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ULTRARELATIVISTIC NUCLEI IN A CRYSTAL CHANNEL: COULOMB SCATTERING, COHERENCE, AND ABSORPTION

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We incorporate the effect of lattice thermal vibrations into the Glauber-theory description of particle and nucleus–crystal Coulomb interactions at high energy. We show that taking the lattice thermal vibrations into account produces a strong absorption effect: the phase shift function of the multiple-diffraction scattering on a chain of N identical atoms acquires a large imaginary part and the radius of the absorption region in the impact parameter plane grows logarithmically with N. Consequences of this observation for the elastic and quasielastic Coulomb scattering are discussed. The practically interesting example of the coherent Coulomb excitation of ultrarelativistic particles and nuclei passing through a crystal is considered in detail.

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1. INTRODUCTION

In this paper, we develop the description of the absorption phenomenon in coherent particle and nucleus–crystal Coulomb interactions at high energy in the Glauber theory framework [1].

As is well known, multi-loop corrections generate an imaginary part of the scattering amplitude even if the tree-level amplitude is purely real. For example, the purely real Born amplitude of the high-energy Coulomb scattering in a crystal acquires an imaginary part due to the multiple scattering (MS) effects. However, in the widely used static/frozen lattice approximation (SL approximation), the account of rescatterings alters only the overall real phase of the full amplitude, thus producing no absorption effect. The latter is related to the creation and annihilation of excited intermediate states of the crystal and as such manifests itself only beyond the SL approximation (see [2] for the analysis of elastic scattering based on the SL approximation). Indeed, the amplitude of small-angle elastic scattering on a chain of N identical atoms in the impact parameter representation is equal to

$$1 - \langle S(b) \rangle,$$

where the scattering matrix placed between the ground states of the crystal is

$$\langle S(b) \rangle = \langle \exp[i\chi(b)] \rangle,$$

with the purely real phase shift function

$$\chi(b) = \sum_{j=1}^{N} \chi_j(b).$$

In the SL approximation,

$$\langle \exp[i\chi(b)] \rangle \approx \exp[i\chi(b)].$$

In general, therefore, the Coulomb phase shift function acquires a nonvanishing imaginary part, which is interpreted as an absorption effect, only with the lattice thermal vibrations taken into account. The imaginary part appears only as a second-order perturbation,

$$\sim \frac{i}{2}[\langle \chi^2 \rangle - \langle \chi \rangle^2].$$

But the strength of the effect is proportional to $\beta^2 N$, where β is the coupling constant. For the coherent scattering of relativistic nuclei (the electric charge Z_1) on the chain of N atoms (the atomic number Z_2) in a crystal, the effective coupling

$$\beta = 2\alpha Z_1 Z_2$$

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is strong and the absorption effect is also strong. The absorption is strong for the impact parameters b smaller than some characteristic value

$$b_a \propto \log(\beta N)$$

and vanishes toward the region of larger b. This phenomenon provides a natural ultraviolet regulator of the theory and enables, in particular, consistent calculation of the coherent elastic scattering cross section. The latter is calculated and turns out to be equal to half the total cross section. As we see in what follows, the absorption effect is also of prime importance for quantitative understanding of the phenomenon of the coherent Coulomb excitation of relativistic particles and nuclei passing through a crystal. A consistent description of this phenomenon is the goal of our paper.

The outline of the paper is as follows. We start with the well-known example of the coherent Coulomb elastic scattering of charged particle/nucleus by a linear chain of N identical atoms in a crystal target (Sec. 2). In Sec. 3, we derive the scattering matrix with absorption and calculate the cross section σ_{el} of the coherent elastic scattering and the cross section σ_{Qel} of the incoherent excitation and break-up of the target. Then we find that in the large-N limit,

$$\sigma_{el} \approx \sigma_{Qel} \approx \frac{1}{2}\sigma_{tot}.$$

In Sec. 4, we discuss the coherent Coulomb excitation of ultrarelativistic particles and nuclei passing through the crystal to the lowest order of perturbation theory. The higher-order effects are considered in Sec. 5, where the cross section of the process is calculated. We finally conclude with a brief summary in Sec. 6.

2. COHERENT ELASTIC SCATTERING AND ABSORPTION

The interatomic distances in a crystal, a, are large compared to the Thomas–Fermi screening radius r_0,

$$a \sim 3\text{–}5\ \text{Å} \gg r_0 = r_B Z_2^{-1/3} \sim 0.1\ \text{Å},$$

where Z_2 is the atomic number of the target atom and r_B is the Bohr radius [3]. The relevant impact parameters b satisfy the condition $b \ll a$ and the amplitudes of scattering by different atomic chains parallel to a given crystallographic axis are incoherent.

