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RELATIVISTIC QUANTUM THEORY OF CYCLOTRON RESONANCE IN A MEDIUM

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The relativistic quantum theory of cyclotron resonance in a medium with arbitrary dispersive properties is presented. The quantum equation of motion for a charged particle in the field of a plane electromagnetic wave and in the uniform magnetic field in a medium is solved in the eikonal approximation. The probabilities of induced multiphoton transitions between the Landau levels in a strong laser field are calculated.

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1. INTRODUCTION

As is known, if a charged particle moves in the field of a transverse electromagnetic (EM) wave in the presence of a uniform magnetic field directed along the wave propagation vector, a resonant effect of the wave on the particle motion is possible. If the interaction takes place in the vacuum, this is the well-known phenomenon of autoresonance [1–3], when the ratio of the Doppler-shifted wave frequency ω' to the cyclotron frequency Ω of the particle is conserved, $\omega'/\Omega = \text{const}$, and the resonance created at the initial moment automatically holds in the course of interaction. But if the interaction takes place in a medium where the phase velocity of an EM wave is larger (plasma-like medium) or smaller (dielectric medium) than the light speed in the vacuum, the picture of the wave–particle interaction is essentially changed. In particular, the autoresonance phenomenon is violated in the medium because of a nonequidistant Stark shift of magnetic sublevels of an electron (Landau levels) in the electric field of an EM wave. As a result, the intensity effect of the wave governs the resonance characteristics, and the particle state essentially depends on the initial conditions and the wave field magnitude at which the nonlinear resonance is achieved [4]. The cyclotron resonance (CR) in a medium was first investigated in the scope of classical theory in papers [3, 5], where oscillating solutions for the particle energy were obtained. However, such behavior is valid only for the EM wave intensity less than some critical value. As shown in [4], at the intensities above that critical value, a nonlinear resonance phenomenon of a threshold nature — the so-called «electron hysteresis» — occurs (the EM wave is turned on adiabatically). If the intensity peak of an actual wave pulse exceeds the mentioned critical value, then significant acceleration of charged particles can be achieved (it is clear that the medium must be plasma-like for this purpose) [6].

Below the threshold intensity of the electron hysteresis, when the linear CR occurs in a medium [3, 5], the free electron laser version has been proposed, based on the combined scheme of CR and Cherenkov radiation in a dielectric–gaseous medium [7].

We note that classical equations of motion for this process in a medium allow an exact solution only in a particular case where the initial velocity of a particle is parallel to the wave propagation direction and the wave has a circular polarization (namely, the electron hysteresis phenomenon has been obtained in this case).

Concerning the quantum description of CR, the relativistic quantum equation of motion allows exact solution only for CR in the vacuum [8] (see [9] and references therein for the description of related quantum electrodynamic processes, such as electron–positron pair production, nonlinear Compton scattering in the presence of uniform magnetic field, etc., by this wave function). We note that the configuration of EM fields with a uniform magnetic field directed along the prop-

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agation of the transverse wave is one of the exotic cases where the relativistic quantum equation of motion in the vacuum allows an exact solution. In a medium, even in the absence of a uniform magnetic field, the relativistic quantum equation of motion for the particle–monochromatic wave interaction reduces to the Mathieu type (in general, Hill type) equation, the exact solution of which is unknown. In this case, obtaining an approximate analytic solution describing the nonlinear process of particle–wave interaction is already problematic [10–12].

The purpose of this paper is to obtain a nonlinear (in the field) approximate solution of the relativistic quantum equation of motion for a charged particle in the plane EM wave in a medium in the presence of a uniform magnetic field, a solution that sufficiently well describes the quantum picture of cyclotron resonance in a medium at high intensities of the external radiation field, in particular, multiphoton stimulated transitions between the Landau levels.

In what follows, the wave function of a charged particle moving in a medium in the field of a transverse EM wave in the presence of a uniform magnetic field directed along the wave propagation direction is obtained. Then the multiphoton CR in a medium is considered and the probabilities of induced multiphoton transitions in a strong circularly polarized EM wave are calculated.

