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WEIBEL INSTABILITY IN PLASMA PRODUCED BY A SUPER-INTENSE FEMTOSECOND LASER PULSE

*V. P. Krainov**

Moscow Institute of Physics and Technology
141700, Dolgoprudny, Moscow Region, Russia

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The Weibel instability increment is analytically derived for plasma produced at the barrier-suppression ionization of atoms and atomic ions by a super-intense femtosecond laser pulse. The cases of linear and circular polarization are considered. Relativistic effects are discussed. It is found that the instability increment is larger for the circular polarization than for the linear polarization. This increment can attain the plasma frequency. Barrier-suppression ionization diminishes the increment compared with the case of tunneling ionization. Relativistic effects also decrease the value of the increment. Estimates of the produced maximum quasistatic magnetic field are given.

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1. INTRODUCTION

Erich Weibel [1] (see also textbook [2]) was first to predict spontaneously growing transverse electromagnetic waves in plasma due to an anisotropic velocity distribution of electrons. The maximum increment of this instability for the wave frequency ω is (in the nonrelativistic approximation)

$$\operatorname{Im}\omega = \frac{u}{c}\omega_p,$$

where ω_p is the plasma frequency and u is the average velocity of electrons in the (longitudinal or transverse) direction along which this velocity has a maximum. This solution is valid under the condition of a strong anisotropy of the velocity distribution in longitudinal and transverse directions.

This approach has been applied in Ref. [3] to electrons produced in the tunneling ionization of atoms by a strong low-frequency linearly polarized laser field. The corresponding average velocities of electrons along the field strength polarization $u_\parallel$ and in the transverse plane $u_\perp$ strongly differ from each other. Their ratio is found in Ref. [4],

$$\frac{u_\perp}{u_\parallel} = \frac{\gamma}{\sqrt{3}},$$

where

$$\gamma = \frac{\omega_0\sqrt{2E_i}}{F}$$

is the Keldysh parameter (the atomic system of units with $e = m = \hbar = 1$ is used in this paper). Here, F is the field strength amplitude and ω_0 is the laser frequency. The quantity $E_i \gg \omega_0$ is the ionization potential of the atom (or atomic ion). In the case of tunneling ionization, we have $\gamma \ll 1$. It was found in Ref. [3] that the maximum instability increment is

$$\operatorname{Im}\omega = \frac{u_\parallel}{c}\omega_p,$$

where, according to Ref. [4],

$$u_\parallel = \frac{\sqrt{3F^3/2}}{\omega_0\,(2E_i)^{3/4}}.$$

In this paper, we consider the barrier-suppression ionization that occurs at irradiation of atoms and atomic ions by the field of a super-intense laser pulse with the peak intensity larger than 10^{16} W/cm^2. The corresponding anisotropic distribution of ejected electrons was obtained in Ref. [5]. We can neglect the collisions of strongly heated ejected electrons with analogous electrons and atomic ions (having in mind, e.g., the cluster plasma [6]) because the Weibel instability is developed during a very short time of the order of

*E-mail: krainov@online.ru

ω_p^{-1}. This process occurs at the peak of the superintense femtosecond laser pulse. We solve the problem in the linear regime only, when the perturbation of the velocity distribution function is smaller than the unperturbed distribution function. We find that the real part of the frequency of the Weibel electromagnetic field is much smaller than the laser frequency. We can therefore consider the Vlasov–Maxwell equations for the Weibel field independent of the Maxwell equation for the external laser field.

2. LINEARLY POLARIZED FIELD

We first assume that the external laser radiation pulse is linearly polarized. With only linear terms of perturbation retained, the Boltzmann transport equation for the Weibel electromagnetic field is of the standard form

$$\frac{\partial f}{\partial t} + \mathbf{v}\frac{\partial f}{\partial \mathbf{r}} = -\left(\mathbf{E} + \frac{1}{c}\mathbf{v}\times\mathbf{B}\right)\frac{\partial f_0}{\partial \mathbf{v}},$$

where $f_0(\mathbf{v})$ is a nonisotropic stationary distribution of electrons, f is a perturbation of the distribution function, and $\mathbf{E}$ and $\mathbf{B}$ represent a perturbation of the electromagnetic field (i.e., Weibel field).

