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COLLECTIVE EXCITATIONS IN EXCITON CRYSTAL

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Crystal phase of indirect excitons formed by spatially separated electrons and holes in coupled quantum wells is analyzed. The collective mode spectrum of the exciton crystal at zero and nonzero magnetic fields is found. The spectrum consists of two optical and two acoustical modes (transverse and longitudinal in each case). We also study changes of the dipole crystal collective excitations at the transition exciton crystal–electron-hole plasma.

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In a bilayer system with spatially separated electrons and holes, the overlapping of wave functions and, consequently, the recombination rate is suppressed and therefore the lifetime of laser pumped electrons and holes can be sufficient for the appearance of different interesting quasi-equilibrium electron-hole phases [1, 2]. In particular, a superfluid phase with anomalous transport and optical properties [3], quasi-Josephson phenomena [4], and nonordinary behavior in strong magnetic fields [5, 6] can be observed (we note that there is strong analogy and some mapping between properties of bilayer electron-hole and electron-electron systems). The essential feature of the system of interwell excitons is the existence of parallel electric dipole moments of the excitons. Dipole-dipole repelling of the excitons suppresses the exchange interaction of the excitons and stabilizes exciton phases [2] in comparison with the three-dimensional case [7]. Modern manufacturing techniques of layer structures enable obtaining high-quality nanostructures and easily varying their parameters. In recent years, very interesting experimental results were obtained in such bilayer electron-electron and electron-hole [8–13] systems. Presently, it is possible to obtain a system with spatially separated electrons and holes of low densities such that the distance between charge carriers in a layer is much greater than the interlayer separation. In this case, formation of indirect excitons (dipoles) [14–20] and the appearance of new exciton phase, the exciton crystal, becomes possible at low temperature in some intermediate region of exciton density [2, 21, 22]. This causes the great interest in the theoretical research of the dipole crystal collective excitations at zero and nonzero normal magnetic fields, which can be used for experimental verification of the exciton crystal existence by optical methods. It is also interesting to study the behavior of the collective excitation spectrum of a dipole crystal at the transition into the phase of the electron-hole plasma.

We consider a double-layer two-dimensional system. The first layer contains an electron channel with the density of carriers N_e and the second layer contains holes with the density N_h. The distance between the layers is D. We are interested the case, where $N_e = N_h$.

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At low temperatures and small concentrations of electrons and holes ($N_{e(h)}a_B^2 \ll 1$, where a_B is the effective radius of indirect excitons along layers), the system is a weakly nonideal excitonic gas of the areal density $N_{exc} = N_e = N_h$ with dipole moments d perpendicular to the layers; in the ground state, $d = eD$ and it increases with the distance between layers. In a spatially separated electron-hole system, contrary to the ordinary (single-layer) electron-hole systems, the direct dipole-dipole repulsion gives the main contribution to the total energy (in contrast with the ordinary single-layer electron-hole system). At the same time, van der Waals attraction between excitons and the exchange interaction are not essential (less than 1 % compared with the contribution of the direct dipole-dipole interaction). At low temperatures, in a certain region of low concentrations $N_{\rm exc}$ and interlayer separations D, the dipole-dipole repulsion leads to the formation of an indirect exciton crystal. The small contribution of the exchange interaction in a spatially separated electron-hole system is connected with an exponentially small probability to tunnel through the barrier of the dipole-dipole interaction.

We start with the exciton crystal phase. The kinetic energy of the system is

$$T_{kin} = \sum_{n,m=-\infty}^{\infty} \left(\frac{m_e \dot{\mathbf{r}}_{enm}^2}{2} + \frac{m_h \dot{\mathbf{r}}_{hnm}^2}{2} \right) = \sum_{n=-\infty}^{\infty} \left(\frac{M}{2} \dot{\mathbf{R}}_{nm}^2 + \frac{\mu}{2} \dot{\boldsymbol{\rho}}_{nm}^2 \right), \quad (1)$$

where $M = m_e + m_h$ is the exciton mass and $\mu = (m_e m_h)/(m_e + m_h)$ is the reduced mass, $\mathbf{R}_{nm} = (m_e \mathbf{r}_{enm} + m_h \mathbf{r}_{hnm}) M^{-1}$ is the coordinate of the center of the exciton mass, and $\boldsymbol{\rho}_{nm} = \mathbf{r}_{enm} - \mathbf{r}_{hnm}$ is the coordinate of relative motion.