2. WAVE FUNCTION OF A CHARGED PARTICLE IN THE PLANE ELECTROMAGNETIC WAVE IN A MEDIUM IN THE PRESENCE OF A UNIFORM MAGNETIC FIELD

Let a charged particle move in a medium in the field of a coherent EM wave and a uniform magnetic field along the wave propagation direction (chosen as the z axis). The four-vector potential of this configuration of the EM field can be represented as

$$A_\mu(x) = A_\mu(x_1) + A_\mu(\tau), \tag{1}$$

where

$$A_\mu(x_1) = (0, x_1 H_0, 0, 0) \tag{2}$$

is the four-vector potential of the uniform magnetic field with the strength $\mathbf{H}_0$ and

$$A_\mu(\tau) = \left\{A_1\left(t - n\frac{x_3}{c}\right), A_2\left(t - n\frac{x_3}{c}\right), 0, 0\right\} \tag{3}$$

is the four-vector potential of a plane transverse EM wave, x is the four-component radius vector, and

$$\tau = t - nx_3/c$$

is the plane wave coordinate. For four-component vectors, we chose the metric $a = (\mathbf{a}, ia_0)$. In (3), $n = n(\omega)$ is the refractive index of the medium and c is the light speed in the vacuum. Hereafter, we take the EM wave to be laser radiation that is quasimonochromatic with high accuracy ($\Delta\omega \ll \omega$, where ω is the carrier frequency),

$$n(\omega) \approx n = \text{const}.$$

We assume that the EM wave is switched on/off adiabatically, and therefore, for the vector potential $A_\mu(\tau)$, we have that

$$A_\mu(\tau) = 0 \quad \text{at } t = \mp\infty.$$

Because we assume the coherent EM wave to be a laser radiation one for which the photon energy is negligibly small compared with the relativistic electron energy, we can neglect the spin interaction, and the Dirac equation in the quadratic form therefore reduces to the Klein–Gordon equation for a charged particle in field (1),

$$\left\{\left(i\hbar\partial_\mu + \frac{e}{c}A_\mu(x)\right)^2 + m^2c^2\right\}\Psi(x) = 0, \tag{4}$$

where m and e are the particle mass and charge, respectively (we assume $e < 0$, with the electron in mind), and

$$\partial_\mu \equiv \frac{\partial}{\partial x_\mu}, \quad \mu = 1, 2, 3, 4,$$

denotes the first derivative of a function over the four-component radius vector x.

The particle quantum motion at $t \to -\infty$, when $A_\mu(\tau) = 0$, is well known and has been the subject of numerous studies (see, e.g., [13]). In the uniform magnetic field, the particle motion is separated into the cyclotron (x_1, x_2) and the longitudinal (x_3) degrees of freedom. Because the coordinate x_2 is cyclic in this case (also in the presence of an EM wave; see (2) and (3)), the cyclotron motion is described by the set of quantum characteristics of the state $\{l, p_2\}$, where the number l labels Landau levels ($l = 0, 1, 2, \ldots$) and p_2 is the x_2 component of the generalized momentum. The longitudinal motion at $t \to -\infty$ is then described by the p_3 component of the particle initial momentum. Concerning the particle transverse initial state, we assume that at $t \to -\infty$, the particle is in the $l = s$ Landau level. Therefore, the wave function of the particle at

$t \to -\infty$ is given by the known formula [13] (with the spin interaction neglected)

$$\Psi(x)|_{t\to-\infty} = = N \exp\left[\frac{i}{\hbar}(p_3 x_3 - E_s(p_3)t)\right] \Phi_{s,p_2}(x_\perp), \quad (5)$$

where N is the normalization constant, $x_\perp = \{x_1, x_2, 0, 0\}$, and

$$\Phi_{s,p_2}(x_\perp) = \exp\left(\frac{i}{\hbar}p_2 x_2\right) U_s \left[\frac{x_1 + p_2 a^2/\hbar}{a}\right], \quad a = \sqrt{\frac{\hbar c}{|e|\, H_0}}, \quad (6)$$

is the wave function corresponding to the cyclotron part of motion. Here, U_s are the Hermit functions and the dispersion law for the particle energy–momentum is

$$E_s^2(p_3) = m^2c^4 + c^2p_3^2 + 2\,|e|\,cH_0\hbar\left(s + \frac{1}{2}\right). \quad (7)$$