Assuming that the first-order quantities $f(\mathbf{v},\mathbf{r},t)$, $\mathbf{E}(\mathbf{r},t)$, and $\mathbf{B}(\mathbf{r},t)$ depend on $\mathbf{r}$ and t only through the factor $\exp(i\omega t + i\mathbf{k}\cdot\mathbf{r})$, we obtain for the Weibel field with the frequency ω and the wave vector $\mathbf{k}$ that

$$i(\omega + \mathbf{k}\cdot\mathbf{v})f = -\mathbf{E}\frac{\partial f_0}{\partial \mathbf{v}} + \frac{1}{c}\mathbf{v}\times\frac{\partial f_0}{\partial \mathbf{v}}\cdot\mathbf{B}. \quad (1)$$

The Maxwell equation

$$\operatorname{rot}\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$$

implies the relation between the electric and magnetic fields,

$$\mathbf{B} = -\frac{c}{\omega}\mathbf{k}\times\mathbf{E}.$$

Substituting this equation in Eq. (1), we find the equation containing only the electric field,

$$i(\omega+\mathbf{k}\cdot\mathbf{v})f = -\mathbf{E}\frac{\partial f_0}{\partial \mathbf{v}} - \frac{1}{\omega}\left[\mathbf{v}\times\frac{\partial f_0}{\partial \mathbf{v}}\right]\cdot[\mathbf{k}\times\mathbf{E}]. \quad (2)$$

We now assume that the wave vector $\mathbf{k}$ is directed along the x axis and the electric field strength $\mathbf{E}$ is directed along the z axis. We then find the function f from Eq. (2) as

$$f = \frac{iE}{\omega + kv_x}\left\{\frac{\partial f_0}{\partial v_z} - \frac{k}{\omega}\left(v_z\frac{\partial f_0}{\partial v_x} - v_x\frac{\partial f_0}{\partial v_z}\right)\right\}. \quad (3)$$

The second Maxwell equation is given by

$$\operatorname{rot}\mathbf{B} = \frac{4\pi}{c}\mathbf{j} + \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t}, \quad (4)$$

where the electric current density is determined by the distribution function,

$$\mathbf{j}(\mathbf{r},t) = \int \mathbf{v} f(\mathbf{v},\mathbf{r},t)\, d^3\mathbf{v}.$$

Substituting this expression and Eq. (3) in Eq. (4), we obtain the Vlasov equation in the form

$$i\mathbf{k}\times\mathbf{B} = \frac{ick^2}{\omega}\mathbf{E} = \frac{4\pi}{c}\int \mathbf{v}\frac{iE}{\omega + kv_x}\times$$
$$\times\left\{\frac{\partial f_0}{\partial v_z} - \frac{k}{\omega}\left(v_z\frac{\partial f_0}{\partial v_x} - v_x\frac{\partial f_0}{\partial v_z}\right)\right\}d\mathbf{v} + \frac{i\omega}{c}\mathbf{E}. \quad (5)$$

Projection of this equation to the z axis gives a dispersion relation between the frequency ω and the wave number k,

$$k^2c^2 - \omega^2 = 4\pi\int_{-\infty}^{\infty} v_z dv_x dv_y dv_z \times$$
$$\times\left\{\frac{\partial f_0}{\partial v_z} - \frac{kv_z}{\omega + kv_x}\frac{\partial f_0}{\partial v_x}\right\}. \quad (6)$$