The potential energy of the system consists of two terms. The first term is the Coulomb interaction of spatially separated electron and hole in one exciton, which in the case where $\rho \ll D$ can be transformed as

$$U_{exc} = -\frac{e^2}{\epsilon\sqrt{\rho^2 + D^2}} \approx -\frac{e^2}{\epsilon D} + \frac{1}{2}\frac{e^2}{\epsilon D^3}\rho^2. \quad (2)$$

Hence, for small oscillation amplitudes, this term can be approximated by the parabolic potential,

$$U_1 = \sum_{n,m} \frac{\mu\omega_0^2 \boldsymbol{\rho}_{nm}^2}{2}, \quad (3)$$

$$\omega_0^2 = \frac{e^2}{\epsilon\mu D^3}.$$

For a real system of finite thickness, the quantity ω_0 must be found using z-dependent wave functions in each layer.

The second term in the potential energy is the interexciton interaction along layers. In the system under consideration, the interaction of indirect excitons is the dipole-dipole repulsion,

$$U_2 = \sum_{n,n',m,m'} \frac{e^2 \boldsymbol{\rho}_{nm}\boldsymbol{\rho}_{n'm'}\mathbf{R}^2 - 3e^2(\boldsymbol{\rho}_{nm}\mathbf{R})(\boldsymbol{\rho}_{n'm'}\mathbf{R})}{\epsilon R^5}. \quad (4)$$

Here, $\mathbf{R}$ is the vector between dipoles at different cells (a is the lattice period),

$$\mathbf{R}^2 = [a(n-n') + (x_n - x_{n'})]^2 + [a(m-m') + (y_m - y_{m'})]^2 + 2[a(n-n') + (x_n - x_{n'})] \times [a(m-m') + (y_m - y_{m'})] \cos\alpha, \quad (5)$$

x_n (y_m) are the displacements to the x- (y-) direction of the dipole (n, m) from the equilibrium, and α is the angle between basis vectors of an arbitrary lattice. We consider a triangular lattice of the dipole (exciton) crystal. It corresponds to a minimum of the potential energy, and the spectrum of its frequencies is stable in the two-dimensional case, in the contrast to other configurations (see, e.g., [21, 23]). Hence, $\alpha = \pi/3$. The corresponding reciprocal hexagonal lattice is presented at Fig. 1.

In the harmonic approximation, we expand the potential energy U_2 to the second order in displacements. As a result, it is easy to obtain the equations of motion for the center-of-mass coordinates x_n and y_m, and also for the coordinates of relative motion u_n, v_m:

$$\ddot{u}_n = -\omega_0^2 u_n + \Omega^2 \sum_{n',m'=-\infty}^{\infty} \frac{\left[(n-n')^2 - 2(m-m')^2\right] u_{n'} - 3\left[(n-n')(m-m')\right] v_{m'}}{\left[(n-n')^2 + (m-m')^2 + (n-n')(m-m')\right]^{5/2}}, \quad (6)$$

$$\ddot{v}_m = -\omega_0^2 v_m + \Omega^2 \sum_{n',m'=-\infty}^{\infty} \frac{\left[(m-m')^2 - 2(n-n')^2\right] v_{m'} - 3\left[(n-n')(m-m')\right] u_{n'}}{\left[(n-n')^2 + (m-m')^2 + (n-n')(m-m')\right]^{5/2}}, \tag{7}$$

$$\ddot{x}_n = -\Omega_1^2 \sum_{n',m'=-\infty}^{\infty} \frac{\left[-3(n-n')^2 + \frac{45}{4}(m-m')^2\right](x_n - x_{n'})}{\left[(n-n')^2 + (m-m')^2 + (n-n')(m-m')\right]^{7/2}} -$$

$$- \Omega_1^2 \sum_{n',m'=-\infty}^{\infty} \frac{\left[15\frac{\sqrt{3}}{2}(m-m')\left[(n-n') + \frac{1}{2}(m-m')\right]\right](y_m - y_{m'})}{\left[(n-n')^2 + (m-m')^2 + (n-n')(m-m')\right]^{7/2}}, \tag{8}$$