Because the EM wave field depends only on the retarded coordinate τ, it is more convenient to pass from the space–time coordinates x_3, t to the wave coordinates

$$\tau = t - nx_3/c, \quad \eta = t + nx_3/c.$$

Then, due to the existence of a certain direction of the wave propagation, the variable η becomes cyclic, and hence the momentum conjugate to the coordinate η is conserved,

$$\frac{1}{2}\left(E_s(p_3) - \frac{c}{n}p_3\right) \equiv \Lambda = \text{const}. \quad (8)$$

This is the known integral of motion in this process according to the classical electrodynamics [4].

The particle wave function can then be sought in the form

$$\Psi(x) = = \exp\left[-\frac{i}{\hbar}\Lambda\eta - \frac{i}{2\hbar}\left(E_s(p_3) + \frac{c}{n}p_3\right)\tau\right] f(x_\perp, \tau), \quad (9)$$

where the unknown function $f(x_\perp, \tau)$ of the variable τ is assumed slowly varying compared with the exponential function of τ in (9). This approximation corresponds to the known eikonal approximation for the particle wave function, in which one can neglect the second derivative of $f(x_\perp, \tau)$ with respect to τ compared with the first-order derivative in the equation of motion (4), which for the function $f(x_\perp, \tau)$ has the form

$$\left\{\frac{\hbar^2}{c^2}(n^2-1)\frac{\partial^2}{\partial\tau^2} + \frac{2i\hbar}{c^2}\widetilde{E}\partial_\tau - \left(i\hbar\partial_\mu^\perp + \frac{e}{c}A_\mu(x)\right)^2 + + \frac{E_s^2(p_3)}{c^2} - m^2c^2 - p_3^2\right\} f(x_\perp, \tau) = 0. \quad (10)$$

Here,

$$\partial_\mu^\perp = \{\partial_1, \partial_2, 0, 0\}, \quad \widetilde{E} = E_s(p_3) - cnp_3.$$

We note that Eq. (10) is already a Hill-type equation even in the absence of a uniform magnetic field, and its exact solution is unknown. We therefore apply the «eikonal approximation», considering $f(x_\perp, \tau)$ a slowly varying function of τ in Eq. (10) (the term with the second derivative of $f(x_\perp, \tau)$ describes the quantum recoil in the interaction of a particle with the EM wave), which is valid under the condition

$$\left|\frac{\hbar(n^2-1)}{2\widetilde{E}}\frac{\partial^2 f}{\partial\tau^2}\right| \ll \left|\frac{\partial f}{\partial\tau}\right|. \quad (11)$$

Such an approximate solution describes the multiphoton interaction of particles with EM fields sufficiently well (for the electron–strong wave interaction in a medium, see [14]). Under condition (11), Eq. (10) implies the following equation for the function $f(x_\perp, \tau)$:

$$\left\{\frac{2i\hbar}{c^2}\widetilde{E}\partial_\tau - \left(i\hbar\partial_\mu^\perp + \frac{e}{c}A_\mu(x)\right)^2 + + \frac{E_s^2(p_3)}{c^2} - m^2c^2 - p_3^2\right\} f = 0. \quad (12)$$

In Eq. (12), the transverse and longitudinal motions are not separated. But after a certain unitary transformation, the variables are separated [9]. The corresponding unitary transformation operator is

$$\widehat{S} = \exp\left\{iK_\mu(\tau)\widehat{P}_{\perp\mu}\right\}, \quad (13)$$

$$\widehat{P}_{\perp\mu} = -i\hbar\partial_\mu^\perp - \frac{e}{c}A_\mu(x_1),$$

$$K_\mu(\tau) = \{K_1(\tau), K_2(\tau), 0, 0\},$$

where $K_\mu(\tau)$ is chosen to separate the cyclotron and longitudinal motions and to satisfy initial condition (5), which is equivalent to the condition

$$K_1 + iK_2 = -\exp\left[-i\frac{ec}{\widetilde{E}}H_0\tau\right] \times \times \int_{-\infty}^{\tau} \frac{ec}{\hbar\widetilde{E}}\left(A_1(\tau') + iA_2(\tau')\right)\exp\left[i\frac{ec}{\widetilde{E}}H_0\tau'\right] d\tau'. \quad (14)$$

