{-1} = \frac{k_0 q_D^6}{\pi^2 m N_i M_i \Theta_D}\left(\frac{T}{\Theta_D}\right)^5 \times$$
$$\times \int_0^{\Theta_D/T} \frac{z^5}{(1 - e^{-z})(e^z - 1)}, \quad (9)$$

where q_D and Θ_D are the Debye wave vector and temperature, respectively. For the object with the N-component Fermi surface under the conditions of the valley uniformity, we obtain $\tau^{-1}_{e-ph} = N\tau_0^{-1}$, where N is a parameter of the model. The effective number N of the FS electron–hole pockets can be estimated using X-ray or electron diffraction experiments, from the amount of the main strong Bragg reflections that satisfy the condition $\mathbf{G} = 2\mathbf{k}_F$ ($\mathbf{G}$ is a vector of the six-dimensional reciprocal lattice) [1], including the multiplicity factor.

In discussing the application of these relations to quasicrystals, one should remember that the electron–phonon interaction can change the electron momentum only by a small amount, of the order T/u. We again note that a large momentum transfer is necessary for the intervalley scattering to occure. Hence, there exists a characteristic temperature $T^* \sim u/a \sim \Theta_D$, below which the phonons are unable to scatter the electrons from one pocket to another, thereby permitting only the intravalley processes. Accordingly, the electron–phonon scattering mechanism is ineffective for the temperatures $T < T^*$ because Eq. (7) does not contain the factor $N \gg 1$. Hence, we can neglect the electron–phonon scattering in the low-temperature region and consider the high-temperature limit of Eq. (9) only. The integral in Eq. (9) is then proportional to $(\Theta/T)^4$ and we find that the electron–phonon relaxation time at high temperatures is given by

$$\tau^{-1}_{e-ph} \approx Nq^4 k_0 (N_i M_i u^2 m)^{-1} T.$$

Because the probability of the electron scattering by phonons is linear in T, it is easy to show, using the

results of Ref. [13], that the temperature dependence of the conductivity at high temperatures must be linear. We note that according to some experimental data, the $\sigma(T)$ dependence at high temperatures for some i-quasicrystals is indeed nearly linear [2, 24, 25].

We finally estimate the conductivity at zero temperature. The relaxation of the electronic momentum on the structural imperfections results in a finite value of σ, in analogy with metallic systems. On the other hand, it was found that $\sigma(0)$ is proportional to the concentration of the structural imperfections, in contrast to the usual metallic Drude-like conductivity [9],

$$\sigma(0) \approx e^2\hbar^{-1}N^2m^2n_{imp}|U_0|^2,$$

where $|U_0|$ is the amplitude of the Born scattering of an electron by the structural defects, m is the electron mass, and n_{imp} is the concentration of defects. To check the agreement of the predicted data with the experimental ones, we numerically estimate the magnitude of $\sigma(0)$. With the rough assumptions $|U_0| \sim 10^{-8}$ cm (atomic radius) and $n_{imp} \sim 10^{22}$ cm^{-3}, we immediately obtain $\sigma(0) \sim 10\ N^2$ [$\Omega^{-1} \cdot$ cm^{-1}]. For N, we can assume a reasonable value of about one hundred [1], which gives the reasonable estimation for $\sigma(0)$. For the imperfection concentration $n_{imp} \sim 10^{-5}$, $\sigma(0)$ is about 1 $\Omega^{-1}\cdot$cm^{-1}, which approximately corresponds to the experimental values for the perfect Al-Pd-Re alloys with the high resistance ratio $\mathcal{R}$ [2].

5. CONCLUSION

We have considered a «scenario» in which the band structure effects, namely the constructive interference of wave functions due to Bragg reflections, are responsible for the localization of the electrons, and consequently, for the electron transport in quasicrystals. The Fermi surface of an atomically ordered perfect icosahedral quasicrystal (such as i-Al-Pd-Re) at zero temperature contains an infinite number of electron–hole pockets where the electrons are localized. In this case, the VRH mechanism of conductivity should be operative, because the intervalley scattering processes with small momentum transfer are available. The Mott law is obtained for the low-temperature region taking the intervalley tunneling transitions into account. We note that the importance of interband tunneling transitions was previously pointed out in Ref. [26]; recently, Krajĉi and Hafner [27] emphasized the significance of these transitions for the electron transport in QC. With increasing the temperature and QC imperfections, the FS effectively contains a finite number of pockets due to smearing, and the conductivity becomes power-law temperature dependent. The estimate made in the framework of the fractional FS model predicts that at high temperatures ($T > \Theta_D$), the conductivity should linearly depend on the temperature. Finally, it is worth noting that we have considered only the role of the band structure effects and did not discuss either the influence of the quantum interference effects, which can be important for a material with a low value of the resistance ratio $\mathcal{R}$, or other possible mechanisms of the conductivity in QC. In addition, we based our consideration only on the experiments carried out for the i-Al-Pd-Re quasicrystal. Although the scattering mechanisms in different types of QC may be different, we believe that our conclusions about the mechanism of conductivity in QC in the framework of the fractional FS model are quite general.

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