We simplify the first term in the right side of this equation by taking the normalization condition for the unperturbed distribution function into account,

$$\int f_0 d\mathbf{v} = n,$$

where n is the number density of free electrons; the above equation then becomes

$$k^2c^2-\omega^2+\omega_p^2 = -4\pi k\int_{-\infty}^{\infty} dv_x dv_y dv_z \frac{v_z^2}{\omega+kv_x}\frac{\partial f_0}{\partial v_x}, \quad (7)$$

where we define the plasma frequency

$$\omega_p = \sqrt{4\pi n}.$$

The inequality $\omega \gg kv_x$ is valid in the tunneling and barrier-suppression ionization regimes. It corresponds to the condition that the longitudinal electron velocity v_z is much larger than the transverse electron velocity v_x. We can then expand the denominator in Eq. (7) as

$$\frac{1}{\omega + kv_x} = \frac{1}{\omega} - \frac{kv_x}{\omega^2} + \ldots$$

Substituting this expansion in Eq. (7), we integrate by parts as

$$k^2c^2 - \omega^2 + \omega_p^2 = \frac{4\pi k}{\omega^2}\int_{-\infty}^{\infty} dv_y \int_{-\infty}^{\infty} v_z^2 dv_z \times$$
$$\times \int_{-\infty}^{\infty} v_x \frac{\partial f_0}{\partial v_x} dv_x = -\frac{4\pi k^2}{\omega^2}\int_{-\infty}^{\infty} dv_y \int_{-\infty}^{\infty} v_z^2 dv_z \int_{-\infty}^{\infty} f_0 dv_x,$$

and finally obtain

$$k^2c^2 - \omega^2 + \omega_p^2 = -\omega_p^2 \frac{k^2}{\omega^2}\left\langle v_z^2\right\rangle, \quad (8)$$

where $\left\langle v_z^2\right\rangle$ is the average square of the longitudinal electron velocity.

Because the most interesting case in dispersion relation (8) is $\omega \ll kc$ (see below), we can neglect the term $-\omega^2$ in the left-hand side of (8). It then follows from Eq. (8) that

$$\omega^2 = -\frac{k^2\left\langle v_z^2\right\rangle}{\omega_p^2 + k^2c^2}\omega_p^2, \quad (9)$$

and therefore, the frequency is a purely imaginary quantity that produces the Weibel plasma instability. The maximum value of this instability increment is achieved at $kc \gg \omega_p$ (the short wavelength limit),

$$\omega^2 = -\frac{\left\langle v_z^2\right\rangle}{c^2}\omega_p^2. \quad (10)$$

In the case of tunneling ionization, the distribution function f_0 is of a Gaussian form [4, 7]. Hence, $\left\langle v_z^2\right\rangle = u_\parallel^2$, where

$$u_\parallel^2 = \frac{3\omega_0}{2\gamma^3}$$

(see the Introduction). Here, ω_0 is the laser frequency and γ is the Keldysh parameter. It therefore follows from Eq. (10) that [3]

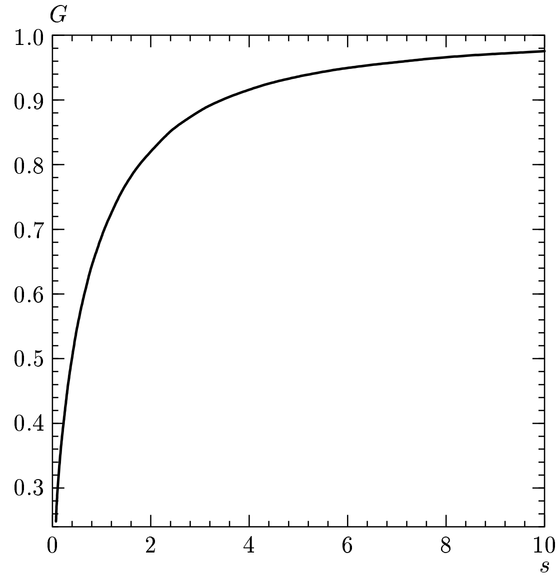
$$\omega^2 = -\frac{u_\parallel^2}{c^2}\omega_p^2 = -\frac{3\omega_0}{2\gamma^3c^2}\omega_p^2 = -\frac{3F^3}{2\omega_0^2(2E_i)^{3/2}c^2}\omega_p^2.$$