The amplitude of small-angle scattering of a charged particle (charge Z_1) by a linear chain of N identical atoms in the eikonal approximation is given by [1]

$$F_{fi}(q) = \frac{ip}{2\pi}\int d^2\mathbf{b}\exp(i\mathbf{q}\cdot\mathbf{b}) \times \\ \times \langle\Psi_f(\{\mathbf{r_j}\})|1 - S(\mathbf{b},\mathbf{s_1},\ldots,\mathbf{s_A})|\Psi_i(\{\mathbf{r}_j\})\rangle, \quad (1)$$

where Ψ_i and Ψ_f are the initial and final state wave functions of the crystal and $\mathbf{q}$ is the two-dimensional vector of the momentum transfer. The incident particle momentum p is assumed to be large enough to satisfy the condition of applicability of the straight-paths approximation, $p/q^2 \gg aN$. The latter condition ensures the coherence of interactions with different atoms.

The elastic scattering corresponds to $i = f$ and the brackets $\langle\ \rangle$ signify that the average is to be taken over all configurations of atoms in the ground state,

$$\langle\Psi(\{\mathbf{r_j}\})|1 - S(\mathbf{b},\mathbf{s}_1,\ldots,\mathbf{s}_N)|\Psi(\{\mathbf{r}_j\})\rangle = \\ = \int d^3\mathbf{r}_1\ldots d^3\mathbf{r}_N|\Psi(\{\mathbf{r}_j\})|^2 \times \\ \times\left[1 - \exp\left(i\sum_{j=1}^{N}\chi(\mu|\mathbf{b}-\mathbf{s}_j|)\right)\right]. \quad (2)$$

In (2), the total scattering phase is the sum of the phase shifts contributed by the individual atoms. The positions of the N atoms that make up the target are defined by the three-dimensional vectors $\mathbf{r}_j$, $j = 1,\ldots,N$. The two-dimensional vectors $\mathbf{s}_j$ are the projections of these vectors on the impact parameter plane. We neglect all position correlations of the atoms and describe the ground state of the crystal by the wave function $|\Psi\rangle$ such that

$$|\Psi(\{\mathbf{r}_j\})|^2 = \prod_{j=1}^{N}|\psi(\mathbf{u}_j)|^2, \quad (3)$$

where the three-dimensional vectors $\mathbf{u}_j$ are defined by

$$\mathbf{r}_j = (j-1)\mathbf{a} + \mathbf{u}_j, \quad j = 1,\ldots,N, \quad \mathbf{a} = (0,0,a)$$

and $\mathbf{u}_j = (\mathbf{s}_j, z_j)$.

From Eq. (2), it follows that

$$F_{ii}(q) = F(q) = \\ = ip\int b\,db\,J_0(qb)\left\{1 - \langle\exp[i\chi(\mu b)]\rangle^N\right\}. \quad (4)$$

Hereafter, $J_{0,1}(x)$ and $K_{0,1}(x)$ are the Bessel functions and the screened Coulomb phase shift function is

$$\chi(\mu b) = -\beta K_0(\mu b) \quad (5)$$

with

$$\beta = 2\alpha Z_1 Z_2, \quad \mu = r_0^{-1}.$$

After integration over longitudinal variables $\{z_j\}$ followed by the azimuthal integration, the term $\langle\exp(i\chi)\rangle$ takes the form

$$\langle\exp(i\chi)\rangle = \int d^2\mathbf{s}\,\rho(s)\exp[i\chi(\mu|\mathbf{b}-\mathbf{s}|)] = \\ = \exp(-\Omega^2 b^2)\int\limits_0^\infty dx\exp(-x)\,\times \\ \times\, I_0(2b\Omega\sqrt{x})\exp[-i\beta K_0(\mu\sqrt{x}/\Omega)]. \quad (6)$$

The two-dimensional vector $\mathbf{s}$ describes the position of the target atom in the impact parameter plane. The one-particle probability distribution $\rho(s)$ is given by

$$\rho(s) = \int dz\,|\psi(\mathbf{s},z)|^2 = (\Omega^2/\pi)\exp\left(-\Omega^2\mathbf{s}^2\right). \quad (7)$$