$$\ddot{y}_m = -\Omega_1^2 \sum_{n',m'=-\infty}^{\infty} \frac{\left[-3(m-m')^2 + 15\left[(n-n') + \frac{1}{2}(m-m')\right]^2\right](y_m - y_{m'})}{\left[(n-n')^2 + (m-m')^2 + (n-n')(m-m')\right]^{7/2}} -$$

$$- \Omega_1^2 \sum_{n',m'=-\infty}^{\infty} \frac{\left[15\frac{\sqrt{3}}{2}(m-m')\left[(n-n') + \frac{1}{2}(m-m')\right]\right](x_n - x_{n'})}{\left[(n-n')^2 + (m-m')^2 + (n-n')(m-m')\right]^{7/2}}. \tag{9}$$

At zero magnetic field, the variables u_n, v_m, describing optical (out-of-phase) vibrations, and x_n, y_m, describing acoustic (in-phase) vibrations, are separated. From Eqs. (6)–(9), we therefore obtain two independent determinants of the second rank giving the spectrum of collective excitations of the two-dimensional dipole crystal.

We search for the normal mode in the form $u_n = u\exp[-i\omega t + ikan]$, $v_m = v\exp[-i\omega t + iqam]$. The final equations for optical modes are

$$\begin{bmatrix} \omega^2 - \omega_0^2 + \Omega^2 f_{11}(k,q) & -3\Omega^2 f_{12}(k,q) \\ -3\Omega^2 f_{21}(k,q) & \omega^2 - \omega_0^2 + \Omega^2 f_{22}(k,q) \end{bmatrix} \times \begin{bmatrix} u \\ v \end{bmatrix} = 0, \tag{10}$$

where $\Omega^2 = 2e^2/\mu\epsilon a^3$, (k,q) are the x-, y-components of wave vector $\mathbf{k}$,

$$f_{11}(k,q) = \sum_{N,J=1}^{\infty} \frac{N^2 - 2J^2}{(N^2+J^2+NJ)^{5/2}} \cos(kaN)\cos(qaJ),$$

$$f_{22}(k,q) = \sum_{N,J=1}^{\infty} \frac{J^2 - 2N^2}{(N^2+J^2+NJ)^{5/2}} \cos(kaN)\cos(qaJ),$$

$$f_{12}(k,q) = f_{21}(k,q) = \sum_{N,J=1}^{\infty} \frac{NJ}{(N^2+J^2+NJ)^{5/2}} \sin(kaN)\sin(qaJ),$$

$$N = n - n', \quad J = m - m'.$$

Correspondingly, for acoustic modes, we obtain

$$\begin{bmatrix} \omega^2 - \Omega_1^2 \widetilde{f}_{11}(k,q) & \Omega_1^2 \widetilde{f}_{12}(k,q) \\ \Omega_1^2 \widetilde{f}_{21}(k,q) & \omega^2 - \Omega_1^2 \widetilde{f}_{22}(k,q) \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix} = 0, \tag{11}$$

where $\Omega_1^2 = 8e^2D^2/M\epsilon a^5$,

$$\widetilde{f}_{11}(k,q) = \sum_{N,J=1}^{\infty} \frac{-3N^2 + 15\frac{3}{4}J^2}{(N^2+J^2+NJ)^{7/2}} \left[1 - \cos(kaN)\cos(qaJ)\right],$$

$$\widetilde{f}_{22}(k,q) =$$

$$= \sum_{N,J=1}^{\infty} \frac{-3J^2+15\left(N+\frac{1}{2}J\right)^2}{(N^2+J^2+NJ)^{7/2}}\Big[1-\cos(kaN)\cos(qaJ)\Big],$$

$$\widetilde{f}_{12}(k,q) = \widetilde{f}_{21}(k,q) =$$

$$= \sum_{N,J=1}^{\infty} \frac{-15\frac{\sqrt{3}}{2}J\left(N+\frac{1}{2}J\right)}{(N^2+J^2+NJ)^{7/2}}\sin(kaN)\sin(qaJ).$$

The dispersion laws $\omega(k,q)$ given by (10) and (11) are presented in Fig. 2 (optical) and Fig. 3 (acoustical). These dispersion laws are obtained, respectively, for the following directions: (a) Γ–X, (b) Γ–L, and (c) L–X, where Γ, L, and X are symmetry points in the first Brillouin zone. The hexagonal reciprocal lattice that corresponds to the triangular lattice in coordinate space and the symmetry directions are shown in Fig. 1. In our calculations, the dimensionless energy units $\omega = \omega/\omega_0$, $\Omega = \Omega/\omega_0 = 0.2$, $\Omega_1 = \Omega_1/\omega_0 = 0.01$ and the dimensionless wave vector units $k = ka$, $q = qa$ are used. Two modes are found in each of the spectra. One of the modes corresponds to a longitudinal (in-phase, or out-of-phase) vibrations and the other to transverse vibrations. The transverse modes must undergo strong changes at crystal melting.