We now consider the case of barrier-suppression ionization. According to Ref. [5], the anisotropic distribution is given by

$$f_0(v_z, v_\perp) \sim n\mathrm{Ai}^2\left\{\frac{2E_i + v_z^2\gamma^2/3 + v_\perp^2}{(2F)^{2/3}}\right\}, \quad (11)$$

where $\mathrm{Ai}(x)$ is the Airy function. This distribution reduces to the tunneling limit [4] under the condition of a weak field (compared with the atomic field),

$$F \ll (2E_i)^{3/2}.$$



Therefore, the deviation for the square of the instability increment from the tunneling limit, Eq. (10), is determined by the ratio

$$\frac{\left\langle v_z^2\right\rangle}{u_\parallel^2} = G(s),$$

and hence

$$\omega^2 = -G(s)\frac{3F^3}{2\omega_0^2(2E_i)^{3/2}c^2}\omega_p^2, \quad (12)$$

where

$$s = \frac{2E_i}{(2F)^{2/3}}.$$

The universal function $G(s)$ is given by

$$G(s) = 4\sqrt{s}\frac{\int_0^\infty dt \int_0^\infty z^2 dz \mathrm{Ai}^2\left(s+t+z^2\right)}{\int_0^\infty dt \int_0^\infty dz \mathrm{Ai}^2\left(s+t+z^2\right)}. \quad (13)$$

We have $G(s) \to 1$ at $s \gg 1$ (the tunneling limit). This function is shown in the Figure.

It can be seen that in the case of barrier-suppression ionization, the increment increases more slowly with the increase of the laser field F than in the case of the tunneling ionization. We can therefore conclude that the electromagnetic field is generated in plasma with the same linear polarization as the initial laser radiation that produced these anisotropic plasma electrons. The frequency of this field (see Eq. (10)) does not contain the real component, and this field is therefore quasistationary, but with an exponentially growing amplitude of the electric and magnetic strengths.

We now briefly discuss the relativistic generalization of the tunneling results. According to Ref. [8], the energy distribution of ejected electrons along the field polarization is given by

$$f_0 \propto \exp\left(-\frac{v_z^2\gamma^3}{3\omega_0}\left(1+\frac{3v_z^2}{4\gamma^2c^2}\right)\right).$$

The second term in the exponent is responsible for the relativistic effect. It diminishes the average longitudinal velocity,

$$u_\parallel^2(rel) = \frac{u_\parallel^2}{1+3u_\parallel^2/(2\gamma c)^2},$$

where $u_\parallel^2 = 3\omega_0/2\gamma^3$ (see the Introduction). This decrease of the instability increment is in agreement with the relativistic results in Ref. [9],

$$\omega^2 = -\frac{u_\parallel^2(rel)}{c^2}\omega_p^2.$$

3. CIRCULARLY POLARIZED FIELD

In this section, we consider the Weibel instability produced in plasma during the tunneling and barrier-suppression ionization of atoms (or atomic ions) by a circularly polarized laser femtosecond pulse. We again direct the wave vector $\mathbf{k}$ of the laser field and of the produced electromagnetic perturbation field along the x axis. The perturbation electric field strength $\mathbf{E}$ is also circularly polarized and rotates in the yz plane. Hence,

$$\mathbf{E} = E\left(\mathbf{i}_z + i\mathbf{i}_y\right)\exp\left(i\omega t + i\mathbf{k}\cdot\mathbf{r}\right),$$

where $\mathbf{i}_z$ and $\mathbf{i}_y$ are unit basis vectors. Equation (2) then becomes

$$i\left(\omega+kv_x\right)f = -\frac{E}{v_\parallel}\left(v_z+iv_y\right)\frac{\partial f_0}{\partial v_\parallel} - \\ -\frac{kE}{\omega}\left(v_z+iv_y\right)\left(\frac{v_x}{v_\parallel}\frac{\partial f_0}{\partial v_\parallel}-\frac{\partial f_0}{\partial v_x}\right),$$

where $v_\parallel$ is the velocity in the polarization plane. We thus obtain the perturbation distribution function

$$f = iE\left(v_z+iv_y\right)\left(\omega+kv_x\right)^{-1}\times \\ \times\left\{\frac{1}{v_\parallel}\left(1+\frac{kv_x}{\omega}\right)\frac{\partial f_0}{\partial v_\parallel} - \frac{k}{\omega}\frac{\partial f_0}{\partial v_x}\right\}.$$