For the most commonly studied elements at room temperature, the ratio μ/Ω varies in a wide range, from $\mu/\Omega\sim 0.1$ to $\mu/\Omega\sim 1$ [3]. We first consider the region of small impact parameters including $b\lesssim 1/\Omega$[1)]. For $b\lesssim 1/2\Omega$, only small s, such that $\mu s\lesssim 1$, contribute. We can then set

$$K_0(\mu s)\approx\log(1/\mu s)$$

and integrate over s,

$$\langle\exp(i\chi)\rangle\approx\left(\frac{\mu}{\Omega}\right)^{i\beta}\exp(-\Omega^2b^2)\,\times \\ \times\int\limits_0^\infty dx\,x^{i\beta/2}\exp(-x)I_0(2b\Omega\sqrt{x}) = \\ = \left(\frac{\mu}{\Omega}\right)^{i\beta}\Gamma\left(1+\frac{i\beta}{2}\right)\Phi\left(-\frac{i\beta}{2};1;-\Omega^2b^2\right), \quad (8)$$

where

$$\Phi(a;b;z) = 1+\frac{a}{b}\frac{z}{1!}+\frac{a(a+1)}{b(b+1)}\frac{z^2}{2!}+\ldots \quad (9)$$

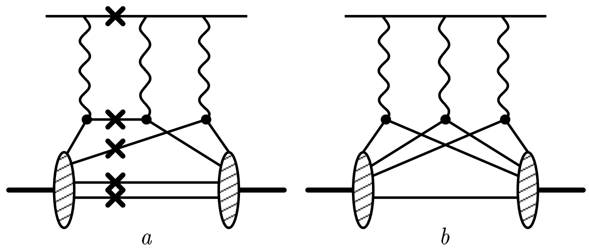
is the confluent hypergeometric function and

$$\Phi(a;b;z) = \exp(z)\Phi(b-a;b;-z).$$

From Eq. (8), it follows that

$$|\langle\exp(i\chi)\rangle|_{b=0} = \left[\frac{\pi\beta}{2\,\mathrm{sh}(\pi\beta/2)}\right]^{1/2}, \quad (10)$$

1) We note that the smallness of the ratio $r_A^2/u^2\sim 10^{-6}$–10^{-5}, where r_A is the nuclear radius and $u=1/\Omega$ is the amplitude of lattice thermal vibrations, allows neglecting the nuclear interactions of the projectile up to $N\sim 10^5$. As we see in what follows, the absorption effect in which we are interested enters the game at much smaller N.



Example of the relevant multiple scattering diagrams to the order β^3. The unitarity cut of the elastic amplitude a that contributes to the absorption is shown by crosses. Diagram b allows cuts only between the projectile-atom blocks and does not contribute to the absorption effect

where the identity

$$|\Gamma(i\beta/2)|^2 = \frac{2\pi}{\beta\,\mathrm{sh}(\pi\beta/2)} \quad (11)$$

has been used. In the weak coupling regime $\beta\ll 1$,

$$|\langle\exp(i\chi)\rangle|_{b=0}\approx 1-\frac{1}{2}(\langle\chi^2\rangle-\langle\chi\rangle^2)$$

and

$$\langle\chi^2\rangle-\langle\chi\rangle^2 = \frac{\pi^2\beta^2}{24}. \quad (12)$$

For $\beta\gtrsim 1$,

$$|\langle\exp(i\chi)\rangle|_{b=0}\approx\sqrt{\pi\beta}\exp(-\pi\beta/4). \quad (13)$$

Therefore, at small impact parameters $b\lesssim 1/2\Omega$, the intensity of outgoing nuclear waves as a function of N exhibits the exponential attenuation. In terms of the unitarity cuts of the elastic scattering amplitude, the imaginary part of the phase shift function comes from the cuts through the multi-photon projectile-atom blocks as shown in Fig. a. The account of diagrams like that in Fig. b, which allows cuts only between projectile-atom blocks, gives the scattering matrix of the form $\exp(iN\langle\chi\rangle)$ and affects only the overall real phase of the amplitude.