In the presence of the perpendicular magnetic field, we add the Lorentz components of acceleration and obtain the system of four equations

$$\begin{bmatrix} \omega^2-\omega_{11}^2 & \omega_{12}^2+i\omega(\omega_e-\omega_h) & 0 & i\omega\dfrac{eB}{\mu c} \\ \omega_{12}^2-i\omega(\omega_e-\omega_h) & \omega^2-\omega_{22}^2 & -i\omega\dfrac{eB}{\mu c} & 0 \\ 0 & -i\omega\dfrac{eB}{Mc} & \omega^2-\omega_{33}^2 & \omega_{34}^2 \\ i\omega\dfrac{eB}{Mc} & 0 & \omega_{34}^2 & \omega^2-\omega_{44}^2 \end{bmatrix} \begin{bmatrix} u \\ v \\ x \\ y \end{bmatrix} = 0, \tag{12}$$

where B is the magnetic field and $\omega_e = eB/m_e c$, $\omega_h = eB/m_h c$ are the respective cyclotron energies of an electron and a hole. Dispersion laws for the exciton crystal in the magnetic field are presented in Fig. 4.

In strong magnetic fields, optical modes approach the electron and hole cyclotron energies. Such behavior occurs because the cyclotron energy of electrons (holes) in this field is much greater than the characteristic Coulomb interactions, and therefore the spectrum of our system is defined by the magnetic field, while the $e-h$ interaction is a small perturbation. This result agrees with [24], where the system of spatially separated electrons and holes at strong magnetic fields was considered. In this case, the frequencies of the acoustic modes decrease (see the inset in Fig. 4) because of magnetic localization electrons and holes in the layers. This behavior is similar to one of the acoustic mode of the Wigner crystal (see, e.g., [25]), with the dispersion law in the region of strong magnetic fields of the form

$$\omega_- \approx \frac{\alpha^{1/2} s k^{3/2}}{\omega_e}, \tag{13}$$

where $\alpha \equiv 2\pi N_e e^2/\epsilon m_e$ and $s^2 = 4e^2/m_e \epsilon a$.

We now discuss the phase transition between the dipole crystal and the electron-hole liquid at which the long-range order in the electron-hole system disappears. More precisely, we are interested in the change of the mode properties under the phase transition. The acoustic transverse mode therefore sharply disappears when the long-range order in the electron-hole system vanishes, and a damping transverse mode appears at wave vectors $k \sim 1/L$, where L is the radius of the short-range crystal order (see, e.g., [26]). The optical transverse mode becomes nondispersive at small wave vectors k. Two others modes (longitudinal optical and acoustic ones) have the following dispersion laws at zero magnetic field:

$$\omega_{op}^2 = \omega_0^2 + \frac{2\pi e^2 N_{exc}}{\epsilon\mu}k, \tag{14}$$

$$\omega_{ac}^2 = \frac{4\pi e^2 N_{exc}^{3/2}}{\epsilon M}(kD)^2. \tag{15}$$

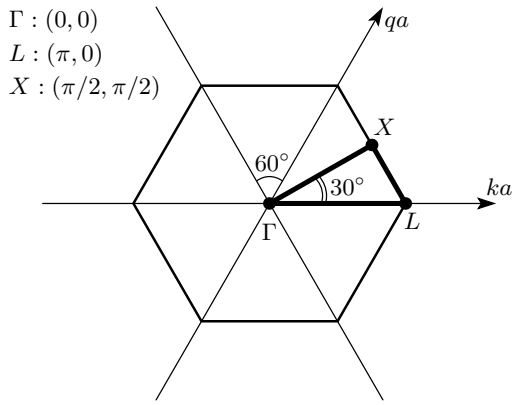

Fig. 1. The hexagonal reciprocal lattice. Shown are the Γ–L, Γ–X, and L–X directions. Presented coordinates of symmetry points are coefficients attached to basis vectors of the oblique-angle coordinate system