Instead of Eq. (6), we find the dispersion relation in the form

$$(k^2c^2-\omega^2)\left(\mathbf{i}_z+i\mathbf{i}_y\right) = 4\pi\int_{-\infty}^{\infty}\mathbf{v}dv_xdv_ydv_z\left(v_z+iv_y\right)\times \\ \times\left\{\frac{1}{v_\parallel}\frac{\partial f_0}{\partial v_\parallel} - \frac{k}{\omega+kv_x}\frac{\partial f_0}{\partial v_x}\right\},$$

or

$$(k^2c^2-\omega^2) = 4\pi^2\int_0^{\infty}v_\parallel^3dv_\parallel\int_{-\infty}^{\infty}dv_x\times \\ \times\left\{\frac{1}{v_\parallel}\frac{\partial f_0}{\partial v_\parallel} - \frac{k}{\omega+kv_x}\frac{\partial f_0}{\partial v_x}\right\}. \quad (14)$$

The unperturbed electron energy distribution function for tunneling ionization is given by (see, e.g., [10])

$$f_0 = \frac{n}{4\pi^2u^2v_0}\exp\left\{-\frac{v_x^2+(v_\parallel-v_0)^2}{2u^2}\right\}, \quad (15)$$

where $v_0 = F/\omega_0$ is the ponderomotive electron velocity and

$$u^2 = \frac{F}{2\sqrt{2E_i}}$$

(F is again the laser field strength amplitude and E_i is the ionization potential of an atom or an atomic ion). Unlike for the linear polarization, dispersions of the average longitudinal and transverse velocities are now equal to each other.

We note that $u \ll v_0$, i.e., the width of the distribution is small compared with its shift in the longitudinal direction. The first term in the right-hand side of Eq. (14) vanishes because the integrand is an odd function of the argument $(v_\parallel - v_0)$. Dispersion relation (14) then becomes

$$(k^2c^2-\omega^2) = -4\pi^2\int_0^{\infty}v_\parallel^3dv_\parallel\int_{-\infty}^{\infty}dv_x\frac{k}{\omega+kv_x}\frac{\partial f_0}{\partial v_x}.$$

We again assume that $\omega \gg kv_x$, i.e., $\omega \gg ku$, and expand the denominator in a Taylor series,

$$\frac{1}{\omega+kv_x} = \frac{1}{\omega} - \frac{kv_x}{\omega^2} + \ldots$$

Integrating by parts, we simplify the dispersion relation as

$$(k^2c^2-\omega^2) = -\frac{4\pi^2k^2}{\omega^2}\int_0^{\infty}v_\parallel^3dv_\parallel\int_{-\infty}^{\infty}f_0dv_x. \quad (16)$$

Substituting Eq. (15) in Eq. (16), we obtain

$$\omega^4 - (kc\omega)^2 - \frac{(\omega_p k v_0)^2}{2} = 0.$$

The solution of this equation is

$$\omega^2 = -\frac{(kc)^2}{2}\left\{\sqrt{1 + 2\left(\frac{\omega_p v_0}{kc^2}\right)^2} - 1\right\} < 0.$$

The frequency ω is therefore a purely imaginary quantity that produces a circularly polarized exponentially increasing electromagnetic wave. Its real part is zero, and the wave is therefore quasistationary. In the short wave limit

$$k \gg \frac{\omega_p v_0}{c^2}, \qquad (17)$$

we simplify this solution by taking into account that $v_0 = F/\omega_0$,

$$\omega^2 = -\frac{1}{2}\left(\frac{v_0}{c}\right)^2 \omega_p^2 = -\frac{1}{2}\left(\frac{F}{\omega_0 c}\right)^2 \omega_p^2. \qquad (18)$$

The condition $\omega \gg ku$ bounds the wave number k from above,

$$k^2 \ll \frac{F\omega_p^2\sqrt{2E_i}}{\omega_0^2 c^2}. \qquad (19)$$

Inequalities (17) and (18) do not contradict each other under the condition

$$F \ll c^2\sqrt{2E_i}$$

which is satisfied up to very high values of the laser field intensities ($c = 137$ a.u.).