The absorption effect becomes weaker toward the region of large impact parameters $b\gtrsim 1/2\Omega$,

$$|\langle\exp(i\chi)\rangle|^N\approx|\langle\exp(i\chi)\rangle|_{b=0}^N\,\times \\ \times\left[1+\frac{N\beta^2}{16}(\Omega b)^4+\ldots\right]. \quad (14)$$

For still larger b, $b\gg 1/2\Omega$, using the asymptotic form

$$I_0(z)\approx(2\pi z)^{-1/2}\exp(z),$$

we obtain

$$\langle \exp(i\chi) \rangle \approx 2\Omega \int \frac{s\,ds}{\sqrt{\pi b s}} \exp[-\Omega^2 (b-s)^2] \times \times \exp[i\chi(\mu s)]. \quad (15)$$

To evaluate the integral in (15), we expand $\chi(\mu s)$ in powers of $(s-b)$,

$$\chi(\mu s) \approx \chi(\mu b) + \omega(s-b).$$

If the frequency ω,

$$\omega = \frac{d\chi}{db} = \mu\beta K_1(\mu b),$$

is small compared to Ω,

$$\omega \ll \Omega, \quad (16)$$

then

$$\langle \chi^2 \rangle - \langle \chi \rangle^2 = \frac{\omega^2}{2\Omega^2}$$

and

$$\langle \exp(i\chi) \rangle \approx \exp[i\chi - i\omega b] \times \times \Omega \int_0^\infty \frac{s\,ds}{\sqrt{\pi b s}} \exp[-\Omega^2(b-s)^2] \exp[i\omega s] \approx \approx \exp(i\chi) \exp[-\omega^2/4\Omega^2]. \quad (17)$$

Condition (16) is satisfied if $b \gg \beta/\Omega$. For the impact parameters from the region

$$\beta/\Omega \ll b \ll 1/\mu,$$

we can write $\omega \approx \beta/b$; for larger b such that $b \gg \mu^{-1}$,

$$\omega \approx \mu\beta \sqrt{\frac{\pi}{2\mu b}} \exp(-\mu b). \quad (18)$$

From the consideration presented above, it follows that the absorption effect in the elastic scattering amplitude is especially strong for impact parameters

$$b \lesssim b_a = \frac{1}{2\mu} \log \frac{\pi \mu^2 \beta^2 N}{4\Omega^2}. \quad (19)$$

For $b \ll b_a$, the atomic chain acts as an opaque «black» disc. Certainly, the value of this finding differs for different observables and for different processes proceeding at different impact parameters. The only thing which is worth noticing here is the representation of the scattering matrix in the form

$$\langle S(b) \rangle \approx \exp\left(iN\chi - \frac{N\omega^2}{4\Omega^2} \right), \quad b \gg \beta\Omega^{-1}. \quad (20)$$

Equation (20) supplemented with the observation that

$$\langle S(b) \rangle \approx (\pi\beta)^{N/2} \exp\left(-\frac{N\pi\beta}{4} \right), \quad b \lesssim \Omega^{-1}, \quad (21)$$

simplifies all further calculations greatly.

3. THE CROSS SECTIONS

3.1. The elastic cross section

Integrating by parts reduces $F(q)$ to the form convenient for evaluation of the total cross section,

$$F(q) = \frac{ip\mu N}{q} \int_0^\infty b\,db\, J_1(qb) \langle i\chi' \exp(i\chi) \rangle \times \times \langle \exp(i\chi) \rangle^{(N-1)}. \quad (22)$$

At small impact parameters $b \ll 1/2\mu$,

$$\langle \chi' \exp(i\chi) \rangle \approx \beta \left(\frac{\mu}{\Omega} \right)^{i\beta - 1} \exp(-\Omega^2 b^2) \times \times \Gamma\left(\frac{i\beta+1}{2} \right) \Phi\left(\frac{i\beta+1}{2}; 1; \Omega^2 b^2 \right). \quad (23)$$

Because of multiple scatterings, only large impact parameters b may contribute to $F(q)$ at large N and small q. Hence,

$$F(q) \approx \frac{ip\mu N}{q} \int_{1/\mu}^\infty b\,db\, J_1(qb) \left[i\chi' - \omega\omega'/2\Omega^2 \right] \times \times \exp(iN\chi) \exp(-N\omega^2/4\Omega^2), \quad (24)$$

where the explicit form of $\langle \exp(i\chi) \rangle$ at large b, Eq. (17), has been used. For large b,

$$\omega^2 \propto \exp(-2\mu b);$$

as b grows, ω^2 decreases much faster than the phase shift function $\chi(\mu b)$, which is proportional to $\exp(-\mu b)$. We see that the leading contribution to the elastic scattering amplitude (24) comes from

$$b \sim \mu^{-1}\xi \gg b_a,$$

where

$$\xi = \log(\beta N).$$

For large N, the second term in the square brackets in (24) is small compared to the first one. Then, for