For the transformed wave function $\widetilde{f} = \widehat{S} f(x_\perp, \tau)$, we then have the equation

$$\left\{ \widehat{P}^2_{\perp\mu} - \frac{E_s^2(p_3)}{c^2} + p_3^2 + m^2c^2 - \right. \\ - i\frac{2\hbar\widetilde{E}}{c^2}\partial_\tau - \frac{e\hbar^2\widetilde{E}}{c^3}F_{\nu\mu}K_\mu\frac{dK_\nu}{d\tau} + \\ \left. + \left(\frac{e\hbar}{c}F_{\mu v}K_\nu - \frac{e}{c}A_\mu(\tau)\right)^2 \right\} \widetilde{f}(x_\perp, \tau) = 0, \quad (15)$$

where $F_{\mu v}$ is the EM field tensor corresponding to the uniform magnetic field $\mathbf{H}_0$. In Eq. (15), the variables are separated; by means of the inverse transformation $f = \widehat{S}^+ \widetilde{f}(x_\perp, \tau)$, we then obtain the solution of the initial equation (4) (taking Eq. (9) into account),

$$\Psi(x) = \\ = N \exp\left[\frac{i}{\hbar}(p_3x_3 - E_s(p_3)t) - \frac{i}{\hbar}\int_{-\infty}^{\tau} Q(\tau')d\tau'\right] \times \\ \times \exp\left[i\frac{e}{c}H_0K_2(x_1 - \frac{\hbar}{2}K_1)\right] \times \\ \times \Phi_{s,p_2}(x_1 - \hbar K_1, x_2 - \hbar K_2), \quad (16)$$

where

$$Q(\tau) = \frac{c^2}{2\widetilde{E}}\left[\left(\frac{e\hbar}{c}F_{\mu v}K_\nu - \frac{e}{c}A_\mu(\tau)\right)^2 - \right. \\ \left. - \frac{e\widetilde{E}\hbar^2}{c^3}F_{\nu\mu}K_\mu\frac{dK_\nu}{d\tau}\right]. \quad (17)$$

The obtained wave function (16) is valid under condition (11), which means that the particle total energy/momentum exchange occuring as a result of the multiphoton interaction with the strong EM wave at the CR in a medium is much smaller than the initial energy/momentum of the particle. This energy/momentum exchange is determined by the full phase of wave function (16) with expressions (6), (14), and (17), which are found and estimated in the next section.

3. THE PROBABILITIES OF MULTIPHOTON TRANSITIONS BETWEEN LANDAU LEVELS

Although the particle motion in a uniform magnetic field is separated into cyclotron (x_1, x_2) and longitudinal (x_3) degrees of freedom, Eq. (5), these motions are not separated in the energy scale due to relativistic effects (7). For not very strong magnetic fields, however, we can separate the energies of longitudinal $(E_\parallel)$ and cyclotron motions,

$$E_s(p_3) \approx E_\parallel + \hbar\Omega\left(s + \frac{1}{2}\right), \quad s\hbar\Omega \ll E_\parallel, \quad (18)$$

$$\Omega = |e|\, cH_0/E_\parallel, \quad E_\parallel = \sqrt{m^2c^4 + c^2p_3^2}.$$

We now consider the concrete case of a circularly polarized quasimonochromatic EM wave with the main frequency ω and the average value $\overline{A}$ of the slowly varying envelope,

$$A_\mu(\tau) = \left\{-\overline{A}\sin(\omega\tau), g\overline{A}\cos(\omega\tau), 0, 0\right\}, \quad (19)$$

which is in resonance with the particle, i.e., the Doppler-shifted wave frequency is close to the cyclotron one,

$$\omega' \equiv (1 - nv_3/c)\,\omega \approx g\Omega, \quad (20)$$

where v_3 is the particle initial longitudinal velocity. In (19), the respective values $g = \pm 1$ correspond to the right- and left-hand circular polarizations of the wave. After the interaction $(t \to +\infty)$, under resonance condition (20), we have from Eq. (14) that

$$K_1 + iK_2 = -\frac{e\overline{A}cT}{\hbar\widetilde{E}}\exp(ig\omega\tau), \quad (21)$$

where T is the coherent interaction time (for a quasimonochromatic wave, $T \to \infty$, and for actual laser radiation, T is the pulse duration).