This result was obtained for a continuous medium (that is, in the long-wavelength limit $k[N_{\rm exc}]^{-1/2} \ll 1$) and for a small distance between the layers $D \ll [N_{\rm exc}]^{-1/2}$. The same result can be obtained by expanding in the small parameter k ($ka \ll 1$) and omitting the summation over the shift coordinates in expressions (10) and (11), which formally means the absence of shear modulus. As follows from formulas (14), (15), and (3), two optical branches go up as the distance between layers decreases, but the acoustic mode goes down.

Transition from the interacting indirect excitons to the electron-hole plasma occurs at further increasing the concentration of spatially separated electrons and holes [27, 28]. Characteristic values of concentration at which such the phase transition occurs are of the order 10^9 cm^{-2}. Changes in the collective excitation spectra have a character of principle at this transition. Systems of the electron-hole plasma have two collective modes (we neglect the tunnelling between layers). One of them is optical, with the square-root dispersion law ($\omega_{op}^{pl} \propto \sqrt{k}$), and the other is acoustic, with the linear law ($\omega_{ac}^{pl} \propto k$) [29, 30]. This means that the frequency of out-of-phase oscillations at $k = 0$ is equal to zero, in contrast to the exciton phase of the system, where out-of-phase branches start from the dipole transition energy ω_0. In a perpendicular magnetic field, the hybrid magnitoplasma excitations are described as

$$\omega_{mp}^{op} = \sqrt{{\omega_{op}^{pl}}^2 + \omega_e^2}, \tag{16}$$

$$\omega_{mp}^{ac} = \sqrt{{\omega_{ac}^{pl}}^2 + \omega_h^2}. \tag{17}$$

We can see that such field dependences differ from those of the dipole crystal (see Fig. 4).

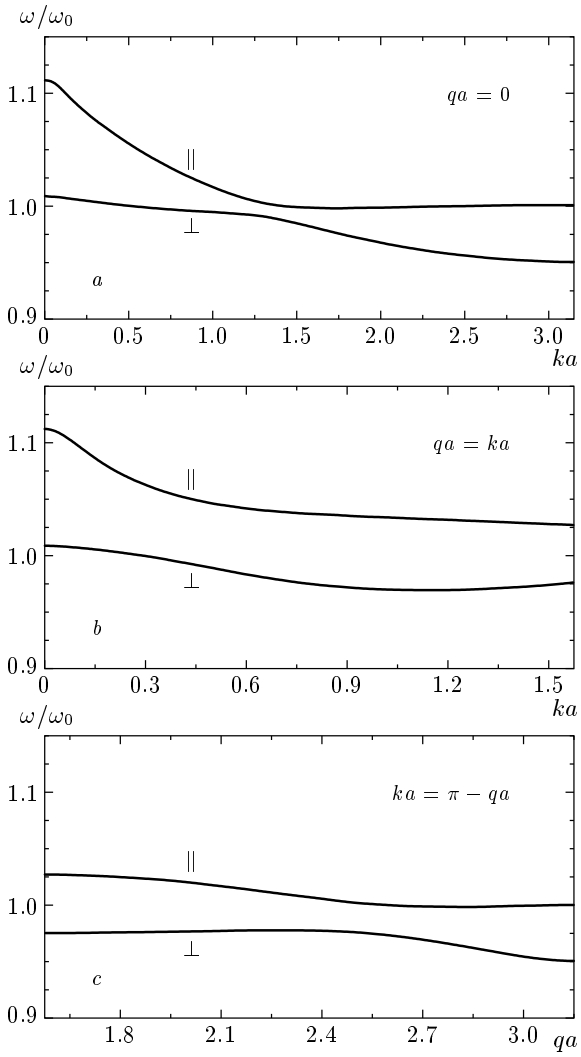

Fig. 2. Dispersion of the optical longitudinal and transverse modes: (a) Γ–L direction, (b) Γ–X, and (c) L–X

In summary, in this work the properties of a dipole crystal, a new exciton phase in a two-layer electron-hole system with low densities of spatially separated electrons and holes, are investigated. Collective modes of the two-dimensional exciton triangular lattice at zero magnetic field are found. The spectrum consists of four branches: two optical (longitudinal and transverse) and two acoustic (longitudinal and transverse). The frequencies of optical modes at zero wave vector are nonzero (in contrast to the Wigner crystal) and the frequencies of the optical modes for $D \ll [N_{exc}]^{-1/2}$ purely in the two-dimensional case are $\omega_0 = \sqrt{e^2/\epsilon\mu D^3}$.

