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ELECTROMAGNETIC ACTIVITY OF A PULSATING PARAMAGNETIC NEUTRON STAR

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The fact that neutron star matter possesses the capability of maintaining a highly intense magnetic field has been and still is among the most debatable issues in pulsar astrophysics. Over the years, there were several independent suggestions that the dominant source of pulsar magnetism is either the field-induced or the spontaneous magnetic polarization of the baryon material. The Pauli paramagnetism of degenerate neutron matter is one of the plausible and comprehensive mechanisms of the magnetic ordering of neutron magnetic moments, promoted by a seed magnetic field inherited by the neutron star from a massive progenitor and amplified by its implosive contraction due to the magnetic flux conservation. Adhering to this attitude and based on the equations of magneto-elastic dynamics underlying continuum mechanics of single-axis magnetic insulators, we investigate electrodynamics of a paramagnetic neutron star undergoing nonradial pulsations. We show that the suggested approach regains a recent finding of Akhiezer, Laskin, and Peletminskii [1] that the spin-polarized neutron matter can transmit perturbations by low-frequency transverse magneto-elastic waves. We found that nonradial torsional magneto-elastic pulsations of a paramagnetic neutron star can serve as a powerful generator of a highly intense electric field producing the magnetospheric polarization charge whose acceleration along the open magnetic field lines leads to the synchrotron and curvature radiation. Analytic and numerical estimates for periods of nonradial torsional magneto-elastic modes are presented and are followed by a discussion of their possible manifestation in currently monitored activity of pulsars and magnetars.

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1. INTRODUCTION

Recent years have seen resurgence of interest in the magnetic properties of neutron star matter [1–4] and of the early advanced hypothesis that considerable contribution to the ultrastrong magnetic field of these compact objects can be attributed to spin polarization of stellar material [5–8]. This development calls into question our understanding of the laws governing continuum mechanics and macroscopic electrodynamics of magnetically ordered nuclear matter. To the best of our knowledge, the first significant step in this direction was made in [1], cited in the abstract and hereafter referred to as the Akhiezer–Laskin–Peletminskii (ALP) model, advocating ferromagnetism of neutron stars. Using equations of the magneto-hydrodynamic type adopted from macroscopic electrodynamics of ferromagnetic dielectrics [9], it was shown that magnetically ordered

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neutron matter can transmit perturbations by low-frequency magneto-elastic waves along with the well-known high-frequency spin waves typical of ferromagnetic solids [9–11]. The observation of these oscillatory motions in currently monitoring neutron stars is crucial, in our opinion, for unambiguous identification of the permanent magnetism of stellar material. This attitude motivates our present work, continuing investigations begun in [1], aimed at searching characteristic features of electromagnetic activity of neutron stars owing its origin to nonradial magneto-elastic pulsations of paramagnetic neutron stars.

The fingerprints of the Pauli mechanism of the field-induced (nonspontaneous) spin polarization of neutron star matter can be traced in the existing scenario of the pulsar birth in a supernova event [12, 13]. The catastrophic collapse of the massive main sequence star exhausting its nuclear fuel implies that implosive contraction of a weakly magnetized massive star is accompanied by intensive neutronization of stellar material due to the inverse β-process

$$e + p \to n + \nu_e$$

responsible for fast cooling of pulsars [12]. Because this urca process is controlled by the weak, parity violating interaction, it is expected that the magnetic anisotropy caused by the presence of a seed magnetic field introduces in the final product of the collapse a tremendous difference between the number of neutrons with spin magnetic moments directed along the seed magnetic field and those with oppositely directed spins, such that the main body of the newly born neutron star mass develops a permanent magnetization of the paramagnetic type. The amplification of the magnetic field in this process is attributed to implosive contraction that proceeds with the preserved magnetic flux.

Following this line of argument, we consider the homogeneous model of a paramagnetic neutron star undergoing nonradial pulsations triggered either by the implosive effect of a supernova event or by gamma-bursting starquakes. In doing this, we utilize a somewhat different, as compared to the ALP model, form of the macroscopic equations governing the motions of magnetically polarized neutron matter, adopted from the macroscopic electrodynamics of single-axis magneto-elastic insulators [14]. One of the purposes is to show that the proposed approach is interesting in its own right because the continuum mechanics of magnetically polarized stellar matter is less studied in astrophysics compared to magneto-hydrodynamics underlying our understanding of the motions of highly conductive stellar matter threaded by a magnetic field. Different aspects of this project have been reported in proceedings of several recent conferences [15–17], and our goal here is to bring them together in extended fashion.

The paper is organized as follows. In Sec. 2, the macroscopic equations of the magneto-elastic dynamics of spin-polarized nuclear matter are introduced and the dispersion equation for the wave transport of magnetization is derived. Section 3 presents a variational calculation of the periods of nonradial torsional pulsations of paramagnetic neutron stars with emphasis on the generation of the magnetospheric polarization charge responsible for the radiation from the star; the obtained analytic estimates are quantified using parameters that are typical of radio pulsars and magnetars. The last section provides a brief summary of the results obtained.