$$q \lesssim q_0 = \mu/\xi$$

and $\xi \gg 1$, the steepest descent from the saddle point

$$b_0 = \mu^{-1}[\xi + i\pi/2] \quad (25)$$

in Eq. (24) yields

$$F(q) \approx \frac{ipb_0}{q} J_1(qb_0). \quad (26)$$

The effect of lattice thermal vibrations at small q appears to be marginal and reduces to the factor $\exp(\mu^2/4\Omega^2 N)$ in (26), which is irrelevant at large N. The amplitude $F(q)$ in Eq. (26) coincides with the elastic scattering amplitude given by the SL approximation [2].

If $q \gtrsim q_0$, the stationary phase approximation gives the elastic scattering amplitude of the form

$$F(q) \approx \frac{-ip\sqrt{\eta}}{\mu q} \exp\left(-\frac{iq\eta}{\mu}\right) \exp\left(-\frac{q^2}{4\Omega^2 N}\right), \qquad (27)$$

where

$$\eta = \log(\mu\beta N/q) \gg 1.$$

The account of the lattice thermal vibrations ensures the convergence of the integral for the coherent elastic scattering cross section,

$$\sigma_{el} = \frac{\pi}{p^2} \int dq^2 |F(q)|^2 \approx \frac{\pi\xi^2}{\mu^2} \int\limits_0^{q_0^2} \frac{dq^2}{q^2} J_1^2\left(\frac{q\xi}{\mu}\right) + \\ + \frac{\pi}{\mu^2} \int\limits_{q_0^2}^{\infty} \frac{dq^2}{q^2} \log\left(\frac{\mu\beta N}{q}\right) \exp\left(-\frac{q^2}{2\Omega^2 N}\right), \qquad (28)$$

which for $\xi \gg 1$ is simply

$$\sigma_{el} \approx \frac{\pi}{\mu^2}\xi^2. \qquad (29)$$

3.2. The quasielastic cross section

In this paper, we focus on coherent nucleus–atom interactions. The incoherent process of ionization of the target atom is suppressed by the factor of the order of Z_2^{-1}. Then the inelastic process, which by unitarity gives rise to attenuation of the elastic amplitude, is the process of the quasielastic scattering (Fig. a). Its cross section is given by [1]

$$p^2 \frac{d\sigma_{Qel}}{d^2\mathbf{q}} = \sum_f |F_{fi}(q)|^2 - |F_{ii}(q)|^2, \qquad (30)$$

where the sum extends over all final states of the crystal in which no particle production occurs. The closure relation then yields

$$\frac{d\sigma_{Qel}}{d^2\mathbf{q}} = \frac{1}{4\pi^2} \int d^2\mathbf{b}\, d^2\mathbf{b}' \exp[i\mathbf{q}(\mathbf{b}-\mathbf{b}')] \times \\ \times \left\{ \langle \exp[i\chi(\mu b) - i\chi^*(\mu b')]\rangle^N - \right. \\ \left. - \langle \exp[i\chi(\mu b)]\rangle^N \langle \exp[-i\chi^*(\mu b')]\rangle^N \right\} \qquad (31)$$

and

$$\sigma_{Qel} = \int d^2\mathbf{b} \left\{ 1 - |\langle \exp[i\chi(\mu b)]\rangle|^{2N} \right\}. \qquad (32)$$

In the SL approximation,

$$|\langle \exp[i\chi(\mu b)]\rangle| = 1$$

and

$$\sigma_{Qel} = 0.$$

From (20) and the discussion of the absorption radius b_a presented above, it follows that for $\xi \gg 1$,

$$1 - |\langle \exp[i\chi(\mu b)]\rangle|^{2N} \approx \theta(2b_a - b) \qquad (33)$$

and

$$\sigma_{Qel} \approx \pi(2b_a)^2 \approx \frac{\pi}{\mu^2}\xi^2. \qquad (34)$$

3.3. The total cross section

From Eq. (26), by means of the optical theorem, we find the total cross section

$$\sigma_{tot} = \frac{4\pi}{p} \operatorname{Im} F(0) \approx \frac{2\pi}{\mu^2}\xi^2. \qquad (35)$$

We therefore conclude that at high energy and in the large-N limit,

$$\sigma_{el} \approx \sigma_{Qel} \approx \frac{1}{2}\sigma_{tot}. \qquad (36)$$

4. COULOMB EXCITATION OF ULTRARELATIVISTIC PARTICLES AND NUCLEI IN A CRYSTAL CHANNEL. THE EXCITATION CROSS SECTION TO THE LOWEST ORDER. THE BORN APPROXIMATION

We now consider the process of the coherent Coulomb excitation of ultrarelativistic particles and nuclei passing through the crystal. This way of the experimental study of rare processes has been proposed in [4–10].