The final state of the particle after the interaction is described by the wave function

$$\Psi_s(x) = N\exp\left[\frac{i}{\hbar}(p_3x_3 + p_2x_2 - E_s(p_3)t)\right] \times \\ \times U_s\left[x_1 + \frac{e\overline{A}cT}{\widetilde{E}}\cos(\omega\tau)\right] \times \\ \times \exp\left[-i\frac{egH_0\hbar}{c}\left(\frac{e\overline{A}cT}{2\hbar\widetilde{E}}\right)^2\sin(2\omega\tau) + \right. \\ \left. + i\frac{eg\overline{A}\Omega TE_\parallel}{\hbar c\widetilde{E}}\left(x_1 + \frac{p_2a^2}{\hbar}\right)\sin(\omega\tau)\right]. \quad (22)$$

Expanding wave function (22) in terms of the complete basis of particle eigenstates (5),

$$\Psi_s(x) = \int dp_2'dp_3'\sum_{s'} C_{ss'}(p_2', p_3')\psi_{s',p_2',p_3'}(x) \quad (23)$$

we find the probabilities of multiphoton induced transitions between the Landau levels.

To calculate the expansion coefficients $C_{ss'}(p'_2, p'_3)$, we use the result of the integration

$$\int_{-\infty}^{\infty} dz \exp(-ikz) U_s(a^{-1}z + ab) U_{s'}(a^{-1}z + ab') = \\ = \exp\{i\mu + i(s-s')\lambda\} I_{ss'}(\zeta), \quad (24)$$

where $I_{ss'}(\zeta)$ is the Lagger function and the characteristic parameters are determined by

$$\mu = \frac{ka^2(b+b')}{2}, \quad \lambda = \mathrm{tg}^{-1}\frac{k}{b'-b}, \\ \zeta = a^2\frac{k^2+(b-b')^2}{2}.$$

We then obtain the transition amplitudes

$$C_{ss'}(p'_2,p'_3) = \delta(p_2-p'_2)\delta(p_3-p'_3-(s-s')g\omega n\hbar c^{-1}) \times \\ \times \exp\left\{-\frac{i}{\hbar}(E_s(p_3) - E_{s'}(p'_3) - (s-s')g\omega\hbar)\,t\right\} \times \\ \times I_{ss'}[\zeta], \quad (25)$$

where $\delta(p)$ is the Dirac δ-function expressing the momentum conservation law and the argument of the Lagger function is

$$\zeta = \frac{e^2\overline{A}^2 T^2 \Omega E_{\|}}{2\hbar\widetilde{E}^2}. \quad (26)$$

According to (25), the transition of the particle from an initial state $\{s, p_2, p_3\}$ to a state $\{s', p'_2, p'_3\}$ is accompanied by emission or absorption of $s-s'$ photons. Consequently, substituting Eq. (25) in Eq. (23) and integrating over the momentum, we can rewrite the particle wave function in another form,

$$\Psi_s(x) = N\sum_{s'} I_{ss'}(\zeta) \times \\ \times \exp\left\{-\frac{i}{\hbar}(E_s(p_3) - (s-s')g\omega\hbar)\,t + \right. \\ \left. +\frac{i}{\hbar}(p_3 - (s-s')g\omega n\hbar c^{-1})x_3 + \frac{i}{\hbar}p_2x_2\right\} \times \\ \times U_{s'}(x_1). \quad (27)$$