2. GOVERNING EQUATIONS FOR MAGNETO-ELASTIC DYNAMICS

In what follows, we assume, as in most of the works cited above, that permanently magnetized baryon matter of a neutron star possesses properties of a degenerate Fermi gas of neutrons condensed by self-gravity to the normal nuclear density $\rho = 2.8 \cdot 10^{14}$ g·cm^{-3}. To describe the equilibrium state of spin-polarized neutron star matter, we use a linear constitutive equation in the form given in [18],

$$\mathbf{M} = \chi\, \mathbf{B}, \tag{2.1}$$

where $\chi > 0$ stands for the average paramagnetic susceptibility of homogeneous neutron star matter, which is estimated to be $\chi \approx 2\chi_F$ at the normal nuclear density [5–8], where χ_F is the Pauli paramagnetic susceptibility of zero-temperature, degenerate, neutron Fermi gas compressed to the nuclear density,

$$\chi_F = \frac{3}{2} n \frac{\mu_n^2}{\epsilon_F} \approx 1.3 \cdot 10^{-4};$$

in (2.1), $\mathbf{B}$ denotes the fossil magnetic field frozen in the neutron star core.

The macroscopic description of motions of neutron star matter in terms of the theory of continuous media implies that the space scale of material displacements is much larger than the spacing between baryons. The basic suggestion underlying continuum models of neutron star material is to identify the behavior of many-component spin polarized baryon matter with that of the spin-polarized neutron degenerate Fermi gas of the equivalent density ρ subjected to the standard continuity equation

$$\frac{d\rho}{dt} + \rho\,\frac{\partial v_k}{\partial x_k} = 0. \tag{2.2}$$

Hereafter,

$$d/dt = \partial/\partial t + \mathbf{v}\cdot\nabla$$

stands for the convective derivative. The second suggestion of particular interest is to consider the magnetization field $\mathbf{m}(\mathbf{r},t)$ (magnetic moment per unit volume) as an independent dynamical variable of motion, on equal footing with the bulk density $\rho(\mathbf{r},t)$ and the elastic displacement velocity $\mathbf{v}(\mathbf{r},t)$. According to [14], the distinguishing feature of mechanical behavior of magneto-elastic insulators is that the dynamics of their intrinsic deformations is controlled by a driving force originating from antisymmetric magnetic stresses τ_{ij} (see also [19]). The dynamical equation of magneto-elasticity is given by

$$\rho\frac{dv_i}{dt} = \frac{\partial \tau_{ik}}{\partial x_k} \quad \tau_{ik} = \frac{1}{2}\left[m_i\,B_k - m_k B_i\right]. \tag{2.3}$$

Thus, the antisymmetric form of the magnetic stress tensor τ_{ij} exhibits a substantially non-Hookean character[1)] of magneto-elasticity, which comes into play only when the direction of the local magnetization $\mathbf{m}$ deviates from the direction of the equilibrium magnetization $\mathbf{M}$. The constitutive equation for the evolution of $\mathbf{m}$ is given by

$$\frac{dm_i}{dt} = \omega_{ik}\,m_k, \quad \omega_{ik} = \frac{1}{2}\left(\frac{\partial v_k}{\partial x_i} - \frac{\partial v_i}{\partial x_k}\right), \tag{2.5}$$

where ω_{ik} is interpreted as the antisymmetric rate-of-deformation tensor [14].

The above equations of dissipation-free magneto-elastic dynamics can be represented in the following equivalent vector form:

$$\frac{d\rho(\mathbf{r},t)}{dt} + \rho(\mathbf{r},t)\,\nabla\cdot\mathbf{v}(\mathbf{r},t) = 0, \tag{2.6}$$

$$\rho\,\frac{d\mathbf{v}(\mathbf{r},t)}{dt} = \frac{1}{2\chi}\nabla\times[\mathbf{m}(\mathbf{r},t)\times\mathbf{M}], \quad \mathbf{M}=\chi\mathbf{B}, \tag{2.7}$$

$$\frac{d\mathbf{m}(\mathbf{r},t)}{dt} = [\boldsymbol{\omega}(\mathbf{r},t)\times\mathbf{M}], \qquad \boldsymbol{\omega}(\mathbf{r},t) = \frac{1}{2}[\nabla\times\mathbf{v}(\mathbf{r},t)]. \tag{2.8}$$

This form accentuates the fact that the magneto-elastic driving force

$$\mathbf{f}(\mathbf{r},t) = \nabla\times\boldsymbol{\tau}(\mathbf{r},t)$$

in Eq. (2.7) is inextricably related to the magnetic torque density

$$\boldsymbol{\tau}(\mathbf{r},t) = \frac{1}{2}[\mathbf{m}(\mathbf{r},t)\times\mathbf{B}];$$

we again see that magneto-elastic effects manifest themselves when the magnetization field $\mathbf{m}$ deviates from the direction of the saturated magnetization $\mathbf{M}=\chi\mathbf{B}$. Equation (2.8) describing differential rotation of the magnetization about the magnetic anisotropy axis is the standard equation of precession under which the direction of $\mathbf{m}$ changes but the magnitude does not. It is noteworthy that similar equations have recently been used in the study of the large-scale motions of a poorly conducting interstellar medium possessing properties of gas-based ferrocolloidal soft matter consisting of tiny ferromagnetic solid grains suspended in a dense magnetically passive and electrically neutral fluid [22].