An ultrarelativistic projectile nucleus (mass number A, charge Z_1, and 4-momentum p) moving along a crystal axis undergoes a correlated series of soft collisions that give rise to diagonal ($A \to A$, $A^* \to A^*$) and off-diagonal ($A \to A^*$, $A^* \to A$) transitions.

In [4, 5, 9], it has been proposed to study the electric dipole transition in ^{19}F, the excitation of the state $|J^\pi = 1/2^-\rangle$ from the ground state $|1/2^+\rangle$. The phenomenological matrix element of the transition $1/2^+ \to 1/2^-$ is [11]

$$\mathcal{M} = \frac{1}{2} d\bar{u}\,(p')\gamma_5\,(\hat{q}\hat{\varepsilon} - \hat{\varepsilon}\hat{q})\,u(p), \qquad (37)$$

where $u(p')$ and $u(p)$ are bispinors of the initial and final states of the projectile, d is the transition dipole moment, and ε is the photon polarization vector. The transverse and longitudinal components of the 4-vector $p - p'$ are denoted by $\mathbf{q}$ and κ, respectively. In what follows, $q = |\mathbf{q}|$. The only phenomenological parameter in the problem is the dipole moment d. The measured life-time of the 110 keV level $^{19}\mathrm{F}(1/2^-)$ is $\tau = (0.853 \pm 0.010) \cdot 10^{-9}$ s [12]; the dipole moment of the $1/2^+ \to 1/2^-$ transition, determined from the width of the level $^{19}\mathrm{F}(1/2^-)$, is $d \approx 5 \cdot 10^{-8}$ keV^{-1} [11]. Then, first, because of the large value of τ, the decay of the excited state inside the target crystal can be safely neglected and, second, due to the smallness of d, the excitation amplitude is much smaller than the elastic Coulomb amplitude for all q up to $q \sim \sqrt{4\pi\alpha}Z_1/d$ and can be considered a perturbation. Thus, the multichannel problem reduces to the one-channel one.

The high-energy helicity-flip Born amplitude of the transition $1/2^+ \to 1/2^-$ in collision of the projectile nucleus with N bound atoms in the crystal is given by

$$F_{ex}^B(\mathbf{q}) = S(\kappa)\frac{p}{2\pi}\frac{g(\boldsymbol{\sigma}\cdot\mathbf{q})}{q^2+\lambda^2}\exp\left(-\frac{q^2}{4\Omega^2}\right), \qquad (38)$$

where $\boldsymbol{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ is the Pauli spin vector, $\{\sigma_i, \sigma_j\} = 2\delta_{ij}$, and the amplitude we are constructing is to be regarded as an operator that transforms the initial helicity state of the projectile into its final state. In the denominator of Eq. (38), $\lambda^2 = \mu^2 + \kappa^2$. In the Glauber approximation, the longitudinal momentum transfer, which determines the coherency length $l_c \sim \kappa^{-1}$, is given by [13]

$$\kappa = \frac{M\Delta E}{p}, \qquad (39)$$

where M is the mass of the projectile and ΔE is the excitation energy[2)].

In the first order in g, the structure factor of the crystal is

$$S(\kappa) = \exp\left[-\frac{\kappa^2}{4\Omega^2}\right]\frac{\sin(\kappa N a/2)}{\sin(\kappa a/2)}. \qquad (40)$$

If the projectile momentum satisfies the resonance condition [4, 5, 7, 9]

$$\frac{M\Delta E}{p} = \frac{2\pi n}{a}, \quad n = 0, 1, 2, \ldots, \qquad (41)$$

then $S(\kappa) \sim N$. In the first order in g and in the zeroth order in β (the Born approximation), the cross section of the coherent excitation of the projectile in scattering on a chain of N atoms in a crystal is

$$\sigma_{ex}^B = \frac{\pi}{p^2}\int dq^2\,|F_{ex}^B(\mathbf{q})|^2 \sim \\ \sim \frac{g^2N^2}{4\pi}\left[\log\left(1+\frac{2\Omega^2}{\lambda^2}\right) - \frac{2\Omega^2}{\lambda^2+2\Omega^2}\right], \qquad (42)$$