The probability of the induced transition $s \to s'$ between the Landau levels is ultimately determined from formula (27):

$$w_{ss'} = I^2_{ss'}\left[\frac{e^2\overline{A}^2 T^2\Omega E_{\|}}{2\hbar\widetilde{E}^2}\right]. \quad (28)$$

Matching resonance condition (20) with formula (27) shows that in the field of a strong EM wave, the Landau levels are excited at the absorption of the wave quanta if $1 - nv_3/c > 0$ and $g = 1$, corresponding to the normal Doppler effect, while in the case where $1 - nv_3/c < 0$ and $g = -1$, which is possible in the refractive medium ($n > 1$), the Landau levels are excited at the emission of the wave quanta due to the anomalous Doppler effect.

We now estimate the average number of emitted (absorbed) photons by the electron at the CR in a medium for high excited Landau levels ($s \gg 1$). In accordance with the chosen approximation, the most probable number of photons in the strong EM wave field corresponds to the semiclassical limit ($|s-s'| \gg 1$), in which multiphoton processes dominate and the nature of the interaction process is very close to the classical one. In this case, the argument of the Lagger function can be represented as

$$\zeta = \frac{1}{4s}\left(\frac{\Delta E_{cl}}{\hbar\omega}\right)^2, \quad \Delta E_{cl} = \frac{e\mathcal{E}v_\perp T}{|1 - nv_3/c|}. \quad (29)$$

Here, ΔE_{cl} is the amplitude of the energy change of the particle according to the classical perturbation theory, $\mathcal{E}$ is the amplitude of the electric field strength of the EM wave, and

$$v_\perp \approx c\sqrt{2\hbar s\Omega/E_{0\|}}$$

is the particle mean transverse velocity. The Lagger function is maximal at

$$\zeta \to \zeta_0 = \left(\sqrt{s'} - \sqrt{s}\right)^2,$$

exponentially falling beyond ζ_0. For the transition $s \to s'$ with $|s-s'| \ll s$, we have

$$\zeta_0 \approx \frac{(s'-s)^2}{4s}.$$

Comparison of this expression with (28) and (29) shows that the most probable transitions are

$$|s-s'| \approx \frac{\Delta E_{cl}}{\hbar\omega}, \quad (30)$$

in accordance with the correspondence principle. Using Eqs. (27) and (30), we can now represent the condition for the eikonal approximation in Eq. (11) as

$$\Delta E_{cl} \ll 2\left|\frac{E_s(p_3) - cnp_3}{n^2-1}\right|.$$

This condition actually restricts the intensity of the EM wave field in accordance with Eq. (29). However, the above condition is practically very weak, and the wave function obtained in (16) describes multiphoton transitions at the CR in strong laser fields with great accuracy.

4. CONCLUSION

In the scope of relativistic quantum theory, a nonlinear (in the field) wave function of the eikonal type of a charged particle in the plane EM wave and a uniform magnetic field in a medium has been obtained neglecting spin interaction and, consequently, quantum recoil of photons (in accordance with the eikonal approximation applied). The eikonal approximation practically does not restrict the applicability of such a wave function in the actual cases of strong radiation fields that are laser fields (with the photon energy much smaller than the electron energy). This wave function well enough describes the quantum picture of CR in a medium at high intensities of the external radiation field, in particular, multiphoton stimulated transitions between the Landau levels.

With this wave function, one can treat a large class of nonlinear quantum electrodynamic processes in strong EM fields with the modifications that a medium brings (e.g., the anomalous Doppler effect), including astrophysical applications, where CR plays a significant role [15]. In addition, one of the advantages of CR in a dielectric medium is that for a moderate relativistic particle beam, one can achieve the CR in the optical region (close to the Cherenkov resonance) by current lasers and existing uniform magnetic fields ($\sim 10^4$ Gs), while in the vacuum, the CR with radio frequencies is possible at the same parameters. Finally, the obtained wave function is especially important for the description of the radiation process by a charged particle at the CR in gaseous media consisting of a superposition of Compton, Cherenkov, and synchrotron radiations.

We note that radiation of a particle at laser-assisted multiphoton transitions at the CR between Landau levels has already been investigated and will be presented elsewhere.

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