2.1. Wave transport of magnetization in paramagnetic neutron star matter

Applying the standard linearization procedure to Eqs. (2.6)–(2.8),

$$\mathbf{v}\to\mathbf{v}_0+\delta\mathbf{v}(\mathbf{r},t), \quad \mathbf{m}\to\mathbf{m}_0+\delta\mathbf{m}(\mathbf{r},t),$$

where

$$\mathbf{v}_0 = 0, \quad \mathbf{m}_0 = \mathbf{M} = \chi\mathbf{B},$$

we obtain

$$\nabla\cdot\delta\mathbf{v}(\mathbf{r},t) = 0, \quad \nabla\cdot\delta\mathbf{m}(\mathbf{r},t) = 0, \tag{2.9}$$

$$\rho\,\frac{\partial\delta\mathbf{v}(\mathbf{r},t)}{\partial t} = \frac{1}{2\chi}\nabla\times[\delta\mathbf{m}(\mathbf{r},t)\times\mathbf{M}], \tag{2.10}$$

$$\frac{\partial\delta\mathbf{m}(\mathbf{r},t)}{\partial t} = \frac{1}{2}[[\nabla\times\delta\mathbf{v}(\mathbf{r},t)]\times\mathbf{M}]. \tag{2.11}$$

This set of coupled equations describes transmission of linear fluctuations in incompressible spin-polarized

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1) The linear elastodynamics of material displacements u_i in an isotropic solid under pure shear deformations that are not accompanied by density fluctuations is described by the Lamé equation [20]

$$\rho\,\frac{\partial^2 u_i}{\partial t^2} = \frac{\partial\sigma_{ik}}{\partial x_k}, \quad \frac{\partial u_k}{\partial x_k} = 0, \qquad \sigma_{ik} = 2\mu\,u_{ik}, \quad u_{ik} = \frac{1}{2}\left(\frac{\partial u_k}{\partial x_i} + \frac{\partial u_i}{\partial x_k}\right), \tag{2.4}$$

where σ_{ik} is the symmetric tensor of elastic stresses, μ is the shear modulus, and u_{ik} is the strain tensor.

baryon matter that are not accompanied by the appearance of the density of magnetic poles (the right-hand sides of Eqs. (2.9)). Substitution of the plane-wave form of the fluctuating variables

$$\delta\mathbf{v} \propto \exp(i\omega t - i\mathbf{k}\cdot\mathbf{r}), \quad \delta\mathbf{m} \propto \exp(i\omega t - i\mathbf{k}\cdot\mathbf{r}) \quad (2.12)$$

in (2.11) leads to the transversality conditions

$$\mathbf{k}\cdot\delta\mathbf{v} = 0, \quad \mathbf{k}\cdot\delta\mathbf{m} = 0.$$

Inserting (2.12) in (2.10) yields

$$\omega\rho\delta\mathbf{v} = -\frac{1}{2\chi}(\mathbf{k}\cdot\mathbf{M})\delta\mathbf{m}.$$

After substitution of (2.12) in (2.11), we obtain that

$$\omega\delta\mathbf{m} = -\frac{1}{2}[(\mathbf{k}\cdot\mathbf{M})\,\delta\mathbf{v} - \mathbf{k}(\delta\mathbf{v}\cdot\mathbf{M})].$$

Taking the scalar product of the last equation with $\mathbf{k} \neq 0$ and considering the above transversality conditions, we obtain

$$\delta\mathbf{v}\cdot\mathbf{M} = 0.$$

Given this, the link between the frequency and the wave vector in the magneto-elastic wave is defined by the coupled equations

$$\begin{aligned} &\omega\rho\delta\mathbf{v} + \frac{1}{2\chi}(\mathbf{k}\cdot\mathbf{M})\delta\mathbf{m} = 0, \\ &\omega\delta\mathbf{m} + \frac{1}{2}(\mathbf{k}\cdot\mathbf{M})\,\delta\mathbf{v} = 0. \end{aligned} \quad (2.13)$$

Eliminating $(\mathbf{k}\cdot\mathbf{M})$, we find that magneto-elastic oscillatory motions satisfy the energy equipartition principle

$$\frac{\rho\,\delta\mathbf{v}^2}{2} = \frac{\delta\mathbf{m}^2}{2\chi}, \quad (2.14)$$

which states that in the magneto-elastic wave, the kinetic energy of fluctuating elastic displacements equals the mean potential energy of fluctuating magnetization. The compatibility of Eqs. (2.13) leads to the dispersion relation of the magneto-elastic wave,

$$\omega^2 = \frac{(\mathbf{k}\cdot\mathbf{M})^2}{4\chi\rho} = \frac{\chi}{4\rho}(\mathbf{k}\cdot\mathbf{B})^2 = V_M^2\,k^2\,\cos^2\theta, \quad (2.15)$$

where θ is the angle between $\mathbf{k}$ and $\mathbf{M}$. It is remarkable that the speed of the wave transport of magnetization

$$V_M = \sqrt{\frac{MB}{4\rho}} \quad (2.16)$$

in paramagnetic neutron matter is proportional to the intensity of the fossil magnetic field $\mathbf{B}$; in ferromagnetic neutron matter, this speed is proportional to the intensity of the spontaneous magnetization $\mathbf{M}$. This is noteworthy because the magneto-elastic wave transport of magnetization is characterized by a dispersion-free law, $\omega \propto k$, in contrast to spin waves that have quadratic dispersion in k, $\omega \propto k^2$. It is therefore expected that under the cooling of paramagnetic neutron star, the temperature variation of the equilibrium magnetization $M(T)$ follows the Curie law

$$\frac{M(T)}{B} = \chi(t) \propto T^{-1},$$

which is due to the dispersion-free nature of magnetophonons, instead of the Bloch law

$$M(0) - M(T)/M(0) \propto T^{3/2}$$

for ferromagnetic dielectrics, which is due to quadratic dispersion of magnons.