where

$$g = \sqrt{4\pi\alpha}dZ_2.$$

The central idea in [4–7, 9, 10], based on the Born approximation, is that the transition rate can be enhanced substantially due to coherency of interactions, which is assumed to sustain over the large-distance scale. The law $\sigma_{ex} \propto N^2$ is expected to hold up to the crystal thicknesses $N = L/a \sim 10^5$–10^6 in a tungsten target. In [10], the Born approximation for the coherent excitation of Σ^+ in high-energy proton–crystal interactions $p\gamma \to \Sigma^+$ was assumed to be valid up to $N \sim 10^8$. However, the account of the initial and final state Coulomb interactions dramatically changes the dependence of σ_{ex} on N. For instance, at $N = 2$, the excitation amplitude is of the form

$$F_{ex}^{(2)}(\mathbf{q}) = \frac{p}{\pi}\int d^2\mathbf{b}\exp(i\mathbf{q}\cdot\mathbf{b})\langle f_{ex}^B\exp(i\chi)\rangle \times \\ \times \langle\exp(i\chi)\rangle. \qquad (43)$$

The first of the two bracketed factors in Eq. (43) corresponds to the nuclear excitation amplitude in scattering on a bound atom. It differs from the excitation amplitude of the Born approximation, $f_{ex}^B(\mathbf{b})$, by phase factor that occurs due to the initial and final state multiple Coulomb scattering. At small impact parameters $b \lesssim 1/2\Omega$,

$$\langle f_{ex}^B\exp(i\chi)\rangle \approx S(\kappa)\frac{g}{2\pi b}(\boldsymbol{\sigma}\cdot\mathbf{n}_b) \times \\ \times \mathrm{sh}\left(\frac{1}{2}\Omega^2b^2\right)\exp\left(-\frac{1}{2}\Omega^2b^2\right). \qquad (44)$$

For large b,

$$\langle f_{ex}^B\exp(i\chi)\rangle = S(\kappa)\frac{g}{4\pi}\int d^2\mathbf{s}\,\rho(s)(\boldsymbol{\sigma}\cdot(\mathbf{n}_b-\mathbf{n}_s)) \times \\ \times \lambda K_1(\lambda|\mathbf{n}_b-\mathbf{n}_s|)\exp\left[i\chi\left(\mu|\mathbf{n}_b-\mathbf{n}_s|\right)\right] \approx \\ \approx S(\kappa)\frac{g}{4\pi}(\boldsymbol{\sigma}\cdot\mathbf{n}_b)\lambda K_1(\lambda b)\exp(i\chi) \times \\ \times \exp\left(-\frac{\omega^2}{4\Omega^2}\right), \qquad (45)$$

2) The Fresnel corrections to the eikonal approximation, which are neglected here, become important at large N or at large q; they diminish the coherency length and additionally suppress coherent processes [14].

where $\mathbf{n}_b = \mathbf{b}/|\mathbf{b}|$, $\mathbf{n}_s = \mathbf{s}/|\mathbf{s}|$, and $b \gtrsim \mu^{-1}$. Because

$$|\langle f_{ex}^B \exp(i\chi)\rangle|^2 \propto \Omega^2 b^2$$

for small b and

$$\mu b K_1^2(\mu b) \propto \exp(-2\mu b)$$

for large impact parameters $b \gtrsim 1/\mu$, the cross section

$$\sigma_{ex}^{(2)} = \int d^2\mathbf{b}\, |\langle f_{ex}^B \exp(i\chi)\rangle|^2 |\langle \exp(i\chi)\rangle|^2 \approx \approx 4\sigma_{ex}^{(1)}\left(1 - \frac{\omega^2}{2\Omega^2}\right) \quad (46)$$

is dominated by $b \sim 1/2\mu$. For the diamond crystal, $\mu/\Omega \approx 0.16$ [3]. Hence,

$$\frac{\omega^2}{2\Omega^2} \approx \frac{2\beta^2\mu^2}{\Omega^2} \sim \frac{1}{20}.$$

This estimate shows that even for the diamond crystal target, the Born approximation is invalid already at $N \gtrsim 10$.