The spectra of in-phase and out-of-phase vibrations in the normal magnetic field are also found. In strong

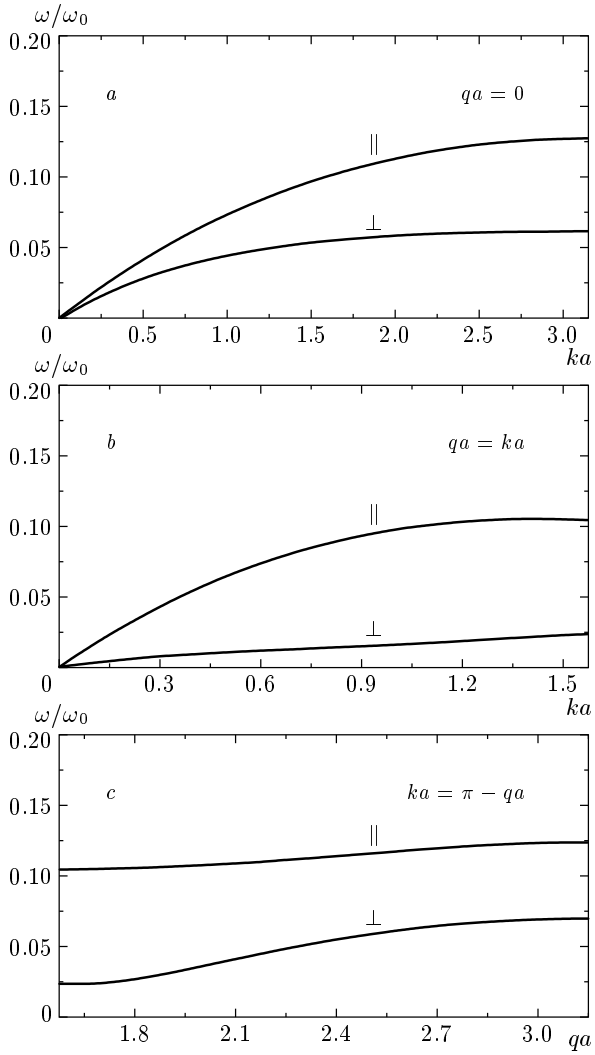

Fig. 3. Dispersion of the acoustical longitudinal and transverse modes: (*a*) Γ–*L* direction, (*b*) Γ–*X*, and (*c*) *L*–*X*

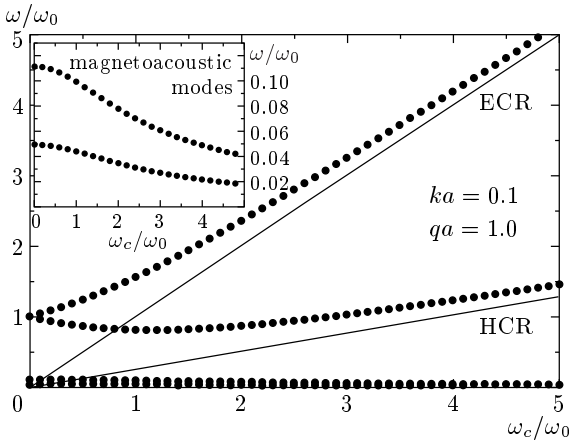

Fig. 4. The dipole crystal collective excitations as a function of the magnetic field. ECR and HCR are the electron and hole cyclotron energies in the GaAs/AlGaAs structure. The inset shows the magnetoacoustic modes in enlarged scale

magnetic fields, the optical modes approach the electron and hole cyclotron modes, and the acoustic ones decrease as the field increases.

We have discussed changes in the collective excitations of a dipole crystal at the transition to the electron-hole plasma phase. Plasma and magnitoplasma vibrations of that phase are considered in the system of two parallel infinite planes.

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