Deserving special comment is the case of the homogeneous spherical mass of paramagnetic matter, which is obviously of particular relevance for neutron stars. In the case of the homogeneous spherical mass of (nonferromagnetic) magnetics, the internal magnetic field is uniform and is expressed by the equations

$$\mathbf{B} + 2\mathbf{H} = 0$$

and

$$\mathbf{B} = \mathbf{H} + 4\pi\mathbf{M},$$

which imply that

$$\mathbf{B} = \frac{8\pi}{3}\mathbf{M};$$

see, for instance, Ref. [21, § 76, problem 2], where it is emphasized that the latter equations hold for solely nonferromagnetic materials. Substituting this latter value of $\mathbf{B}$ in (2.16), we find

$$V_M = \sqrt{\frac{2\pi}{3}\frac{M^2}{\rho}}.$$

This form of the speed of the magneto-elastic wave is very similar to that found in [1]. On this ground, we can conclude that magneto-elastic waves is a feature generic to the permanent magnetization of neutron star matter of both ferromagnetic and paramagnetic types. For condensed media possessing a highly pronounced property of magnetic polarizability, the considered magneto-elastic dynamic wave has the same physical significance as the Alfvén magneto-hydrodynamic wave does for incompressible magnetoactive plasma.

Quantitatively, the speed of a magneto-elastic wave in paramagnetic neutron matter compressed to the normal nuclear density with the magnetic field strength $B \sim 10^{12}$–10^{14} (cm/s)·G, typical of pulsars and magnetars, falls into the interval $10^5 < V_M < 10^6$ cm/s; for comparison, the speed of the zero-temperature longitudinal sound wave is

$$c_s = \sqrt{\frac{v_F^2}{3}} \approx 10^9 \text{ cm/s}.$$

The transverse magneto-elastic wave is therefore a slowly propagating exitation in spin-polarized neutron star matter possessing properties of the degenerate paramagnetic Fermi gas of neutrons.

3. NONRADIAL MAGNETO-ELASTIC PULSATIONS OF A PERMANENTLY MAGNETIZED NEUTRON STAR

The purpose of the remainder of this paper is to elucidate the character of mechanical distortions of a neutron star caused by strong coupling between fluctuations of the local magnetization and material displacement and their effect on electromagnetic activity of a paramagnetic neutron star. In doing this, we focus on nonradial magneto-elastic pulsations, which are of particular interest in pulsar astrophysics [23–25]. Circumstantial evidence for the neutron star pulsations is the coherence of millisecond micropulses inferred in [26].

The eigenfrequencies of nonradial magneto-elastic pulsations can be computed on the basis of the energy variational principle. The starting point of this method is the energy balance equation

$$\frac{\partial}{\partial t}\int \frac{\rho\delta\mathbf{v}^2}{2}\,dV = \int [\delta\mathbf{m}\times\mathbf{B}]\cdot\delta\boldsymbol{\omega}\,dV, \qquad \delta\boldsymbol{\omega} = \frac{1}{2}[\nabla\times\delta\mathbf{v}(\mathbf{r},t)], \tag{3.1}$$

which is obtained by taking the scalar product of (2.10) with $\delta\mathbf{v}$ and integrating by parts over the star volume; the surface integral is then dropped because the crustal material of a neutron star possesses properties of a magnetoactive solid-state plasma in which the magnetic ordering effects are heavily suppressed. The left-hand side of (3.1) exhibits a substantially rotational character of motions accompanying magneto-elastic pulsations of a permanently magnetized neutron star. The next step is to use the factorized representation of the velocity and vorticity fields

$$\delta\mathbf{v}(\mathbf{r},t) = \mathbf{a}(\mathbf{r})\dot\alpha(t), \quad \delta\boldsymbol{\omega}(\mathbf{r},t) = \boldsymbol{\phi}(\mathbf{r})\dot\alpha(t), \qquad \boldsymbol{\phi}(\mathbf{r}) = \frac{1}{2}\,[\nabla\times\mathbf{a}(\mathbf{r})], \tag{3.2}$$

where $\mathbf{a}(\mathbf{r})$ is the field of instantaneous displacements and $\alpha(t)$ defines the temporal evolution of fluctuations. Inserting (3.2) in (2.11) and eliminating time derivatives, we obtain

$$\delta\mathbf{m}(\mathbf{r},t) = \boldsymbol{\mu}(\mathbf{r})\alpha(t), \qquad \boldsymbol{\mu}(\mathbf{r}) = [\boldsymbol{\phi}(\mathbf{r})\times\mathbf{M}] = \frac{1}{2}[[\nabla\times\mathbf{a}(\mathbf{r})]\times\mathbf{M}]. \tag{3.3}$$

Substitution of (3.2) and (3.3) in (3.1) leads to

$$\frac{dH}{dt} = 0, \quad H = \frac{M\dot\alpha^2}{2} + \frac{K\alpha^2}{2} \to \ddot\alpha + \omega^2\alpha = 0, \qquad \omega^2 = \frac{K}{M}. \tag{3.4}$$

where the inertia M and the stiffness K of magneto-elastic vibrations are given by

$$M = \int \rho\,\mathbf{a}^2\,dV, \qquad K = \chi^{-1}\int \boldsymbol{\mu}^2\,dV = \frac{1}{4\chi}\int [[\nabla\times\mathbf{a}]\times\mathbf{M}]^2\,dV. \tag{3.5}$$

Thus, computing the frequency of the magneto-elastic mode requires specifing the field $\mathbf{a}$ of instantaneous displacements that have the differentially rotational character, as follows from the expression for the coefficient K of the restoring force of magneto-elastic pulsations.