In the case of barrier-suppression ionization by a circularly polarized field, the unperturbed distribution function is given by [5]

$$f_0(v_x, v_\parallel) \sim n\mathrm{Ai}^2\left\{\frac{2E_i + \left(v_\parallel - v_0\right)^2 + v_x^2}{(2F)^{2/3}}\right\}.$$

Substituting this expression in Eq. (16), we obtain the same dispersion relation as in the case of tunneling ionization. Therefore, the maximum increment of the Weibel instability is again determined by Eq. (18) also for barrier-suppression ionization.

In the nonrelativistic limit, we have

$$v_0 = \frac{F}{\omega_0} \ll c.$$

Hence, the Weibel increment is small compared with the plasma frequency. The anisotropic relativistic distribution of ejected electrons was obtained in Ref. [10]. Most electrons are ejected not in the polarization plane of circularly polarized laser radiation, but at the angle θ with respect to this polarization plane determined from the relation [11]

$$\mathrm{tg}\,\theta = \frac{F}{2\omega_0 c}.$$

The normalized unperturbed relativistic distribution function is given by [10]

$$f_0(p_x, p_\parallel) = \frac{n}{4\pi^2 u_r^2 v_0} \times \\ \times \exp\left\{-\frac{\left(p_x - v_0\,\mathrm{tg}\,\theta\right)^2 + (p_\parallel - v_0)^2}{2u_r^2}\right\}, \qquad (20)$$

where p_x and $p_\parallel$ are the respective momentum components of the ejected electron along the wave vector and in the polarization plane. The relativistic width u_r of the distribution is given by [10]

$$u_r^2 = \frac{\left(1 + (F/\omega_0 c)^{1/2}\right)^2}{1 + (F/\omega_0 c)^2} u^2,$$

where (see above)

$$u^2 = \frac{F}{2\sqrt{2E_i}}$$

is the nonrelativistic width.

Substituting Eq. (20) in Eq. (16), we find instead of Eq. (18) that

$$\omega^2 = -\frac{1}{2}\frac{\left\langle v_\parallel^2\right\rangle}{c^2}\omega_p^2,$$

where $\sqrt{\left\langle v_\parallel^2\right\rangle}$ is the average relativistic velocity in the polarization plane, to be compared with Eq. (10) for the linearly polarized field. This quantity can be expressed through the corresponding relativistic momentum

$$p_\parallel = v_0 = F/\omega_0$$

and the relativistic energy

$$E_{rel} = c\sqrt{p_\parallel^2 + p_x^2 + c^2}$$

due to narrow peaks in unperturbed distribution (20),

$$\frac{\sqrt{\left\langle v_\parallel^2\right\rangle}}{c} = \frac{p_\parallel c}{E_{rel}} = \frac{v_0}{v_0^2/2c + c}.$$

Thus, we finally obtain

$$\omega = -\frac{i}{\sqrt{2}}\frac{v_0}{v_0^2/2c + c}\omega_p. \qquad (21)$$

It follows that relativistic effects diminish the Weibel increment for a circularly polarized field similarly to the case of linear polarization (see above). The maximum value of the increment is achieved at $v_0 = F/\omega_0 = c\sqrt{2}$,

$$\omega_{max} = -\frac{i}{2}\omega_p.$$

4. CONCLUSIONS

We have found that the plasma instability produces a quasistatic magnetic field B (the frequency does not contain the real part). The corresponding quasistatic electric field E is much smaller in the short-wave limit $kc \gg \omega$,

$$E = \frac{\omega}{kc}B \ll B.$$

We estimate the maximum value of this field for the circularly polarized field. Our derivation is valid in the linear approximation where $f \ll f_0$. In accordance with the results in the previous Section, we rewrite this inequality for a circularly polarized field as

$$\frac{E}{\omega^3}k^2 v_0 \ll 1,$$

or

$$\frac{kBv_0}{\omega^2 c} \ll 1.$$

Substituting $\omega \sim \omega_p(v_0/c)$, we find the maximum magnetic field

$$B_{max} \sim \omega_p c.$$

It follows that the magnetic field is determined only by the number density of plasma electrons and can be very large.

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