5. MULTIPLE SCATTERING EFFECTS AND ABSORPTION IN THE COHERENT COULOMB EXCITATION PROCESSES

The transition amplitude on a chain of N identical atoms including all the multi-photon t-channel exchanges is given by

$$F_{ex}(\mathbf{q}) = \frac{p}{\pi}\int d^2\mathbf{b}\exp(i\mathbf{q}\cdot\mathbf{b})\langle f_{ex}^B \exp(i\chi)\rangle \times \times \langle \exp(i\chi)\rangle^{N-1}. \quad (47)$$

Because of both the multiple scattering effect and absorption, only large impact parameters $b \gg \mu^{-1}$ may contribute to $F_{ex}(\mathbf{q})$. Evaluation of $F_{ex}(q)$ then gives

$$F_{ex}(\mathbf{q}) \approx \frac{gp}{2\pi} S(\kappa)(\boldsymbol{\sigma}\cdot\mathbf{n}_q)\int\limits_{1/\mu}^{\infty} b\,db\, J_1(qb) \times \times \lambda K_1(\lambda b)\exp(iN\chi)\exp(-N\omega^2/4\Omega^2), \quad (48)$$

where $\mathbf{n}_q = \mathbf{q}/|\mathbf{q}|$. The contribution of the domain $q \lesssim q_0 = \mu/\xi$ to the excitation cross section can be neglected because $F_{ex} \propto q$ in this region. If $q \gg q_0$ and $\xi \gg 1$, the stationary phase approximation gives the coherent excitation amplitude of the form

$$F_{ex}(\mathbf{q}) \approx \frac{ipg(\boldsymbol{\sigma}\cdot\mathbf{n}_q)}{2\pi\beta}\frac{S(\kappa)}{N}\frac{\lambda}{\mu}\sqrt{\eta}\exp(-\delta\eta) \times \times \exp\left(-\frac{iq\eta}{\mu}\right)\exp\left(-\frac{q^2}{4\Omega^2 N}\right). \quad (49)$$

We see that the helicity-flip dynamics removes the factor $1/q$ from elastic amplitude (27), thus making the UV regularization of the excitation cross section indispensable. This cross section is evaluated as

$$\sigma_{ex} = \frac{\pi}{p^2}\int dq^2\, |F_{ex}(\mathbf{q})|^2 \sim \sim \frac{g^2 N^{1-\delta}}{8\pi} C \log\left(\frac{N}{\delta\gamma}\right), \quad (50)$$

where

$$C = \gamma^\Delta \Delta^2 \Gamma(\Delta), \quad \gamma = 2\Omega^2/\beta^2\mu^2,$$

$$\Delta = \lambda/\mu, \quad \delta = \Delta - 1 \sim \kappa^2/2\mu^2 \ll 1.$$

In (50), we simply set $S(\kappa) = N$. Thus, the account of multiple scatterings and absorption turns the Born approximation cross section

$$\sigma_{ex} \propto N^2$$

into

$$\sigma_{ex} \propto N^{1-\delta}\log N.$$

In the limit as $p \to \infty$ and $\delta \to 0$,

$$\sigma_{ex} \sim \frac{g^2 N}{8\pi}\gamma \log\left(\frac{N}{\gamma}\right). \quad (51)$$

The dependence of σ_{ex} on N differs from that of the fully unitarized elastic cross section,

$$\sigma_{el} \propto \log^2 N.$$

The reason is that in σ_{ex}, we sum the eikonal diagrams to all orders in β but only to the first order in g. Such a unitarization procedure is, of course, incomplete, but this is not important for practical purposes because the smallness of $d^2\Omega^2$ makes the next-to-leading-order terms negligibly small up to

$$N \sim \alpha Z_1^2/\delta\Omega^2 d^2 \sim 10^{12}.$$

6. SUMMARY

The main goal we pursued in this paper was a consistent description of the coherent Coulomb excitation of ultrarelativistic particles and nuclei passing through the aligned crystal. We started with the discussion of the elastic scattering and found that the account of lattice thermal vibrations within the Glauber multiple scattering theory gives rise to a strong absorption effect. The radius of the absorption region in the impact parameter space appeared to grow logarithmically as the crystal thickness grows. We derived convenient representation for the scattering matrix with absorption and calculated the coherent elastic and the incoherent quasielastic cross sections. Suppression of scattering amplitudes in the absorption region was shown to serve as a natural UV regulator and to enable consistent calculation of the cross section σ_{ex} of the coherent nuclear excitation. The dependence of σ_{ex} on the crystal thickness is found. The multiple scattering effects are shown to become numerically important already at $N \gtrsim 1$, thus leaving no room for the Born approximation widely used in early analyses of the problem.

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