3.1. Comments on nonradial elastic pulsations of a solid star

The eigenmodes of neutron stars associated with deformation properties of incompressible baryon material, highly robust to mechanical distortions, can be specified, as was first suggested in [23], by spheroidal and torsional modes of shear elastic vibrations of a solid sphere. This terminology is due to Lamb [27], who first tackled the latter problem and gave its solution for substantially radial spheroidal and torsional elastic vibrations of a solid sphere (see, e.g., [28]). In the meantime, the case of nonradial pulsations, which is of particular interest in the astrophysics of compact stars, has not been considered in the literature on elasticity and therefore deserves a special analysis. Essentially, the problem is as follows. From classical equations of elastodynamics (2.4), it follows that the field of material displacements

$$\mathbf{u}(\mathbf{r}, t) = \mathbf{a}(\mathbf{r})\alpha(t)$$

corresponding to standing elastic waves of pure shear satisfying the Helmholtz equation

$$\nabla^2 \mathbf{u} + k^2 \mathbf{u} = 0.$$

Clearly, this equation holds for the solenoidal field of instantaneous displacements,

$$\nabla^2 \mathbf{a} + k^2 \mathbf{a} = 0,$$

where

$$k^2 = \omega^2 / c_t^2$$

and

$$c_t^2 = \mu/\rho$$

is the speed of elastic shear waves in solid bulk. The poloidal solution

$$\mathbf{a}_p = A_p(L)\, \nabla \times \nabla \times [\mathbf{r}\, j_L(kr)\, P_L(z)]$$

describes even-parity spheroidal modes. The toroidal solution

$$\mathbf{a}_t = A_t(L)\, \nabla \times\, [\mathbf{r}\, j_L(kr)\, P_L(z)]$$

describes odd-parity torsional modes; hereafter, $j_L(kr)$ is the spherical Bessel function and $P_L(z)$ ($z = \cos\theta$) is the Legendre polynomial of the multipole degree L. General properties of solenoidal vector fields, both the toroidal and the poloidal ones, can be found in [29]. The arbitrary constants and the frequencies of these modes are customarily found from the boundary condition of a stress-free surface,

$$n_k \sigma_{ik}|_{r=R} = 0$$

(where n_i are components of the unit vector normal to the surface), which leads to a transcendent dispersion equation whose roots are determined by the nodal structure of Bessel functions. In the case of low-frequency nonradial substantially long wavelengths, $\lambda \to \infty$, with

$$k = \omega/c_t = 2\pi/\lambda \to 0,$$

the Helmholtz equation of standing shear waves is reduced to the vector Laplace equation for the solenoidal field of elastic displacements,

$$\nabla^2 \mathbf{a} = 0, \quad \nabla \cdot \mathbf{a} = 0. \tag{3.6}$$

The poloidal and toroidal solutions of (3.6) are given by [24]

$$\mathbf{a}_p = N_p(L)\, \nabla \times \nabla \times [\mathbf{r}\, r^L\, P_L(z)] = \\ = N_p(L+1)\, \nabla\, r^L\, P_L(z) \tag{3.7}$$

$$\mathbf{a}_t = N_t(L)\, \nabla \times [\mathbf{r}\, r^L\, P_L(z)]. \tag{3.8}$$

From the standpoint of Lamb's solutions for the fields of displacements, the spherical Bessel function $j_L(kr)$ determining the radial dependence of $\mathbf{a}(r, \theta)$ asymptotically tends in the long wavelength limit to the function r^L that has no nodes in the interval $0 < r < R$; from this, the term nonradial vibrations is derived. The frequencies of nonradial shear modes can be computed from the above expanded energy variational principle. Taking the scalar product of Lamé equation (2.4) with

$$u_i(\mathbf{r}, t) = a_i(\mathbf{r})\, \alpha(t)$$

and integrating over the volume, we obtain

$$M\ddot{\alpha} + K\alpha = 0, \quad M = \int \rho\, a_i\, a_i\, dV, \\ K = \frac{\mu}{4} \int \left(\frac{\partial a_i}{\partial x_j} + \frac{\partial a_j}{\partial x_i} \right)^2 dV. \tag{3.9}$$

Substituting in (3.9) the poloidal and the toroidal displacement fields in respective Eqs. (3.7) and (3.8) allows us to analytically express the respective frequences of nonradial spheroidal and torsional shear modes $\omega_s(L)$ and $\omega_t(L)$ of a spherical mass of an elastic solid through the multipole degree L as

$$\omega_s(L) = \omega_E[2(2L+1)(L-1)]^{1/2}, \\ \omega_t(L) = \omega_E[(2L+3)(L-1)]^{1/2}, \quad \omega_E = \frac{c_t}{R}, \tag{3.10}$$

where $\omega_E = [\mu/(\rho\, R^2)]^{1/2}$ is the natural unit of frequency of elastic shear vibrations. Equations (3.10) were obtained in recent works [30] in a somewhat different context. The goal of this short comment was to demonstrate the efficiency of the energy variational principle in the study of nonradial vibrations, which allows computing the frequency of both the even-parity s-mode and the odd-parity t-mode of the solid sphere on an equal footing. It is also noteworthy that the problem of inertial waves in a uniformly rotating solid, which, in our opinion, is of particular interest in the study of pulsations of rotating neutron stars, was only recently considered and solved in [31].

The fact that spin-polarized neutron matter can transmit perturbations by transverse waves indicates that the magnetic field penetrated into the body of the star imparts to stellar material a supplementary portion of elasticity generic to solids. In computing periods of nonradial magneto-elastic pulsations of a permanently magnetized neutron star, it therefore seems natural to use the fields of instantaneous displacements corresponding to nonradial spheroidal and torsional shear vibrations of a solid sphere. In doing this, we note that the poloidal vector field associated with spheroidal nonradial pulsations is irrotational,

$$\nabla \times \mathbf{a}_p = 0.$$

This implies that a paramagnetic neutron star does not support nonradial spheroidal pulsations (because the coefficient of the restoring force K in Eq. (3.5) vanishes), but solely supports nonradial torsional shear pulsations coupled with fluctuations in magnetization.

3.2. Periods of torsional magneto-elastic pulsations

Under the global nonradial differentially rotational vibrations of a neutron star, the velocity field of torsional material displacements is described by [24, 25]

$$\begin{aligned} &\delta\mathbf{v}(\mathbf{r},t) = \dot{\mathbf{u}}(\mathbf{r},t) = \mathbf{a}_t(\mathbf{r})\dot{\alpha}(t) = \\ &\quad = \frac{1}{2}[\delta\boldsymbol{\omega}(\mathbf{r},t)\times\mathbf{r}], \\ &\delta\boldsymbol{\omega}(\mathbf{r},t) = N_t \nabla r^L\, P_L(z)\dot{\alpha}(t). \end{aligned} \tag{3.11}$$

The constant N_t is eliminated from the boundary condition

$$\delta\mathbf{v}\big|_{r=R} = \frac{1}{2}[\boldsymbol{\Omega}(t)\times\mathbf{R}], \quad \boldsymbol{\Omega}(t) = \dot{\alpha}(t)\,\nabla P_L(z),$$

as

$$N_t = \frac{1}{R^{L-1}}. \tag{3.12}$$

The dipole field, with $L = 1$, corresponds to the rigid-body rotation of the star, because the angular velocity becomes a homogeneous vector. The differentially rotational deformations of the star corresponding to quadrupole, $L = 2$, and octupole, $L = 3$, overtones of nonradial torsional pulsations are illustrated in Fig. 1. In spherical polar coordinates, the components of the toroidal field of instantaneous displacements $\mathbf{a}_t(\mathbf{r})$ in the star undergoing torsional nonradial pulsations about polar axis are given by

$$\begin{aligned} &a_r = 0, \quad a_\theta = 0, \\ &a_\phi = N_t r^L (1-z^2)^{1/2}\frac{dP_L(z)}{d\mu}. \end{aligned} \tag{3.13}$$

Computed with this field, the parameter of inertia as a function of the multipole degree of vibration is given by

$$M(L) = \int \rho\mathbf{a}_t^2 dV = 4\pi\rho R^5 \frac{L(L+1)}{(2L+1)(2L+3)}. \tag{3.14}$$

It is easy to see that at $L = 1$, this parameter equals the moment of inertia of rigid sphere,

$$M(L=1) = \frac{2}{5}\mathcal{M}R^2,$$

where

$$\mathcal{M} = \frac{4\pi}{3}\rho\, R^3$$

is the star mass.

In the general case, the direction of the equilibrium magnetic anisotropy $\mathbf{M}$ can be tilted to the polar axis about which the torsional pulsations of the star occur,

$$\begin{aligned} &M_r = M\left[(1-z^2)^{1/2}\cos\phi\,\sin\beta + z\cos\beta\right], \\ &\quad z = \cos\theta, \\ &M_\theta = M\left[z\cos\phi\,\sin\beta - (1-z^2)^{1/2}\cos\beta\right], \\ &M_\phi = -M\,\sin\phi\,\sin\beta, \end{aligned} \tag{3.15}$$

where β is the inclination angle between the polar axis z and the vector $\mathbf{M}$. After simple, but fairly tedious calculation of integrals, we obtain the following analytic form of the stiffness:

$$\begin{aligned} K(L) &= \frac{1}{4\chi}\int [[\nabla\times\mathbf{a}_t]\times\mathbf{M}]^2\, dV = \\ &= \pi MBR^3\frac{L(L^2-1)(L+1)}{4L^2-1}\times \\ &\quad \times\cos\beta\left[1+\frac{3L-1}{2(L-1)}\operatorname{tg}\beta\right]. \end{aligned} \tag{3.16}$$

The frequency of a nonradial torsional magneto-elastic mode is given by

$$\begin{aligned} &\omega^2(L) = \omega_M^2(L^2-1)\frac{2L+3}{2L-1}\cos\beta\times \\ &\quad\times\left[1+\frac{3L-1}{2(L-1)}\operatorname{tg}\beta\right], \quad \omega_M^2 = \frac{V_M^2}{R^2}, \end{aligned} \tag{3.17}$$

where ω_M is the natural unit of frequency and $V_M = [MB/4\rho]^{1/2}$ is the speed of the magneto-elastic wave in bulk. This mode can be considered as a magneto-elastic counterpart of Walker's mode for spherical homogeneous mass of a ferromagnetic solid [11]. For the adopted constitutive equation of paramagnetic matter $B = \chi^{-1}M$, this frequency is given by

$$\omega_M^2 = \frac{M^2}{4\chi\rho\, R^2}. \tag{3.18}$$

Fig. 1. Geometrical illustration of torsional deformations of a neutron star undergoing quadrupole (left) and octupole (right) nonradial pulsations

For an ideal homogeneous magnetic sphere, with $B = (8\pi/3)M$, this frequency is given by

$$\omega_M^2 = \frac{2\pi}{3}\frac{M^2}{\rho R^2}. \quad (3.19)$$

The corresponding period is $P_M = (2\pi\omega_M)^{-1}$. This mode, which is said to be the magneto-torsional or m/t mode in what follows, is unique to the permanent magnetization of neutron star matter and is an axial or abnormal parity mode. In the case where $\beta = 0$ (the model of the aligned magnetic torsator), the frequency of the m/t mode is given by

$$\omega(L) = \omega_M \, [(L^2-1)(2L+3)/(2L-1)]^{1/2}$$

(see [16]). This equation implies that the asymptotic shortening of the period $P(L) = (2\pi\omega(L))^{-1}$ as $L \to \infty$ is inversely proportional to the multipole degree of vibrations, $P(L) \propto 1/L$. On the other hand, this indicates the lengthening of periods as the multipole degree of vibration L decreases. It seems quite plausible that under the implosive effect of a supernova event or starquake, the permanently magnetized core of the nascent neutron star can show a highly restless oscillatory behavior characterized by sufficiently large values of L, whereas a mature object becomes quieter and its transition to lower overtones of magneto-elastic pulsations is accompanied by lengthening of periods.

3.3. Application to pulsars and magnetars

To estimate the timing of magneto-elastic pulsations, we here evaluate periods of the m/t mode for

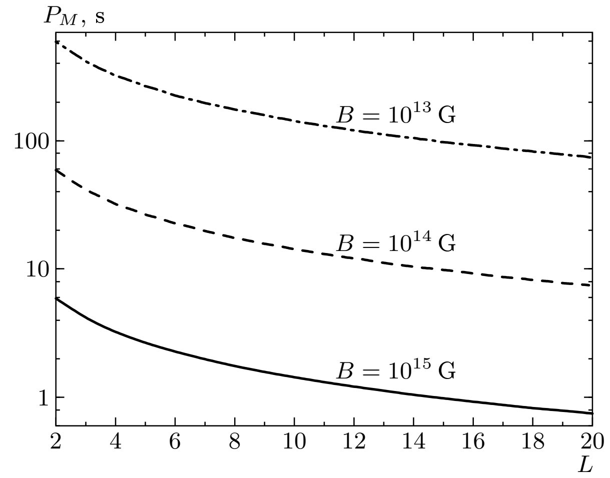

Fig. 2. The period P_M of nonradial torsional magneto-elastic pulsations (in seconds) of a neutron star as a function of the multipole degree L of vibration

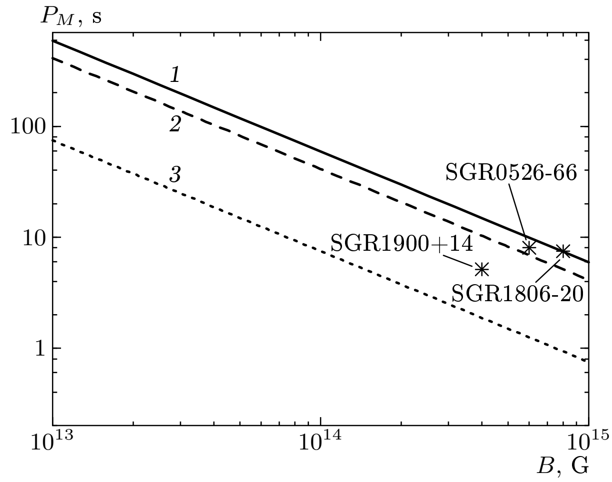

Fig. 3. The period of nonradial torsional magneto-elastic pulsations (in seconds) versus the magnetic field intensity B and data on periods of pulsed gamma emission of soft gamma repeaters taken from [38]; $M = 1.4 M_{sun}$, $R = 12$ km; $L = 2$ (*1*), 3 (*2*), 20 (*3*)

a homogeneous model of a paramagnetic neutron star with the standard parameters, the mass $M = 1.4\,M_{\odot}$ and the radius $R = 12$ km, and with the magnetic susceptibility taken from the model of the degenerate paramagnetic Fermi gas of neutrons condensed to the normal nuclear density, which corresponds to the homogeneous neutron star model with the above parameters. In Fig. 2, we plot the period $P(L)$ as a function of the multipole degree of vibration L, computed in the model of the aligned magnetic torsator for the magnetic field intensity typical of both radio pulsars, $B \sim 10^{11}$–10^{13}, and supermagnetic anomalous X-ray pulsars and soft gamma repeaters $B \sim 10^{14}$–10^{15}, dubbed magnetars [34]. For a neutron star with the magnetic field of Crab-pulsar, the expected period of the m/t mode is $P \sim 3$–5 min. It is remarkable that the computed periods are close to those for pulsed gamma emission of currently monitored soft gamma repeaters (see, e.g., [35]). One of the salient features of the soft gamma repeater radiation activity is that they do not display radiation in the radio region. The pulsed gamma emission of soft gamma repeaters becomes well discernible just after highly energetic gamma bursts [36], which are presumably associated with irregular starquakes [37]. In Fig. 3, the period of torsional magneto-elastic pulsations of a paramagnetic neutron star is pictured in juxtaposition with data on the periodic pulsed radiation of soft gamma repeaters. Bearing in mind that the computed periods fall into the realm of pulsed gamma emission of magnetars, we conjecture that the detected 5–10 s periodicity of their pulsed gamma-activity is powered by nonradial torsional magneto-elastic vibrations exhibiting permanent magnetization of this class of neutron stars.

3.4. Magnetosphere of a permanently magnetized neutron star

One more remarkable inference of the model under consideration is that a paramagnetic neutron star undergoing nonradial torsional magneto-elastic pulsations is capable of generating a periodically oscillating electric field inducing the magnetospheric effect that has many features in common with the Goldreich–Julian effect [32, 33]). This can be readily seen from the Minkowski equation describing the dielectric polarizability $\mathbf{D}$ in moving permanently magnetized matter of nonferromagnetic type (see, e.g., [21])

$$\mathbf{D} = \epsilon\mathbf{E} + \frac{4\pi(2\epsilon+1)}{3c}\,[\mathbf{v}\times\mathbf{M}], \tag{3.20}$$

under the assumption that the dielectric permeability of spin-polarized baryon matter is infinitely large, $\epsilon \to \infty$, as in metallic solids. For a linear, small-amplitude, differentially rotational fluctuations of such matter around the equilibrium state with $\mathbf{v}_0 = 0$, Eq. (3.20) is reduced to

$$\delta\mathbf{D} = \epsilon\delta\mathbf{E} + \frac{4\pi(2\epsilon+1)}{3c}[\delta\mathbf{v}\times\mathbf{M}], \tag{3.21}$$

Fig. 4. Cross-section of the paramagnetic neutron star structure according to expected electromagnetic properties of stellar matter

and as $\epsilon \to \infty$, the last equation becomes[2)]

$$\delta \mathbf{E} = -\frac{8\pi}{3c}[\delta \mathbf{v} \times \mathbf{M}], \quad \delta \mathbf{v} = \frac{1}{2}[\delta \boldsymbol{\omega} \times \mathbf{r}]. \tag{3.22}$$

Identifying the angular velocity magnitude with the frequency of magneto-torsional pulsations, we find the intensity of equatorial electric field $E \sim 10^{10}$–$10^{12}\ P^{-1} \cdot \mathrm{V} \cdot \mathrm{cm}^{-1}$. This field pulls off the charged particles from the star surface and accelerates them along the open magnetic field lines frozen into the neutron star; the electric force $F_e \sim eE$ is much greater

[2)] Equation (3.22) has the same physical meaning as the equation

$$\mathbf{E} = -\frac{1}{c}[\mathbf{u} \times \mathbf{B}], \quad \mathbf{u} = \frac{1}{2}[\boldsymbol{\omega} \times \mathbf{r}],$$

in the Goldreich–Julian theory [32] of pulsar magnetosphere resulting from the perfect conductivity condition $\sigma \to \infty$ in the Ohm law:

$$\mathbf{j} = \sigma\left(\mathbf{E} + \frac{1}{c}[\mathbf{v} \times \mathbf{B}]\right).$$

than the Newtonian force of gravitational attraction $F_g \sim m\,g$: the ratio $F_e/F_g \sim 10^8$–10^{10}. The density of the resultant magnetospheric polarization charge is given by

$$\delta\rho = \frac{1}{4\pi}\nabla\cdot\delta\mathbf{E} = -\frac{4}{3c}(\delta\boldsymbol{\omega}\cdot\mathbf{M}). \qquad (3.23)$$

Numerically, the particle density of the polarization charge $\delta n_c = |\delta\rho/e|$ is of the order $10^{-2}B\cdot P_M^{-1}\cdot\text{cm}^{-3}$. We can expect that magneto-elastic pulsations causing periodic fluctuations of the open magnetic field lines frozen into the star core should affect the electromagnetic (synchrotron and/or curvature [39]) radiation by periodic deviations of the beam direction. For neutron stars with the magnetic field intensity typical of radio pulsars, the above periodicity manifests itself as a long periodic modulation of the main pulse train. In searching for this effect, the satellite-based telescopes seem to be more promising, because proper rotation of the Earth highly limits the monitoring time of radio pulsars by stationary Earth-based telescopes. Understandably, this discussion is suggestive rather than conclusive.

4. SUMMARY AND CONCLUSION

While the magnetic flux conservation in the process of contraction of the main sequence star, predicted in [40], serves as a sufficiently reliable guide in estimating the surface magnetic field for both pulsars and magnetars, the electrodynamics of neutron star matter responsible for the long-term stability of such highly intense fields remains one of the challenges in astrophysics of compact stars (e.g., [41]). One of the plausible explanations is that the fossil magnetic field of a collapsed massive star, amplified by processes of catastrophic implosion, resides in the star interior by causing strong spin polarization of baryon matter in the neutron star core such that the main body of the neutron star mass comes into gravitational equilibrium in the state of permanent magnetization promoted by the Pauli paramagnetism. The resultant structure of the paramagnetic neutron star relevant to this scenario, pictured in Fig. 4, is thought of as a dense magnetic core (composed of spin-polarized baryon matter) covered by a magnetoactive solid-state plasma (composed of highly mobile electrons and the crystallized structure of immobilized protons and nuclei). It is noteworthy that the presence of a magnetic core provides a natural justification of the magnetoplasma processes in the neutron star crust like Alvfén waves [42] and helicons [43].

To explore characteristic features of electromagnetic activity of a neutron star owing its origin to the permanent magnetization of stellar material, we have considered a highly idealized model of a homogeneous paramagnetic star undergoing global nonradial magneto-elastic pulsations. Highlighted are magneto-elastic dynamics equations adopted from the macroscopic theory of poorly conducting magnetics; it was shown that this theory can be efficiently utilized in the study of motions of permanently magnetized stars associated with large-scale transport of magnetization in an incompressible magnetically ordered stellar matter. What is newly disclosed here is that a permanently magnetized neutron star can support torsional nonradial magneto-elastic pulsations generating the electric field responsible for the neutron star magnetosphere. The net outcome of this paper is that the paramagnetic magnetization of neutron star matter is not inconsistent with the available data on electromagnetic activity of both pulsars and magnetars.

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